Solution

Section A

1. H.C.F and L.C.M of two numbers can be same only if two numbers are equal. Hence their difference will be zero. (1)

2. Let roots of equation: $ax^2 + bx + c = 0$ are α and $-\alpha$, then sum of roots

$$\therefore \quad \alpha + (-\alpha) = -\frac{b}{a}$$

$$\Rightarrow \quad 0 = \frac{-b}{a}$$

$$\Rightarrow \quad b = 0$$ (1)

OR

Using factorization method of splitting the middle term, we can solve the quadratic equation as follows:

$$2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 - 4x + 3x - 6 = 0$$

$$\Rightarrow \quad (2x^2 - 4x) + (3x - 6) = 0$$

$$\Rightarrow \quad 2x(x-2) + 3(x-2) = 0$$ [½]

$$\Rightarrow \quad (x-2)(2x+3) = 0$$

$$\Rightarrow \quad x - 2 = 0 \text{ or } 2x + 3 = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = -\frac{3}{2}$$

Thus, $x = 2$ and $x = -\frac{3}{2}$ are the two roots of the $2x^2 - x - 6 = 0$ equation. [½]

Option (b) is correct.

3. Here, $a_1 = -1, b_1 = 2$ and $a_2 = \frac{1}{2}, b_2 = -\frac{1}{4}$.

$$\therefore \quad \frac{a_1}{a_2} = \frac{-1}{\frac{1}{2}} = -2, \frac{b_1}{b_2} = \frac{2}{-\frac{1}{4}} = -8$$

$$\Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ (1)

$\Rightarrow$ The given pair of linear equations has a unique solution.

Hence, the given statement is true.

OR

We have, $x = 2$ and $y = 3$

Let us substitute the above values in the equation $2x + 3y - 13 = 0$, we get

$$\text{LHS} = (2 \times 2) + (3 \times 3) - 13 = 4 + 9 - 13 = 13 - 13 = 0$$

RHS = 0

Since, LHS = RHS, the required linear equation is $2x + 3y - 13 = 0$.

Hence, the correct option is (a). (1)

4. We have

l = last term = 185

and d = common difference

$= 9 - 5 = 4$ (½)

$\Rightarrow$ 9th term from the end $= 185 + (9-1)(-4)$

$[\because$ nth term from the end $= l + (n-1)(-d)]$

$= 185 + 8 \times (-4)$

$= 185 - 32 =$ **153** (½)

5. It is not possible to construct a pair of tangents from a point P situated at a distance of 3 cm from the centre of a circle of radius 3.5 cm.

$\because$ $3 < 3.5$ = radius of circle (1)

6. When two dice are thrown together, then the possible outcomes of the experiment are listed in the table below:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

So, the number of possible outcomes = $6 \times 6 = 36$

The outcome favourable to the event "Getting the same number on both dice" is denoted by A, are (1,1), (2,2), (3,3), (4,4), (5, 5), (6,6) (see table), i.e., the number of outcomes favourable to A = 6. (1)

Thus, $P(A) = \frac{6}{36} = \mathbf{\frac{1}{6}}$

Section B

7. True, because $n(n+1)(n+2)$ will always be divisible by 6, as at least one of the factors will be divisible by 2 and at least one of the factors will be divisible by 3. (1+1)

OR

The first number that is divisible by 8 between 200 and 500 is 208 and the last number that is divisible by 8 are 496.

So, the sequence will be 208, 216, 224 .... 496.

Common difference $d = 8$

First term $a = 208$ (1)

Let there be n terms is the sequence

Using the formula $a_n = a+(n-1)d$

Where $a_n = 496,\ a = 208$ and $d = 8$

$$496 = 208+(n-1)(8)$$

$$(n-1)8 = 288$$

$$n-1 = 36$$

$$n = 37$$

Hence, between 200 and 500 there are 37 integers that are divisible by 8. [1]

8. Let α, β be the zeros of the polynomial $f(x) = x^2 - 8x + k$. Then,

$$\alpha + \beta = -\left(\frac{-8}{1}\right) = 8$$

and $$\alpha\beta = \frac{k}{1} = k$$ (1)

It is given that

$$\alpha^2 + \beta^2 = 40$$

$$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow 8^2 - 2k = 40 \quad [\because \alpha + \beta = 8 \text{ and } \alpha\beta = k]$$

$$\Rightarrow 2k = 64 - 40$$

$$\Rightarrow 2k = 24$$

$$\Rightarrow k = 12$$ (1)

OR

Let the polynomial be

$$p(x) = ax^2 + bx + c$$

Its given that, Sum of zeroes = $\frac{21}{8} \Rightarrow \frac{-b}{a} = \frac{21}{8}$

Assuming $a = 1$,

$$\Rightarrow \quad b = -\frac{21}{8}$$

We also know that, Product of zeroes = $\frac{5}{16}$

$$\Rightarrow \quad \frac{c}{a} = \frac{5}{16}$$

Assuming $a = 1$,

$$\Rightarrow \quad c = \frac{5}{16}$$

Now, $a = 1, b = -\frac{21}{8}, c = \frac{5}{16}$ (1)

