CBSE X 2019

Chapter and Topic-Wise

Solved Papers

2011-2018

Mathematics

Includes
2 CBSE Sample Papers

Career Launcher

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CONTENTS

Mathematics

PREFACE

Class X Board Exams are a race against time. You must know how to manage time efficiently if you want to ace your exams. At Career Launcher, we understand the struggle of attempting such a crucial examination for the first time and the pressure that comes along with it. Which is why, our Chapter and Topic-Wise Solved Papers for Mathematics have been designed to help you become acquainted with the exam pattern and hone your time management skills, both at the same time.

Exclusively designed for the students of CBSE Class X by highly experienced teachers, the book provides answers to all actual questions of Mathematics Board Exams conducted from 2011 to 2018. The solutions have been prepared exactly in coherence with the latest marking pattern; after a careful evaluation of previous year trends of the questions asked in Class X Boards and actual solutions provided by CBSE.

The book follows a three-pronged approach to make your study more focused. The questions are arranged Chapter-wise so that you can begin your preparation with the areas that demand more attention. These are further segmented topic-wise and eventually the break-down is as per the marking scheme. This division will equip you with the ability to gauge which questions require more emphasis and answer accordingly. Apart from this, several value-based questions have also been included.

At the end of the book, two sample papers are provided for you to practice, and you can get the solutions on the Career Launcher CBSE Board Exams App. The App further provides important formulae, examination pattern, marking scheme and syllabus as well as the date sheet.

We hope the book provides the right exposure to Class X students so that you not only ace your Boards but mold a better future for yourself. And as always, Career Launcher's school team is behind you with its experienced gurus to help your career take wings.

Let's face the Boards with more confidence!

Wishing you all the best,

Team CL

Blueprint & Marks Distribution

Class 10th Mathematics 2018 Analysis Unit Wise

Unit No.	Name	No. of Periods	Marks
I	Number Systems	15	6
II	Algebra	45	20
III	Coordinate Geometry	14	6
IV	Geometry	31	15
V	Trigonometry	33	12
VI	Mensuration	24	10
VII	Statistics & Probability	28	11
	Total		**80**
	Internal Assessment		**20**
Grand Total			**100**

No. of Questions of Various Forms	Total Marks
Very Short Answer (1 mark each) – 6	6
Short Answer (2 marks each) – 6	12
Short Answer (3 marks each) – 10	30
Long Answer (4 marks each) – 8	32
Total	**80**

UNIT I: NUMBER SYSTEMS (15) PERIODS

1. REAL NUMBERS

Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of results - irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, decimal expansions of rational numbers in terms of terminating/non-terminating recurring decimals.

UNIT II: ALGEBRA (45) PERIODS

1. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and their graphical solution. Geometric representation of different possibilities of solutions/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination and by cross multiplication method. Simple situational problems must be included. Simple problems on equations reducible to linear equations.

3. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2+bx+c=0$, (a   0). Solution of the quadratic equations (only real roots) by factorization, by completing the square and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first *n* terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY (14) PERIODS

1. LINES (IN TWO-DIMENSIONS)

Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division). Area of a triangle.

UNIT IV: GEOMETRY (31) PERIODS

1. TRIANGLES

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.
6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right traingle.

2. CIRCLES

Tangents to a circle motivated by chords drawn from points coming closer and closer to the point.

i. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

ii. (Prove) The lengths of tangents drawn from an external point to circle are equal.

3. CONSTRUCTIONS

i. Division of a line segment in a given ratio (internally).

ii. Tangent to a circle from a point outside it.

iii. Construction of a triangle similar to a given triangle.

UNIT V: TRIGONOMETRY (33) PERIODS

1 . INTRODUCTION TO TRIGONOMETRY

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios, whichever are defined at 0° and 90°. Values (with proofs) of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given. Trigonometric ratios of complementary angles.

3. HEIGHTS AND DISTANCES

Simple and believable problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, 60°.

UNIT VI: MENSURATION (24) PERIODS

1. AREAS RELATED TO CIRCLES

Motivate the area of a circle; area of sectors and segments of a circle. Problems based on areas and perimeter/circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle should be taken).

2. SURFACE AREAS AND VOLUMES

(i) Problems on finding surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Frustum of a cone.

(ii) Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken).

UNIT VII: STATISTICS AND PROBABILITY (28) PERIODS

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.

2. PROBABILITY

Classical definition of probability. Simple problems on single events (not using set notation).

Be Board Ready

MASTER MATHEMATICS

The theories and principles of Mathematics apply to different fields of life. But despite the relevance, for many students, Mathematics remains the most difficult subject of the curriculum. It may be slightly tricky, but there's no denying that Mathematics is one of the highest scoring papers. If one has a clear understanding of the basic concepts of all chapters, a good practice regimen and a command over formulae,scoring well in Maths is not difficult for anyone.

We have come up with a few smart strategies to help you perform well in your Mathematics paper.

These will reduce your stress and ease your preparation:

1. **Work on your speed:** Often, students aren't able to attempt questions they know because of lack of time. They thus, end up scoring a lot less than what they should have. Improving speed is a must. To do so, practice a variety of questions from all topics. Keep a check on the amount of time spent on solving questions from various sections. Ideally, you should spend 3-5 minutes on questions carrying lesser marks. That way, you will be able to afford more time for the 6-markers, which are typically lengthy/tricky questions.

2 **Focus on accuracy:** Never give accuracy a miss. Do calculations very carefully. Be accurate in applying any formula. Remember, even a lapse in change of sign from (+) to (-), or vice versa, can lead to losing marks. Being aware of the chapter-wise marking

scheme is beneficial, as it helps increase your level of preparedness. Give extra attention to topics with higher marks weightage.

3. **Stick to NCERT for basic questions:** It has been observed that around 90% of the questions in Board exams are either taken directly from NCERT textbooks or are their replicas. Hence, for basic concepts-and for getting an idea of the type of questions-you should stick to NCERT and prescribed CBSE books. Practice every question that is there in the textbooks, including the solved examples. However, for a comprehensive preparation, a good reference book is a must.
4. **Practice previous year papers and sample papers:** Doing time-bound solved papers practice will prove to be of great help in understanding the flair and pattern of the exam. For better practice, solve the sample papers (especially the ones released by CBSE) and model papers within the specified time frame.
5. **Write down the steps neatly**: You must already be aware that writing the steps down carries marks. So, even if you do not know the complete solution to a question, you should try to write the steps you are sure of.

Follow these strategies, and boost your chances of scoring a 90+ in the exam!

SAIL THROUGH SCIENCE

Science is considered a tough subject by many, yet it is the most sought afterstream for students after Class X. For students, their performance in Class X Boards exams forms the basis for stream selectionin Class XI. Many schools also have 'minimum marks criteria' to be met by students who want to take up Science in Classes XI & XII.

You need to understand it is not the complexity of the subject that acts as a roadblock to success, but simply the mindset. In fact, when one works hard and prepares smartly, acing the Science Board paper becomes easy. What you need to do is follow a correct strategy while attempting the paper, and learn some hacks.

1. **Specialized strategies for each subject:** For scoring overall good marks in Science, your individual performance in Physics, Chemistry and Biology must be excellent. This will only happen when you adopt the right preparation strategy for individual sections.

   Let's discuss a few pro tips, shared by our experts:

   - **Physics:** To score well in Physics, you should attempt all NCERT questions and additional exercises as many times as possible. Practice numerical; and solve at least five previous years' papers. You will also need to correctly memorize all formulae, practice derivations and use the right SI and other conversion units in your answers.
   - **Biology:** Biology doesn't consist of practical questions and numerical but the devil here is in the details. Answer questions in the form of points, as it presents a neat and summarized appearance.Underline key words; and draw large diagrams (where needed) with the right labeling.

- **Chemistry:** Regularly practice numerical-based questions, as well as balancing chemical equations. Revise names and examples of reactions, word problems,tables and periods from periodic classification, valency, metals and non-metals, and important points from acids, bases, and salts. Write crisp, concise answers and you will fetch good marks.

2. **Be aware of examination pattern and marking scheme:** Being familiar with the examination pattern and marking scheme goes a long way in ensuring a good performance in boards. Your level of preparedness improves a great deal if you are also aware of chapter-wise marking scheme. Pay special attention to topics with higher marks weightage. Practice writing long answers for 3-5 markers and limit to one to two lines in case of 1-2 markers.
3. **Stick to NCERT for basic questions:** Time and again, it has been seen that around 90% ofquestions in Board exams are taken directly from NCERT textbooks. It only makes sense to stick to NCERT and CBSE books for basic concepts and types of questions.
4. **Practice previous years' papers and sample papers:** Sample papers (especially the ones released by CBSE) and model papers should be solved within the specified timeframe, for better practice.
5. **At the exam, use reading time wisely:** In the allotted 15 minutes, make sure you read the question paper thoroughly; and select the questions you wish to attempt first. If there is any topic you are not comfortable with, attempt it towards the end. Also, make sure you spare at least 10 minutes for revision in the end.
6. **Focus on basic concepts and study important topics in detail:** While it goes without saying that your basics need to be clear, merely reading the summary of a topic will not help. You need to study each topic in detail. The Science paper of 2018 saw tricky questions which required in-depth reading of topics.

SUCCEED IN SOCIAL SCIENCE

Social Science paper, as a medium to score high marks is often overlooked. Most students feel that too much theory will not fetch them good marks. On the contrary, with a few smart tips, you can easily score over 95 in your Social Science paper.

First, and foremost, prepare properly for the examination. If you are sure about your learning and knowledge, then you have no reason to be worried.

1. **At the exam, use reading time wisely:** Once you are handed the question paper, use the allotted reading time to study the questions carefully. Go through the entire paper, a silly mistake made in understanding a question can prove fatal and will lead to loosing marks. If you are not confident about a topic or a question, do not panic. Ask before you start writing answers.

2. **Divide time judiciously:** Check how lengthy the question paper is. This will help you estimate how much time you will have for each section and accordingly divide time.
3. **Answer Sequentially:** Try to solve questions in the given sequence. Do not ponder over any problem if you do not know the solution. Instead, skip the question, and come back to it later; or, answer it in the best way you can.
4. **Answer clearly and precisely:** Give precise answers that are simple and easy to follow. Avoid using words just to make the answer seem lengthy. Rather than using words you have mugged up, it is always preferable to explain a concept in your own words. Make sure your answers do not appear to be haphazard or unorganized. Well-structured answers will fetch you more marks. Highlight the important information, key words, and headings in every answer, so that they attract attention immediately. It will also help you score good marks.
5. **Write in points:** Try writing answers in form of points, instead of paragraphs. This will ensure that you save time for difficult questions.Try to attempt all questions. Even if you write some related facts, and not exactly what has been asked for, you might score some marks.

Bonus Tips:

- Mark out the map work neatly.
- Ensure that the respective question number is written against every answer.
- Draw and properly label all diagrams.

HANDLE HINDI WITH EASE

One of the biggest apprehensions for many CBSE Board examinees is how to obtain a good score in the Literature and Language papers, especially Hindi. With a total of 55 marks being devoted to the Language section (15 for Grammar, 20 for an unseen passage, and 20 marks for creative writing, i.e., essay and letters), what follows is that you must be ready to let your analytical andcreative juices flow, because memorizing the textbook will just not do.

Listed below are some tips and tricks on how to go about your Hindi syllabus, so that you score well in the Hindi Board examination.

First and foremost, read all the chapters in your textbooks; and revise them regularly. Questions from textbooks will comprise almost 50% of the question paper.

1. **Grammar (Vyakaran):** It is quite easy to score high marks in Hindi, but only if you are clear on basics. Have complete clarity on grammatical rules related to topics prescribed in your syllabus. Do not waste your time mugging up definitions from every chapter. Rather, understand concepts in depth and read solved exercises provided at the back of chapters. Practice answering questions on grammar as much as you can. Be aware of not only the rules of grammar, but also the exceptions. Read the NCERT textbook thoroughly.

2. **Descriptive answers (Essay and Letter Writing):** Have thorough knowledge on the format for essay and letter writing (both formal and informal). Focus more on the style of writing formal letters, as it will not be possible for you to use fine professional language during the exam. While attempting descriptive questions, first jot down in a rough space, the points you will cover;and then proceed to writing answers. This will help you structure your answers properly, and will also fetch you good marks. Add a lot of facts and figures to make your essay look attractive and authentic. Ensure your handwriting is neat.
3. **Reading Comprehension:** There is a widespread, but incorrect notion among students that unseen passages do not require any preparation. In reality, you are advised to practice such passages as well. While attempting questions, read the passage carefully; and underline the important words and phrases. Understand the type of questions that have been asked. Do not devote more than 40 minutes to this section.
4. **Literature:** Undeniably, Literature, with its chapters and poems, is the toughest section to attempt. However, if you know your NCERT chapters thoroughly, chances are, you will not face a lot of trouble. You can get a good score, if you have a clear understanding of chapters. Learn the name of the authors and the titles of the poems. Make sure all your answers are properly structured-having an introduction,body, and a conclusion.

There will be internal choices for questions, but no overall choice. Carefully, and wisely, choose the questions you want to answer. Attempt only those questions in which you are totally confident.

EXCEL IN ENGLISH

English is the only subject that is compulsory across various streams and classes. It is also a high-scoring subject. Class X English may seem easy, but it's important to understand that one cannot gain mastery over the subject in a few days or weeks. It can only be done over a period of time!

The English paper is divided into four sections: Reading, Writing, Grammar, and Literature & Long Reading texts. To get good marks overall, you will need to score well in each of these.

Now, let's take a look at the ideal study plan for these sections, individually:

1. **Reading:** The first section of the paper comprises unseen passages. The most common, and effective way to improve reading-comprehension is by reading an unknown passage and summarizing it point-wise. Read anything and everything you can find; and before you realize it, you will find yourself gaining a strong hold over this section! While reading, keep a check on the time spent on a passage, and in solving all the questions of that particular passage. Ideally, not more than 10-12 minutes should be spent on a moderately difficult passage.

   To improve your vocabulary, make notes of the words, whose meaning you are not aware of.

2. **Writing:** This section tests your writing skills. Questions based on letter writing, formal letters(Complaints/Inquiry/Placing Order/Letter to the Editor), articles, applications, etc., will be asked in the exam. Hence, you will need to improve your writing skills. Try to summarize what you write. Practice a different writing composition every day; and while doing so, make sure to stay within word-limit. You should also be comfortable with common, general topics, as well as social issues, as articles based on these can be asked in the examination.
3. **Grammar:** This section tests your knowledge on topics like tenses, modals (have to/had to, must, should, need, ought to; and their negative forms), use of passive voice, subject-verb agreement, reporting (commands and requests, statements, questions), clauses, determiners, conjunctions, articles, and prepositions. Revise concepts of basic grammar, while focusing on usage. Remember, questions asked can be in the form of sentence correction, gap filling, editing or omission, etc.; so, you need to practice all of these.
4. **Literature & Long Reading:** Questions on long reading texts are usually in-depth; so, you will need to go through the entire chapter. Do note that questions can be from any part of the chapter. In case of poems, you need to have complete clarity on the context of lines. Usually, questions on theme/plot are asked in case of a long text. Practice writing character sketches, as it will help you answer better.

Follow this study plan, and the tips and tricks discussed above; and we are sure nothing can stop you from excelling in the English paper!

Make the Most of Your Time

The Class X Board exams play a crucial role in shaping your career, irrespective of the stream(Science, Commerce, or Humanities) you opt in Class XI. A crucial factor that decides how you fare in the X board exams is time management. However, most of us don't realize its importance. Yes, we do design a timetable, while preparing for the Class X Boards, but seldom adhere to it. The results are often disappointing.

A simple strategy of effective time management can work wonders, and guarantee your success. After all, the exam is nothing but a race against time; and to survive in this race, you need to have a fool-proof time-management strategy.

Here are some tips to propel your chances of success:

1. **Organize your tasks:** Organization not only helps eliminate stress, but also leads to devising an effective strategy. Start early to stay ahead. Commencing your preparation just 2-3 months before your Board exams won't help.
2. **Avoid being over-enthusiastic:** Set realistic goals; and plan your preparation for the upcoming weeks and months. This will help you get a clear picture of how much time you have,and how to allocate that time among the various subjects and topics.
3. **Analyze your requirements:** To begin with, analyze how much time you need for studying, sleeping, and for other miscellaneous activities. This way, you will avoid confusion, have clarity, and gain the motivation to shift gears, as and when required.

4. **Study the way you are comfortable in:** Some of us are morning birds, while others are night owls. Know when you are happy and your mind is refreshed; and reserve this time for the most difficult and tricky topics and theoretical revisions. When you feel dull and sleepy, start doing practice problems, make mind maps, or pick up the easier topics. Active engagement will help your brain remain switched on and alert.
5. **Dedicate a place to yourself:** Having a fixed place to study helps your mind to re-position itself to the study mode as soon as you sit down. The place can also prove to be a confidence booster a week or so before your exams.
6. **Prioritize your subjects:** Utilizing your time in the best-possible manner is the key to your success. Prioritize, as per your strong and weak areas, in each chapter or topic. Devote less time to your strong areas; more to the weaker areas; and the remainder to your daily activities. Do not skip any topic during preparation. Over-confidence can get the better of you during the exam.
7. **Prepare a schedule:** Draw up a timetable based on your priorities. Do not get over-ambitious. Stick to a balanced and simple schedule. Remember to include sleep time, break time, and other recreational activities.
8. **Adhere to the schedule:** The biggest challenge is not to prepare a timetable, but to follow it. If you don't, all your effort will go in vain. Be sincere and honest. Remember, there is no shortcut to success. While preparing the schedule, ask yourself, how it is going to benefit you. Manage time accordingly.

Advantages of time management

- It reduces tension; and calms you down, even in stressful moments during preparing for an exam.
- It increases your productivity; and hence, leads to better performance in exams.
- It gives you confidence and motivation.

Every famous personality has faced success and failure in their careers. However, their strategy would always have been to put to use wisely the available time and resources. Similarly, without proper time management in your preparation for the Class X Board exams, success will be impossible to achieve.

Scope of Choosing Different Streams After Class X

As a student, you are expected to make an informed choice, while selecting a specialization, post Class X. ***The current CBSE system offers students three streams to lay the foundation of their career:***

1. Commerce
2. Science
3. Humanities

Each of these fields opens up a wide range of lucrative careers. However, the final choice a student makes should depend on his/her own area of interest.The scope of each of these streams is discussed below, in brief.

1. COMMERCE

The field of Commerce today is producing leaders and successful personalities like never before. It is being chosen by a large number of students after completion of their 10th standard due to its worth interms of finding early employment; and also as the first step towards entrepreneurship. A student taking up Commerce starts to inculcate knowledge of business from the very beginning. He/she also attains knowledge of business, trade, basics of the economy, fiscal policies, sharemarket, stock markets, etc., by studying subjects like Business Studies, Economics, and Accountancy.

Those who study Commerce can opt for a Bachelor's and Master's Degree in Economics, Business Administration, CA, CS, CFA, and so on.

2. SCIENCE

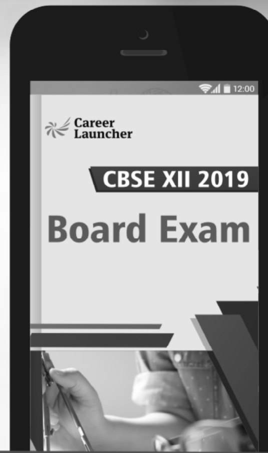
Science, as a stream, has always been prestigious and is highly preferred by students. Its relevance and contribution to society keeps it at the top of the game. Over the years, Science stream has given us successful doctors, engineers, and scientists; who have brought about revolutionary changes in the society, by improving the way things work. When you opt for Science stream, interms of selection of subjects, Physics and Chemistry are compulsory; but, you do get the option to choose between Mathematics and Biology. Then again, there is the option of choosing PCMB combination (i.e.,choosing both Math and Biology, along with Physics and Chemistry), as well.

3. HUMANITIES

Earlier, it was a common notion that toppers never opt for Arts/Humanities. But, the realm of academia has seen this perception change significantly over the last decade. After all, as they say, change is the only constant.

Nowadays, the mantra is to pursue **what you love**; and build a career in the field of your interest. Life is a void without dancers, painters, singers, poets, and artists; which you only get to be by studying Arts & Humanities.

Humanities also help us understand others through their languages, histories, and cultures. Arts & Humanities is an extremely diverse stream; and students mainly deal with subjects like Literature, Sociology, Psychology, Political Science, etc. As astudent of Humanities, you can opt for a wide range of careers, such as teaching, social work, law, politics, business, media, fashion, journalism, etc.



Career Launcher's CBSE Board Exams App

You can now prepare for the Class X Boards with Career Launcher's CBSE Board Exams App, available for free download on Google Play Store.

