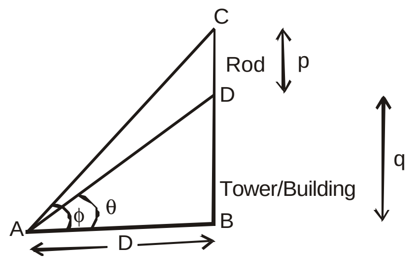
IIFT-2006 (Quant solutions)

1. (A,D)



In the right triangle ABC,

$$\frac{p+q}{D} = \tan\phi \qquad \ldots(1)$$

In the right triangle ABD,

$$\frac{q}{D} = \tan\theta \qquad \ldots(2)$$

From (1) and (2)

$$\frac{p}{q} + 1 = \frac{\tan\phi}{\tan\theta}$$

or $\frac{p}{q} = \frac{\tan\phi - \tan\theta}{\tan\theta}$

A. is correct as height of tower is

$$q = \frac{p\tan\theta}{\tan\phi - \tan\theta}$$

B is wrong as height of rod is

$$p = q\frac{(\tan\phi - \tan\theta)}{\tan\theta}$$

C. is wrong
D. is correct.
So, A and D are correct options.

2. (A,C) $px^2 + qx + r = 0$. Let roots are α and β.

Sum of the roots = $\alpha + \beta = \frac{-q}{p}$

product of roots = $\alpha \times \beta$

Sum of the squares of the reciprocals of the roots $= \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{-q}{p}\right)^2 - 2\left(\frac{r}{p}\right)}{\left(\frac{-q}{p}\right)^2}$$

Given that

$$\frac{-q}{p} = \frac{\left(\frac{-q}{p}\right)^2 - 2\left(\frac{r}{p}\right)}{\left(\frac{-q}{p}\right)^2}$$

or $2p^2r = pq^2 + qr^2$

$$\frac{2p^2r}{pqr} = \frac{p.q^2}{pqr} + \frac{qr^2}{pqr} \text{ (divide by pqr)}$$

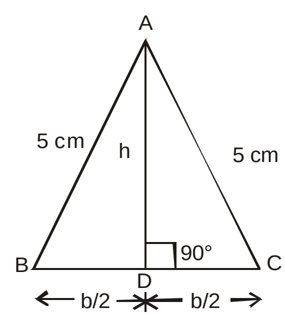
$$\Rightarrow \frac{2p}{q} = \frac{q}{r} + \frac{r}{p}$$

(A) clearly $\frac{r}{p}, \frac{p}{q}$ and $\frac{q}{r}$ are in A.P. (A) is correct

(B) As $\frac{q^2}{p^2} \neq \left(\frac{p}{r}\right) \times \left(\frac{r}{q}\right) = \frac{p}{q}$. (B) is incorrect.

(C) As $\frac{r}{p}, \frac{p}{q}$ and $\frac{q}{r}$ are in A.P., their reciprocals are in H.P.
(D) is wrong as these terms as in H.P., not in A.P.
Hence A and C are correct.

3. (B,C)



$$\text{Area}(\Delta ABC) = \frac{1}{2} \times b \times h = 12\text{ sq.cm}$$

$$\Rightarrow b \times h = 24 \qquad \ldots(1)$$

Also, in right triangle ΔABD,

$$\Rightarrow (5)^2 = \left(\frac{b}{2}\right)^2 + h^2$$

or $b^2 + 4h^2 = 100$ …(2)

From (1) and (2)

Put $h = \frac{24}{b}$ in (2)

so, $b^2 + 4 \times \left(\frac{24}{b}\right)^2 = 100$

or $b^2 + \left(\frac{48}{b}\right)^2 = 100$ …(3)

put b = 4 cm in (3) does not satisfy.
Hence (A) is wrong
put b = 6 cm in (3), does satisfy.
Hence (B) is right.
put b = 8 cm in (3), satisfies.
Hence (C) is right.
put b = 9 cm in (3), does not satisfy.
Hence (D) is wrong.
So, B and C are correct.

4.(A,C,D) Initial mixture = 40 kg
Sand (S): Cement (C) ratio is 1 : 4

$\Rightarrow$ S by weight $= \frac{1}{5} \times 40 = 8kg$

and C by weight $= \frac{4}{5} \times 40 = 32kg$

Let he mixes x kg of sand to the 40 kg mixture, so that the ratio becomes 4 : 3. We should have

$$\frac{8+x}{40+x} = \frac{3}{3+4} = \frac{3}{7} \Rightarrow x = 16kg$$

$\Rightarrow$ he mixed 16 kg of sand to the 40 kg mixture.
Option (A): Weight of second mixture = 40 + 16 = 56 kg which is $\frac{56}{40} = 1.4$ times and not 1.5 times heavier. Hence (A) is incorrect.
Option (B): Correct. x = 16 kg, as solved above.
Option (C): If the original mixture was in 8 : 3 ratio, the weight of sand would have been

$$\frac{3}{8+3} \times 40 = 10.9\,kg \neq 12kg$$

Hence, (C) is incorrect.

Option (D): The mixture weights 56 kg. After selling 7 kg of it, he is left with 49 kg of the mixture. In 11 kg of new mixture (7 : 4 ratio)

Sand is $\frac{4}{7+4} \times 11 = 4kg$

and Cement is $\frac{7}{7+4} \times 11 = 7kg$

In the final mixture

Cement = $\frac{4}{7} \times (49) + 7$

= 28 + 7 = 35 kg

Sand $\frac{3}{7} \times (49) + 4 = 25kg$

Cement : Sand ratio = $\frac{35}{25} = \frac{7}{5}$

Hence (D) is correct.
In all B and D options are correct.