Hence, the required quadratic polynomial = $ax^2 + bx + c$

Substituting the values of *a*, *b* and *c* in the above equation, we get,

$$x^2 - \frac{21}{8}x + \frac{5}{16}$$

Multiply the equation by 16.

$$\Rightarrow \quad 16x^2 - 42x + 5$$

Hence, the required quadratic polynomial is $16x^2 - 42x + 5$ (1)

9. Lets the ten's digit be x and unit's digit = y

$$\text{Number} = 10x + y$$
$$\therefore \quad 10x + y = 4(x + y)$$
$$\Rightarrow \quad 6x = 3y$$
$$\Rightarrow \quad 2x = y \quad (1)$$
$$\text{Again} \quad 10x + y = 3xy$$
$$10x + 2x = 3x(2x)$$
$$\Rightarrow \quad 12x = 6x^2$$
$$\Rightarrow \quad x = 2 \quad (\text{rejecting } x = 0)$$
$$2x = y$$
$$\Rightarrow \quad y = 4 \quad (1)$$

$\therefore$ The required number is 24

10. Given $x = a\cos\theta, y = b\sin\theta$ (1)

$$b^2x^2 + a^2y^2 - a^2b^2 = b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2 - a^2b^2$$
$$= a^2b^2\cos^2\theta + a^2b^2\sin^2\theta - a^2b^2$$
$$= a^2b^2(\sin^2\theta + \cos^2\theta) - a^2b^2$$
$$= a^2b^2 - a^2b^2 = 0 \quad (\because \sin^2\theta + \cos^2\theta = 1) \quad (1)$$

11. Since $\angle C = 90°$

$\therefore \quad \angle A + \angle B = 180° - \angle C = 90°$ (1)

Now, $\sin^2 A + \sin^2 B = \sin^2 A + \sin^2(90° - A)$

$= \sin^2 A + \cos^2 A = 1$ (1)

12. Given, total surface area of solid hemisphere = 462 cm^2

$\Rightarrow \quad 3\pi r^2 = 462 \text{ cm}^2$ (1)

$$3 \times \frac{22}{7} \times r^2 = 462$$

$$r^2 = 49$$

$\Rightarrow \quad r = 7\text{cm}$

Volume of solid hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$= 718.66 \text{ cm}^3$ (1)

Section C

13. For the maximum number of columns, we have to find the HCF of 616 and 32.

Now, since 616 > 32, we apply division lemma to 616 and 32. (1)

We have, $616 = 32 \times 19 + 8$

Here, remainder $8 \neq 0$. (1)

So, we again apply division lemma to 32 and 8.

We have, $32 = 8 \times 4 + 0$

Here, remainder is zero. So, HCF (616, 32) = 8

Hence, maximum number of columns is 8. (1)

14. Let a – d, a and a+d be the zeros of the polynomial f(x). Then,

Sum of the zeros $= \frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$ (1)

$$\Rightarrow \quad (a - d) + a + (a + d) = -\frac{(-p)}{1}$$

$$\Rightarrow \quad 3a = p$$

$\Rightarrow \quad a = \frac{p}{3}$ (1)

Since a is a zero of the polynomial f(x). Therefore,

$$f(a) = 0$$

$$\Rightarrow \quad a^3 - pa^2 + qa - r = 0$$

$$\Rightarrow \left(\frac{p}{3}\right)^3 - p\left(\frac{p}{3}\right)^2 + q\left(\frac{p}{3}\right) - r = 0$$

$$\Rightarrow \quad p^3 - 3p^3 + 9pq - 27r = 0 \qquad \left[\because a = \frac{p}{3}\right]$$

$$\Rightarrow \quad 2p^3 - 9pq + 27r = 0, \text{ which is the required condition} \qquad (1)$$

15. The given system of equations may be written as

$$ax + by - (a - b) = 0$$
$$bx + ay - (a + b) = 0$$

By cross-multiplication, we have

$$\frac{x}{\begin{matrix} b & -(a-b) \\ -a & -(a+b) \end{matrix}} = \frac{-y}{\begin{matrix} a & -(a-b) \\ b & -(a+b) \end{matrix}} = \frac{1}{\begin{matrix} a & b \\ b & -a \end{matrix}} \qquad (1)$$

$$\Rightarrow \quad \frac{x}{b\times-(a+b)-(-a)\times-(a-b)} = \frac{-y}{a\times-(a+b)-b\times-(a-b)} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad \frac{x}{-b(a+b)-a(a-b)} = \frac{-y}{-a(a+b)+b(a-b)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-b^2-a^2} = \frac{-y}{-a^2-b^2} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-(a^2+b^2)} = \frac{y}{(a^2+b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad -\frac{(a^2+b^2)}{-(a^2+b^2)} = 1 \text{ and } y = \frac{(a^2+b^2)}{-(a^2+b^2)} = -1 \qquad (2)$$

Hence, the solution of the given system of equations is $x = 1$, $y = -1$.