The App provides free sample papers and their solutions, along with the marking scheme & syllabus prescribed by CBSE. All solutions have been designed by experts in coherence with CBSE exam syllabus; after a careful evaluation of previous year question pattern. Other than this, the app provides value based content which will act as supplement to your exam preparation. You can quickly revise important formulae and key concepts on the go. We've also included examination pattern, marking scheme, syllabus as well as date-sheet within the app.

In addition, to help you with selection of streams after Class X, the Board Exams App offers 'Stream Selector', wherein you will be able to identify suitable stream, based on details filled out in a form.

Make the most of the App; and boost your Class X exam preparation.

Stream Selector: The Career Assessment Test

Before choosing a career path, you need to figure out what ignites the fire in you. Future Map is a one-stop, reliable, career-guidance program from Career Launcher. It helps you choose the best and the most exciting career on the basis of your Aptitude, Personality, and Interest.

Future Map is conducted through three consecutive online quizzes: Aptitude quiz, Personality quiz, and Interest quiz. These quizzes are administered over a period of three days.

Steps to take the Future Map test:

1. To take the test, you need to register for the program first. Register for Future Map at http://www.futuremap.in/index.jsp.
2. You will receive your FutureMap login credentials through an email on your registered mail ID.
3. To login, key in the user id and password received.
4. Click on the 'Take the Test' button on the right-hand side.
5. You will be directed to a page asking for payment of a token amount.
6. If you have a Discount Code, key in the details; and then take the test.

We hope you are able to find the right career based on what your personality and interests are. At Career Launcher, we are there to guide you in becoming the ***best YOU*** at every step of your education!

CHAPTER 1

Real Numbers

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Proving Irrational number	2 marks	2 marks				
Question based on H.C.F. and L.C.M.	1, 3 marks	1, 3 marks				

[TOPIC 1] Euclid's Division Lemma and Fundamental Theorem of Arithmetic

Summary

Euclid's Division Lemma

Dividend = divisor × quotient + remainder.

Given two positive integers a and b. There exist unique integers q and r satisfying

$a = bq + r$ where $0 \leq r < b$

where a is dividend, b is divisor, q is quotient and r is remainder.

- ***If a = bq + r, then every common divisor of a and b is a common divisor of b and r also.***

Euclid's Division Algorithm

To obtain the HCF of two positive integers, say c and d, with $c > d$, follow the steps below:

Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.

Step 2: If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step 3: Write $d = er + r_1$ where $0 < r_1 < r$

Step 4: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

- *Odd integers of the form **6q + 1, 6q + 3 or 6q + 5** shows that 6 is the divisor of given integer*
- *Any positive integer can be of the form 3m, 3m + 1, 3m + 2. Such that its cube would be of the form 9q + r.*

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes and this expression is **unique**, except from the order in which the prime factors occur.

- *HCF is the lowest power of common prime and LCM is the highest power of primes.*
- *HCF (a, b) × LCM (a, b) = a × b.*
- *Any number ending with zero must have a factor of 2 and 5.*

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. L.C.M. of $2^3 \times 3^2$ and $2^3 \times 3^3$ is:

(a) 2^3 (b) 3^3

(c) $2^3 \times 3^3$ (d) $2^2 \times 3^2$

[TERM 1, 2012]

2. If p and q are two co-prime numbers, then HCF (p, q) is:

(a) p (b) q

(c) pq (d) 1

[TERM 1, 2013]

3. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$, then HCF (a, b) is equal to:

(a) 900 (b) 180

(c) 360 (d) 540

[TERM 1, 2013]

4. The HCF of two numbers is 27 and their LCM is 162, if one of the number is 54, find the other number.

[TERM 1, 2017]

5. What is the HCF of the smallest prime number and the smallest composite number?

[TERM 1, 2017]

2 Marks Questions

6. Show that 8^n cannot end with the digit zero for any natural number n.

[TERM 1, 2011]

7. Euclid's algorithm, find the HCF of 240 and 228.

[TERM 1, 2012]

8. Explain why $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5$ is a composite number?

[TERM 1, 2014]

9. Find the least positive integer which on diminishing by 5 is exactly divisible by 36 and 54.

[TERM 1, 2015]

10. Express 5050 as product of its prime factors. Is it unique?

[TERM 1, 2016]

3 Marks Questions

11. Show that square of any positive integer is either of the form $3m$ or $(3m+1)$ for some integer m.

[TERM 1, 2011]

12. Find the LCM and HCF of 336 and 54 and verify that LCM × HCF = Product of the two numbers.

[TERM 1, 2012]

13. Using Euclid's division algorithm, find whether the pair of numbers 847, 2160 are co-primes or not.

[TERM 1, 2012]

14. Find HCF and LCM of 180, 252 and 324.

[TERM 1, 2013]

15. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

[TERM 1, 2014]

16. Explain whether the number $3 \times 5 \times 13 \times 46 + 23$ is a prime number or a composite number.

[TERM 1, 2015]

17. Find the greatest number of six digit number exactly divisible by 18, 24 and 36.

[TERM 1, 2016]

18. Using division algorithm find quotient and remainder dividing $x^3 + 13x^2 + x - 2$ by $2x + 1$

[TERM 1, 2016]

19. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

[TERM 1, 2017]

4 Marks Question

20. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or $3m + 1$ for some integer m.

[TERM 1, 2012]

Solutions

1. Given, $2^3 \times 3^2$ and $2^2 \times 3^3$

We know, LCM is the product of terms containing highest powers of

$$(2,3) \Rightarrow 2^3 \times 3^3$$

Hence, the correct option is (c). [1]

2. LCM of the given number $= pq$

$$\text{HCF} = \frac{\text{product of numbers}}{\text{LCM of numbers}} = \frac{p \times q}{pq} = 1$$

Two integers are co prime when they have no common factor other than 1.

Therefore the H.C.F is 1.

Hence the correct option is (d). [1]

3. The HCF of a and b $= \left(2^2 \times 3^2 \times 5\right)$

$$= (4 \times 9 \times 5)$$

$$= (36 \times 5)$$

$$= (180)$$ [1]

4. HCF (54, b) = 27 and LCM (54, b) = 162

According to the formula,

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$ [½]

$$\Rightarrow 27 \times 162 = 54 \times b$$

$$\Rightarrow b = \frac{27 \times 162}{54}$$

$$\Rightarrow b = \frac{162}{2}$$

$$\Rightarrow b = 81$$

So, the other number is 81. [½]

5. The smallest prime number is 2.

And the smallest composite number is 4.

Factors of $2 = 1 \times 2$

$4 = 1 \times 2 \times 2.$ [½]

So the HCF of the smallest prime number and the smallest composite number is 2. [½]

6. The prime factorization should have 2 and 5 as a common factor for a number to end with the digit zero. [1]

$8^n = (2 \times 2 \times 2)^n$ does not have 5 in its prime factorization.

Hence, 8^n cannot end with the digit zero for any natural number *n*. [1]

7. We know, by Euclid's Division Lemma,

$a = bq + r,\ 0 \le r < b$

Applying Euclid's Lemma,

Step 1 : Since 240 > 228, we apply the division lemma to 240 and 228, to get $240 = 228 \times 1 + 12$ [1]

Step 2 : Since the remainder $12 \ne 0$, we apply the division lemma to 228 and 12, to get $228 = 12 \times 19 + 0$

The remainder has now become zero.

Since the divisor at this stage is 12, the HCF is 12. [1]

8. We can write $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5$ as

$\Rightarrow 5(1 \times 2 \times 3 \times 4 \times 6 \times 7 + 1)$

$\Rightarrow 5(1 \times 2 \times 3 \times 4 \times 6 \times 7 + 1) = 5 \times 1009$ [1]

Hence we can say that the given number has at least one factor other than 1 and number itself.

$\Rightarrow (5, 1009, 1, 5045)$

Therefore $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5$ is a composite number. [1]

9. Finding the LCM of 36 and 54,

$36 = 2 \times 2 \times 3 \times 3$

$54 = 2 \times 3 \times 3 \times 3$

$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 3 = 108$ [1]

Now it is given that the number is diminished by 5.

This means the least positive will be:

$5 + (\text{LCM of 36 and 54})$

$= 5 + 108$

$= 113$ [1]

Hence, 113 is the least positive integer which on diminishing by 5 is exactly divisible by 36 and 54.

10. 5050 can be factored as,

$5050 = 2 \times 5 \times 5 \times 101$

We can write it as $2 \times 5^2 \times 101$

Here all the factors are prime numbers and can be expressed as product of its prime numbers.

So, Yes it is unique. [2]

11. Let c be any positive number and $d = 3$

Then $c = 3q + r$ for $q \ge 0$

Also, $r = 0, 1, 2$ as $0 \le r \le 3$ [1]

Thus, $c = 3q$ or $c = 3q + 1$ or $c = 3q + 2$

$\Rightarrow c^2 = (3q)^2$ or $(3q+1)^2$ or $(3q+2)^2$

$\Rightarrow c^2 = 3 \times (3q^2)$ or $9q^2 + 6q + 1$ or $9q^2 + 12q + 4$

$\Rightarrow c^2 = 3 \times (3q^2)$ or $3(3q^2 + 2q) + 1$ or $3(3q^2 + 4q + 1) + 1$ [1]

$\Rightarrow c^2 = 3m_1$ or $3m_2 + 1$ or $3m_3 + 1$ where

$m_1 = 3q^2, m_2 = 3q^2 + 2q$ and $m_3 = 3q^2 + 4q + 1$

Hence, square of any positive integer is either of $3m$ or $(3m + 1)$ for some integer m. [1]

12. Find the factors of 336 and 54.

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$ [1]

HCF of 336 and $54 = 2 \times 3 = 6$

LCM of

336 and $54 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$ [1]

Product of two numbers $= 336 \times 54 = 18144$

Hence verified. [1]

13. $a = 2160,\ b = 847$

By Euclid's lemma, given positive integers a and b, there exist unique integers q and r satisfying

$a = bq + r,\ 0 \le r < b.$ [1]

As $2160 > 847$, we apply the division lemma to 2160 and 847, to get $2160 = 847 \times 2 + 466$

Since the remainder $466 \ne 0$, we apply the division lemma to 847 and 466, and continue the same process till we get remainder 0. [1]

$847 = 466 \times 1 + 381$

$466 = 381 \times 1 + 85$

$381 = 85 \times 4 + 41$

$85 = 41 \times 2 + 3$

$41 = 3 \times 13 + 2$

$3 = 2 \times 1 + 1$

$2 = 1 \times 1 + 1$

$1 = 1 + 0$

As 1 is the HCF of 847 and 2160. 847 and 2160 are the co-primes. [1]

14. Consider 252 and 324. Let, $a = 324$ and $b = 252$ by Euclid's division lemma-

$a = bq + r,\ 0 < \ or = \ r < b$

$324 = 252 \times 1 + 72$

$252 = 72 \times 3 + 36$

$72 = 36 \times 2 + 0$ [1]

Therefore, HCF $(252,\ 324) = 36$

Now consider 36 and 180,here $a = 180$ and $b = 36$. By Euclid's division

$a = bq + r,\ 0 < \ or \ = \ r \ < \ b$ [1]

$180 = 36 \times 5 + 0$

Therefore, $HCF(180,\ 36) = 36$ [1]

15. Pens are sold in pack of 8 and notepads are sold in pack of 12,

LCM of 8 and 12 is:

$8 = 2^3$ and $12 = 2^2 \times 3$ [1]

LCM $= 2^3 \times 3 = 8 \times 3 = 24$

Least number of pack of pen $= \frac{24}{8} = 3$ [1]

Least number of pack of notepads $= \frac{24}{12} = 2$

Hence, 3 packs of pen and 2 packs of notepads one should buy to get 24 pens and notepads. [1]

16. $3 \times 5 \times 13 \times 46 + 23$

It can be re-written as:

$3 \times 5 \times 13 \times 2 \times 23 + 23$

$= 23(3 \times 5 \times 13 \times 2 + 1)$

$= 23 \times 391$

$= 8993$ [1]

Here 8993 is written as the product of two different numbers 23×391. [1]

It means it has 23 and 391 as its factors other than 1 and 8993.

Hence, it is a composite number. [1]

17. Greatest number of 6 digits is 999999

The numbers given are 18, 24 and 36.

Here LCM of 18, 24, 36.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$ [1]

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

Now,

The LCM of 18, 24 and 36 $= 2^3 \times 3^2 = 72$

Now dividing 999999 by 72 [1]

$\frac{999999}{72} = 13888$ with remainder 63

And,

$999999 - 63 = 999936$

Thus 999936 is the greatest number 6-digit number divisible by18, 24 and 36. [1]

18. $x^3 + 13x^2 + x - 2$ can be divided by $2x + 1$ as

$$
\begin{array}{r|l}
 & \frac{1}{2}x^2 + \frac{25}{4}x - \frac{21}{8} \\
\hline
2x+1 & x^3 + 13x^2 + x - 2 \\
 & x^3 + \frac{1}{2}x^2 \\
 & - \quad - \\
\hline
 & \frac{25}{2}x^2 + x - 2 \\
 & \frac{25}{2}x^2 + \frac{25}{4}x \\
 & - \quad - \\
\hline
 & -\frac{21}{4}x - 2 \\
 & -\frac{21}{4}x - \frac{21}{8} \\
 & + \quad + \\
\hline
 & \frac{5}{8}
\end{array}
$$

[1]

[1]

Here quotient is $\frac{1}{2}x^2 + \frac{25}{4}x - \frac{21}{8}$ and remainder is $\frac{5}{8}$. [1]

19. The prime factors of:

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$404 = 2 \times 2 \times 101$ [1]

Therefore the HCF = Product of smallest power of each common prime factor = 2 × 2 = 4 [1]

And LCM = Product of greatest power of each prime factor = 2 × 2 × 2 × 2 × 2 × 3 × 101 = 9696

To prove:

HCF × LCM = 101 × 96

Here, HCF × LCM = 4 × 9696 = 38784

$101 \times 96 = 38784$

Hence proved, HCF × LCM = Product of the two given numbers. [1]

20. We know, By Euclid's Division Lemma,

If a and b are two positive integers, then

$a = bq + r$ where $0 \le r < b$ ...(1) [1]

Let a be any positive integer and $b = 3$, using equation 1, we get,

$a = 3q + r$ where $0 \le r < 3$

We know can be either 0, 1 or 2 [1]

If $r = 0$	If $r = 1$	If $r = 2$
The equation becomes, $a = 3q + 0$ $\Rightarrow a = 3q$ Squaring both sides, $a^2 = (3q)^2$ $\Rightarrow a^2 = 9q^2$ $\Rightarrow a^2 = 3(3q^2)$ Let $m = 3q^2$ $\Rightarrow a^2 = 3m$	The equation becomes, $a = 3q + 1$ Squaring both sides, $a^2 = (3q+1)^2$ $\Rightarrow a^2 = 9q^2 + 6q + 1$ $\Rightarrow a^2 = 3(3q^2 + 2q) + 1$ Let $m = 3q^2 + 2q$ $\Rightarrow a^2 = 3m + 1$	The equation becomes, $a = 3q + 2$ Squaring both sides, $a^2 = (3q+2)^2$ $\Rightarrow a^2 = 9q^2 + 12q + 4$ $\Rightarrow a^2 = 9q^2 + 12q + 3 + 1$ $\Rightarrow a^2 = 3(3q^2 + 4q + 1) + 1$ Let $m = 3q^2 + 4q + 1$ $\Rightarrow a^2 = 3m + 1$

[2]

Hence, square of any positive number can be expressed of the form $3m \text{ or } 3m + 1$ for some integer m.

Hence proved.

[TOPIC 2] Irrational Numbers, Terminating and Non-Terminating Recurring Decimals

Summary

Irrational Numbers

All real numbers which are not rational are called irrational numbers. $\sqrt{2}$, $\sqrt[3]{3}$, $-\sqrt{5}$ are some examples of irrational numbers.

There are decimals which are non-terminating and non-recurring decimal.

Example: 0.303003000300003…

Hence, we can conclude that

An irrational number is a non-terminating and non-recurring decimal and cannot be put in the form $\frac{p}{q}$ where p and q are both co-prime integers and $q \neq 0$.

Decimal Representation of Rational Numbers

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of q is of the form $2^n \times 5^m$ where m, n are non-negative integers then x has a decimal representation which terminates.

For example : $0.275 = \frac{275}{10^3} = \frac{5^2 \times 11}{2^3 \times 5^3} = \frac{11}{2^3 \times 5} = \frac{11}{40}$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers, then x has a decimal expansion which is non-terminating repeating.

For example : $\frac{5}{3} = 1.66666...$

Rational number	Form of prime factorisation of the denominator	Decimal expansion of rational number
$x = \frac{p}{q}$, where p and q are coprime and $q \neq 0$	$q = 2^m 5^n$ where n and m are non-negative integers	terminating
	$q \neq 2^m 5^n$ where n and m are non-negative integers	non-terminating

- *If the denominator is of the form $2^m \times 5^n$ for some non negative integer m and n, then rational number has terminating decimal otherwise non terminating.*

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. The prime factorization of the denominator of the rational number expressed as $46.\overline{123}$ is:

(a) $2^m \times 5^n$ Where m and n are integers

(b) $2^m \times 5^n$ Where m and n are positive integers

(c) $2^m \times 5^n$ Where m and n are rational numbers

(d) Not of the form $2^m \times 5^n$ where m and n are non-negative integers.

[TERM 1, 2011]

2. The decimal expansion of $\frac{6}{1250}$ will terminate after how many places of decimal?

(a) 1

(b) 2

(c) 3

(d) 4

[TERM 1, 2011]

3. Decimal expansion of $\frac{23}{(2^3 5^2)}$ will be:

(a) Terminating
(b) Non- terminating
(c) Non terminating and repeating.
(d) Non-terminating and non-repeating

[TERM 1, 2012]

2 Marks Questions

4. What can you say about the prime factorization of the denominator of the rational number 0.134 when written in the form $\frac{p}{q}$. Is it of form $2^m \times 5^n$? If yes, write the values of m and n.

[TERM 1, 2013]

5. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

[TERM 1, 2011]

6. Show that $\left(\sqrt{3}+\sqrt{5}\right)^2$ is an irrational number.

[TERM 1, 2015]

7. Write down the decimal expansion of $\frac{76}{6250}$, without actual division.

[TERM 1, 2016]

8. Find how many integers between 200 and 500 are divisible by 8.

[TERM 1, 2017]

9. Given that $\sqrt{2}$ is irrational, prove that $\left(5+3\sqrt{2}\right)$ is an irrational number.

[TERM 1, 2017]

10. Prove that $\sqrt{3}+\sqrt{2}$ is irrational.

[TERM 1, 2011]

4 Marks Question

11. Define irrational number and prove that $3+\sqrt[2]{5}$ is an irrational number.

[TERM 1, 2017]

Solutions

1. As the decimal expansion $46.\overline{123}$ is a non-terminating repeating, the given number is a rational number of the form $\frac{p}{q}$ where q is not of the form $2^m \times 5^n$.

Let $x = 46.\overline{123}$ ...(1)

$1000x = 46123.\overline{123}$ ...(2)

$(2)-(1) \Rightarrow \frac{46077}{999} = x$

Hence, the correct option is (d). [1]

2. Express 6 and 1250 as a product of prime factors.

$$\frac{6}{1250} = \frac{2\times 3}{2\times 5^4}$$

$$\Rightarrow \frac{6}{1250} = \frac{2\times 3}{2\times 5^4}\times\frac{2^3}{2^3} = \frac{48}{5^4\times 2^4}$$

$$\Rightarrow \frac{6}{1250} = \frac{48}{(5\times 2)^4} = \frac{48}{10000} = 0.0048$$

Hence, decimal expansion terminates after 4 places of decimal.

The correct option is (d). [1]

3. We know by a theorem that,

If $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is in the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates. [½]

Hence, Decimal expansion of $\frac{23}{2^3 5^2}$ will be terminating [½]

So, the correct option is (a).