5.(A,B,C,D) 4 different countries have to be assigned four different years. The correct option will have exactly one country assigned (or matched) to a single year of the four given.
There is only one set of assignment of countries to the years that is correct. So; of all the possible ways of matching the countries to different years, only one is correct.
Total no. of ways of randomly answering (randomly matching) the question is given by:-
= 4 × 3 × 2 × 1 = 24 ways.
Now:- Option (D) P (X = 4) = P(All four matches are correct)

= P(Answer is marked correctly) $= \frac{1}{24}$

Hence, (D) is correct)
Option (C) $P(X = 3) \equiv P$ (Exactly three matches are correct & fourth is not correct)
If 3 countries are matched, correctly, fourth must be correct as well.
So P(X = 3) event will never occur.
Hence P(X = 3) = 0 is also correct.
Option (B) P(X = 1)
WG – 66
I – 82
F – 90
E – 98
Let's assume, the above given match is the correct match. We have to count, all the possible ways of matches where exactly one is correct and all three are wrong.
Beginning with Italy:- Italy $\rightarrow 82$ is a correct match as above. For a wrong match,
WG can be assigned 90 or 98
& $F \rightarrow 98 or 66$

$E \rightarrow 90 or 66$

Case 1:- When F is assigned 98; The other two can be assigned in only one way $(E \rightarrow 66 \,\&\, WG \rightarrow 90)$

Case 2:- Similarly, when F is assigned 66; the other two countries can be assigned in only one way $(E \to 90 \; \& \; WG \to 98)$

So; given that the only correct match is that of Italy, there are exactly two ways of marking the other countries, wrongly.

$$\Rightarrow P(X=1) = \frac{4 \times (2)}{24} = \frac{8}{24}$$

$$P(X = 1) = \frac{1}{3}$$

$\Rightarrow$ Option (B) is wrong.

Option (A) $P(X \geq 1)$

$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$P(X = 2)$ going as in option (B)

$$P(X = 2) = \frac{({}^4C_2 \times 1)}{24} = \frac{6}{24}$$

$$P(X = 2) = \frac{1}{4}$$

$$\Rightarrow P(X \geq 1) = \frac{1}{3} + \frac{1}{4} + 0 + \frac{1}{24} = \frac{8+6+1}{24}$$

$$= \frac{15}{24}$$

$$P(X \geq 1) = \frac{5}{8}$$

$\Rightarrow$ Option(A) is correct.

$\Rightarrow$ In all options (A), (C) & (D) are correct.

6. (A,B,C,D) Joshi's Total cost price $= 20{,}000 + 8{,}000 + 2{,}000$ = Rs. 30,000/-

Joshi's selling price = Wadhwa's cost price

$$= 30{,}000\left(1 + \frac{20}{100}\right) = \text{Rs. } 36{,}000/-$$

And then Wadhwa sells it back to Joshi.

Option A:-Wadhwa lost Rs. 7200/- when selling the shop, back to Joshi

$\Rightarrow$ Wadhwa's selling price = 36000 – 7200

= Rs. 28800/-

$$\Rightarrow \text{Wadhwa's loss} = \frac{28800 - 36000}{36000}$$

= 20% exactly

$\Rightarrow$ Option (A) is correct.

Option B:- Joshi's new total cost price

= 14000 + 8000 + 4000 = Rs. 24000/-

Joshis selling price is still the same = 36000

$$\Rightarrow \text{Joshi's profit} = \frac{36}{24} = 1.5$$

$\Rightarrow$ Profit is 50%

Option (B) is correct.

Option C:- At a profit of 40% on his total cost price of Rs. 36000, Joshi's monetary gain is:-

$$= 30000 \times \frac{40}{100} = \text{Rs. } 12000/-$$

Hence, Option (C) is correct.

Option D:- If Joshi sold to Wadhwa at 40% profit then Wadhwa's cost price = 42,000/-

Wadhwa sells it back to Joshi at a loss of 40%

i.e. at a price of $= 42000 \times \left(1 - \frac{40}{100}\right) = 25{,}000.$

As, Joshi had made a monetary gain of Rs. 12,000 in selling the same shop to Wadhwa earlier, his net investment in taking the shop back is:-

(Rs. 25200 – Rs.12,000) = Rs. 13,200/-

$\Rightarrow$ Option (D) is correct.

$\Rightarrow$ All the 4 options are correct.

7. (A,B,C,) Given $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = K$, say

Let the base of the logarithm be "B".

then $\log_B x = K(b-c) \Rightarrow x = B^{K(b-c)}$

$\log_B y = K(c-a) \Rightarrow y = B^{K(c-a)}$

$\log_B z = K(a-b) \Rightarrow z = B^{K(a-b)}$

Adding, we get

$\log_B x + \log_B y + \log_B z$

$= \{K(b-c) + K(c-a) + K(a-b)\}$

or $\log_B(xyz) = K\{b - c + c - a + a - b\} = 0$

$\Rightarrow (xyz) = B^{(0)} = 1$

or xyz = 1

Option (A): xyz = 1, is correct.

Option (B): $x^a.y^b.z^c$

$$= \left[B^{K(b-c)}\right]^a \times \left[B^{K(c-a)}\right]^b \times \left[B^{K(a-b)}\right]^c$$

$$= B^{K[a(b-c) + b(c-a) + c(a-b)]}$$

$$= B^{K(0)} = B^0 = 1$$

Option (B) is correct.

Option (C): $x^{b+c}.y^{c+a}.z^{a+b}$

$$\left[B^{k(b+c)(b-c)}\right] \times \left[B^{k(c+a)(c-a)}\right] \times \left[B^{k(a+b)(a-b)}\right]$$

$= B^{K\left[(b^2-c^2)+(c^2+a^2)+(a^2+b^2)\right]}$

$= B^0 = 1$

Option (C) is correct.
Option (D): is wrong as the expression evaluates to 1 as in (C) and not zero.
In all options A, B and C are correct.