OR

Clearly, the given equations are not linear equations in the variables u and v but can be reduced to linear equations by an appropriate substitution.

If we put $u = 0$ in either of the two equations, we get $v = 0$.

So, $u = 0$, $v = 0$ form a solution of the given system of equations.

To find the other solutions, we assume that $u \neq 0$, $v \neq 0$.

Now, $u \neq 0, v \neq 0 \Rightarrow uv \neq 0$.

On dividing each one of the given equations by uv, we get

$$\frac{6}{v} + \frac{3}{u} = 7 \qquad \ldots(i)$$

$$\frac{3}{v}+\frac{9}{u} = 11 \quad \text{...(ii)}$$

Taking $\frac{1}{u} = x$ and $\frac{1}{v} = y$, the above equations become

$$3x + 6y = 7 \quad \text{...(iii)}$$

$$9x + 3y = 11 \quad \text{...(iv)} \quad (1)$$

Multiplying equation (iv) by 2, the above system of equations becomes

$$3x + 6y = 7 \quad \text{...(v)}$$

$$18x + 6y = 22 \quad \text{...(vi)}$$

Substracting equation (vi) from equation (v), we get

$$-15x = -15$$

$$\Rightarrow \quad x = 1$$

Putting x = 1 in equation (iii), we get

$$3 + 6y = 7$$

$$\Rightarrow \quad y = \frac{4}{6} = \frac{2}{3} \quad (1)$$

Now, $x = 1$

$$\Rightarrow \quad \frac{1}{u} = 1$$

$$\Rightarrow \quad u = 1$$

and, $y = \frac{2}{3}$

$$\Rightarrow \quad \frac{1}{v} = \frac{2}{3}$$

$$\Rightarrow \quad v = \frac{3}{2}$$

Hence, the given system of equations has two solutions given by

(i) u = 0, v = 0 (ii) u = 1, v = 3/2

16. Clearly, the given sequence is an A.P. with first term a(=54) and common differenced d(= –3). Let the sum of n terms be 513. Then,

$$S_n = 513$$

$$\Rightarrow \quad \frac{n}{2}\{2a + (n-1)d\} = 513 \quad (1)$$

$$\Rightarrow \frac{n}{2}[108 + (n-1) \times -3] = 513$$

$$\Rightarrow \quad n^2 - 37n + 342 = 0$$

$$\Rightarrow \quad (n-18)(n-19) = 0$$

$$\Rightarrow \qquad n = 18 \text{ or } 19 \qquad (1)$$

Here, the common difference is negative. So, 19th term is given by

$$a_{19} = 54 + (19 - 1) \times -3 = 0 \qquad (1)$$

Thus, the sum of 18 terms as well as that of 19 terms is 513.

17. Let $A = (x_1, y_1) = (t, t-2)$, $B = (x_2, y_2) = (t+2, t+2)$ and $C = (x_3, y_3) = (t+3, t)$ be the vertices of a given triangle. Then,

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| \qquad (1)$$

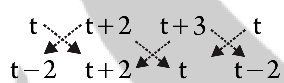
$$\Rightarrow \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|\{t(t+2-t)+(t+2)(t-t+2)+(t+3)(t-2-t-2)\}\right|$$

(1)

$$\Rightarrow \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|\{(2t+2t+4-4t-12)\}\right| = |-4| = 4 \text{ sq. units} \qquad (1)$$

Clearly, area of ΔABC is independent of t.

Alternative Method : We have,



$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|\begin{array}{l}\{t(t+2)+(t+2)t+(t+3)(t-2)\} \\ \quad -\{(t+2)(t-2)+(t+3)(t+2)+t\times t\}\end{array}\right| \qquad (1)$$

$$\Rightarrow \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|\left(t^2+2t+t^2+2t+t^2+t-6\right)-\left(t^2-4+t^2+5t+6+t^2\right)\right|$$

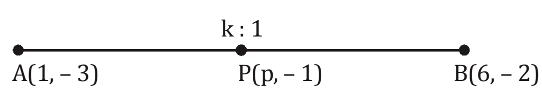
$$\Rightarrow \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|\left(3t^2+5t-6\right)-\left(3t^2+5t+2\right)\right| \qquad (1)$$

$$\Rightarrow \quad \text{Area of } \Delta ABC = \frac{1}{2}\left|-6-2\right|$$

$$\Rightarrow \quad \text{Area of } \Delta ABC = 4 \text{ sq. units} \qquad (1)$$

Hence, Area of ΔABC is independent of t.