4. Let x = 0.134 ....(1)

Now, $100x = 134.134$ ....(2) [1]

Subtract eqn (1) from (2) We get,

$999x = 134$

$$x = \frac{134}{999}$$

$$x = \frac{134}{9(111)}$$

$$x = \frac{134}{3^2(111)}$$

The above expression can-not be written as $2^m \times 5^n$. [1]

5. Decimal expansion of a any rational number terminates if the denominator of the rational number is in the form 2^n5^m

Let the number multiplied by $\frac{1}{7}$ be x,

$$\frac{1}{7} \times x = \frac{1}{2^n5^m}$$

$$\therefore x = \frac{7}{2^n5^m}$$ [1]

Now here when $n = 2$ and $m = 0$

$$x = \frac{7}{2^2 5^0} = \frac{7}{4}$$

When $n = 0$, $m = 2$

Now if we put $n = 2$ and $m = 2$,

We have $x = \frac{7}{2^2 5^2} = \frac{7}{100}$

Hence we can see that $\frac{7}{100}$ is smallest possible rational number we multiply by $\frac{1}{7}$ so that the decimal expansion will terminate after 2 decimal places. [1]

6. Let $\left(\sqrt{3} + \sqrt{5}\right)^2$ is a rational number.

$\Rightarrow \left(\sqrt{3} + \sqrt{5}\right)^2 = \frac{p}{q}$ Where p, q are co-prime

Using $(a+b)^2 = a^2 + b^2 + 2ab$ we get,

$$\left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right) + 2\sqrt{3}\sqrt{5} = \frac{p}{q}$$ [1]

$$\Rightarrow 3 + 5 + 2\sqrt{15} = \frac{p}{q}$$

$$\Rightarrow 8 + 2\sqrt{15} = \frac{p}{q}$$

$$\Rightarrow 2\sqrt{15} = \frac{p}{q} - 8$$

$$\Rightarrow \sqrt{15} = \frac{1}{2}\left(\frac{p}{q} - 8\right)$$

$$\Rightarrow \sqrt{15} = \left(\frac{p}{2q} - 4\right)$$

The RHS is the difference of two rational numbers.

Therefore LHS will also be rational.

But we know that $\sqrt{15}$ is irrational.

So our assumption is wrong. [1]

Hence, $\left(\sqrt{3} + \sqrt{5}\right)^2$ is an irrational number.

7. $\frac{76}{6250} = \frac{76}{5^5 \times 2}$

Here,

$\frac{76}{6250}$ is in the form of $\frac{p}{q}$ and q is in the form of 2^n5^m where n and m are non - negative integers. [1]

Hence $\frac{76}{6250}$ has terminating decimal expression.

Now,

$$\frac{76}{6250} = \frac{76}{5^5 \times 2} = \frac{76 \times 2^4}{5^5 \times 2 \times 2^4} = \frac{76 \times 16}{10^5} = \frac{1216}{100000}$$

$= 0.01216$

Thus the decimal expansion of $\frac{76}{6250}$ is 0.01216.

[1]

8. The first number that is divisible by 8 between 200 and 500 is 208 and the last number that is divisible by 8 are 496.

So, the sequence will be 208, 216, 224 .... 496.

Common difference $d = 8$

First term $a = 208$ [1]

Let there be n terms is the sequence

Using the formula $a_n = a + (n-1)d$

Where $a_n = 496$, $a = 208$ and $d = 8$

$496 = 208 + (n-1)(8)$

$(n-1)8 = 288$

$n - 1 = 36$

$n = 37$

Hence, between 200 and 500 there are 37 integers that are divisible by 8. [1]

9. Suppose $\left(5+3\sqrt{2}\right) = \frac{p}{q}$

Now assume $\left(5+3\sqrt{2}\right)$ is a rational number.

Therefore p and q should be co-prime numbers. [1]

$\left(5+3\sqrt{2}\right) = \frac{p}{q}$

$\Rightarrow \frac{p}{q} - 5 = 3\sqrt{2}$

$\Rightarrow \frac{p}{3q} - \frac{5}{3} = \sqrt{2}$

$\Rightarrow \frac{p-5}{3q} = \sqrt{2}$

Since $\sqrt{2}$ is irrational number.

Thus the assumption is incorrect and hence $\left(5+3\sqrt{2}\right)$ is an irrational number.

Hence proved. [1]

10. Let $\sqrt{3}$ is a rational number. So, two integers a and b can be found so that $\sqrt{3} = \frac{a}{b}$

Assume that a and are co-prime.

$\Rightarrow a = \sqrt{3}b$

Squaring both the sides,

$\Rightarrow a^2 = 3b^2$ [1]

So, a^2 is divisible by 3 and it can be said that a is divisible by 3.

Let $a^2 = 3c$, where c is an integer.

$a^2 = 3b^2$

$\Rightarrow (3c)^2 = 3b^2$

$\Rightarrow b^2 = 3c^2$

So, b^2 is divisible by 3 and it can be said that b is divisible by 3.

This means that a and b have 3 as a common factor which is a contradiction to fact that *a* and *b* are co-prime.

Hence, $\sqrt{3}$ cannot be expressed as $\frac{p}{q}$ or $\sqrt{3}$ is irrational.

Similarly, $\sqrt{2}$ is irrational. The sum of two irrational numbers is an irrational number.

$\sqrt{3}+\sqrt{2}$ is sum of two irrational numbers, hence it is an irrational number.

Hence proved. [1]

11. **Irrational numbers:** are those numbers that cannot be written in form $\frac{p}{q}$ where *p* and *q* are integers and $q \neq 0$. In other words, these are the numbers whose decimal expansion is non-terminating and non-repeating.

Let $3+\sqrt[2]{5}$ be a rational number [1]

$\therefore$ We can find two integers *a*, *b* $(b \neq 0)$ such that

$3+\sqrt[2]{5} = \frac{a}{b}$ [1]

$\sqrt[2]{5} = \frac{a}{b} - 3$

Since *a* and *b* are integers, $\frac{a}{b} - 3$ is also a rational number and hence $\sqrt[2]{5}$ should be rational. [1]

This contradicts the fact that $\sqrt[2]{5}$ is irrational.

Therefore, our assumption s wrong and hence, $3+\sqrt[2]{5}$ is an irrational number. [1]

Smart Notes ....

Smart Notes ....

CHAPTER 2

Polynomials

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Solving Equations				4 marks	3, 4 marks	3, 4 marks
Zeroes of Polynomial	3 marks	3 marks				

[TOPIC 1] Zeroes of a Polynomial and Relationship between Zeroes and Coefficients of Quadratic Polynomials

Summary

Polynomials

An expression $p(x)$ of the form $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n$ where all $a_0, a_1, a_2, ..., a_n$, are real numbers and n is a non-negative integer, is called a polynomial.

The degree of a polynomial in one variable is the greatest exponent of that variable.

$a_0, a_1, a_2, ..., a_n$ are called the co-efficients of the polynomial $p(x)$.

a_n is called constant term.

Degree of a Polynomial

The exponent of the term with the highest power in a polynomial is known as its degree.

$f(x) = 8x^3 - 2x^2 + 8x - 21$ and $g(x) = 9x^2 - 3x + 12$ are polynomials of degree 3 and 2 respectively.

Thus, $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is a polynomial of degree n, if $a_0 \neq 0$.

On the basis of degree of a polynomial, we have following standard names for the polynomials.

A polynomial of degree 1 is called a **linear polynomial**. Example: $2x+3, \frac{1}{3}u+7$ etc.

A polynomial of degree 2 is called a **quadratic polynomial**. Example: $x^2 + 2x + 3$, $y^2 - 9$ etc.

A polynomial of degree 3 is called a **cubic polynomial**. Example: $x^3 + 7x - 3$, $-x^3 + x^2 + \sqrt{3}x$ etc.

A polynomial of degree 4 is called a **biquadratic polynomial**. Example: $3u^4 - 5u^3 + 2u^2 + 7$.

Value of a Polynomial

If $f(x)$ is a polynomial and α is any real number, then the real number obtained by replacing x by α in $f(x)$ is called the value of $f(x)$ at $x = \alpha$ and is denoted by $f(\alpha)$.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The graph of the polynomial $p(x)$ intersects the x-axis three times in distinct points, then which of the following could be an expression for $p(x)$:

   (a) $4 - 4x - x^2 + x^3$ (b) $3x^2 + 3x - 3$

   (c) $3x + 3$ (d) $x^2 - 9$

   [TERM 1, 2011]

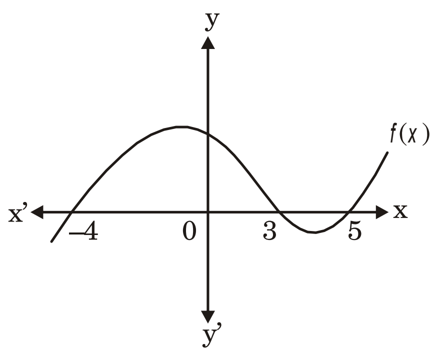
2. The polynomial whose zeroes are –5 and 4 is:

   (a) $x^2 - 5x + 4$ (b) $x^2 + 5x - 4$

   (c) $x^2 + x - 20$ (d) $x^2 - 9x - 20$

   [TERM 1, 2012]

3. In the given figure, the number of zeroes of the polynomial $f(x)$ are:



   (a) 1 (b) 2

   (c) 3 (d) 4

   [TERM 1, 2013]

2 Marks Questions

4. Divide $6x^3+13x^2+x-2$ by $2x+1$, and find quotient and remainder.

[TERM 1, 2011]

5. Find a quadratic polynomial whose zeroes are $2+\sqrt{3}$ and $2-\sqrt{3}$.

[TERM 1, 2012]

6. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

[TERM 1, 2012]

3 Marks Questions

7. Find the zeroes of the following quadratic polynomial and verify the relationship between the zeroes and the coefficients.

$2x^2-3+5x$

[TERM 1, 2012]

8. Solve for x:

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}, x\neq 1,2,3$$

[TERM 1, 2014]

Solutions

1. It is given that graph of the polynomial $p(x)$ intersects the x-axis three times in distinct points, so the number of zeroes for the polynomial is 3 and it is a third degree polynomial of the form

$ax^3+bx^2+cx+d=0$ where $a\neq 0$.

From the given options, the one option matching with the standard equation of a third degree polynomial is $4-4x-x^2+x^3=x^3-x^2-4x+4$.

Hence, the correct option is (a). [1]

2. Let $a=-5$ and $b=4$

Sum of zeroes

$\Rightarrow a+b=-5+4=-1$ ...(1)

Product of zeroes

$\Rightarrow a\times b=-5\times 4=-20$ ...(2) [½]

The general equation of the quadratic is:

$x^2-(a+b)x+ab=0$

Substituting values from equation 1 and 2, we get,

$x^2+x-20=0$

Hence, The polynomial whose zeroes are –5 and 4 is (c). [½]

3. From the graph we can say that the number of times the graph touches the x-axis is 3.

So, the number of zeroes of the polynomial is 3.

Hence the correct option is (c). [1]

4. Use long division to divide $6x^3+13x^2+x-2$ by $2x+1$.

$$\begin{array}{r|l} & 3x^2+5x-2 \\ 2x+1 & 6x^3+13x^2+x-2 \\ & -(6x^3+3x^2) \\ \hline & 10x^2+x-2 \\ & -(10x^2+5x) \\ \hline & -4x-2 \\ & -(-4x-2) \\ \hline & 0 \end{array}$$

Hence, quotient is $3x^2+5x-2$ and remainder is 0. [2]

5. Let the zeroes of a quadratic polynomial be

$a=2+\sqrt{3}$ *and* $b=2-\sqrt{3}$

We know that,

The quadratic polynomial with a and b as zeroes is $x^2-(a+b)x+ab$

Substituting values of a and b in the above equation, we get,

$\Rightarrow x^2-(2+\sqrt{3}+2-\sqrt{3})x+(2+\sqrt{3})(2-\sqrt{3})$ [1]

Applying, $(a+b)(a-b)=a^2-b^2$

$\Rightarrow x^2-4x+(4-3)$

$\Rightarrow x^2-4x+1$

Hence, the required quadratic polynomial is x^2-4x+1 [1]

6. Let the polynomial be

$p(x)=ax^2+bx+c$

Its given that, Sum of zeroes = $\frac{21}{8}$

$\Rightarrow \frac{-b}{a}=\frac{21}{8}$

Assuming $a=1$,

$\Rightarrow b=-\frac{21}{8}$

We also know that, Product of zeroes = $\frac{5}{16}$

$\Rightarrow \frac{c}{a} = \frac{5}{16}$

Assuming $a = 1$,

$\Rightarrow c = \frac{5}{16}$

Now, $a = 1, b = -\frac{21}{8}, c = \frac{5}{16}$ [1]

Hence, the required quadratic polynomial

$= ax^2 + bx + c$

Substituting the values of a, b and c in the above equation, we get,

$x^2 - \frac{21}{8}x + \frac{5}{16}$

Multiply the equation by 16.

$\Rightarrow 16x^2 - 42x + 5$

Hence, the required quadratic polynomial is $16x^2 - 42x + 5$ [1]

7. Let $p(x) = 2x^2 + 5x - 3$

Zero of the polynomial is the value of x where $p(x) = 0$. So, equate the given expression with 0, we get:

$2x^2 + 5x - 3 = 0$

Compare the given equation with the general equation $ax^2 + bx + c$, we get:

$a = 2, b = 5, c = -3$ [1]

Let, α and β ne the roots of the given equation.

Find the roots of the given equation by using quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$\frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm \sqrt{49}}{4} = \frac{-5 \pm 7}{4}$$

So, the roots will be:

$$\alpha = \frac{-5 + 7}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\beta = \frac{-5 - 7}{4} = \frac{-12}{4} = -3$$

Therefore, $\alpha = \frac{1}{2}$ and $\beta = -3$ are the roots/zeroes of the given equation. [1]

Now, we verify the relationship between the zeroes and the coefficients. We know that:

Sum of zeroes = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{b}{a}$

LHS

$\alpha + \beta = \frac{1}{2} - 3 = -\frac{5}{2}$

RHS

$-\frac{b}{a} = -\frac{5}{2}$

Therefore, LHS = RHS

Product of zeroes = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \alpha \times \beta = \frac{c}{a}$

LHS

$\frac{1}{2} \times -3 = -\frac{3}{2}$

RHS

$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-3}{2} = -\frac{3}{2}$

Therefore, LHS = RHS.

As in both the conditions, LHS = RHS, therefore the relationship between coefficient and the zeroes is verified. [1]

8. Solving for x, using common denominator

$$\Rightarrow \frac{x - 3 + x - 1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow \frac{2x - 4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$ [1]

Cancel out like terms

$$\Rightarrow \frac{2(x - 2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

Cross multiply

$\Rightarrow 3 = (x-1)(x-3)$ [1]

$\Rightarrow x^2 - 3x - x + 3 = 3$

$\Rightarrow x^2 - 4x = 0$

$\Rightarrow x(x - 4) = 0$

Hence, $x = 0$ and $x - 4 = 0$

Therefore, $x = 0$ and $x = 4$. [1]

[TOPIC 2] Problems on Polynomials

Summary

Zeros of a Polynomial

A real number α is a zero of polynomial $f(x)$ if $f(\alpha) = 0$.

The zero of a linear polynomial $ax + b$ is $-\frac{b}{a}$.

i.e. $-\frac{\text{Constant term}}{\text{Coefficient of } x}$

Geometrically zero of a polynomial is the point where the graph of the function cuts or touches x-axis.

When the graph of the polynomial does not meet the x-axis at all, the polynomial has no real zero.

Signs of Coefficients of a Quadratic Polynomial

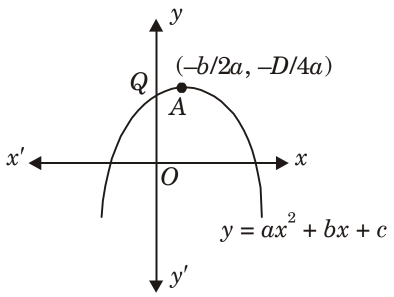
The graphs of $y = ax^2 + bx + c$ are given in figure. Identify the signs of a, b and c in each of the following:

(i) We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a < 0$. We observe that the turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola is in first quadrant where $D = b^2 - 4ac$

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \; -b < 0$

$\Rightarrow \; b > 0$ $\qquad [\because a < 0]$



Parabola $y = ax^2 + bx + c$ cuts y-axis at Q. On y-axis, we have $x = 0$.

Putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of Q are (0, c). As Q lies on the positive direction of y-axis. Therefore, $c > 0$.

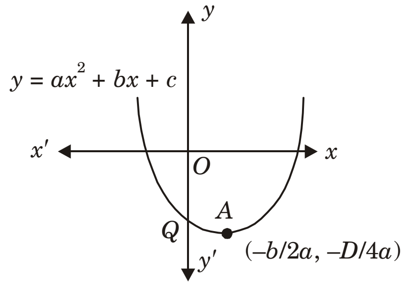
Hence, $a < 0$, $b > 0$ and $c > 0$.

(ii) We find that $y = ax^2 + bx + c$ represents a parabola opening upwards. Therefore, $a > 0$. The turning point of the parabola is in fourth quadrant.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \; -b > 0$

$\Rightarrow \; b < 0$



Parabola $y = ax^2 + bx + c$ cuts y-axis at Q and y-axis. We have $x = 0$. Therefore, on putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of Q are (0, c). As Q lies on negative y-axis. Therefore, $c < 0$.

Hence, $a > 0$, $b < 0$ and $c < 0$.

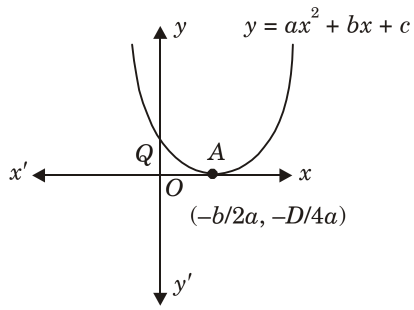
(iii) Clearly, $y = ax^2 + bx + c$ represents a parabola opening upwards.

Therefore, $a > 0$. The turning point of the parabola lies on positive direction of x-axis.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \; -b > 0$

$\Rightarrow \; b < 0$



The parabola $y = ax^2 + bx + c$ cuts y-axis at Q which lies on positive y-axis. Putting $x = 0$ in $y = ax^2 + bx + c$ we get $y = c$. So, the coordinates of Q are (0, c). Clearly, Q lies on OY.

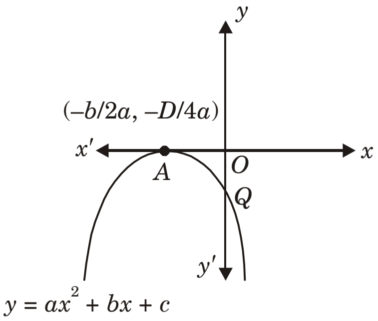
$\therefore \quad c > 0$.

Hence, $a > 0$, $b < 0$, and $c > 0$.

(iv) The parabola $y = ax^2 + bx + c$ opens downwards. Therefore, $a < 0$. The turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola is on negative x-axis,

$\therefore \quad -\frac{b}{2a} < 0$

$\Rightarrow \quad b < 0 \qquad [\because a < 0]$



Parabola $y = ax^2 + bx + c$ cuts y-axis at $Q\,(0, c)$ which lies on negative y-axis. Therefore, $c < 0$.

Hence, $a < 0$, $b < 0$ and $c < 0$.

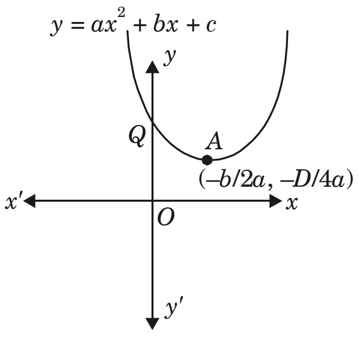
(v) We notice that the parabola $y = ax^2 + bx + c$ opens upwards. Therefore, $a > 0$.

Turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola lies in the first quadrant.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \quad \frac{b}{2a} < 0$

$\Rightarrow \quad b < 0 \qquad [\because a > 0]$



As $Q\,(0, c)$ lies on positive y-axis. Therefore, $c > 0$.

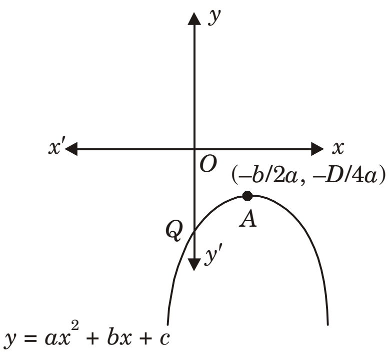
Hence, $a > 0$, $b < 0$ and $c > 0$.

(vi) Clearly, $a < 0$ Turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola lies in the fourth quadrant.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \quad \frac{b}{2a} < 0$

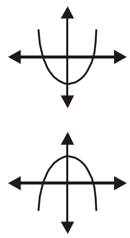
$\Rightarrow \quad b > 0 \qquad [\because a < 0]$



As $Q\,(0, c)$ lies on negative y-axis. Therefore, $c < 0$.

Hence, $a < 0$, $b > 0$ and $c < 0$.

- ***The graph of quadratic polynomial is a parabola.***
- ***If a is +ve, graph opens upward.***
- ***If a is –ve, graph opens downward.***
- ***If D > 0, parabola cuts x-axis at two points i.e. it has two zeros.***



If D = 0, parabola touches x-axis at one point i.e. it has one zero.

If D < 0, parabola does not even touch x-axis at all i.e. it has no real zero.