8.(A,B,C) In the pot there are x tickets of "Knife Throwing" game and y tickets of Talking Dolls game. Starting from Ajay, one ticket out of a total of (x + y) is drawn. If the ticket is one of the x tickets for the Knife Throwing game, the drawing of tickets is stopped. If a Talking Dolls ticket is drawn, it is put back in the pot.
Probability of Ajay, first drawing the Knife Throwing ticket is
P_A = (Ajay gets it in first draw)
+ (Ajay gets it in third draw)
+ (Ajay gets it in fifth draw)
+ …… ∞

$$P_A = \frac{x}{x+y} + \left(\frac{y}{x+y}\right)\times\left(\frac{y}{x+y}\right)\times\left(\frac{x}{x+y}\right) + \left(\frac{y}{x+y}\right)\times\left(\frac{y}{x+y}\right) \times\left(\frac{y}{x+y}\right)\left(\frac{y}{x+y}\right)\times\left(\frac{x}{x+y}\right) + \ldots\infty$$

$$= \left(\frac{x}{x+y}\right) + \left(\frac{x}{x+y}\right)\times\left(\frac{y}{x+y}\right)^2 + \left(\frac{x}{x+y}\right)\times\left(\frac{y}{x+y}\right)^4 \ldots\infty$$

This is an infinite GP.

First term $= \frac{x}{x+y}$

Common ratio = $\left(\frac{y}{x+y}\right)^2 \qquad \left\{0 \le \frac{y}{x+y} < 1\right\}$

P_A = Sum of the GP

$$P_A = \frac{\frac{x}{x+y}}{1-\left(\frac{y}{x+y}\right)^2} = \frac{x+y}{x+2y}$$

$$P_A = \frac{x+y}{x+2y}$$

For Mohan to get the first Knife throwing ticket, let the probability is P_M
P_M = (Mohan gets it in second draw)
+ (Mohan gets it in fourth draw)
+ (Mohan gets it in sixth draw)
+ …… ∞

$$P_A = \left(\frac{y}{x+y}\right)\times\left(\frac{x}{x+y}\right) + \left(\frac{y}{x+y}\right)\times\left(\frac{y}{x+y}\right)\times\left(\frac{y}{x+y}\right)\times\left(\frac{x}{x+y}\right) + \left(\frac{y}{x+y}\right)\left(\frac{y}{x+y}\right)\left(\frac{y}{x+y}\right)\left(\frac{y}{x+y}\right)\times\left(\frac{y}{x+y}\right)\times\left(\frac{x}{x+y}\right) + \ldots\infty$$

$$= \frac{\frac{xy}{(x+y)^2}}{1-\left(\frac{y}{x+y}\right)^2}$$

$$P_M = \frac{y}{x+2y}$$

Option A: Given $P_A = 4\ P_M$

i.e. $\frac{x+y}{x+2y} = \frac{4y}{x+2y}$

$\Rightarrow \frac{x}{y} = \frac{3}{1}$

Hence, (A) is correct.

Option (B): Given $P_A = 5\ P_M$

$\Rightarrow \frac{x}{y} = \frac{4}{1}$

Hence (B) is correct.

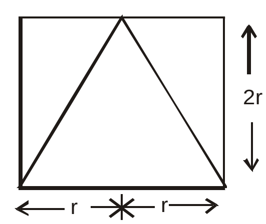
Option C: $P_A = 2\ P_M$

$\Rightarrow \frac{x}{y} = \frac{1}{1}$

Hence (C) is correct.

Option D: $P_M < P_A$ always. $\{\because y < (x+y)\}$
Here (D) cannot be a valid statement.
In all options. (A), (B) and (C) are correct.

9.(A,C) Memento is a right circular cone. The circumference of the cone's base touches each side of the base of the box. The vertex of the cone touches the opposite face of the box.
⇒ height of the cone = diameter of the cone
= side of the cube.



Volume of the memento = $718\frac{2}{3}$ cc

or $\frac{1}{3}\times\pi\times r^2(2r) = 718\frac{2}{3}$

or $\frac{1}{3}\times\frac{22}{7}\times 2\times(r^3) = \frac{2156}{3}$

$\Rightarrow r = 7cm$

$\Rightarrow$ The box is a cube of side 14 cm.

$\Rightarrow$ The total surface area of the box = 6 × $(14)^2$
= 1176 cm^2

$\Rightarrow$ expenditure incurred in packing the box
= 1176 × 1.5 = 1764 Rs.

Option (A) Madan's total expenditure on the box = (cost of the box) + (expenditure in the covering)
= 500 + 1764 = Rs. 2264.
Hence, option (A) is correct.

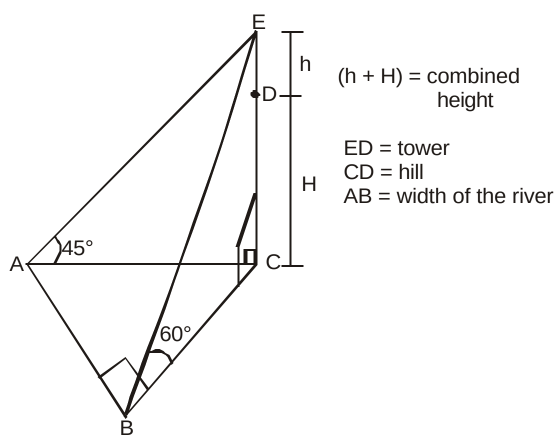
Option (B): is wrong as the expenditure was Rs. 1764 .

Option (C): is correct as the area = $(14)^2$
= 196 sq. cm

Option (D): Volume of the box = $(14)^3$
= 2744 cc.