18. Let P (p, –1) divide the line segment joining the points A(1,–3) and B(6,2) in the ratio k : 1 i.e. AP : PB = k : 1.



(½)

$$\therefore \quad \text{Coordinates of P are } \left(\frac{k\times 6+1\times 1}{k+1}, \frac{k\times 2+1\times(-3)}{k+1}\right) \text{i.e} \left(\frac{6k+1}{k+1}, \frac{2k-3}{k+1}\right) \qquad (½)$$

But P is $(p, -1)$

$$\Rightarrow \quad \frac{2k-3}{k+1} = -1$$

$$\Rightarrow \quad 2k - 3 = -k - 1$$

$$\Rightarrow \quad 3k = 2$$

$$\Rightarrow \quad k = \frac{2}{3}. \qquad (1)$$

$\therefore$ The required ratio is $\frac{2}{3}:1$ i.e. 2:3 (internally).

Also $\quad \frac{6k+1}{k+1} = p \qquad \ldots(i)$

Putting $k = \frac{2}{3}$ in (i), we get

$$p = \frac{6\times\frac{2}{3}+1}{\frac{2}{3}+1} = \frac{5}{\frac{5}{3}} = \frac{5}{1}\times\frac{3}{5} = 3. \qquad (1)$$

Hence, $p = 3$

OR

Let the point $M(11, y)$ divides the line segment joining the points $P(15, 5)$ and $Q(9, 20)$ in the ratio $K : 1$.

Then the coordinates of M are $\left(\frac{9K+15}{K+1}, \frac{20K+5}{K+1}\right) \qquad (1)$

But the coordinates of M are given as $(11, y)$.

$$\therefore \quad \frac{9K+15}{K+1} = 11$$

and $\quad \frac{20K+5}{K+1} = y$

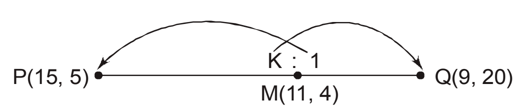
Consider, $\quad \frac{9K+15}{K+1} = 11$

$$\Rightarrow \quad 9K + 15 = 11K + 11$$

$$\Rightarrow \quad 11K - 9K = 15 - 11$$

$$\Rightarrow \quad 2K = 4$$

$$\Rightarrow \quad K = 2 \qquad (1)$$



$\Rightarrow$ M divides the line segment PQ in the ratio 2 : 1.

Substituting K = 2 in $y = \frac{20K+5}{K+1}$, we get (1)

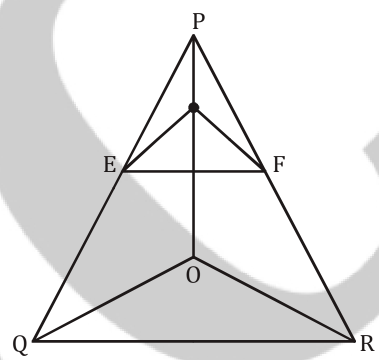
$$y = \frac{20(2)+5}{2+1}$$

$$\Rightarrow \quad y = \frac{40+5}{3}$$

$$= \frac{45}{3}$$

$$\Rightarrow \quad y = 15.$$ (1)

19.



In ΔPOQ, we have

$DE \parallel OQ$ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \text{... (i)}$$ (1)

Similarly, in ΔPOR, we have

$DF \parallel OR$ (Given)

$$\therefore \quad \frac{PD}{DO} = \frac{PF}{FR} \quad \text{... (ii)}$$ (1)

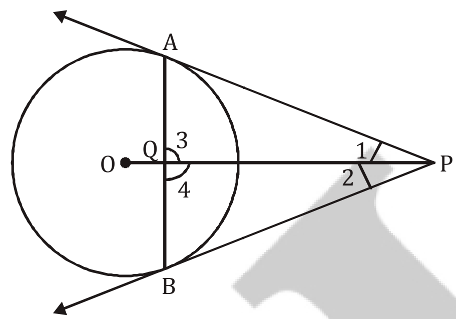
Now, from (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$$\Rightarrow \quad EF \parallel QR$$ (1)

[Applying the converse of Basic Proportionality Theorem in ΔPQR]

20.



Mark the angles as shown in the figure.

Let OP meet AB at Q.

In ΔAQP and ΔBQP,

$PA = PB$ (lengths of tangents from P)

$\angle 1 = \angle 2$ ($\because$ Tangents are equally inclined to OP) (1)

$QP = QP$ (common)

$\therefore$ $\Delta AQP \cong \Delta BQP$ (SAS congruence rule)

$\therefore$ $AQ = BQ$ (c.p.c.t.)

$\angle 3 = \angle 4$ (c.p.c.t.) (1)

But $\angle 3 = \angle 4 = 180°$ (linear pair)

$\Rightarrow$ $2\angle 3 = 180°$ ($\because \angle 4 = \angle 3$)

$\Rightarrow$ $\angle 3 = 90°$.