Division Algorithm for Polynomials

Let $p(x)$ and $g(x)$ be polynomials of degree n and m respectively such that $m \le n$. Then there exist unique polynomials $q(x)$ and $r(x)$ where $r(x)$ is either zero polynomial or degree of $r(x) <$ degree of $g(x)$ such that $p(x) = q(x) \,.\, g(x) + r(x)$.

$p(x)$ is dividend, $g(x)$ is divisor.

$q(x)$ is quotient, $r(x)$ is remainder

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

2 Marks Question

1. Find the quotient and remainder when polynomial $x^4-6x^3-26x^2+138x-35$ is divided by $x^2-2x-35$.

[TERM 1, 2015]

3 Marks Questions

2. On dividing x^3-2x^2+x-2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$ respectively. Find $g(x)$.

[TERM 1, 2011]

3. Find all the zeroes of $x^3+11x^2+23x-35$, if two of its zeroes are 1 and –5.

[TERM 1, 2012]

4. If two zeroes of the polynomial $x^4+3x^3-20x^2-6x+36$ are $\sqrt{2}$ and $-\sqrt{2}$, find all the zeroes of the polynomial.

[TERM 1, 2013]

5. If x^3-4x^2+5x-k is completely divisible by $x-4$, then find the value of k.

[TERM 1, 2014]

6. What should be added in the polynomial $3x^4-4x^3-6x^2+4$ so that it is completely divisible by x^2-2.

[TERM 1, 2015]

4 Marks Questions

7. Find all the zeroes of $2x^4+7x^3-19x^2-14x+30$ given that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

[TERM 1, 2011]

8. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by x^2-2x+k, the remainder comes out to be $x+a$ find k and a.

[TERM 1, 2012]

Solutions

1. Using the long division we get,

$$
\begin{array}{r|l}
 & x^2-4x+1 \\
x^2-2x-35 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-2x^3-35x^2 \\
 & -\quad +\quad + \\
\hline
 & -4x^3+9x^2+138x-35 \\
 & -4x^3+8x^2+140x \\
 & +\quad -\quad - \\
\hline
 & x^2-2x-35 \\
 & x^2-2x-35 \\
 & -\quad +\quad + \\
\hline
 & 0
\end{array}
$$

Hence the quotient is x^2-4x+1 and the remainder is 0. [2]

2. Dividend = Divisor × Quotient + Remainder

Here, dividend $=x^3-2x^2+x-2$

Quotient $=x-2$ and

Remainder $=-2x+4$

Divisor $=g(x)$

$\Rightarrow x^3-2x^2+x-2=g(x)\times(x-2)+(-2x+4)$ [1]

Subtracting $-2x+4$ from both sides of above equation,

$\Rightarrow x^3-2x^2+x-2+2x-4=g(x)\times(x-2)$

$\Rightarrow x^3-2x^2+3x-6=g(x)\times(x-2)$ [1]

Divide the above equation by $(x-2)$,

$$g(x)=\frac{x^3-2x^2+3x-6}{x-2}$$

$$\Rightarrow g(x)=\frac{x^2(x-2)-3(x-2)}{x-2}$$

$$\Rightarrow g(x)=\frac{(x-2)(x^2-3)}{x-2}=x^2-3$$

Hence, the value of $g(x)$ is x^2-3. [1]

3. Consider the polynomial $x^3+11x^2+23x-35$

Let, α and β are zeroes of the quadratic polynomial so,

$\alpha=1$ and $\beta=-5$

$\alpha+\beta=-5+1=-4$

$\alpha\beta=-5\times1=-5$ [1]

So, the general equation can be written as:

$x^2-(\alpha+\beta)x+\alpha\beta=0$

Or, $x^2-(-4)x-5=0$

$\Rightarrow x^2+4x-5=0$ ....(i) [1]

Divide the given polynomial by (i)

$$\begin{array}{r|l} & x+7 \\ x^2+4x-5 & x^3+11x^2+23x-35 \\ & -(x^3+4x^2-5x) \\ \hline & 7x^2+28x-35 \\ & 7x^2+28x-35 \\ \hline & 0 \end{array}$$

Therefore, we get another polynomial $x+7$

Put the polynomial equal to zero, we get;

$x+7=0$

$\Rightarrow x=-7$

Hence, all the zeroes of the polynomial 1, –5 are –7. [1]

4. Let $p(x)=x^4+3x^3-20x^2-6x+36$

given $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of the polynomial.

Hence $(x+\sqrt{2})$ and $(x-\sqrt{2})$ are the factors of the given polynomial.

$(x+\sqrt{2})(x-\sqrt{2})=x^2-2$ [1]

On dividing $(x^4+3x^3-20x^2-6x+36)$ with (x^2-2) we get the quotient as $(x^2+3x-18)$

$(x^4+3x^3-20x^2-6x+36)=(x^2-2)(x^2+3x-18)$

$=(x^2-2)(x^2+6x-3x-18)$

$=(x^2-2)\left[x(x+6)-3(x+6)\right]$ [1]

$\Rightarrow (x^2-2)(x+6)(x-3)$

$\Rightarrow (x+6)(x-3)=0$

$\Rightarrow (x+6)=0 \; and (x-3)=0$

$\Rightarrow x=-6$ and $x=3$

Therefore the other zeroes are 3 and –6. [1]

5. If x^3-4x^2+5x-k is completely divisible by $x-4$, Then using long division

$$\begin{array}{r|l} & x^2+5 \\ x-4 & x^3-4x^2+5x-k \\ & x^3-4x^2 \\ & -\quad + \\ \hline & 0 \quad 0 \quad +5x-k \\ & 5x-20 \\ & -\quad + \\ \hline & -k+20=0 \end{array}$$ [2]

Here the remainder is zero,

Hence,

$-k+20=0$

$k=20$ [1]

6. Solving using long division we get,

$$\begin{array}{r|l} & 3x^2-4x \\ x^2-2 & 3x^4-4x^3-6x^2+4 \\ & 3x^4 \quad -6x^2 \\ & -\quad + \\ \hline & -4x^3 \quad +4 \\ & -4x^3 \quad +8x \\ & +\quad - \\ \hline & -8x+4 \end{array}$$ [2]

Here the remainder is $-8x+4$ which can become zero to make the given polynomial divisible by x^2-2 on adding its compliment i.e. on adding to the given polynomial.

Hence, $8x-4$ must be added to the polynomial $3x^4-4x^3-6x^2+4$ so that it is completely divisible by x^2-2. [1]

7. The factors of $2x^4+7x^3-19x^2-14x+30$ are $x-\sqrt{2}$ and $x+\sqrt{2}$.

$\Rightarrow (x-\sqrt{2})(x+\sqrt{2})$ is a factor of

$2x^4+7x^3-19x^2-14x+30$. [1]

$\Rightarrow x^2 - 2$ is a factor of $2x^4 + 7x^3 - 19x^2 - 14x + 30$.

Divide $2x^4 + 7x^3 - 19x^2 - 14x + 30$ by $x^2 - 2$ using long division,

$$\begin{array}{r|l} & 2x^2 + 7x - 15 \\ \hline x^2 - 2 & 2x^4 + 7x^3 - 19x^2 - 14x + 30 \\ & -(2x^4 \qquad\quad - 4x^2) \\ \hline & 7x^3 - 15x^2 - 14x + 30 \\ & -(7x^3 \qquad\quad - 14x) \\ \hline & -15x^2 \qquad\quad + 30 \\ & -(-15x^2 \qquad\quad + 30) \\ \hline & 0 \end{array}$$ [1]

$$2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$$

Factorize $(2x^2 + 7x - 15)$,

$$\Rightarrow 2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 10x - 3x - 15)$$

$$\Rightarrow 2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x(x + 5) - 3(x + 5))$$

$$\Rightarrow 2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)((x + 5)(2x - 3))$$ [1]

Equating to zero,

$x^2 - 2 = 0, x + 5 = 0$ and $2x - 3 = 0$

$\Rightarrow x = \pm\sqrt{2}, x = -5$ and $x = \frac{3}{2}$

Hence, the zeros are $\sqrt{2}$, $-\sqrt{2}$, -5 and $\frac{3}{2}$ [1]

8. Let $p(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$

And $q(x) = (x^2 - 2x + k)$

Divide $p(x)$ *by* $q(x)$

$$\begin{array}{r|l} & x^2 - 4x + 8 - k \\ \hline x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\ & x^4 - 2x^3 + kx^2 \\ \hline & -4x^3 + 16x^2 - kx^2 - 25x + 10 \\ & -4x^3 + 8x^2 \qquad - 4kx \\ \hline & 8x^2 - kx^2 - 25x + 4kx + 10 \\ & 8x^2 \qquad - 16x \qquad + 8k \\ \hline & -kx^2 - 9x + 4kx + 10 - 8k \\ & -kx^2 \qquad + 2kx \qquad - k^2 \\ \hline & -9x + 2kx + 10 - 8k + k^2 \end{array}$$ [2]

Hence, Remainder $= -9x + 2kx + 10 - 8k + k^2$

$\Rightarrow (2k - 9)x + k^2 - 8k + 10$

Its given, Remainder = $x + a$

On comparing the coefficients of x, we get,

$2k - 9 = 1$

$\Rightarrow k = 5$ [1]

Also, $k^2 - 8k + 10 = a$

Put $k = 5$ in above equation, we get,

$a = 25 - 40 + 10$

$a = -5$

Hence, $k = 5$ and $a = -5$. [1]

Smart Notes ....

CHAPTER 3

Linear Equation

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Word Problem based on Boat and Stream	4 marks	4 marks	4 marks			4 marks
Word Problem based on Time and Work				4 marks		
Word Problem based on Speed, Time and Distance	3 marks	3 marks			4, 4 marks	
Word Problem based on Rectangle	2 marks	2 marks				

[TOPIC 1] Linear Equations (Two Variables)

Summary

Pair of Linear Equations in Two Variables

1. The equation of the form $ax = b$ or $ax + b = 0$, where a and b are two real numbers such that $x \neq 0$ and x is a variable is called a linear equation in one variable.
2. The general form of a linear equation in two variables is $ax + by + c = 0$ or $ax + by = c$ where a, b, c are real numbers and $a \neq 0$, $b \neq 0$ and x, y are variables.
3. The graph of a linear equation in two variables is a straight line.
4. The graph of a linear equation in one variable is a straight line parallel to x-axis for $ay = b$ and parallel to y-axis for $ax = b$, where $a \neq 0$.
5. A pair of linear equations in two variables is said to form a system of simultaneous linear equations.
6. The value of the variable x and y satisfying each one of the equations in a given system of linear equations in x and y simultaneously is called a solution of the system.

Graphical Method of Solution of a Pair of Linear Equations

1. Read the problem carefully to find the unknowns (variables) which are to be calculated.
2. Depict the unknowns by x and y etc.
3. Use the given conditions in the problem to make equations in unknown x and y.
4. Make the proper tables for both the equations.
5. Draw the graph of both the equations on the same set of axis.
6. Locate the co-ordinates of point of intersection of the graph, if any.
7. Coordinates of point of intersection will give us the required solution.

System of Two Simultaneous Linear Equations in X and Y

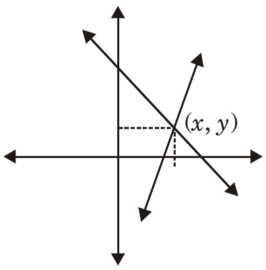
Consistent system: A system of two linear equations is said to be consistent if it has at least one solution.

Inconsistent system: A system of two linear equations is said to be inconsistent if it has no solution.

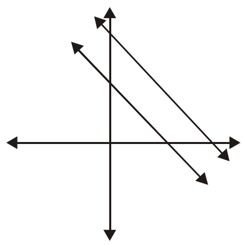
Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a system of two linear equations.

The following cases occur :

(i) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, it has a unique solution. The graph of lines intersects at one point. The system is independent consistent.

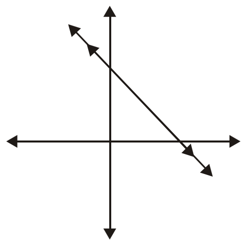


(ii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. It has no solution. The graph of both lines is parallel to each other. The system is inconsistent



(iii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. It has infinite many solutions.

Every solution of one equation is a solution of other also. The graph of both equations is coincident lines. The system is dependent consistent.



PREVIOUS YEARS'
EXAMINATION QUESTIONS
TOPIC 1

1 Mark Questions

1. Which of the following is not a solution of the pair of equations $3x-2y=4$ and $6x-4y=8$?

(*a*) $x=2, y=1$ (*b*) $x=4, y=4$

(*c*) $x=6, y=7$ (*d*) $x=5, y=3$

[TERM 1, 2011]

2. $x=2$, $y=3$ is a solution of the linear equation:

(*a*) $2x+3y-13=0$ (*b*) $3x+2y-31=0$

(*c*) $2x-3y+13=0$ (*d*) $2x+3y+13=0$

[TERM 1, 2012]

3. The pair of equations $y = 0$ *and* $y = -5$ has

(*a*) One solution (*b*) Two solution

(*c*) Three solution (*d*) No solution

[TERM 1, 2013]

4. Find whether the lines representing the following pair of linear equation intersect at a point, are parallel or coincident:

$$\frac{3}{2}x+\frac{5}{3}y=7 \text{ and } \frac{3}{2}x+\frac{2}{3}y=6$$

[TERM 1, 2016]

2 Marks Questions

5. For what value of k, $2x+3y=4$ and $(k+2)x=3k+2$ will have infinitely many solutions.

[TERM 1, 2011]

6. For what value of c, the pair of equations $x-4y=2$ *and* $3x+cy=10$ has no solution?

[TERM 1, 2013]

3 Marks Questions

7. For which value of k will the following pair of linear equations have no solution?

$$3x+y=1,\ (2k-1)x+(k-1)y=2k+1$$

[TERM 1, 2012]

8. Draw the graphs of the equations $x-y+1=0$ *and* $3x+2y-12=0$. Write the co-ordinates of the vertices of the triangle formed by these lines and the x-axis and shade the corresponding triangular region.

[TERM 1, 2013]

9. Check graphically whether the following pair of linear equations is consistent. If yes, solve it graphically

$2x-5=0$ and $x+y=0$

[TERM 1, 2014]

10. Determine graphically whether the following pair of linear equations

$2x-3y=8$

$4x-6y=16$

has

(i) A unique solution

(ii) Infinitely many solutions

(iii) No solution

[TERM 1, 2015]

4 Marks Questions

11. Draw the graphs of the following equations:

$x + y = 5$

$x - y = 5$

(i) Find the solution of the equations from the graph.

(ii) Shade the triangular region formed by the lines and the y-axis.

[TERM 1, 2011]

12. Check graphically, whether the pair of sequence is consistent. If so, then solve them graphically.

$x + 3y = 6$

$2x - 3y = 12$

[TERM 1, 2012]

13. Solve the following pair of linear equations graphically $6x - y + 4 = 0$ and $2x - 5y = 8$. Shade the region bounded by the lines and y-axis.

[TERM 1, 2016]

14. Find the graphically solution of $x-2y=0$ and $3x+4y=20$

[TERM 1, 2017]

Solutions

1. Check every option one by one.

(a) The value of y for $x = 2$ is

$$\frac{3}{2}x-2=\frac{3}{2}(2)-2=1 \text{ which is true.}$$

(b) The value of y for $x = 4$ is

$$\frac{3}{2}x-2=\frac{3}{2}(4)-2=4 \text{ which is true.}$$

(c) The value of y for $x = 6$ is $\frac{3}{2}x-2=\frac{3}{2}(6)-2=7$ which is true.

(d) The value of y for $x = 5$ is

$\frac{3}{2}x-2=\frac{3}{2}(5)-2=\frac{11}{2}$ which is not equal to given value is of .

Hence, the correct option is (d). [1]

2. We have,

$x = 2$ and $y = 3$

Let us substitute the above values in the equation $2x+3y-13=0$, we get

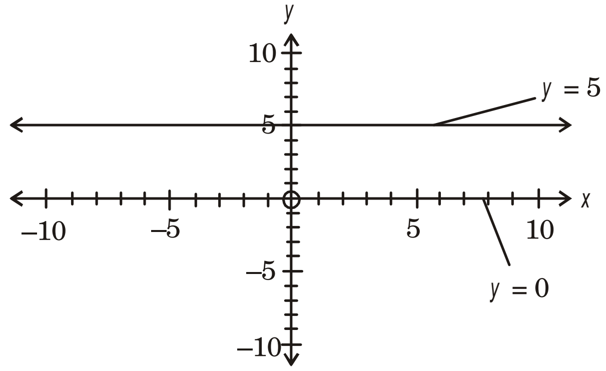
$$\text{LHS}=(2\times 2)+(3\times 3)-13=4+9-13=13-13=0$$

RHS = 0

Since, LHS = RHS, the required linear equation is $2x+3y-13=0$.

Hence, the correct option is (a). [1]

3. Both the lines $y = 0$ and $y = –5$ represent parallel lines. Hence they do not intersect and don't have a solution.



Hence, the correct option is (d). [1]

4. Compare given equations with $ax+by=c$,

$$\frac{3}{2}x+\frac{5}{3}y=7$$

$$\therefore a_1=\frac{3}{2}, b_1=\frac{5}{3} \text{ and } c_1=7$$

$$\frac{3}{2}x+\frac{2}{3}y=6$$

$$\therefore a_2=\frac{3}{2}, b_2=\frac{2}{3} \text{ and } c_2=6$$ [½]

Now,

$$\frac{a_1}{a_2}=\frac{\frac{3}{2}}{\frac{3}{2}}=1 \text{ and } \frac{b_1}{b_2}=\frac{\frac{5}{3}}{\frac{2}{3}}=\frac{5}{2}$$

Here,

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Thus,the lines formed by following pair of linear equation intersect at a point. [½]

5. The given equations are $2x+3y=4$.......(*i*) and

$(k+2)x=3k+2$.......(*ii*)

For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{c_1}{c_2}$$

Here, $a_1=2, a_2=k+2, c_1=4$ and $c_2=3k+2$.

$$\frac{2}{k+2}=\frac{4}{3k+2}$$

$$\Rightarrow 2(3k+2)=4(k+2)$$ [1]

Using the distributive property $a(b+c)=ab+ac$

$$\Rightarrow 6k+4=4k+8$$

$$\Rightarrow 2k=4$$

Divide the above equation by 2,

$k = 2$

Hence, for $k = 2$, $2x+3y=4$ and

$(k+2)x=3k+2$ will have infinitely many solutions. [1]

6. Given equations are

$x - 4y = 2$ *and* $3x + cy = 10$

Comparing the equations with $ax + by = c$, we get

$a_1 = 1, a_2 = 3, b_1 = -4$ and $b_2 = c$ [1]

For equation to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{3} = \frac{-4}{c}$$

$$\Rightarrow c = -4 \times 3$$

$$\Rightarrow c = -12$$

Hence, the value of c for equations to have no solution is –12. [1]

7. Consider $3x + y = 1$ and

$(2k-1)x + (k-1)y = 2k+1$

Compare the above two equations with $ax + by = c$, we get,

$a_1 = 3, b_1 = 1, c_1 = 1$

$a_2 = 2k-1, b_2 = k-1, c_2 = 2k+1$ [1]

We know that for no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Or, $\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$

Consider

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$\Rightarrow 3(k-1) = 2k-1$ [1]

Applying distributive property,

Or $3k - 3 = 2k - 1$

Subtract 2k on both sides and then, we get,

$$\Rightarrow 3k - 2k - 3 = 2k - 2k - 1$$

$$\Rightarrow k - 3 = -1$$

Now add 3 on both sides,

$$\Rightarrow k - 3 + 3 = -1 + 3$$

$k = 2$

Hence, for $k = 3$ the equations have no solution.