Hence option (D) is incorrect.
In all, option (A) and (C) are correct.

10.(A,B,D)



See the above figure
Water does not move in the lake.
In the right triangle ACE;

$\angle A = 45° \Rightarrow EC = AC = (h+H)$
= 300 m ...(1)

In the right triangle BCE;

$\angle B = 60° \Rightarrow \frac{EC}{BC} = \tan 60° = \sqrt{3}$ ...(2)

and in right triangle ABC, $AC^2 = AB^2 + BC^2$ ...(3)
speed of boat = 2 km / hr.
Option (A):- AC = 300 m

$\frac{EC}{BC} = \sqrt{3} \Rightarrow BC = \frac{300\times\sqrt{3}}{3} = 100\sqrt{3}m$

$BC = 100\sqrt{3}m$

$\Rightarrow AB = \sqrt{AC^2 - BC^2}$

$AB = 100\sqrt{6}$ m is the breadth of the river.

$\Rightarrow$ time taken by boat to move from A to B.

$time_{A\to B} = \frac{100\sqrt{6}\times 60}{2\times 1000} = 3\sqrt{6}$ minutes

Option (A) is correct.

Option (B):- The breadth of the river is $100\sqrt{6}$m, as calculated above.

Option (C):- is incorrect as Rajan took $3\sqrt{6}$ minutes.

Option (D):- h + H = 450 m
speed of boat = 1 km / hr.
As all the angles remain the same, we will get 3 new right triangles which are correspondingly similar to the earlier ones.

Breadth of the river $= \frac{450}{300}\times(100\sqrt{6})$ m

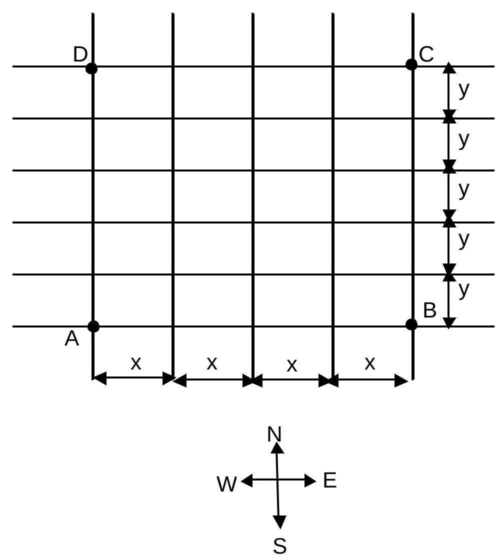
$= 150\sqrt{6}$ m

$\Rightarrow$ timetaken $= \frac{150\sqrt{6}m}{1km/hr} = 9\sqrt{6}$ minutes

$\Rightarrow$ Option (D) is correct.

In all; options; (A), (B) & (D) are correct.

11.(ACD)



Option (A) Sunil has to go from A to C. There are 4 x's and 5 y's which have to be arranged in order to reach C, from A. For example, A can take 4 x's consecutively, thus reaching the last line (North-South) and then take 5 y's along this line to reach C. Any arrangement of 4 x's and 5 y's will have the same distance. The total number of such routes can be found by arranging 9 objects (4 x's + 5 y's) out of which 4 are alike of one kind and 5 are alike of another kind. Thus, the total number of routes :

$\frac{9!}{4!\ 5!}$ = 126 routes.

Option (B) The total number of ways includes the shortest possible routes as well. There can be infinitely possible routes as retracing of the paths will not be restricted. Hence (B) is wrong.

Option (C) A square is a special case of rectangle. To form a rectangle, we have to choose 2 N-S lines from the given 5 and 2 W-E lines, out of 6.

$\Rightarrow$ The total number of rectangles = ${}^6C_2 \times {}^5C_2$ = 150.

Option (C) is correct.

Option (D): As in option (A) we have 11 objects now, out of which 5 are similar, of one kind and 6 are similar of another kind. Total number of routes:-

$= \frac{11!}{5! \times 6!}$ = 336

Option (D) is correct.

$\Rightarrow$ In all options (A), (C) and (D) are correct.

12.(A,C,D) Laxman takes the first train which is the slower one. Bharat takes the faster train.

Let the trains be A and B respectively.

Speed of the faster train, train B =

$\frac{180km}{3hr} = 60km/hr$

$V_B = 60km/hr$

As train A takes twice the time, so

$V_A = 30km/hr$

Speed of train B, w.r.t. Laxman (when he is sitting in the train A) is 60 – 30 = 30 km/hr

Laxman observes the train B, pass by him in 12 seconds. If L_B were the length of the faster train then,

$30km/hr = \frac{L_B}{12seconds}$

$\Rightarrow L_B = \frac{30 \times 1000}{3600} \times 12$ m

$\Rightarrow L_B = 100m$

Option A: $30km/hr = \frac{L_A + L_B}{30seconds}$

{L_A = Length of the slower train}

$\Rightarrow L_A + L_B = \frac{30 \times 1050}{3600} \times 30$

$L_A + L_B = 250m$

$\Rightarrow L_A = (250 - 100)m$ OR L_A = 150 m

So, $L_A - L_B$ = 50 m

option (A) is correct.

Option (B) If V_B = 60 × 2 = 120 km/hr

V_A = 30 km/hr, as before.

To overtake, train A; train B has to cover its length, L_A. As we cannot determine the length of the slower train, we cannot find the time taken in overtake.

Hence, (B) is not correct.

Option (C): V_A = 30 km/hr

V_B = 60 km/hr

$30km/hr = \frac{L_A + L_B}{24}$

$\Rightarrow L_A + (100) = 30 \times \frac{1000}{3600} \times 24$

L_A = 200 – 100

L_A = 100 m

Option (C) is correct.