Hence, OP is the right bisector of AB. (1)

21. Given, sum of frequencies = 120

$\Rightarrow$ $17 + f_1 + 32 + f_2 + 19 = 120$

$\Rightarrow$ $f_1 + f_2 = 52$ ...(i) (½)

Given, mean of distribution is 50

$$\Rightarrow \frac{\sum f_i x_i}{\sum f_i} = 50$$

$$\Rightarrow \frac{17\times 10 + f_1 \times 30 + 32\times 50 + f_2 \times 70 + 19\times 90}{120} = 50 \quad (½)$$

$\Rightarrow$ $170 + 30f_1 + 1600 + 70f_2 + 1710 = 120 \times 50$

$\Rightarrow$ $30f_1 + 70f_2 = 6000 - 170 - 1600 - 1710$

$\Rightarrow$ $30f_1 + 70f_2 = 2520$

$\Rightarrow$ $3f_1 + 7f_2 = 252$ ... (ii)

Multiplying (i) by 3, we get

$3f_1 + 3f_2 = 156$ ... (iii) (1)

Subtracting (iii) from (ii), we get

$4f_2 = 96$

$\Rightarrow \quad f_2 = 24$ (1)

Substituting this value of f_2 in (i), we get

$f_1 + 24 = 52$

$\Rightarrow \quad f_1 = 28.$

Hence, $f_1 = 28$ and $f_2 = 24$.

OR

Ordinary Frequency Distribution

Daily income (in ₹)	Frequency (f_i)	Class-mark (x_i)	$u_i = \frac{x_i - 250}{100}$	$f_i u_i$
0–100	12	50	–2	–24
100–200	16 = (28 – 12)	150	–1	–16
200–300	6 = (34 – 28)	250	0	0
300–400	7 = (41 – 34)	350	1	7
400–500	9 = (50 – 41)	450	2	18
Total	$n = \Sigma f_i = 50$			$\Sigma f_i u_i = -15$

(2)

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 250 + \frac{(-15)}{50} \times 100$$

$$= 250 - 30$$

$$= 220$$ (1)

Hence the average daily income is 220. (1)

22. A coin is tossed three times. Sample space is S = {HHT, HTH, THH, HTT, THT, TTH, HHH, TTT}

There are 8 possible outcomes. n(S) = 8

The game consists of tossing a coin 3 times.

If one or two heads show. Sweta gets her entry fee back.

If she throws 3 heads, she receives double the entry fees.

If she gets TTT, she loses the entry fees.

(i) Loses the entry.

Out of 8 possible outcomes, only one (TTT) is favorable.

$\therefore \quad P(\text{loses entry}) = \frac{1}{8}$ (1)

(ii) gets double entry fee.

Out of 8 possible outcomes, only one (HHH) is favorable.

$$P(\text{double entry fees}) = \frac{1}{8} \quad (1)$$

(iii) Let E b event that she just gets her entry fee. Then

E = {HHT, HTH, THH, HTT, THT, TTH}

$\therefore$ n(E) = 6

$$P\ (\text{She just gets her entry fee}) = \frac{n(E)}{n(S)} = \frac{6}{8} = \frac{3}{4} \quad (1)$$

OR

Let there be b blue, g green and w white marbles in the jar. Then,

$$b + g + w = 54 \quad \text{...(i)} \quad (1)$$

$$\therefore \quad P(\text{Selecting a blue marble}) = \frac{b}{54}$$

It is given that the probability of selecting a blue marbles is $\frac{1}{3}$.

$$\therefore \quad \frac{1}{3} = \frac{b}{54}$$

$$\Rightarrow \quad b = 18 \quad (1)$$

We have,

$$P(\text{Selecting a green marble}) = \frac{4}{9}$$

$$\frac{g}{54} = \frac{4}{9}$$

$$\left[\because P\ (\text{Selecting a green marble}) = \frac{4}{9}\ (\text{Given})\right]$$

$$\Rightarrow \quad g = 24$$

Substituting the values of b and g in (i), we get

$$18 + 24 + w = 54$$

$$\Rightarrow \quad w = 12$$

Hence, the jar contains 12 white marbles. (1)

SECTION D

23. Let total time be n minutes

Total distance covered by thief = 100 n meters

Total distance covered by policeman = 100 + 110 + 120 + ... + (n – 1) terms (½)

$$\therefore \quad 100n = \frac{n-1}{2}[100(2) + (n-2)10] \qquad (½)$$

$$\Rightarrow \quad 200n = (n-1)(180 + 10n) \qquad (1)$$

$$\Rightarrow \quad 10n^2 - 30n - 180 = 0$$

$$\Rightarrow \quad n^2 - 3n - 18 = 0$$

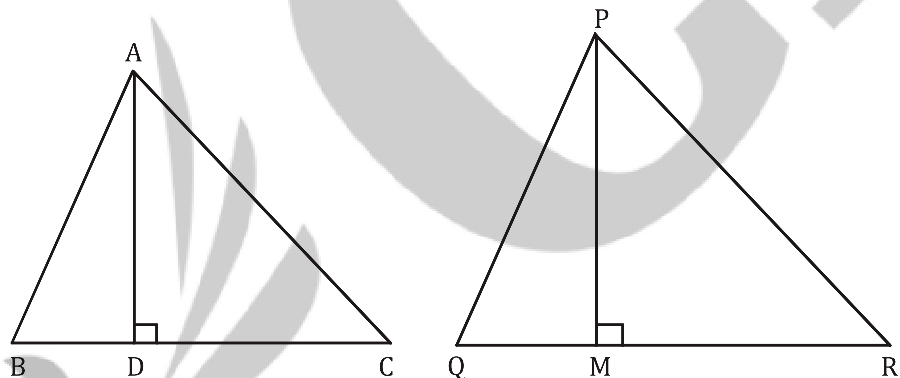
$$\Rightarrow \quad (n-6)(n+3) = 0$$

$$\Rightarrow \quad n = 6 \qquad (1)$$

Policeman took (n – 1) = (6 – 1) = 5 minutes to catch the thief.