[1]

8. $x - y + 1 = 0$

Or

$x = y - 1$

X	0	1	2
Y	1	2	3

$3x + 2y - 12 = 0$

$\Rightarrow 3x = 12 - 2y$

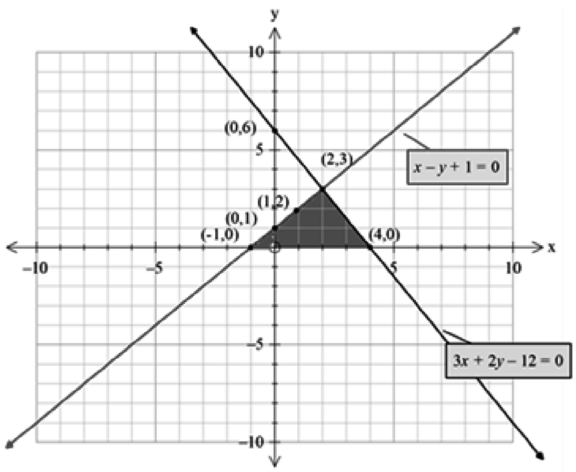
Divide the above equation by 3,

$$x = \frac{12 - 2y}{3}$$

x	4	2	0
y	0	3	6

[1]

Hence, the graphic representation is as follows:



[1]

From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and x- axis at (–1, 0) and (4, 0). Therefore the vertices of the triangle are (2, 3), (–1, 0) and (4, 0) [1]

9. To plot $2x - 5 = 0$, set of points we have,

$$x = \frac{5}{2}$$

for all values of y

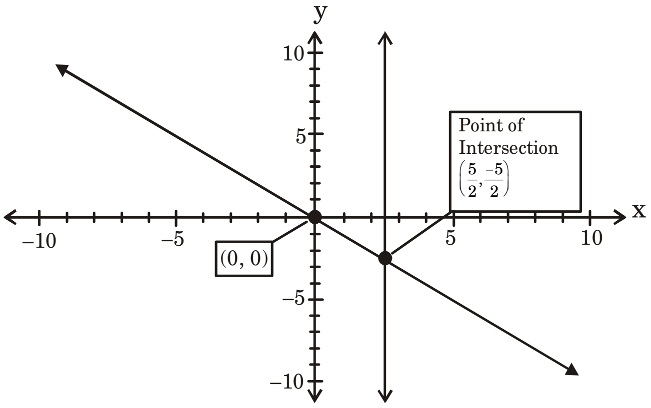
To plot $x + y = 0$, set of points we have,

x	0	1	–2
y	0	–1	2

[1]

Plot the points to obtain the graph.

Graph we have is:



[1]

As we can see from the graph the lines intersect each other at point, hence they are consistent and have a unique solution $\left(\frac{5}{2}, -\frac{5}{2}\right)$.

Hence the system is consistent and the solution is $x = \frac{5}{2}, y = -\frac{5}{2}$. [1]

10. $2x - 3y = 8$

$\Rightarrow 3y = 2x - 8$

$\Rightarrow y = \frac{2x - 8}{3}$

Substituting arbitrary values of x to get corresponding y values we get,

x	y
4	0
0	$-\frac{8}{3}$
7	2

[1]

Now $4x - 6y = 16$

$\Rightarrow 6y = 4x - 16$

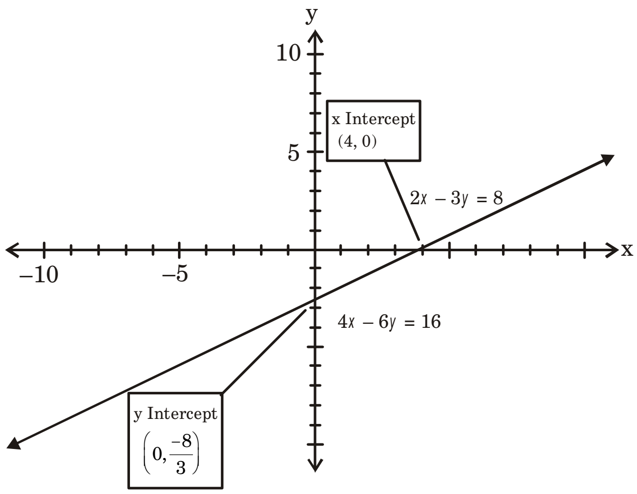
$\Rightarrow y = \frac{4x - 16}{6}$

Again substituting arbitrary values of x to get corresponding y values we get,

x	y
4	0
0	$-\frac{8}{3}$
7	2

[1]

Plotting these points we obtain the graph as:



Since the graph is a pair of coincident lines. Each point on the lines is a solution and so the pair of equations have infinitely many solutions. Hence, the correct option is (ii). [1]

11. (i) Plot the equations of the two lines $x + y = 5, x - y = 5$.

For $x + y = 5$,

x	y
0	5
5	0
1	4

[1]

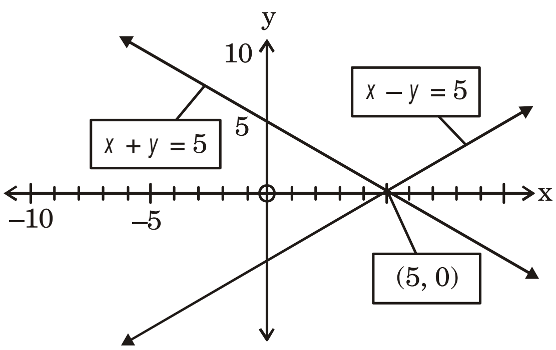
For $x - y = 5$,

x	y
0	−5
5	0
6	1

[1]

Plot the points on graph.

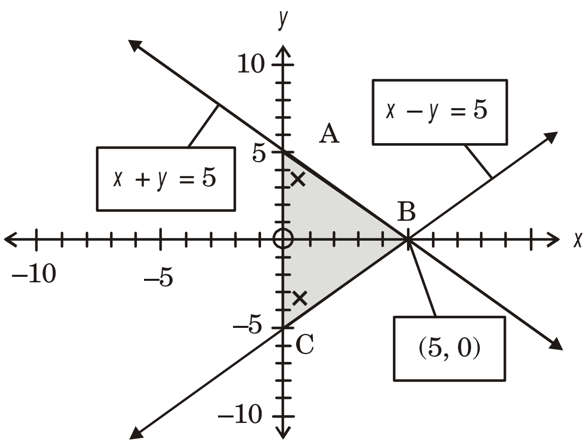
The solution of equations is point of intersection of two lines.



[1]

Hence, the solution is (5, 0).

(ii) The triangular region formed by the lines and the y axis is shown below.



ΔABC is the required region. [1]

12. $x+3y=6$ (1)

$2x-3y=12$ (2)

Solving equation 1,

We have $x+3y=6$

$\Rightarrow 3y=6-x$ [1]

When $x = 0$, we have, $y = 2$

When $x = 3$, we have, $y = 1$

Solving equation 2,

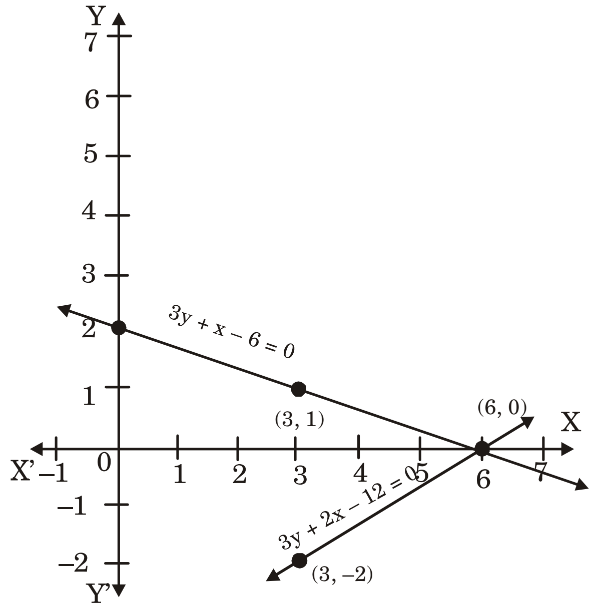
We have, $2x-3y=12$

$\Rightarrow y=\dfrac{2x-12}{3}$ [1]

When $x = 3$, we have, $y = -2$

When $x = 6$, we have, $y = 0$

Hence, plotting both equations on the graph,



[1]

Since, from the graph it is clear that both the lines intersect at a common point (6, 0).

Hence, the given equations are consistent and the solution is (6, 0). [1]

13. Consider $6x-y+4=0$

$\Rightarrow y=6x+4$

When $x = 0$ then $y = 6(0) + 4 = 4$

When $x = 1$, then $y = 6(1) + 4 = 10$

The table gives points for $6x-y+4=0$

x	0	1
y	4	10

[1]

For, $2x-5y=8$

$\Rightarrow 5y=2x-8$

$\Rightarrow y=\dfrac{2x-8}{5}$

When $x = 4$, then $y=\dfrac{2(4)-8}{5}=0$

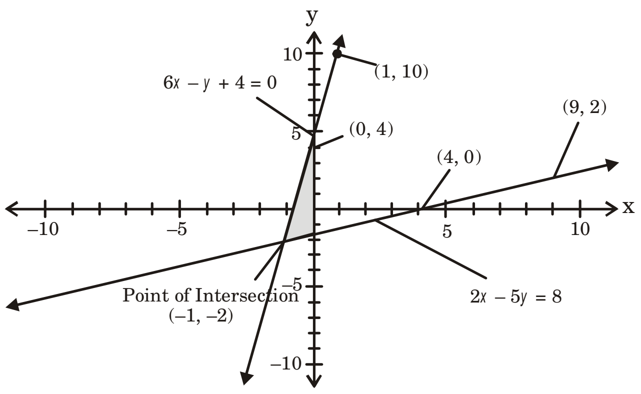
When $x = 9$, then $y=\dfrac{2(9)-8}{5}=\dfrac{10}{5}=2$

The table gives points for $2x - 5y = 8$

x	4	9
y	0	2

[1]

So the desired graph is:



[1]

The region bounded by the lines and the y axis is shaded in the graph.

The point of intersection of two equations is (–1, –2) . Hence the solution is (–1, –2). [1]

14. Here, $x-2y=0$ ......(i)

On simplifying equation (i), we get

$x=2y$

Substitute value of y to get value of x

x	0	2	4
y	0	1	2

[1]

Also, $3x+4y=20$ ....(ii)

On simplifying Eqn. (ii), we get

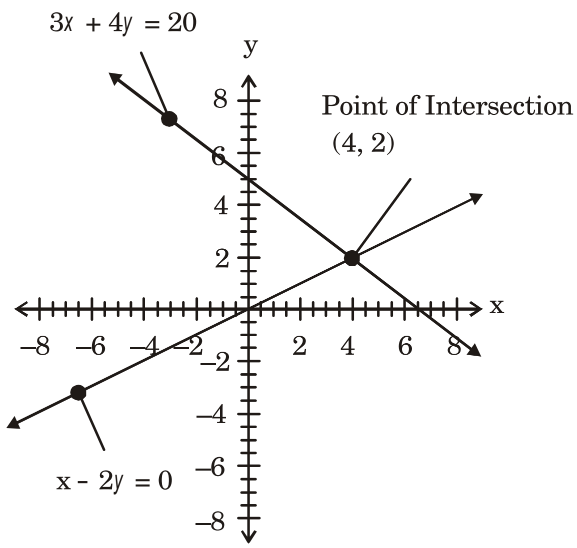
$$x=\frac{20-4y}{3}$$

Substitute value of y to get value of x

x	8	4	0
y	–1	2	5

[1]

Plot the points on a graph.



[1]

As the both the lines intersect at (4, 2), therefore the solution of $x-2y=0$ and $3x+4y=20$ is (4, 2). [1]

[TOPIC 2] Different Methods to Solve Quadratic Equations

Summary

Methods of Solving Linear Equation

ELIMINATION METHOD (Eliminating One variable by making the coefficient equal to get the value of one variable and than put it in any equation to find other variable).

1. First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.
2. Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to step 3.
3. Solve the equation in one variable (x or y) so obtained to get its value.
4. Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

- *If equations are of the following form:*

$$ax + by = cxy$$

$$dx + ey = fxy$$

Then, trivial solutions $x = 0$, $y = 0$ is one solution and the other solution can be obtained by elimination method.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

SUBSTITUTION METHOD (Find the value of any one variable in terms of other and than use it to find other variable from the second equation).

1. Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.
2. Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.
3. Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

COMPARISON MEHTOD (Find the value of one variable from both the equation and equate them to get the value of other variable).

Let any pair of linear equations in two variables is of the form

$$a_1x + b_1y + c_1 = 0 \quad ...(i)$$

$$a_2x + b_2y + c_2 = 0 \quad ...(ii)$$

1. Find the value of one variable, say y in terms of other variable, i.e. x from equation (i), to get equation (iii).
2. Find the value of the same variable (as in step 1) in terms of other variable from equation (ii) to get equation (iv).
3. By equating the variable from equation (iii) and (iv) obtained in above two steps. We get the value of second variable.
4. Substituting the value of above said variable in equation (iii), we get the value of another variable.

CROSS MULTIPLICATION METHOD

Let the equation

$$a_1x + b_1y + c_1 = 0 \quad ...(i)$$

$$a_2x + b_2y + c_2 = 0 \quad ...(ii)$$

To obtain the values of x and y, we follow these steps:

1. Multiply Equation (i) by b_2 and (ii) by b_1, to get

$$b_2a_1x + b_2b_1y + b_2c_1 = 0 \quad ...(iii)$$

$$b_1a_2x + b_1b_2y + b_1c_2 = 0 \quad ...(iv)$$

2. Subtracting Equation (iv) from (iii), we get:

$$(b_2a_1 - b_1a_2)x + (b_2b_1 - b_1b_2)y + (b_2c_1 - b_1c_2) = 0$$

i.e. $(b_2a_1 - b_1a_2)x = b_1c_2 - b_2c_1$

So, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, if $a_1b_2 - a_2b_1 \neq 0 \quad ...(v)$

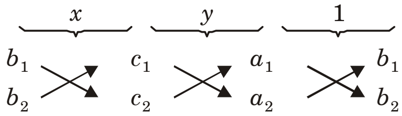
3. Substituting this value of x in (i) or (ii), we get

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad ...(vi)$$

We can write the solution given by equations (v) and (vi) in the following form:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad ...(vii)$$

In remembering the above result, the following diagram may be helpful :



...(viii)

For solving a pair of linear equations by this method, we will follow the following steps:

1. Write the given equations in the form (i) and (ii).
2. Taking the help of the diagram above, write equations as given in (viii).
3. Find x and y.

- ***For the equations like*** $a_1x+b_1y=c_1$ ***and*** $a_2x+b_2y=c_2$. ***The solution set can be calculated by using***

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{-1}{a_1b_2-a_2b_1}$$

(Please note position of c_1 ***and*** c_2 ***with equality sign and subsequent change in the third term i.e. –1)***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

2 Marks Question

1. Seema can row downstream 20km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

[TERM 1, 2017]

3 Marks Questions

2. Solve for x and y.

$$99x+101y=1499$$
$$101x+99y=1501$$

[TERM 1, 2011]

3. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other they meet in 1 hour. What are the speeds of the two cars?

[TERM 1, 2011]

4. The sum of digits of a two-digit number is 7. If the digits are reversed, the new number decreased by 2 equals twice the original number. Find the number.

[TERM 1, 2012]

5. Solve for x and y

$$\frac{5}{x-1}+\frac{1}{y-2}=2;\frac{6}{x-1}-\frac{3}{y-2}=1;\begin{bmatrix}x\neq 1\\ y\neq 2\end{bmatrix}$$

[TERM 1, 2013]

6. Solve the following pair of equations:

$$49x+51y=499 \text{ and } 51x+49y=501$$

[TERM 1, 2014]

7. Solve the equation, $\frac{4}{x}-3=\frac{5}{2x+3}; x\neq 0,\frac{-3}{2}$, for x.

[TERM 1, 2014]

8. Solve for x and y:

$$\frac{2}{x-1}-\frac{1}{y-1}=4$$
$$\frac{4}{x-1}-\frac{1}{y-1}=10$$

[TERM 1, 2015]

9. Solve for x:

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0,\ x\neq 3,-\frac{3}{2}$$

[TERM 1, 2016]

10. Solve by elimination

$$3x = y + 5 \text{ and } 5x - y = 11$$

[TERM 1, 2016]

4 Marks Questions

11. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}$. Find the fraction.

[TERM 1, 2012]

12. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

[TERM 1, 2012]

13. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

[TERM 1, 2014]

14. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

[TERM 1, 2015]

15. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of one chair and one table separately.

[TERM 1, 2015]

16. Raghav scored 70 marks in a test getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test?

Which value would Raghav violate if he resorts to unfair means?

[TERM 1, 2015]

17. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

[TERM 1, 2017]

18. The ratio of income of two persons is 9 : 7 and the ratio of their expenditure is 4 : 3 if each of them manages to save Rs. 2000/month. Find their monthly incomes.

[TERM 1, 2017]

19. The sum of the digits of two digits number is 9. Also 9 times the number is twice the number obtain by reversing the order of digits. Find the numbers.

[TERM 1, 2017]

20. Solve for x & y.

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4} \text{ and } \frac{1}{2(3x+y)}-\frac{1}{(3x-y)}=\frac{-1}{8}$$

[TERM 1, 2017]

Solutions

1. Assume the speed of current = y km/h

Speed of boat in still water = x km/h

So, Speed of boat in downstream = $x + y$ km/h

And, Speed of boat in upstream = $x - y$ km/h

According to the question,

$$\frac{20}{x+y}=2\left(\because \text{Time}=\frac{\text{Distance}}{\text{speed}}\right)$$

$\Rightarrow x+y=10$ .....(*i*)

Also, $\Rightarrow x+y=10$ .....(*i*)

$\Rightarrow x-y=2$ ......(*ii*)

Adding equations (i) and (ii) [1]

$\Rightarrow x+y+x-y=10+2$

$\Rightarrow 2x=12$

Divide the above equation by 2,

$\Rightarrow x=6$

Substituting value of x in Eqn. (i)

$\Rightarrow 6+y=10$

Subtract 6 from both sides of the equation,

$\Rightarrow y=4$

So, speed of rowing in still water is 6 km/hr and speed of current is 4 km/hr. [1]

2. Add the two given equations,

$$\begin{array}{l} 99x+101y=1499 \\ 101x+99y=1501 \\ \hline 200x+200y=3000 \end{array}$$ [1]

Divide by 200,

$x+y=15$ ....(i)

Subtract the two given equations,

$$\begin{array}{l} 99x+101y \quad =1499 \\ -(101x+99y)=-1501 \\ \hline -2x+2y \quad = \quad -2 \end{array}$$

Divide by – 2,

$x-y=1$

$\Rightarrow x=y+1$ [1]

Substitute the above value of x in (i)

$y+1+y=15$

$2y+1=15$

$\Rightarrow 2y=14$

Divide by 2,

$y=\frac{14}{2}=7$

Also, $x = y + 1$

$\Rightarrow x=7+1=8$ [1]

The speed of cureent = 7 km/hr

The speed of boat is still water = 8 km/hr

3. Assume the speeds of the cars to be x km/hr and y km/hr

When the cars are traveling in the same direction

Relative speed = $x - y$

Distance = 100 *km*

Apply $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$$\text{Time} = \frac{100}{x-y} = 5 \; hrs$$

$$\Rightarrow x - y = \frac{100}{5}$$

x – y = 20 ....(i) [1]

When the cars are going in the opposite direction

Relative speed = $x + y$

$$\text{Time} = \frac{100}{x+y} = 1$$

$$\Rightarrow x + y = 100 \quad ...(ii)$$ [1]

Add equations (i) and (ii),

$2x = 120$

Divide by 2,

$x = 60$

Substitute in (ii),

$$60 + y = 100$$
$$\Rightarrow y = 100 - 60$$
$$\Rightarrow y = 40$$

Hence the speeds of the cars are 60 km/hr and 40 km/hr [1]

4. Let the digit at tens place be x and digit at ones place be y, so number is given by

Given that the sum of digits of a two-digit number is 7. So,

$$x + y = 7 \quad ....(i)$$

Subtract x on both the sides of equation, we get;

$$y = 7 - x \quad ....(ii)$$ [1]

Also, if the digits are reversed, the new number decreased by 2 equals twice the original number. Therefore,

$$(10y + x) - 2 = 2(10x + y)$$

$$\Rightarrow 10y + x - 2 = 20x + 2y$$

Subtract $10y + x$ from both the sides of the equation. We get:

$$-2 = 19x - 8y$$

$$\Rightarrow 19x - 8y = -2 \quad ....(iii)$$ [1]

Put value of y from equation (ii) in equation (iii)

$$\Rightarrow 19x - 8(7 - x) = -2$$
$$\Rightarrow 19x - 56 + 8x = -2$$

Or, $27x - 56 = -2$

Add 56 on both the sides of equation;

$$\Rightarrow 27x = 54$$

Divide both sides by 27,

$x = 2$

Also, put value of x in equation (ii), we get;

$y = 7 - 2$

$\Rightarrow y = 5$

We know that the number is $10x + y$. Put the value of x and y in this equation, we get;

10 × (2) + 5 = 20 + 5 = 25 [1]

So, the number is 25.