Option (D): $V_A = 30 \times \frac{3}{2} = 45km/hr$

$V_B = 60km/hr$

$\Rightarrow 15km/hr = \frac{L_B}{t} = \frac{100}{t}m$

$\Rightarrow t = \frac{100m \times 3600}{15 \times 1000}$

= 24 seconds

Hence (D) is correct.

13.(A,B,D) 20 men can finish the work in 10 days and 15 women can finish it in 20 days.
From here we can say that

$\Rightarrow 20 \times 10\ M = 15 \times 20\ W$

$\Rightarrow 2M = 3W \quad \ldots(i)$

Hence 10 Men and 10 Women together can finish the work in N days such that
{N = Number of Days taken working together.}

$\Rightarrow 20 \times 15 = 25 \times N$

$\Rightarrow N = 12$

Option (A) Total wage for Men = 12 × 50 × 10
= Rs. 6000
Total wage for Women = 12 × 10 × 45
= Rs. 5400
$\therefore$ Total wage bill = Rs. 11400
Hence, option (A) is correct.

Option (B) Total wage for Men = 12 × 10 × 45
= Rs. 5400
Total wage for Women = 12 × 10 × 40
= Rs. 4800
$\therefore$ Total wage bill = Rs. 10200
Hence, option (B) is correct.

Option (C) Total wage for Men = 12 × 10 × 40
= Rs. 800
Total wage for Women = 12 × 10 × 40
= Rs. 4800
$\therefore$ Total wage bill = Rs. 9600
Hence, option (C) is correct.

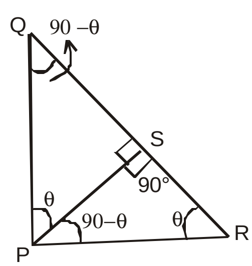
Option (D) If 20 Men and 30 Women are employed. Then, together they way finish the work in N days such that

$\Rightarrow 20 \times 15 = 60 \times N \quad \{\because 20\ M = 30\ W\}$

$\Rightarrow N = 5$

Total wage for Men = 5 × 20 × 40 = Rs. 4000
Total wage for Women = 5 × 30 × 35 = Rs. 5250
$\therefore$ Total wage bill = Rs. 9250
Hence, option (D) is correct.

14.



Statement I: ΔPQS has angles

$\angle P = \theta,\ \angle Q = 90 - \theta,\ \angle S = 90°$

ΔRPS has angles

$\angle R = \theta,\ \angle P = 90 - \theta$ and $\angle S = 90°$

As all the corresponding angles are equal,

$\Delta PQS \sim \Delta RPS$

Hence, statement I is correct.

Statement II:

ΔPSQ, angles $\angle P = \theta,\ \angle S = 90°$ and $\angle Q = 90 - \theta$

ΔRSP, angles $\angle R = \theta$, $\angle S = 90$, $\angle P = 90 - \theta$

$\Rightarrow \Delta PSQ \sim \Delta RSP$

but $\Delta PSQ \not\cong \Delta RSP$ similar but not congruent.
Hence, II is incorrect.

Statement III:

ΔPSQ, angles $\angle P = \theta,\ \angle S = 90°$ and $\angle Q = 90 - \theta$

and

ΔRPQ, angles $\angle R = \theta,\ \angle P = 90°$ and $\angle Q = 90 - \theta$

Hence, statement III is correct.
Hence, statement (I) and (III) are correct.
None of the option (A), (B), (C) and (D) are correct.

The problem has ERRATA.

15.(D)

$$S = \frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \ldots.$$

$$S = \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \ldots.\left(\frac{1}{(n)^2} - \frac{1}{n+1)^2}\right)$$

or

$$S = \left(1 - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} + \ldots + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)\right)$$

$$= 1 - \frac{1}{(n+1)^2}$$

$$= \frac{(n+1)^2 - 1}{(n+1)^2} = \frac{(n^2 + 2n + 1) - 1}{(n+1)^2}$$

$$S = \frac{n^2 + 2n}{(n+1)^2}$$

$$\Rightarrow \frac{1}{S} = \frac{(n+1)^2}{n^2 + 2n}$$

Hence, option (d) is the correct answer.

16.(D) $(5+\sqrt{2})x^2-(4+\sqrt{5})x+(8+2\sqrt{5})=0$

Let the roots be α and β.

Let H be the harmonic mean of α and β. Then

$\frac{1}{\alpha}, \frac{1}{H}, \frac{1}{\beta}$ are in an A.P..

$$\Rightarrow \frac{2}{H}=\frac{1}{\alpha}+\frac{1}{\beta}$$

$$=\frac{\alpha+\beta}{\alpha\beta}$$

$$=\frac{\frac{4+\sqrt{5}}{5+\sqrt{2}}}{\frac{8+2\sqrt{5}}{5+\sqrt{2}}}=\frac{4+\sqrt{5}}{8+2\sqrt{5}}$$

or $\frac{2}{H}=\frac{1}{2}\Rightarrow H=4$

$\Rightarrow \sqrt{H}=\pm 2$

Hence, option (D) is correct.