Value : We should never indulge in theft habits. (1)

24.



Given. $\Delta ABC \sim \Delta PQR$.

To Prove. $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$ (1)

Construction. Draw AD ⊥ BC and AM ⊥ QR.

Proof. We know that the area of triangle = $\frac{1}{2}$ base × height.

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2}BC \times AD}{\frac{1}{2}QR \times PM} = \frac{BC}{QR} \times \frac{AD}{PM} \qquad \ldots (i)$$

Given $\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ... (ii)

and $\angle B = \angle Q$ ... (iii)

In ΔABD and ΔPQM,

$\angle B = \angle Q$ (from (iii))

and $\angle ADB = \angle PMQ$ (each = 90°)

$\therefore \quad \Delta ABD \sim \Delta PQM$ (AA similarity criterion)

$\therefore \quad \frac{AD}{PM} = \frac{AB}{PQ}$ ... (iv)

(2)

From (i), (ii) and (iv), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$$

$$= \left(\frac{AB}{PQ}\right)^2 \quad ... (v)$$

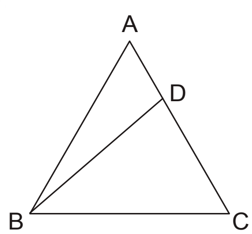
From (v) and (ii), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$

$$= \left(\frac{BC}{QR}\right)^2$$

$$= \left(\frac{CA}{RP}\right)^2.$$ (1)

OR



Given: ΔABC in which AB = AC and D is a point on the side AC such that

$BC^2 = AC \times CD$

To Prove: BD = BC (1)

Construction : Join BD

PROOF : We have,

$$BC^2 = AC \times CD$$

$$\Rightarrow \quad \frac{BC}{CD} = \frac{AC}{BC} \quad ...(i)$$

Thus, in ΔABC and ΔBDC, we have

$$\frac{AC}{BC} = \frac{BC}{CD} \quad \text{[From (i)]}$$

and, $\angle C = \angle C$ [Common] (2)

$\therefore$ $\Delta ABC \sim \Delta BDC$ [By SAS criterion of similarity]

$$\Rightarrow \quad \frac{AB}{BD} = \frac{BC}{DC}$$

$$\Rightarrow \quad \frac{AC}{BD} = \frac{BC}{CD} \quad [\because AB = AC]$$

$$\Rightarrow \quad \frac{AC}{BC} = \frac{BD}{CD} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{BC}{CD} = \frac{BD}{CD}$$

$\Rightarrow$ BD = BC (1)

25. In order to show that,

$$\frac{1}{(\operatorname{cosec} x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\operatorname{cosec} x - \cot x)}$$

It is sufficient to show

$$\frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x} \quad (2)$$

$$\frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{2}{\sin x} \quad ...(i) \quad (1)$$

Now, LHS of above is

$$\frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{(\operatorname{cosec} x - \cot x) + (\operatorname{cosec} x + \cot x)}{(\operatorname{cosec} x - \cot x)(\operatorname{cosec} x + \cot x)} \quad (1)$$

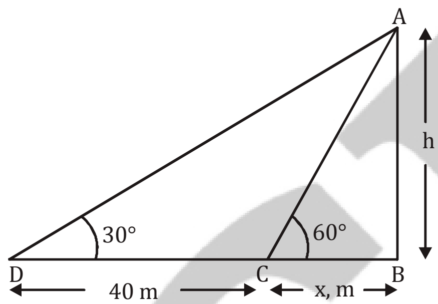
$$= \frac{2 \operatorname{cosec} x}{\operatorname{cosec}^2 x - \cot^2 x} \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{2 \operatorname{cosec} x}{1} = \frac{2}{\sin x} = \text{RHS of (i)} \quad (2)$$

Hence
$$\frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x} \quad (1)$$

or
$$\frac{1}{(\operatorname{cosec} x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\operatorname{cosec} x - \cot x)}. \quad (1)$$

26. Let AB be the tree of height h metres standing on the bank of a river. Let C be the position of man standing on the opposite bank of the river such that BC = x m. Let D be the new position of the man. It is given that CD = 40 m and the angles of elevation of the top of the tree at C and D are 60° and 30°, respectively, i.e.,