5. We have,

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \quad ...(1)$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad ...(2)$$ [1]

Multiply equation 2 by 3, we get,

$$3\left(\frac{5}{x-1} + \frac{1}{y-2}\right) = 2 \times 3$$

$$\Rightarrow \frac{15}{x-1} + \frac{3}{y-2} = 6 \quad ...(3)$$

Adding equation (2) and equation (3), we get,

$$\begin{array}{r} \frac{6}{x-1} - \frac{3}{y-2} = 1 \\ + \frac{15}{x-1} + \frac{3}{y-2} = 6 \\ \hline \frac{6}{x-1} + \frac{15}{x-1} = 7 \end{array}$$

$$\Rightarrow \frac{21}{x-1} = 7$$ [1]

Solving for x, we get,

$$21 = 7(x-1)$$

Divide the above equation by 3,

$$\Rightarrow x - 1 = 3$$

Add 1 to both sides of above equation,

$\Rightarrow x = 4$

Putting $x = 4$ in equation 2, we get,

$$\frac{6}{4-1} - \frac{3}{y-2} = 1$$

$$\Rightarrow 2 - \frac{3}{y-2} = 1$$

Subtract 2 from both sides of above equation,

$\Rightarrow y = 5$

Hence, the solution is

$x = 4$ and $y = 5$ [1]

6. $49x + 51y = 499$ (1)

$51x + 49y = 501$ (2)

Multiplying eq (1) by 51 and eq(2) by 49

$\Rightarrow 49x \times 51 + 51y \times 51 = 499 \times 51$

$\Rightarrow 51 \times 49x + 51^2 y = 499 \times 51$ ....(3)

$\Rightarrow 51x \times 49 + 49y \times 49 = 501 \times 49$

$\Rightarrow 49 \times 51x + 49^2 y = 501 \times 49$ ....(4) [1]

Subtracting eq(4) from eq(3)

$\Rightarrow 51 \times 49x - 49 \times 51x + 51^2 y - 49^2 y = 499 \times 51 - 501 \times 49$

$\Rightarrow 51^2 y - 49^2 y = 25449 - 24549$

$\Rightarrow y(51^2 - 49^2) = 25449 - 24549$

$\Rightarrow y(51+49)(51-49) = 900$

(Using $a^2 - b^2 = (a+b)(a-b)$)

$\Rightarrow y(100)(2) = 900$

$\Rightarrow 200y = 900$

$\Rightarrow y = \frac{9}{2} = 4.5$ [1]

Now substituting the value of y in eq(1),

$\Rightarrow 49x + 51 \times 4.5 = 499$

$\Rightarrow 49x + 229.5 = 499$

$\Rightarrow 49x = 499 - 229.5$

$\Rightarrow 49x = 269.5$

$\Rightarrow x = \frac{269.5}{49} = 5.5$

Hence solution for the given equation is $x = 5.5$ & $y = 4.5$. [1]

7. $\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, \frac{-3}{2}$

Solving the given equation

$\frac{4-3x}{x} = \frac{5}{2x+3}$ [1]

$(4-3x)(2x+3) = 5x$

$-6x^2 + 8x - 9x + 12 = 5x$

$-6x^2 - 6x + 12 = 0$

Divide the above equation by –6, [1]

$x^2 + x - 2 = 0$

Splitting the middle term,

$x^2 + 2x - x - 2 = 0$

$(x+2)(x-1) = 0$

$(x+2) = 0$

or $(x-1) = 0$

$\Rightarrow x = -2$ or $x = 1$

Thus, the solution of the given equation is $x = -2$ or $x = 1$. [1]

8. Let $\frac{1}{x-1} = u$ and $\frac{1}{y-1} = v$

Then we have,

$2u - v = 4$ (i)

$4u - v = 10$ (ii)

Subtract (i) from (ii) we have,

$4u - v - (2u - v) = 10 - 4$

$\Rightarrow 4u - v - 2u + v = 6$

$\Rightarrow 2u = 6$

$\Rightarrow u = \frac{6}{2} = 3$ [1]

Substitute $u = 3$ in (i) we have,

$2(3) - v = 4$

$\Rightarrow 6 - v = 4$

$\Rightarrow 6 - 4 = v$

$\Rightarrow v = 2$ [1]

Back Substituting $\frac{1}{x-1} = u$ we get,

$\frac{1}{x-1} = 3$

$\Rightarrow 3(x-1) = 1$

$\Rightarrow 3x - 3 = 1$

$\Rightarrow 3x = 4$

$\Rightarrow x = \frac{4}{3}$

Also, $\frac{1}{y-1} = v$

$\Rightarrow \frac{1}{y-1} = 2$

$\Rightarrow 2(y-1) = 1$

$\Rightarrow 2y - 2 = 1$

$\Rightarrow 2y = 3$

$\Rightarrow y = \frac{3}{2}$

Hence, $x = \frac{4}{3}$ and $y = \frac{3}{2}$. [1]

9. $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \frac{2x(2x+3)+(x-3)+3x+9}{(x-3)(2x+3)} = 0$ [1]

$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$\Rightarrow 4x^2 + 10x + 6 = 0$

Solving by splitting the middle term,

$\Rightarrow 4x^2 + 4x + 6x + 6 = 0$

$\Rightarrow 4x(x+1) + 6(x+1) = 0$

$\Rightarrow (x+1)(4x+6) = 0$ [1]

$\Rightarrow x + 1 = 0$ or $4x + 6 = 0$

$\Rightarrow x = -1, \frac{-3}{2}$

As given in the question, $x \neq \frac{-3}{2}$

Hence, the solution of the given equation is $x = -1$. [1]

10. $3x = y + 5$

$\Rightarrow 3x - y = 5$ ...(i)

And

$5x - y = 11$ ...(ii) [1]

Subtracting (i) from (ii)

$$\begin{array}{r} 5x - y = 11 \\ 3x - y = 5 \\ - \quad + \quad - \\ \hline 2x = 6 \end{array}$$ [1]

$\Rightarrow x = 3$

Substituting in (ii)

$5(3) - y = 11$

$15 - y = 11$

$-y = 11 - 15$

$-y = -4$

$y = 4$

Thus, the solution is given by

$x = 3$ and $y = 4$ [1]

11. Let the denominator of the fraction be x. Then, the numerator of the fraction will be $x - 3$.

Thus, the fraction is $\frac{x-3}{x}$. [1]

When 1 is added to the denominator, the fraction gets decreased by $\frac{1}{15}$.

$\therefore \frac{x-3}{x} - \frac{1}{15} = \frac{x-3}{x+1}$

$\Rightarrow \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$ [1]

$\Rightarrow \frac{(x-3)[x+1-x]}{x(x+1)} = \frac{1}{15}$

$\Rightarrow \frac{x-3}{x^2+x} = \frac{1}{15}$

$\Rightarrow x^2 + x = 15x - 45$

$\Rightarrow x^2 - 14x + 45 = 0$

$\Rightarrow (x-9)(x-5) = 0$

$\Rightarrow x = 9$ or $x = 5$ [1]

When $x = 9$, the fraction will be

$$\frac{x-3}{x} = \frac{9-3}{9} = \frac{6}{9}$$

When $x = 5$, the fraction will be

$$\frac{x-3}{x} = \frac{5-3}{5} = \frac{2}{5}$$

The fractions are $\frac{6}{9}$ and $\frac{2}{5}$. [1]

12. Let us suppose that the average speed of the flight is x km/h.

Total distance covered by the flight in the journey = 2800 km

$\therefore$ The duration of the flight $= \frac{2800}{x}$ hr [1]

Now, it is given that, the average speed is reduced by 100 km/h and the duration of the flight is increased by 30 minutes, due to bad weather in the journey.

$\therefore$ New speed = $x - 100$ km/h

$\Rightarrow$ New duration of the flight $= \frac{2800}{x-100}$ hrs [1]

Now, according to the question,

New duration of the flight - Original duration of the flight = 30 minutes = $\frac{1}{2}$ hr

$$\Rightarrow \frac{2800}{x-100} - \frac{2800}{x} = \frac{1}{2}$$

$$\Rightarrow 2800\left[\frac{1}{x-100} - \frac{1}{x}\right] = \frac{1}{2}$$

$$\Rightarrow 2800\left[\frac{x - x + 100}{(x-100)x}\right] = \frac{1}{2}$$

$$\Rightarrow 2800\left[\frac{100}{x^2 - 100x}\right] = \frac{1}{2}$$

$$\Rightarrow \frac{100}{x^2 - 100x} = \frac{1}{5600}$$

$$\Rightarrow x^2 - 100x = 560000$$

$$\Rightarrow x^2 - 100x - 560000 = 0$$

$$\Rightarrow (x-800)(x+700) = 0$$ [1]

Equating each factor to zero,

$\Rightarrow x = 800$ or $x = -700$

Now. Since speed can't be negative, so we ignore $x = -700$

Therefore, the original average speed of the flight = 800 *km/h*

Thus, the original duration of the flight

$$= \frac{2800}{800} = 3.5 \text{ hours}$$

Hence, original duration of flight is 3.5 hours. [1]

13. Let the two natural numbers ne N and $N + 5$.

The difference of their reciprocals is $\frac{1}{10}$.

$$\Rightarrow \frac{1}{N} - \frac{1}{N+5} = \frac{1}{10}$$

$$\Rightarrow \frac{N+5-N}{N(N+5)} = \frac{1}{10}$$

$$\Rightarrow \frac{5}{N(N+5)} = \frac{1}{10}$$ [1]

$$\Rightarrow N(N+5) = 50$$

$$\Rightarrow N^2 + 5N - 50 = 0$$

Splitting the middle term,

$$\Rightarrow N^2 + 10N - 5N - 50 = 0$$

$$\Rightarrow N(N+10) - 5(N+10) = 0$$

$$\Rightarrow (N-5)(N+10) = 0$$ [1]

Applying zero product rule property,

$\Rightarrow (N-5) = 0$ Or $(N+10) = 0$

$\Rightarrow N = 5$ or $N = -10$ [1]

N cannot be negative since it is a natural number.

$\therefore\ N = 5$

Hence the numbers N and $N + 5$ are 5 and 10. [1]

14. Let x be the initial speed of the bus for a distance of 54 km.

Then $x + 6$ is the speed of bus for a distance of 63 km.

We know that, $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ [1]

The total time taken for the journey is 3 hours.

So, $\frac{54}{x} + \frac{63}{x+6} = 3$

$$\frac{54(x+6)+63x}{x(x+6)}=3$$

$\Rightarrow 54(x+6)+63x=3x(x+6)$ [1]

$\Rightarrow 54x+324+63x=3x^2+18x$

$\Rightarrow 3x^2-99x-324=0$

Divide the above equation by 3,

$\Rightarrow x^2-33x-108=0$ [1]

Splitting the middle term,

$\Rightarrow x^2-36x+3x-108=0$

$x(x-36)+3(x-36)=0$

$\Rightarrow (x+3)(x-36)=0$

$\Rightarrow x=36$ or $x=-3$

Speed cannot be in negative, hence ignore $x=-3$.

Hence, the first speed is 36 km/hr. [1]

15. Let cost of 1 chair be Rs x.

Let cost of 1 table be Rs y. [1]

Given:

4 chairs and 3 tables cost Rs 2100

$4x+3y=2100$ ....(i)

5 chairs and 2 tables cost Rs 1750

$5x+2y=1750$ ....(ii)

Solving equation (i) and (ii)

$4x+3y=2100$ ....(i)$\times 2$

$\Rightarrow 8x+6y=4200$ ...(iii)

$5x+2y=1750$ ....(ii)$\times 3$

$\Rightarrow 15x+6y=5250$ ....(iv) [1]

Now, (iii) – (iv)

$\Rightarrow (8x+6y=4200)-(15x+6y=5250)$

$\Rightarrow 8x+6y-15x-6y=4200-5250$

$\Rightarrow -7x=-1050$

Dividing both sides by 7

$$\Rightarrow \frac{7x}{7}=\frac{1050}{7}$$

$\Rightarrow x=150$

Put x = 150 in (ii),

$5x+2y=1750$ ....(ii)

$\Rightarrow 5\times150+2y=1750$

$\Rightarrow 750+2y=1750$ [1]

Subtracting both sides by 750

$\Rightarrow 750+2y-750=1750-750$

$\Rightarrow 2y=1000$

Dividing both sides by 2

$$\Rightarrow \frac{2y}{2}=\frac{1000}{2}$$

$\Rightarrow y=500$

Therefore, Cost of 1 chair Rs x = Rs 150 and cost of 1 table is Rs y = Rs 500 [1]

16. Let numbers of incorrect answered question = y

Let numbers of correct answered question = x

Given, [1]

Raghav scored 70 marks in a test getting 4 marks for each right answer and losing 1 mark for each wrong answer.

$4x-y=70$ ....(i)

Had 5 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks.

$5x-2y=80$ ....(ii)

Now multiply (i) by 2

$8x-2y=140$ ....(iii)

Subtract (i) from (iii)

$(8x-2y=140)-(5x-2y=80)$

$8x-5x-2y+2y=140-80$

$3x=60$ [1]

Divide both sides by 3

$$\frac{3x}{3}=\frac{60}{3}$$

$x=20$

Put value x = 20 in (i)

$4\times20-y=70$

80 – y = 70

80 – 70 = y

y = 10 [1]

Total number of questions in the test

= x + y = 20 + 10 = 30

The total number of questions in test are 30.

Raghav would violate the principle of honesty if he resorts to unfair means. [1]

17. Assume the speed of the boat to be 15 km/h

$\therefore$ Upstream speed of boat = Speed of boat in still water – Speed of the stream = (15 – x) km/h

Also, Downstream speed of boat = Speed of boat in still water + Speed of the stream = (15 + x) km/h

Apply $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$ [1]

Time for upstream $= \dfrac{30}{15-x}$

Time for downstream $= \dfrac{30}{15+x}$ [1]

4 hours 30 minutes $= 4\dfrac{1}{2}$ hours

Total time $= \dfrac{30}{15-x} + \dfrac{30}{15+x} = 4\dfrac{1}{2}$

$$\Rightarrow 30\left[\frac{15+x+15-x}{(15-x)(15+x)}\right] = \frac{9}{2}$$

Divide the above equation by 3,

$$\Rightarrow 10\left[\frac{30}{225-x^2}\right] = \frac{3}{2}$$ [1]

Divide the above equation by 3,

$$\Rightarrow 10\left[\frac{10}{225-x^2}\right] = \frac{1}{2}$$

$\Rightarrow 200 = 225 - x^2$

$\Rightarrow x^2 = 225 - 200 = 25$

Taking square root,

$\Rightarrow x = \pm 5$

The speed cannot be negative. So, the speed of stream is 5 km/h. [1]

18. Given ratio of income of two persons = 9 : 7

The ratio of their expenditure = 4 : 3

Let income of person A = 9x, income of person B = 7x

And expenditure of person $A = 4y$, expenditure of person $B = 3y$

We know that Income – Expenditure = Saving

According to the question,

$9x - 4y = 2000$ .....(i)

$7x - 3y = 2000$ .....(ii) [1]

Solve both the equations by elimination method

Multiply equation (i) by 3

$\Rightarrow 3(9x - 4y = 2000)$

$\Rightarrow 27x - 12y = 6000$ ....(iii) [1]

Multiply equation (ii) by 4

$\Rightarrow 4(7x - 3y = 2000)$

$\Rightarrow 28x - 12y = 8000$ .....(iv)

Subtracting Eqn. (iv) from (iii)

$$\begin{array}{rcrcl} 27x & - & 12y & = & 6000 \\ 28x & - & 12y & = & 8000 \\ (-) & & (+) & & (-) \\ \hline -x & & & = & -2000 \end{array}$$ [1]

$\Rightarrow x = 2000$

Substituting value of x in Eqn. (i)

$9(2000) - 4y = 2000$

$\Rightarrow 18000 - 4y = 2000$

$\Rightarrow 4y = 16000$

Divide above equation by 4,

$\Rightarrow y = 4000$

Now, Income of person A = 9×2000 = Rs.18,000

And Income of person B = 7×2000 = Rs.14,000

Hence, monthly income of two persons are Rs.18,000 and Rs. 14,000 [1]

19. Let the one's digit of the number = x

And the ten's digit of the number = y

Sum of the digits of two digits number is 9

$x + y = 9$

$\Rightarrow y = 9 - x$ .....(i) [1]

Now, the value of number will be = 10 (9 – x) + x

After reversing the digits, the number will be = 10 (x) + (9 – x)

According to the question,

$9\{10(9-x) + x\} = 2\{10(x) + (9-x)\}$

We will proceed with simplifying the equation,

$\Rightarrow 9(90 - 10x + x) = 2(10x + 9 - x)$

$\Rightarrow 9(90 - 9x) = 2(9x + 9)$ [1]

$\Rightarrow 810 - 81x = 18x + 18$

$\Rightarrow 810 - 18 = 18x + 81x$

$\Rightarrow 792 = 99x$

Divide the above equation by 99,

$\Rightarrow x = \frac{792}{99}$ [1]

$\Rightarrow x = 8$

Substituting value of x in Eqn. (i), we get,

$y = 9 - 8 = 1$

So the required number is 18. [1]

20. Let $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$

Now, the reduced equations are

$p + q = \frac{3}{4}$

$\Rightarrow 4p + 4q = 3$ .....(*i*)

Also,

$\frac{p}{2} - q = \frac{-1}{8}$ [1]

$\Rightarrow 4p - 8q = -1$ ....(*ii*)

We will use elimination method to solve these Eqn. (i) and (ii)

Subtracting Eqn. (ii) from (i)

$$\begin{array}{r} 4p + 4q = 3 \\ 4p - 8q = -1 \\ (-) \quad (+) \quad (+) \\ \hline 12q = 4 \end{array}$$

$\Rightarrow q = \frac{4}{12}$

$\Rightarrow q = \frac{1}{3}$ [1]

Substituting value of q in Eqn. (i)

$p + \frac{1}{3} = \frac{3}{4}$

$\Rightarrow p = \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

Now we need to find out the values of x and y,

As assumed, $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$

$\Rightarrow \frac{1}{3x+y} = \frac{5}{12}$ and $\frac{1}{3x-y} = \frac{1}{3}$

$\Rightarrow 3x + y = \frac{12}{5}$ .....(*iii*) and $3x - y = 3$ ......(*iv*)

Using elimination method, adding equation (iii) and (iv).

$$\begin{array}{l} 3x + y = \frac{12}{5} \\ 3x - y = 3 \\ \hline 6x \quad = \frac{27}{5} \end{array}$$

$\Rightarrow x = \frac{9}{10}$ [1]

Putting the value of x in Eqn. (iv), we get

$\Rightarrow 3\left(\frac{9}{10}\right) - y = 3$

$\Rightarrow \frac{27}{10} - y = 3$

$\Rightarrow y = \frac{27}{10} - 3$

$\Rightarrow y = -\frac{3}{10}$

So, the value of x is $\frac{9}{10}$ and value of y is $-\frac{3}{10}$.

[1]

Smart Notes ....

Smart Notes ....

CHAPTER 4

Quadratic Equations

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Relation between Zeroes and Coefficient of Quadratic Equation				2 marks	2 marks	
Finding Value of constant by putting the values of zeroes in Quadratic Equation	1 mark	1 mark				
Relationship between discriminant and nature of roots			2, 3 marks	3 marks		3 marks
Finding roots of Quadratic Equation			2 marks			

[TOPIC 1] Basic Concept of Quadratic Equations

Summary

Quadratic Equations

A polynomial of degree 2 (i.e. $ax^2 + bx + c$) is called a quadratic polynomial where $a \neq 0$ and a, b, c are real numbers.

Any equation of the form, $p(x) = 0$, where $p(x)$ is a polynomial of degree 2, is a quadratic equation. Therefore, $ax^2 + bx + c = 0$, $a \neq 0$ is called the standard form of a quadratic equation.

e.g. $2x^2 - 3x + 7$, $8x^2 + x - \sqrt{19}$

CLASSIFICATION OF A QUADRATIC EQUATION

It is classified into two categories:

(i) Pure quadratic equation (of the form $ax^2 + c = 0$ i.e., $b = 0$ in $ax^2 + bx + c = 0$)

e.g. $x^2 - 4 = 0$ and $3x^2 + 1 = 0$ are pure quadratic equations.

(ii) Affected quadratic equation (of the form $ax^2 + bx + c \neq 0$.

e.g. $x^2 - 2x - 8 = 0$ and $5x^2 + 3x - 2 = 0$ are affected quadratic equations.

ZEROS OF QUADRATIC POLYNOMIAL

For a quadratic polynomial $p(x) = ax^2 + bx + c$, those values of x for which $ax^2 + bx + c = 0$ is satisfied, are called zeros of quadratic polynomial $p(x)$, i.e. if $p(\alpha) = a\alpha^2 + b\alpha + c = 0$, then α is called the zero of quadratic polynomial.

ROOTS OF QUADRATIC EQUATION

If α, β are zeros of polynomial $ax^2 + bx + c$, then α, β are called roots (or solutions) of corresponding equation $ax^2 + bx + c = 0$ which implies that $p(\alpha) = p(\beta) = 0$.

i.e., $a\alpha^2 + b\alpha + c = 0$ and $a\beta^2 + b\beta + c = 0$.

Solution of a Quadratic Equation by Factorisation

Consider the following products.