17.(A) Given,

$\tan\alpha$ is the G.M. of $\sin\alpha$ and $\cos\alpha$

$$\Rightarrow (\tan\alpha)^2=(\sin\alpha)(\cos\alpha)$$

or $\left(\frac{\sin\alpha}{\cos\alpha}\right)^2=(\sin\alpha)(\cos\alpha)$

or $\frac{\sin^2\alpha}{\cos^2\alpha}=(\sin\alpha)(\cos\alpha)$

or $(\sin\alpha)=(\cos\alpha)^3$

{$\because \alpha\neq n\pi$, $\sin\alpha\neq 0$, So we can divide both sides of the equation by $\sin\alpha$ }

Squaring on both sides

$$(\sin\alpha)^2=\left[(\cos\alpha)^3\right]^2$$

or $\sin^2\alpha=\left[\cos^2\alpha\right]^3=\left[1-\sin^2\alpha\right]^3$

or

$$\sin^2\alpha=(1)^3+(-\sin^2\alpha)^3+3(1)^2(-\sin^2\alpha)+3(1)(-\sin^2\alpha)^2$$

or $\sin^2\alpha=1-\sin^6\alpha-3\sin^2\alpha+3\sin^4\alpha$

or $1-4\sin^2\alpha+3\sin^4\alpha-\sin^6\alpha=0$

Adding 1, on both sides,

$$2-4\sin^2\alpha+3\sin^4\alpha-\sin^6\alpha=1$$

Hence, option (A) is correct.

18.(A,D) $\frac{1}{\sqrt{5}+\sqrt{6}+\sqrt{11}}$

$$=\frac{1}{\left(\sqrt{5}+\sqrt{6}\right)+\sqrt{11}}\times\frac{\left(\sqrt{5}+\sqrt{6}\right)-\sqrt{11}}{\left(\sqrt{5}+\sqrt{6}\right)-\sqrt{11}}$$

$$=\frac{\sqrt{5}+\sqrt{6}-\sqrt{11}}{\left(\sqrt{5}+\sqrt{6}\right)^2-\left(\sqrt{11}\right)^2}$$

$$=\frac{\left(\sqrt{5}+\sqrt{6}\right)-\sqrt{11}}{\left(5+6+2\sqrt{5\times 6}\right)-11}$$

$$=\frac{\sqrt{5}+\sqrt{6}-\sqrt{11}}{(11+2\sqrt{30}-11)}$$

$$=\frac{\sqrt{5}+\sqrt{6}-\sqrt{11}}{2\sqrt{30}}$$

$$=\frac{\left(\sqrt{5}+\sqrt{6}-\sqrt{11}\right)\times\sqrt{30}}{2\left(\sqrt{30}\right)^2}$$

$$=\frac{\sqrt{5\times 30}+\sqrt{6\times 30}-\sqrt{11\times 30}}{2\times 30}$$

$$=\frac{\sqrt{5\times 6\times 5}+\sqrt{6\times 5\times 6}-\sqrt{330}}{60}$$

$$=\frac{5\sqrt{6}+6\sqrt{5}-\sqrt{330}}{60}$$

Option (A) and (B) both have this value. Hence both are correct. (Although, the section had only one answer questions).

19.(B) $x^2+px+q=0$

Let roots be α and α^2.

We have

$\alpha+\alpha^2=-p$

and $(\alpha)\times\left(\alpha^2\right)=q$

or $\alpha+\alpha^2=-p$ …(1)

and $\alpha^3=q$ …(2)

cubing both sides of equation (1)

$$\left(\alpha+\alpha^2\right)^3=(-p)^3$$

or

$(\alpha)^3 + (\alpha^2)^3 + 3(\alpha)^2 \times (\alpha^2) + 3(\alpha)(\alpha^2)^2 = -p^3$

or $\alpha^3 + \alpha^6 + 3\alpha^4 + 3\alpha^5 = -p^3$

or $q + q^2 + 3\alpha^3(\alpha + \alpha^2) = -p^3$

or $q + q^2 + 3(q)(-p) = -p^3$

or $p^3 - 3pq + q + q^2 = 0$

or $p^3 - q(3p - 1) + q^2 = 0$

of the given option, only (B) is correct.

20.(A) $x^2 + mx + 1 = 0$ roots are α and β

$\Rightarrow \alpha + \beta = -m$

$\alpha.\beta = 1$

And $x^2 + nx + 1 = 0$ roots are γ and δ

$\Rightarrow \gamma + \delta = -n$

$\gamma.\delta = 1$

Expression:

$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= (\alpha - \gamma)(\beta + \delta)(\beta - \gamma)(\alpha + \delta)$

$= [\alpha\beta + \alpha\cdot\gamma - \gamma\beta - \gamma\delta][\alpha\beta + \beta\delta - \gamma\alpha - \gamma\delta]$

$= [1 - \alpha.\delta - \gamma\beta - 1][1 + \beta\delta - \gamma\alpha - 1]$

$= (\alpha\delta - \gamma\beta)(\beta\delta - \gamma\alpha)$

$= \alpha\beta\delta^2 - \alpha^2\delta\gamma - \beta^2\gamma\delta + \gamma^2\alpha\beta$

$= 1.\delta^2 - \alpha^2.1 - \beta^2.1 + \gamma^2.1$

$= \delta^2 - \alpha^2 - \beta^2 + \gamma^2$

$= \left(\delta^2 + \gamma^2\right) - \left(\alpha^2 + \beta^2\right)$

$= \left[(\delta + \gamma)^2 - 2\delta.\gamma\right] - \left[(\alpha + \beta)^2 - 2\alpha\beta\right]$

$= \left[(-n)^2 - 2.1\right] - \left[(-m)^2 - 2.1\right]$

$= n^2 - m^2$

Hence, option (A) is correct.

21.(C) Area of square = 484 cm^2

$\Rightarrow$ side of the square $= \sqrt{484}$ cm $= 22$cm

$=$ length of the wire = 4 × (side)

= 4 × (22)

original length of wire = 88 cm

After cutting the wire

longer part $= \frac{3}{4} \times 88 = 66$cm

shorter part = 88 – 66 = 22 cm.