(1)

$$\angle ACB = 60°$$

and $$\angle ADB = 30°.$$

In $\angle ACB$, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \qquad (1)$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \text{...(i)}$$

In $\angle ADB$, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$\Rightarrow \quad \sqrt{3}h = x + 40 \qquad \text{...(ii)}$$

(1)

Substituting $x = \frac{h}{\sqrt{3}}$ in equation (ii), we get

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 40$$

$$\Rightarrow \quad \sqrt{3}h - \frac{h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad \frac{3h - h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad \frac{2h}{\sqrt{3}} = 40$$

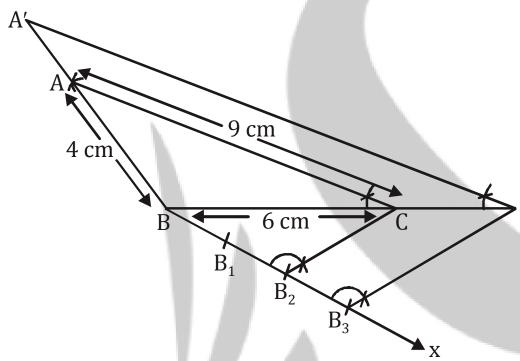
$$\Rightarrow \quad h = \frac{40 \times \sqrt{3}}{2}$$

$$\Rightarrow \quad h = 20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$$ (1)

Substituting h in equation (i), we get $x = \frac{20\sqrt{3}}{\sqrt{3}} = 20$ metres.

Hence, the height of the tree is 34.64m and the width of the river is 20m.

27. (2)



Steps of Construction

Step I : Draw a line segment BC = 6 cm.

Step II : With centre B and radius 4 cm draw an arc.

Step III : With centre C and radius 9 cm draw another arc which intersects the previous at A.

Step IV : Join BA and CA. ABC is the required triangle.

Step V : Through B, draw an acute angle ∠CBX on the side opposite to vertex A.

Step VI : Locate three arcs B_1, B_2 and B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$.

Step VII : Join B_2C.

Step VIII Draw $B_3C' \parallel B_2C$ intersecting the extended line segment BC at C′.

Step IX : Draw $C'A' \parallel CA$ intersecting the extended line segment BA to A′.

Thus, ΔA′BC′ is the required triangle (ΔA′BC′ ~ ΔABC). (1)

Justification:

$\because$ $B_3C' \parallel B_2C$

$$\frac{BC}{CC'} = \frac{2}{1}$$

Now,
$$\frac{BC'}{BC} = \frac{BC+CC'}{BC}$$

$$= 1+\frac{CC'}{BC} = 1+\frac{1}{2}=\frac{3}{2}$$

Again, $CC' \parallel CA$

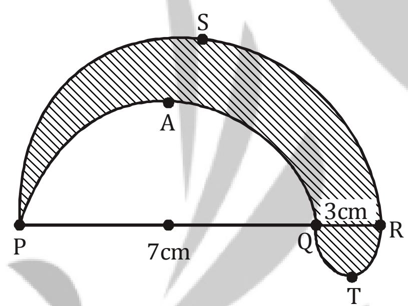
$\Delta ABC \sim \Delta A'BC'$

$$\therefore \quad \frac{A'B}{AB} = \frac{BC'}{BC}$$

$$= \frac{CC'}{AC} = \frac{3}{2}$$ (1)

The 2 triangles are not congruent because the sides of first are 4, 6, 9 while of 2nd are 6, 9 and 13.5

28. (1)



Radius of semicircle PSR $= \frac{1}{2} \times 10\text{cm} = 5\text{cm}$

Radius of semicircle RTQ $= \frac{1}{2} \times 3\text{cm} = 1.5\text{cm}$

Radius of semicircle PAQ $= \frac{1}{2} \times 7\text{cm} = 3.5\text{cm}$ (1½)

Perimeter of shaded region

= Circumference of semicircle PSR

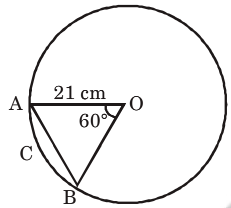
+ Circumference of semicircle RTQ

+ Circumference of semicircle PAQ.

$$= \left[\frac{1}{2}\times 2\pi \times 5+\frac{1}{2}\times 2\pi \times 1.5\frac{1}{2}\times 2\pi \times 3.5\right]$$

$= \pi[5 + 1.5 + 3.5] = 3.14 \times 10 = 31.4$ cm (1½)

OR



The radius of circle = 21 cm.