$6 \times 0 = 0$; $-b \times 0 = 0$, $0 \times a = 0$

Above example illustrate that whenever the product is 0, at least one of the factors is 0.

- ***If a and b are numbers, then ab = 0, iff a = 0 or b = 0.***

Above principle is used in solving quadratic equation by factorisation. Let the given quadratic equation be $ax^2 + bx + c = 0$. Let the quadratic polynomial be expressed as product of two linear factors i.e. $(px + q)$ and $(rx + s)$, where p, q, r, s are real numbers and $p \neq 0$, $r \neq 0$,

Then, $ax^2 + bx + c = 0$

$\Rightarrow \quad (px + q)(rx + s) = 0$

$\therefore$ Either $(px + q) = 0$ or $(rx + s) = 0$

$px = -q$ or $rx = -s$

$x = -\dfrac{q}{p}$ or $x = -\dfrac{s}{r}$

Following steps are involved in solving a quadratic equation by factorisation.

- Transform the equation into standard form, if necessary.
- Factorise $ax^2 + bx + c$.
- Put each factor containing variable = 0.
- Solve each of the resulting equation

Solution of a Quadratic Equation by Completing the Square

Following steps are involved in solving a quadratic equation by quadratic formula

- Consider the equation $ax^2 + bx + c = 0$, where $a \neq 0$

- Dividing throughout by 'a', we get

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

- Add and subtract $\left(\frac{1}{2}\text{ coefficient of } x\right)^2$, we get

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0,$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

- If $b^2-4ac\geq 0$ taking square root of both sides, we obtain

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

Therefore $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

The Quadratic Formula: Quadratic equation, $ax^2+bx+c=0,$ where a, b, c are real number and $a\neq 0$, has the roots as

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The roots of the equation $x^2 + x - p(p + 1) = 0$ where p is a constant, are

(*a*) $p, p + 1$ (*b*) $-p, p + 1$

(*c*) $p, -(p + 1)$ (*d*) $-p, -(p + 1)$

[TERM 2, 2011]

2. The roots of the quadratic equation $2x^2-x-6=0$ are

(*a*) $-2, \frac{3}{2}$ (*b*) $2, -\frac{3}{2}$

(*c*) $-2, -\frac{3}{2}$ (*d*) $2, \frac{3}{2}$

[TERM 2, 2012]

3. Solve the quadratic equation $2x^2+ax-a^2=0$ for x.

[TERM 2, 2014]

4. If $x=-\frac{1}{2}$, is a solution of the quadratic equation $3x^2+2kx-3=0$, find the value of k.

[TERM 2, 2015]

2 Marks Questions

5. Form a quadratic polynomial whose zeroes are $\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$

[TERM 2, 2013]

6. Solve the following quadratic equation for :

$$4\sqrt{3}x^2+5x-2\sqrt{3}=0$$

[TERM 2, 2013]

7. Find the quadratic polynomial whose zeroes are $\sqrt{3}+\sqrt{5}$ and $\sqrt{5}-\sqrt{3}$.

[TERM 2, 2014]

8. Solve the following quadratic equation for x:

$$4x^2-4a^2x+\left(a^4-b^4\right)=0$$

[TERM 2, 2015]

9. Solve the following quadratic equations for x:

$$4x^2+4bx-\left(a^2-b^2\right)=0$$

[TERM 2, 2015]

10. If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b.

[TERM 2, 2016]

11. Find the roots of the quadratic equation $\sqrt{2}x^2+7x+5\sqrt{2}=0$

[TERM 2, 2017]

12. If the sum of two natural numbers is 8 and their product is 15, find the numbers.

[TERM 2, 2012]

13. Solve the following quadratic equations for x: $x^2 - 4ax - b^2 + 4a^2 = 0$

[TERM 2, 2012]

3 Marks Questions

14. Solve for x:

$$\frac{16}{x} - 1 = \frac{15}{x+1}; x \neq 0, -1$$

[TERM 2, 2014]

15. Solve for x:

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

[TERM 2, 2015]

16. Solve for x, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7.$

[TERM 2, 2017]

17. Solve for x, $x^2 - (2b-1)x + (b^2 - b - 20) = 0$

[TERM 2, 2017]

4 Marks Questions

18. A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream to the same spot. Find the speed of the stream.

[TERM 2, 2011]

19. Solve the following for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

[TERM 2, 2013]

20. Sum of the areas of two squares is 400cm^2. If the difference of their perimeters is 16, find the sides of the two squares.

[TERM 2, 2013]

21. Solve for x:

$$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}; x \neq 3, 5$$

[TERM 2, 2014]

22. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

[TERM 2, 2014]

23. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

[TERM 2, 2015]

24. Solve for x:

$$\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}, x \neq 0, -1, 2$$

[TERM 2, 2015]

25. If α and β are the zeros of the polynomial $f(x) = x^2 - 6x + k$, find the value of k for $\alpha^2 + \beta^2 = 40$.

[TERM 2, 2015]

26. Solve for x:

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}, x \neq -1, -2, -4$$

[TERM 2, 2016]

27. A motor boat whose speed is 24km/h in still water takes 1 hour more to 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

[TERM 2, 2016]

28. Solve for x:

$$\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$$

[TERM 2, 2017]

29. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

[TERM 2, 2017]

Solutions

1. We have, $x^2 + x - p(p+1) = 0$

$\Rightarrow x^2 + (p+1-p)x - p(p+1) = 0$

$\Rightarrow x^2 + (p+1)x - px - p(p+1) = 0$

$\Rightarrow x(x+p+1) - p(x+p+1) = 0$ [½]

$\Rightarrow (x+p+1)(x-p) = 0$

$\Rightarrow (x+p+1)=0$ or $(x-p)=0$

$\Rightarrow x=-(p+1)$ or $x=p$.

The roots of the given quadratic equation are p and $-(p+1)$.

Hence, the correct option is (c). [½]

2. Using factorization method of splitting the middle term, we can solve the quadratic equation as follows:

$2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow (2x^2-4x)+(3x-6)=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$ [½]

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0$ or $2x+3=0$

$\Rightarrow x=2$ or $x=-\frac{3}{2}$

Thus, $x=2$ and $x=-\frac{3}{2}$ are the two roots of the $2x^2-x-6=0$ equation. [½]

Option (b) is correct.

3. Given $2x^2+ax-a^2=0$

Comparing the given equation with the standard quadratic equation $(ax^2+bx+c=0)$,

we get $a=2$, $b=a$ and $c=-a^2$

Using the quadratic formula,

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get: [½]

$$x=\frac{-a\pm\sqrt{a^2-4\times 2(-a^2)}}{2\times 2}$$

$$=\frac{-a\pm\sqrt{9a^2}}{4}$$

$$x=\frac{-a+3a}{4}=\frac{a}{2} \text{ and } x=\frac{-a-3a}{4}=-a$$

So, the solutions of the given quadratic equation are

$x=\frac{a}{2}$ and $x=-a$ [½]

4. It is given that $x=-\frac{1}{2}$ is the solution of the quadratic equation $3x^2+2kx-3=0$.

It means it will satisfy the given equation.

Substitute $x=-\frac{1}{2}$ in $3x^2+2kx-3=0$ we get,

$3\left(-\frac{1}{2}\right)^2+2k\left(-\frac{1}{2}\right)-3=0$ [½]

$\Rightarrow \frac{3}{4}-k-3=0$

$\Rightarrow \frac{3-4k-12}{4}=0$

$\Rightarrow -9-4k=0$

$\Rightarrow -4k=9$

$\Rightarrow k=-\frac{9}{4}$

Hence, the value of k is $-\frac{9}{4}$. [½]

5. $\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$ are the zeroes of the quadratic polynomial.

Let $\alpha=\frac{3-\sqrt{3}}{5}$ and $\beta=\frac{3+\sqrt{3}}{5}$ [½]

Sum of zeroes $=\alpha+\beta$

$$\Rightarrow \frac{3-\sqrt{3}}{5}+\frac{3+\sqrt{3}}{5}=\frac{3-\sqrt{3}+3+\sqrt{3}}{5}=\frac{6}{5}$$

Product of zeroes $=\alpha\beta$

$\Rightarrow \left(\frac{3-\sqrt{3}}{5}\right)\left(\frac{3+\sqrt{3}}{5}\right)=\frac{9-3}{25}=\frac{6}{25}$ [½]

The quadratic equation whose zeroes are α and β are:

$\Rightarrow x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-\left(\frac{6}{5}\right)x+\left(\frac{6}{25}\right)=0$

Multiply both sides by 25

$25x^2-30x+6=0$

Therefore, the required polynomial is

$25x^2-30x+6=0$ [1]

6. $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

$4\sqrt{3}x^2+8x-3x-2\sqrt{3}=0$

Factoring out the common terms,

$4x\left(\sqrt{3}x+2\right)-\sqrt{3}\left(\sqrt{3}x+2\right)=0$

$\left(4x-\sqrt{3}\right)\left(\sqrt{3}x+2\right)=0$ [1]

Equating both the factors to

$4x-\sqrt{3}=0$

$\Rightarrow x=\frac{\sqrt{3}}{4}$

And $\sqrt{3}x+2=0$

$\Rightarrow x=\frac{-2}{\sqrt{3}}$ [1]

7. Quadratic polynomial in terms of x, where coefficient of x is sum of zeros and constant term will be product of zeros.

$\Rightarrow x^2-\left(\sqrt{3}+\sqrt{5}+\sqrt{5}-\sqrt{3}\right)x+\left(\sqrt{3}+\sqrt{5}\right)\left(\sqrt{5}-\sqrt{3}\right)$ [1]

$\Rightarrow x^2-\left(2\sqrt{5}\right)x+(5-3)$

$\Rightarrow x^2-2\sqrt{5}x+2$

Hence the required quadratic polynomial is

$\Rightarrow x^2-2\sqrt{5}x+2$ [1]

8. $4x^2-4a^2x+\left(a^4-b^4\right)=0$

This equation is of the form

$ax^2+bx+c=0$

Here $a=4, b=-4a^2, c=\left(a^4-b^4\right)$

The quadratic formula to solve for x is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{4a^2\pm\sqrt{\left(-4a^2\right)^2-4(4)\left(a^4-b^4\right)}}{2(4)}$$

$$\Rightarrow x=\frac{4a^2\pm\sqrt{16a^4-16a^4+16b^4}}{8}$$ [1]

$$\Rightarrow x=\frac{4a^2\pm\sqrt{16b^4}}{8}$$

$$\Rightarrow x=\frac{4a^2\pm 4b^2}{8}$$

Dividing the numerator and denominator by 4 we get,

$$x=\frac{a^2\pm b^2}{2}$$

$$\Rightarrow x=\frac{a^2+b^2}{2} \text{ or } x=\frac{a^2-b^2}{2}$$

Hence, the values of x are $\frac{a^2+b^2}{2}$ and $\frac{a^2-b^2}{2}$.

[1]

9. The given quadratic equation is

$4x^2+4bx-\left(a^2-b^2\right)=0$

So, $a=4, b=4b, c=-\left(a^2-b^2\right)$

Quadratic formula to find the roots is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4b\pm\sqrt{(4b)^2-4\times4\times\left\{-\left(a^2-b^2\right)\right\}}}{2\times4}$$

$$x=\frac{-4b\pm\sqrt{16b^2+16\left(a^2-b^2\right)}}{2\times4}$$ [1]

$$x=\frac{-4b\pm\sqrt{16b^2+16a^2-16b^2}}{8}$$

$$x=\frac{-4b\pm\sqrt{16a^2}}{8}$$

$$x=\frac{-4b\pm4a}{8}$$

$$x=\frac{-b\pm a}{2}$$

$$x=\frac{-b+a}{2},\frac{-b-a}{2}$$ [1]

10. The roots of the quadratic equation $ax^2 + 7x + b = 0$ are -3 and $\frac{2}{3}$.

The sum of roots $= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

$\Rightarrow -3 + \frac{2}{3} = \frac{-(7)}{a}$

$\Rightarrow \frac{-9+2}{3} = \frac{-7}{a}$ [1]

$\Rightarrow \frac{-7}{3} = \frac{-7}{a}$

$\Rightarrow a = 3$ ...(i)

Also, we know that the product of roots

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow -3 \times \frac{2}{3} = \frac{b}{a}$

$\Rightarrow -2 = \frac{b}{3}$ (Using (i))

$\Rightarrow b = -6$ [1]

Hence, the values of a and b are 3 and –6 respectively.

11. Factorizing the equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$, we'll get

$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$ [1]

Either $(\sqrt{2}x + 5) = 0$ or $(x + \sqrt{2}) = 0$

$\therefore x = -\frac{5}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}$ or $x = -\sqrt{2}$

Hence, the roots of the given quadratic equation are $-\frac{5\sqrt{2}}{2}$ and $-\sqrt{2}$. [1]

12. Let one natural number be x. Therefore, another natural number will be $8 - x$.

Now, It is given that the product of these two natural numbers is 15.

Thus,

$\Rightarrow 8x - x^2 = 15$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 3x - 5x + 15 = 0$

$\Rightarrow x(x-3) - 5(x-3) = 0$ [1]

$\Rightarrow (x-3)(x-5) = 0$

$\Rightarrow x - 3 = 0$ or $x - 5 = 0$

$\Rightarrow x = 3$ or $x = 5$

Therefore, if first natural number is 3, the second number will be 5. And, if first natural number is 5, the second number will be 3. [1]

13. We have, $x^2 - 4ax - b^2 + 4a^2 = 0$

$\Rightarrow (x^2 - 4ax + 4a^2) - b^2 = 0$

$\Rightarrow (x - 2a)^2 - b^2 = 0$ [1]

$\Rightarrow (x - 2a + b) = 0$ or $(x - 2a - b) = 0$

$\Rightarrow (x - 2a + b)(x - 2a - b) = 0$

$\Rightarrow x = 2a - b$ or $x = 2a + b$

Thus, $2a - b$ and $2a + b$ are the two roots of the equation $x^2 - 4ax - b^2 + 4a^2 = 0$. [1]

14. Consider the equation:

$\frac{16}{x} - 1 = \frac{15}{x+1}$

$\Rightarrow \frac{16}{x} - \frac{15}{x+1} = 1$ [1]

$\frac{16(x+1) - 15x}{x(x+1)} = 1$

$\Rightarrow 16x + 16 - 15x = x(x+1)$

$\Rightarrow x + 16 = x^2 + x$ [1]

$\Rightarrow x^2 = 16$

Taking square root,

$x = \pm 4$

Therefore the solutions are $x = \pm 4$. [1]

15. The given quadratic equation is

$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

So,

$a = \sqrt{3}, b = -2\sqrt{2}, c = -2\sqrt{3}$ [1]

The quadratic formula to find the root is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x=\frac{-\left(-2\sqrt{2}\right)\pm\sqrt{\left(-2\sqrt{2}\right)^2-4\times\sqrt{3}\times\left(-2\sqrt{3}\right)}}{2\times\sqrt{3}}$$

$$x=\frac{2\sqrt{2}\pm\sqrt{8+24}}{2\sqrt{3}}$$ [1]

$$x=\frac{2\sqrt{2}\pm\sqrt{32}}{2\sqrt{3}}$$

$$x=\frac{2\sqrt{2}\pm4\sqrt{2}}{2\sqrt{3}}$$

$$x=\frac{2\sqrt{2}+4\sqrt{2}}{2\sqrt{3}},\frac{2\sqrt{2}-4\sqrt{2}}{2\sqrt{3}}$$

$$x=\frac{6\sqrt{2}}{2\sqrt{3}},\frac{-2\sqrt{2}}{2\sqrt{3}}$$

$$x=\frac{3\sqrt{2}}{\sqrt{3}},\frac{-\sqrt{2}}{\sqrt{3}}$$ [1]

16. We will first simplify the given equation,

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$$

We will proceed with taking the LCM of LHS,

$$\Rightarrow\frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$$ [1]

$$\Rightarrow\frac{x-7-x-4}{(x+4)(x-7)}=\frac{11}{30}$$

$$\Rightarrow\frac{-11}{(x+4)(x-7)}=\frac{11}{30}$$

After cross multiplication of both the fraction, we have,

$$\Rightarrow -11\times30=11(x+4)(x-7)$$

$$\Rightarrow -30=x(x-7)+4(x-7)$$

$$\Rightarrow -30=x^2-7x+4x-28$$

$$\Rightarrow -30=x^2-3x-28$$

$$\Rightarrow -30-x^2+3x+28=0$$ [1]

After simplification, we have a quadratic equation

$$\Rightarrow x^2-3x+2=0$$

Factorising the equation,

$$\Rightarrow x^2-2x-x+2=0$$

$$\Rightarrow x(x-2)-1(x-2)=0$$

$$\Rightarrow (x-1)(x-2)=0$$

$$\Rightarrow x=1,\ 2$$ [1]

17. Here, the given equation is

$$x^2-(2b-1)x+(b^2-b-20)=0$$

Finding the discriminant

$$D=b^2-4ac$$

$$D=\left(-(2b-1)\right)^2-4\times1\times\left(b^2-b-20\right)$$

$$D=4b^2+1-4b-4\left(b^2-b-20\right)$$ [1]

$$\left[\text{using }(a-b)^2=a^2+b^2-2ab\right]$$

$$D=4b^2+1-4b-4b^2+4b+80$$

$$D=81$$ [1]

Now,

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{-(-(2b-1)\pm\sqrt{81}}{2}$$

$$x=\frac{2b-1\pm9}{2}$$

$$x=\frac{2b-1+9}{2},\frac{2b-1-9}{2}$$

$$x=\frac{2b+8}{2},\frac{2b-10}{2}$$

$$x=b+4,\ b-5$$ [1]

18. Let the speed of the stream be x km/h,

Let the distance of the spot be d = 48km,

Speed of boat in upstream will be km/h

Speed of boat in downstream will be $(20-x)$ km/h

[1]

Difference in time taken by the boat in upstream and downstream is 1 hr

$$\therefore\frac{d}{20-x}-\frac{d}{20+x}=1$$

$$\Rightarrow\frac{48}{20-x}-\frac{48}{20+x}=1$$

$$\Rightarrow\frac{48(20+x-20+x)}{(20-x)(20+x)}=1$$ [1]

$$\Rightarrow 96x=400-x^2$$

$$\Rightarrow x^2+96x-400=0$$

$$\Rightarrow x^2+100x-4x-400=0$$ [1]

$\Rightarrow x(x+100)-4(x+100)=0$

$\Rightarrow (x+100)(x-4)=0$

$x=4$ *or* -100

Speed of stream cannot be negative

Hence speed of the stream is 4km/h [1]

19. $\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

$\frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$ [1]

$\Rightarrow \frac{2x-2a-b-2x}{(2x)(2a+b+2x)}=\frac{(2a+b)}{2ab}$

$\Rightarrow \frac{-(2a+b)}{(2x)(2a+b+2x)}=\frac{(2a+b)}{2ab}$ [1]

$\Rightarrow -2ab=(2x)(2a+b+2x)$

$\Rightarrow -2ab=4ax+2bx+4x^2$

$\Rightarrow 4x^2+2bx+4ax+2ab=0$

$\Rightarrow 2x(2x+b)+2a(2x+b)=0$ [1]

$\Rightarrow (2x+2a)(2x+b)=0$

$2x+2a=0 \Rightarrow x=-a$

And $2x+b=0 \Rightarrow x=\frac{-b}{2}$

Hence, $x=-a, \frac{-b}{2}$ [1]

20. Let us assume the side of first square to be x cm and for the second one be y cm.

Hence, the area of square 1 = x^2 and, the area of square 2 = y^2

Similarly, the perimeter of square 1 = $4x$ and, the perimeter of square 2 = $4y$ [1]

According to the question,

$x^2+y^2=400$ (1)

and, $4x-4y=16$

Dividing both sides by 4

$x-y=4$

$\Rightarrow x=y+4$ (2)

Substituting the value of x in equation (1),

$(y+4)^2+y^2=400$

$\Rightarrow y^2+16+8y+y^2=400$

$\Rightarrow 2y^2+8y-384=0$ [1]

$\Rightarrow y^2+4y-192=0$

Splitting up the middle term,

$\Rightarrow y^2+16y-12y-192=0$

$\Rightarrow y(y+16)-12(y+16)=0$

$\Rightarrow (y-12)(y+16)=0$

$\Rightarrow y=-16$ or $y=12$ [1]

As we know that y cannot be negative, hence, $y=12$.