Radius of the circle, formed

$= \frac{66}{2\pi} = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm

Side of the square formed $= \frac{22}{4} = \frac{11}{2}$ cm

Area of the circle

$= \pi\left(\frac{21}{2}\right)^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2}$ sq.cm

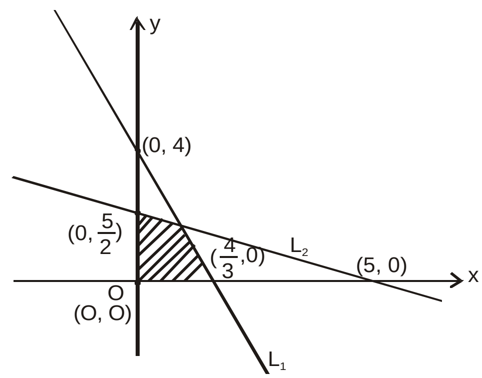
Area of the square $= \left(\frac{11}{2}\right)^2 = \frac{121}{4}$ sq.cm

Total area enclosed by both

$= \left(\frac{693}{2} + \frac{121}{4}\right)$sqcm $= 376.75cm^2$

Hence (c) is correct.

22.(C)



In the x-y plane, straight line L_1 passes through $(0, 4)$ and $\left(\frac{4}{3}, 0\right)$: It's equation is

$\frac{y - 4}{x - 0} = \frac{0 - 4}{\frac{4}{3} - 0} = -3$

or $y + 3x - 4 = 0$ …(1)

and the equation of line L_2 is

$x + 2y - 5 = 0$ …(2)

The shaded region includes x-axis (y = 0) and y-axis (i.e. x = 0) as its boundaries and bounds only the positive quadrant of the x-y plane.

$\Rightarrow x \geq 0$ and $y \geq 0$ …(3)

Put (0, 0) in equations (1) and (2)

in (1) $0 + 3(0) - 4 = -4 < 0$

in (2) $0 + 2(0) - 5 = -5 < 0$

$\Rightarrow$ the shaded region is given by

$y + 3x - 4 < 0$ ...(4)

and $x + 2y - 5 < 0$ ...(5)

As the region is bounded by both the lines as well,

we have $y + 3x - 4 = 0$ ...(6)

and $x + 2y - 5 = 0$ ...(7)

From (1), (4), (5), (6) and (7).

The region bounded is

$3x + y \le 4,\ x + 2y \le 5,\ x \ge 0,\ y \ge 0$

Hence option (c) is correct.

23.(B) Here we can find after 6 hours, how much part was filled by the three pipes.

$$= \frac{16}{12} + \frac{4}{18} - \frac{6}{36} = \frac{1}{2} + \frac{2}{9} - \frac{1}{6}$$

$$= \frac{9+4-3}{18} = \frac{10}{18} = \frac{5}{9}$$

Remaining part = $\frac{1-5}{9} = \frac{4}{9}$

Since, this part is to be filled by A and B together. Further, A and B together can fill in one hour $= \frac{1}{12} + \frac{1}{10}$ part $= \frac{5}{36}$ part.

$\therefore \frac{4}{9}$ part will be filled by A and B together in time $= 3\frac{1}{5}$ h.

$\therefore$ Total time required = 6h + 3h + 12 min

= 9 hrs and 12 min.

Hence, option (B) is correct.

24.(A) $$y = \frac{1}{\{\log_{10}(3-x)\}} + \sqrt{x+7}$$

(i) $\log_{10}(3-x) \ne 0 \Rightarrow 3 - x \ne 1$

or $x \ne 2$ ...(1)

(ii) $3 - x > 0 \Rightarrow x < 3$ ...(2)

(iii) $x + 7 \ge 0 \Rightarrow x \ge -7$ ... (3)

The domain is

$-7 \le x < 3,\ x \ne 2$

or $[-7,3) \sim \{2\}$

There is a misprint in the options.

The correct answer is $[-7,3) \sim \{2\}$.

Apparently, option (A) should have had this answer, if there was no misprint.

Hence, (A) is correct.

25.(A) The angle between $\vec{a}$ and $\vec{b}$ is θ.

$(\vec{a} + 3\vec{b}).(7\vec{a} - 5\vec{b}) = 0$ ... (1)

and $(\vec{a} - 4\vec{b}).(7\vec{a} - 2\vec{b}) = 0$ ... (2)

From (1)

$(\vec{a}).(7\vec{a}) + (\vec{a}).(-5\vec{b}) + (3\vec{b}).(7\vec{a}) + (3\vec{b})(-5\vec{b}) = 0$

or

$$7|\vec{a}|^2 \cos 0^\circ - 5|\vec{a}||\vec{b}|\cos\theta + 21|\vec{a}||\vec{b}|\cos\theta - 15|\vec{b}|^2 \cos 0^\circ = 0$$

or $7a^2 - 5ab\cos\theta + 21ab\cos\theta - 15b^2 = 0$

or $7a^2 - 15b^2 + 16ab\cos\theta = 0$ ...(3)

Similarly, solving the dot product in (2), we get

$7a^2 + 8b^2 - 30ab\cos\theta = 0$ ...(4)

Subtracting (3) from (4)

$23b^2 = 46ab\cos\theta$

$$\Rightarrow \frac{b}{a} = 2\cos\theta \Rightarrow \cos^2\theta = \frac{b^2}{4a^2}$$

or

$$\Rightarrow \sin^2\theta = 1 - \frac{b^2}{4a^2}$$

$$\Rightarrow \tan^2\theta = 4\left(\frac{a^2}{b^2}\right) - 1$$

From equation (3) and (4)

$7a^2 - 15b^2 = -16ab\cos\theta$

$7a^2 + 8b^2 = 30ab\cos\theta$

$$\Rightarrow \frac{7a^2 - 15b^2}{7a^2 + 8b^2} = \frac{-16}{30}$$

or $$\frac{7\left(\frac{a^2}{b^2}\right) - 15}{7\left(\frac{a^2}{b^2}\right) + 8} = \frac{-8}{15}$$

$$\Rightarrow \frac{a^2}{b^2} = 1$$

$\Rightarrow \tan^2\theta = 4(1) - 1$

$\tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm\sqrt{3}$

Option (A) has one of the two possible values i.e. $\sqrt{3}$.