An arc ACB subtends an angle of 60° at the centre

$$OA = OB = 21 \text{ cm}$$ (1)

$$\therefore \quad \angle OAB = \angle OBA$$

$$= \frac{1}{2}(180° - 60°) = 60°$$

$$\therefore \quad \Delta OAB \text{ is equilateral,}$$

(i) $$\text{Length of the arc} = \frac{60°}{360°} \times \text{circumference}$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 \text{ cm} = 22 \text{ cm}$$

(ii) $$\text{Area of the sector} = \frac{60}{360} \times \text{area of the circle}$$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 \text{sq. cm} = 231 \text{ sq. cm}$$

(iii) Area of segment

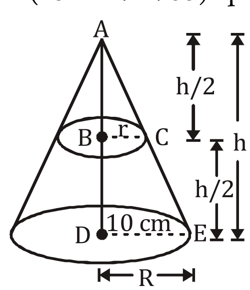
= Area of sector – Area of equilateral ΔAOB of side 12 cm

$$= \left(231 - \frac{\sqrt{3}}{4} \times 21 \times 21\right) \text{ sq. cm}$$

$$= \left(231 - \frac{1.732 \times 441}{4}\right) \text{ sq. cm}$$

$$= (231 - 190.953) \text{sq. cm} = 40.047 \text{ sq. cm}$$

29. (1)



Let BC = r cm,

DE = 10cm

Since, B is the mid-point of AD and BC is parallel to DE, therefore C is the mid-point of AE.

$\therefore \quad AC = CE$

Also, $\quad \Delta ABC \sim \Delta ADE$

$$\therefore \quad \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$= \frac{1}{2} \quad (1)$$

i.e., $\quad BC = \frac{1}{2} DE$

$$= \frac{1}{2} \times 10 = 5 \text{ cm}$$

or $\quad r = 5$ cm

Now
$$\frac{\text{Volume of cone}}{\text{Volume of the frustum}} = \frac{\frac{1}{3}\pi r^2 (AB)}{\frac{1}{3}\pi (BD)[R^2 + r^2 + Rr]} \quad (1)$$

$$= \frac{\frac{1}{3}\pi (5)^2 \left(\frac{h}{2}\right)}{\frac{1}{3}\pi \left(\frac{h}{2}\right)[(10)^2 + (5)^2 + 10 \times 5]}$$

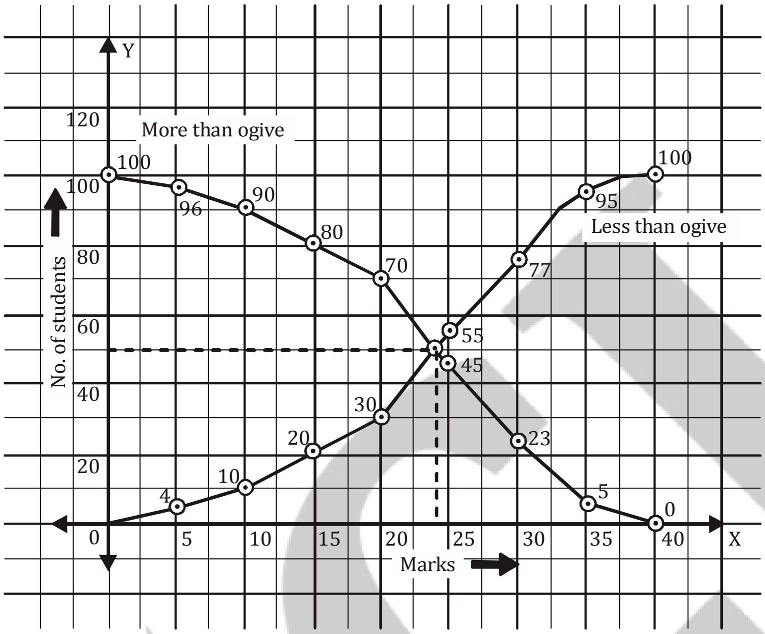
$$= \frac{25}{100 + 25 + 50}$$

$$= \frac{25}{175} = \frac{1}{7} \quad (1)$$

$\therefore$ The required ratio = 1:7. (1)

30.

Marks	Cumulative Frequency	Marks	Cumulative Frequency
Less than 5	4	More than 0	100
Less than 10	10	More than 5	96
Less than 15	20	More than 10	90
Less than 20	30	More than 15	80
Less than 25	55	More than 20	70
Less than 30	77	More than 25	45
Less than 35	95	More than 30	23
Less than 40	100	More than 35	5



(3)

Hence, median marks = 24 *OR* Let the assumed mean be A = 25 and h = 5.

Calculation of mean

Variate x_i	Frequency f_i	Deviations $d_i = x_i - 25$	$u_i = \frac{x_i - 25}{5}$	f_iu_i
5	20	–20	–4	–80
10	43	–15	–3	–129
10	75	–10	–2	–150
20	67	–5	–1	–67
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
40	9	15	3	27
45	8	20	4	32
50	6	25	5	30
$n = \Sigma f_i = 384$				$\Sigma f_iu_i = -214$

We have, N= 384, A = 25, h = 5 and $\sum f_iu_i = -214$

$\therefore$ Mean = $\overline{X} = A + h\left\{\frac{1}{N}\sum f_iu_i\right\}$

$\Rightarrow$ Mean $= 25 + 5 \times \left(\frac{-214}{384}\right)$

$= 25 - 2.786 = 22.214$ (4)