Hence, (A) is correct.

26. (C) Before the merger
X Ltd. had no. of employees = 4
Y Ltd. had no. of employees = 3
Z Ltd. had no. of employees = 5
After the merger, there are quarrels amongst the employees of the erstwhile X ltd, Y ltd, & Z ltd. An employee does not quarrel with any of the employees from his parent company.

Company	X	Y	Z
Employees	4	3	5

Any employees in X can have a maximum possible no. of quarrels with the employees of Y ltd. & Z ltd. as given by:-
1 × 3 + 1 × 5
As there are 4 employees in the merged company, who came from X; So; the total number of quarrels involving employees of X company is
4 × [1 × 3 + 1 × 5]
= 4 × 8
= 32 quarrels. ...(1)
Further employees of Y & Z ltd. will be involved with quarrels within each other. The total number of such quarrels.
= 3 × [1 × 5]
= 5 × [1 × 3]
= 15 quarrels ...(2)
From (1) and (2)
The total number of quarrels between the employees of three erstwhile companies is
= 32 + 15 = 47 quarrels.
Hence (C) option is correct.

27. (C)
$$\frac{1-c}{1+c}=\frac{1-\tan\left(\frac{\alpha}{2}\right)\times\tan\left(\frac{\beta}{2}\right)}{1+\tan\left(\frac{\alpha}{2}\right)\times\tan\left(\frac{\beta}{2}\right)}$$
$$=\frac{\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\times\sin\left(\frac{\beta}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)+\sin\left(\frac{\alpha}{2}\right)\times\sin\left(\frac{\beta}{2}\right)}$$
$$=\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \quad \ldots(i)$$
$\because \sin\alpha+\sin\beta=a$
$\&\cos\alpha+\cos\beta=b$

Squaring and adding up both equations
$\Rightarrow \sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta=a^2$
$\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta=b^2$
$\Rightarrow 1+1+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=a^2+b^2$
$2+2\{\cos(\alpha-\beta)\}=a^2+b^2$
$\Rightarrow 1+\cos(\alpha-\beta)=\frac{a^2+b^2}{2} \quad \ldots(ii)$
$\because \cos\alpha+\cos\beta=b \quad \ldots(iii)$
Dividing equation (iii) by (ii)
$$\Rightarrow \frac{\cos\alpha+\cos\beta}{1+\cos(\alpha-\beta)}=\frac{2b}{a^2+b^2}$$
$$\Rightarrow \frac{2\cos\frac{(\alpha+\beta)}{2}\times\cos\frac{(\alpha-\beta)}{2}}{2\cos^2\frac{(\alpha-\beta)}{2}}=\frac{2b}{a^2+b^2}$$
$$\Rightarrow \frac{\cos\frac{(\alpha+\beta)}{2}}{\cos\frac{(\alpha-\beta)}{2}}=\frac{2b}{a^2+b^2}$$
From equation (i) we get
$$\Rightarrow \frac{1-c}{1+c}=\frac{2b}{a^2+b^2}$$
Option C is correct.

28.(C)
$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$
$\because a\sin^{-1}x=c+b\cos^{-1}x$
$\Rightarrow \frac{a}{b}\sin^{-1}x=\frac{c}{b}+\cos^{-1}x \quad \ldots(I)$
or $b\cos^{-1}x=a\sin^{-1}x-c$
$\Rightarrow \frac{b}{a}\cos^{-1}x=\sin^{-1}x-\frac{c}{a} \quad \ldots(II)$
Adding equation (I) and (II)
$$\Rightarrow \frac{a}{b}\sin^{-1}x+\frac{b}{a}\cos^{-1}x=(\sin^{-1}x+\cos^{-1}x)+\frac{c}{b}-\frac{c}{a}$$
$$=\left(\frac{\pi}{2}\right)+c\left\{\frac{1}{a}-\frac{1}{b}\right\}$$
$$=\left(\frac{\pi}{2}\right)+\frac{c(b-a)}{ab}$$
$$\frac{\pi ab+2c(b-a)}{2ab}$$
Hence, option (c) is the correct answer.

29. (A)

$$\frac{2\sin\theta}{1+\sin\theta+\cos\theta}=\frac{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\cos^2\frac{\theta}{2}-1}$$

$$=\frac{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}\left\{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right\}}$$

$$\Rightarrow K=\frac{2\sin\frac{\theta}{2}}{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)} \qquad \ldots(I)$$

Now

$$\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$$

$$=\frac{1-1+2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}$$

$$=\frac{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}$$

$$=\frac{2\sin\frac{\theta}{2}}{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)} \qquad \ldots(II)$$

$\because$ Equation (I) and (II) are same.
Hence, the value is equal to K and option (A) is correct.

30. (A) The probability that Sumit actually sees a Shark, given that he claimed to have seen one.

$$=\frac{\text{P(He actually sees the shark \& reports truth)}}{\text{P(He claims of seeing a shark)}}$$

$$=\frac{\text{P(Sees the shark)} \times \text{P(Reports Truth)}}{\left(\begin{array}{c}\text{P(sees the shark)} \times \text{P(reports truth)}\\ + \text{P(Doesn't see)} \times \text{P(reports false)}\end{array}\right)}$$

$$=\frac{\frac{1}{8}\times\frac{1}{6}}{\frac{1}{8}\times\frac{1}{6}+\frac{7}{8}\times\frac{5}{6}}$$

$$=\frac{\frac{1}{48}}{\frac{36}{48}}=\frac{1}{36}$$

$\Rightarrow$ Option (A) is the correct answer.