Now substituting the value of y in (2),

$x=y+4=12+4=16$

Hence, side of square 1 = 16 cm and, side of square 2 = 12 cm [1]

21. Consider the equation:

$\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}$

$\Rightarrow \frac{(x-2)(x-5)+(x-4)(x-3)}{(x-3)(x-5)}=\frac{10}{3}$

$\Rightarrow \frac{x^2-5x-2x+10+x^2-3x-4x+12}{x^2-3x-5x+15}=\frac{10}{3}$ [1]

$\Rightarrow \frac{2x^2-14x+22}{x^2-8x+15}=\frac{10}{3}$

$\Rightarrow 3(2x^2-14x+22)=10(x^2-8x+15)$

$\Rightarrow 6x^2-42x+66=10x^2-80x+150$ [1]

$\Rightarrow 10x^2+150-6x^2-66-80x+42x=0$

$4x^2-38x+84=0$

$2(2x^2-19x+42)=0$

$2x^2-19x+42=0$ [1]

Splitting the middle term,

$2x^2-12x-7x+42=0$

$2x(x-6)-7(x-6)=0$

$(x-6)(2x-7)=0$

$x-6=0$ or $2x-7=0$

$x=6$ or $x=\frac{7}{2}$ [1]

22. Let us suppose that the speed of the stream is x km/h.

Speed of motorboat in still water is 18 km/h

Speed of motor boat upstream will be $(18-x)$ km/h

Speed of motor boat downstream will be $(18+x)$ km/h [1]

Distance travelled by the motorboat on one side is

$$\frac{24}{18-x}-\frac{24}{18+x}=1$$

$$24\left[\frac{1}{18-x}-\frac{1}{18+x}\right]=1$$ [1]

$$24\left[\frac{(18+x)-(18-x)}{(18-x)(18+x)}\right]=1$$

$$\frac{18+x-18+x}{(18-x)(18+x)}=\frac{1}{24}$$

$$\frac{2x}{(18-x)(18+x)}=\frac{1}{24}$$

$$48x=(18-x)(18+x)$$

$$48x=324+18x-18x-x^2$$ [1]

$$x^2+48x-324=0$$

$$x^2+54x-6x-324=0$$

$$x(x+54)-6(x+54)=0$$

$$(x+54)(x-6)=0$$

$$x+54=0 \quad \text{or} \quad x-6=0$$

$$x=-54 \quad \text{or} \quad x=6$$

Speed cannot be negative so –54 will not be considered.

Thus the speed of the stream is 6 km/h. [1]

23. Let the denominator be x.

Now since numerator of the fraction is 3 less than its denominator,

Therefore numerator is $x-3$.

And if 2 is added to both numerator and denominator,

New fraction is

$$\frac{x-3+2}{x+2}=\frac{x-1}{x+2}$$ [1]

Sum of original and new fraction is $\frac{29}{20}$,

Hence, $\frac{x-3}{x}+\frac{x-1}{x+2}=\frac{29}{20}$.

$$\Rightarrow \frac{(x-3)(x+2)+x(x-1)}{x(x+2)}=\frac{29}{20}$$

$$\Rightarrow \frac{x^2-3x+2x-6+x^2-x}{x(x+2)}=\frac{29}{20}$$

$$\Rightarrow \frac{2x^2-2x-6}{x(x+2)}=\frac{29}{20}$$ [1]

$$\Rightarrow 20(2x^2-2x-6)=29x(x+2)$$

$$\Rightarrow 40x^2-40x-120=29x^2+58x$$

$$\Rightarrow 40x^2-29x^2-40x-58x-120=0$$

$$\Rightarrow 11x^2-98x-120=0$$ [1]

Now solving this quadratic equation by splitting the middle term,

$$\Rightarrow 11x^2-110x+12x-120=0$$

$$\Rightarrow 11x(x-10)+12(x-10)=0$$

$$\Rightarrow (11x+12)(x-10)=0$$

Hence, x = 10 or $x=\frac{-12}{11}$

Since x can only be a whole number, hence denominator is 10.

And the numerator is $(x-3)=7$.

Hence, the original fraction is $\frac{7}{10}$. [1]

24. Solving for x,

$$\Rightarrow \frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}$$

$$\Rightarrow \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)}=\frac{23}{5x}$$ [1]

$$\Rightarrow \frac{4x-8+3x+3}{2(x^2+x-2x-2)}=\frac{23}{5x}$$

$$\Rightarrow \frac{7x-5}{2(x^2-x-2)}=\frac{23}{5x}$$ [1]

Cross multiplying both sides,

$\Rightarrow 5x(7x-5) = 46\left(x^2 - x - 2\right)$

$\Rightarrow 35x^2 - 25x = 46x^2 - 46x - 92$

Re-arranging the terms,

$\Rightarrow 11x^2 - 21x - 92 = 0$ [1]

Solving the above quadratic equation by splitting the middle term,

$\Rightarrow 11x^2 - 44x + 23x - 92 = 0$

$\Rightarrow 11x(x-4) + 23(x-4) = 0$

$\Rightarrow (11x+23)(x-4) = 0$

Hence $x = 4$ and $x = -\dfrac{23}{11}$ [1]

25. Given $f(x) = x^2 - 6x + k$

$\therefore\ a = 1, b = -6, c = k$

And $\alpha^2 + \beta^2 = 40$ [1]

Sum of roots

$(\alpha + \beta) = \left(\dfrac{-b}{a}\right) = \dfrac{-(-6)}{1} = 6$

$(\alpha \times \beta) = \dfrac{c}{a}$

$= \dfrac{k}{1} = k$ [1]

We have ,

$(\alpha + \beta)^2 = \left(\alpha^2 + \beta^2\right) + 2\alpha\beta$

$\Rightarrow (6)^2 = (40) + 2k$ [1]

$\Rightarrow 36 = (40) + 2k$

$\Rightarrow 36 - 40 = 2k$

$\Rightarrow 2k = -4$

$\Rightarrow k = -2$ [1]

26. L.C.M of all the denominators is .

Multiply throughout by the L.C.M we get,

$(x+1)(x+2)(x+4)$

$(x+1)(x+2)(x+4)\left(\dfrac{1}{x+1} + \dfrac{2}{x+2}\right)$ [1]

$= (x+1)(x+2)(x+4)\left(\dfrac{4}{x+4}\right)$

$(x+2)(x+4) + 2(x+1)(x+4)$

$= 4(x+1)(x+2)$

$(x+4)(x+2+2x+2)$ [1]

$= 4\left(x^2 + 2x + x + 2\right)$

$(x+4)(3x+4) = 4x^2 + 12x + 8$

$3x^2 + 4x + 12x + 16 = 4x^2 + 12x + 8$

$x^2 - 4x - 8 = 0$ [1]

Now,

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{16 + 32}}{2}$

$= \dfrac{4 \pm \sqrt{48}}{2}$

$x = \dfrac{4 \pm 4\sqrt{3}}{2}$

Or, $x = 2 \pm 2\sqrt{3}$ [1]

27. Let speed of stream $= x$ km/h

Speed of boat in still water $= 24$ km/h

Speed of boat downstream $= (24 + x)$ km/h

Speed of boat upstream $= (24 - x)$ km/h [1]

Distance $= 32$ km

From Question,

$\dfrac{32}{24 - x} - \dfrac{32}{24 + x} = 1$

$\Rightarrow \dfrac{32[24 + x - (24 - x)]}{(24 - x)(24 + x)} = 1$ [1]

$\Rightarrow \dfrac{32[2x]}{576 - x^2} = 1$

$\Rightarrow \dfrac{64x}{576 - x^2} = 1$

$\Rightarrow 64x = 576 - x^2$

$\Rightarrow x^2 + 64x - 576 = 0$

Splitting the middle term,

$\Rightarrow x^2 + 72x - 8x - 576 = 0$

$\Rightarrow x(x+72) - 8(x+72) = 0$ [1]

$\Rightarrow (x-8)(x+72) = 0$

$\Rightarrow (x-8) = 0$ or $(x+72) = 0$

$\Rightarrow x = 8$ or $x = -72$

Speed of stream cannot be negative.

Therefore, Speed of stream is $8\,\text{km/h}$ [1]

28. $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$

$\Rightarrow \frac{5x+1+3x+3}{(x+1)(5x+1)} = \frac{5}{x+4}$

$\Rightarrow (x+4)(8x+4) = 5(x+1)(5x+1)$ [1]

$\Rightarrow (x+4)(8x+4) = 5(x+1)(5x+1)$

$\Rightarrow 8x^2 + 32x + 4x + 16 = 5(5x^2 + x + 5x + 1)$

$\Rightarrow 8x^2 + 36x + 16 = 25x^2 + 30x + 5$ [1]

$\Rightarrow 17x^2 - 6x - 11 = 0$

$\Rightarrow 17x^2 - 17x + 11x - 11 = 0$

$\Rightarrow 17x(x-1) + 11(x-1) = 0$ [1]

$\Rightarrow (17x+11)(x-1) = 0$

$\Rightarrow 17x + 11 = 0$ *or* $x - 1 = 0$

$\Rightarrow x = -\frac{11}{17}$ *or* $x = 1$ [1]

29. Let in x one pipe fills the cistern.

Other pipe will fill in $(x+3)$ hours

Time taken by both pipes, running together

$= 3\frac{1}{13}$ hours $= \frac{40}{13}$ Hours x [1]

Cistern filled by one pipe in 1 hour $= \frac{1}{x}$

Cistern filled by other pipe in 1 hour $= \frac{1}{x+3}$ [1]

Cistern filled by both pipes, running together for 1 hour $= \frac{1}{x} + \frac{1}{x+3}$

$\therefore \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow \frac{2x+3}{x^2+3x} = \frac{13}{40}$

$\Rightarrow 13x^2 + 39x = 80x + 120$ [1]

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0$

$\Rightarrow (x-5)(13x+24) = 0$

$\Rightarrow x - 5 = 0$ *or* $13x + 24 = 0$

$\Rightarrow x = 5$ *or* $x = -\frac{24}{13}$

As time cannot be negative.

Time taken by one pipe to fill cistern = 5 hours

Time taken by other pipe to fill cistern

$= 5 + 3 = 8$ hours [1]

[TOPIC 2] Roots of a Quadratic Equation

Summary

Nature of Roots

In previous section, we have studied that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A quadratic equation $ax^2 + bx + c = 0$ has

- **Two distinct real roots if $b^2 - 4ac > 0$.**
  If $b^2 - 4ac > 0$, we get two distinct real roots $-\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$ and $-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$
- **Two equal roots, if $b^2 - 4ac = 0$.**
  If $b^2 - 4ac > 0$, then $x = -\frac{b \pm 0}{2a}$
  i.e. $x = -\frac{b}{2a}$
  So, the roots are both $-\frac{b}{2a}$
- **No real roots, if $b^2 - 4ac < 0$**
  If $b^2 - 4ac < 0$, then there is no real number whose square is $b^2 - 4ac$.
- ***(b^2 − 4ac) determines whether the quadratic equation ax^2 + bx + c = 0 has real roots or not, hence (b^2 − 4ac) is called the discriminant of quadratic equation. It is denoted by D.***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Question

1. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots,
   Then find the value of p.
   [TERM 2, 2015]

2 Marks Questions

2. Find the value of p so that the quadratic equation $px(x - 3) + 9 = 0$ has two equal roots.
   [TERM 2, 2017]
3. Find the value of p for which the roots of the equation $px(x - 2) + 6 = 0$, are equal.
   [TERM 2, 2012]
4. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.
   [TERM 2, 2014]
5. If −5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.
   [TERM 2, 2016]
6. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.
   [TERM 2, 2017]

3 Marks Questions

7. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.
   [TERM 2, 2015]
8. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.
   [TERM 2, 2017]
9. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.
   [TERM 2, 2017]
10. Is it possible to design a rectangular park of perimeter 80m and area 400m^2? If so find its length and breadth.
    [TERM 2, 2017]

4 Marks Questions

11. Find the values of k for which the quadratic equation $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots. Also find these roots.
    [TERM 2, 2014]

Solutions

1. The given quadratic equation is

$px^2 - 2\sqrt{5}\,px + 15 = 0$

So, $a = p, b = -2\sqrt{5}\,p, c = 15$

When the roots of the quadratic equation are equal, then its discriminant will be zero.

$D = b^2 - 4ac = 0$ [½]

$\Rightarrow \left(-2\sqrt{5}\,p\right)^2 - 4 \times p \times 15 = 0$

$\Rightarrow 20p^2 - 60p = 0$

$\Rightarrow 20p(p-3) = 0$

$\Rightarrow 20p = 0$ or $(p-3) = 0$

$\Rightarrow p = 0$ or $p = 3$

$px^2 - 2\sqrt{5}\,px + 15 = 0$ is given a quadratic equation, so, p cannot be equal to 0.

Hence, $p = 3$ [½]

2. We have, $px(x-3) + 9 = 0$

$\Rightarrow px^2 - 3px + 9 = 0$

For two equal roots, $D = 0$

$\Rightarrow b^2 - 4ac = 0$ [1]

$\Rightarrow (-3p)^2 - 4p(9) = 0$

$\Rightarrow 9p^2 - 36p = 0$

$\Rightarrow 9p(\text{p}-4) = 0$

$\Rightarrow 9p = 0$ *and* $(\text{p}-4) = 0$

$\Rightarrow p = 0$ and $p = 4$

Here, value of p cannot be 0.

Therefore, the value of p is 4. [1]

3. The given equation is

$px(x-2) + 6 = 0$

$\Rightarrow px^2 - 2px + 6 = 0$

Here, $a = p, b = -2p$ and $c = 6$. [½]

$\therefore D = b^2 - 4ac$

$= (-2p)^2 - 4 \times p \times 6$

$= 4p^2 - 24p$ [½]

The given equation will have equal roots, if $D = 0$

$\Rightarrow 4p^2 - 24p = 0$

$\Rightarrow 4p(p-6) = 0$

$\Rightarrow p = 0$ *or* $p = 6$

Here, $p = 0$ is not possible because putting $p = 0$ in the equation will eliminate the term containing x^2.

Thus, the value of p is 6. [1]

4. Consider the equation:

$4x^2 + px + 3 = 0$

For roots to be equal, discriminant is equal to zero.

$b^2 - 4ac = 0$

Here $a = 4,\ b = p,\ c = 3$ [1]

Substitute the values in the equation

$\Rightarrow p^2 - 4(4)(3) = 0$

$\Rightarrow p^2 - 48 = 0$

$\Rightarrow p^2 = 48$

$\Rightarrow p = \sqrt{48} = \mathbf{4\sqrt{3}}$ [1]

5. –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ so it will satisfy the equation.

$2 \times (-5)^2 + p \times (-5) - 15 = 0$

$50 - 5p - 15 = 0$

$35 - 5p = 0$

$p = 7$

By putting the value of p in the equation $p\left(x^2 + x\right) + k = 0$, we get

$7x^2 + 7x + k = 0$ [1]

This equation has equal roots so its discriminant will be zero

$D = b^2 - 4ac = 0$

Here, $a = 7, b = 7$ and $c = k$

$7^2 - 4 \times 7 \times k = 0$

$49 - 28k = 0$

$49 = 28k$

$k = \frac{49}{28} = \frac{7}{4}$

The value of k is $\frac{7}{4}$ [1]

6. Simplifying the given equation:

$$x^2 + k(2x + k - 1) + 2 = 0$$

$$x^2 + 2xk + k^2 - k + 2 = 0$$

$$x^2 + 2kx + (k^2 - k + 2) = 0$$

$$a = 1; b = 2k; c = (k^2 - k + 2)$$ [1]

A quadratic equation has real and equal roots if: $D = 0$ i.e. discriminant is zero]

Applying the above condition in given equation:

$$D = b^2 - 4ac$$

$$(2k)^2 - 4 \times 1 \times (k^2 - k + 2) = 0$$

$$4k^2 - 4k^2 + 4k - 8 = 0$$

$$4k - 8 = 0$$

$$\Rightarrow 4k = 8$$

$$\Rightarrow k = \frac{8}{4} = 2$$

So value of $k = 2$ [1]

7. Given the quadratic equation

$$kx^2 + 1 - 2(k-1)x + x^2 = 0 \ .$$

It can be written as:

$$(k+1)x^2 - 2(k-1)x + 1 = 0$$

This equation is of the form

$$ax^2 + bx + c = 0$$

Discriminant is given by the formula,

$$D = b^2 - 4ac$$

$$D = [-2(k-1)]^2 - 4(k+1)(1)$$

$$= 4(k^2 + 1 - 2k) - 4k - 4$$

$$= 4k^2 - 12k$$

$$= 4k(k-3)$$ [1]

For this quadratic equation to have zero roots the discriminant is zero.

$$\Rightarrow 4k(k-3) = 0$$

$$\Rightarrow 4k = 0 \text{ or } k - 3 = 0$$

$$\Rightarrow k = 0 \text{ or } k = 3$$

But it is given that k is non-zero.

$$\Rightarrow k = 3$$

To find the roots of the equation substitute $k = 3$ in the given quadratic equation we get,

$$\Rightarrow (3+1)x^2 - 2(3-1)x + 1 = 0$$

$$\Rightarrow 4x^2 - 4x + 1 = 0$$ [1]

On splitting the middle term we get,

$$4x^2 - 2x - 2x + 1 = 0$$

$$\Rightarrow 2x(2x-1) - 1(2x-1) = 0$$

$$\Rightarrow (2x-1)(2x-1) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ and } x = \frac{1}{2}$$

The value of k is 3 and the roots of the equation are $\frac{1}{2}$ and $\frac{1}{2}$. [1]

8. We have,

$$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$$

$$\text{a} = (a^2 + b^2), \text{b} = 2(ac + bd), c = (c^2 + d^2)$$ [1]

$$D = b^2 - 4ac$$

Therefore,

$$D = [2(ac + bd)]^2 - 4 \times (a^2 + b^2) \times (c^2 + d^2)$$

$$\Rightarrow [4(a^2c^2 + b^2d^2 + 2\text{abcd})]$$

$$-4 \times (a^2 + b^2) \times (c^2 + d^2)$$

$$\Rightarrow [4(a^2c^2 + b^2d^2 + 2\text{abcd})]$$

$$-4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$$

$$\Rightarrow 4\begin{bmatrix}(a^2c^2 + b^2d^2 + 2\text{abcd}) \\ -(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)\end{bmatrix}$$

$$\Rightarrow 4\left[\begin{pmatrix}a^2c^2 + b^2d^2 + 2\text{abcd} - \\ a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2\end{pmatrix}\right]$$ [1]

$$\Rightarrow 4[(2\text{abcd} - a^2d^2 - b^2c^2)]$$

$$\Rightarrow -4[(a^2d^2 + b^2c^2 - 2\text{abcd})]$$

$$\Rightarrow -4(ad - bc)^2$$

Given $ad \neq bc$

Therefore,

$$(ad - bc) \neq 0$$

$$\Rightarrow (ad - bc)^2 > 0$$

$$\Rightarrow -4(ad - bc)^2 < 0$$

$$\Rightarrow D < 0$$

So, the given equation has no real roots. Hence, proved. [1]

9. For the given equation

$$\left(1+m^2\right)x^2+2mcx+c^2-a^2=0$$

$a=\left(1+m^2\right),\ b=2mc,\ c=c^2-a^2$

We know that when quadratic equation has equal roots, its discriminant is zero.

i.e. $D=b^2-4ac=0$ [1]

Putting the values, we'll get

$\Rightarrow D=b^2-4ac$

$=(2mc)^2-4\times\left(1+m^2\right)\times\left(c^2-a^2\right)=0$

$\Rightarrow 4m^2c^2-4\times\left(c^2-a^2+c^2m^2-a^2m^2\right)=0$

$\Rightarrow 4c^2m^2-4c^2+4a^2-4c^2m^2+4a^2m^2=0$ [1]

$\Rightarrow -4c^2+4a^2+4a^2m^2=0$

$\Rightarrow 4a^2+4a^2m^2=4c^2$

$\Rightarrow 4a^2\left(1+m^2\right)=4c^2$

$\Rightarrow c^2=a^2\left(1+m^2\right)$

Hence proved. [1]

10. Let the length of the park = y m

Given perimeter of the park is 80m. So,

$2\left(y+\text{width}\right)=80$

$\Rightarrow y+\text{width}=40$

$\Rightarrow \text{width}=40-y$ [1]

According to the question, the area of the park is $400m^2$. So,

$\text{length}\times\text{width}=400$

$\Rightarrow y(40-y)=400$

$\Rightarrow 40y-y^2=400$ [1]

$\Rightarrow y^2-40y+400=0$

$\Rightarrow y^2-20y-20y+400=0$

$\Rightarrow y(y-20)-20(y-20)=0$

$\Rightarrow \left(y-20\right)\left(y-20\right)=0$

$\Rightarrow y=20,20$

As the roots are real, given situation is possible but the roots are equal, so it is a square with the length of each side equal to 20m. [1]

11. It is given that the quadratic equation has equal roots, so its discriminant will be zero.

$\therefore D=b^2-4ac=0$

Compare given quadratic equation with standard form of quadratic equation $ax^2+bx+c=0$,

$a=k+4, b=k+1, c=1$

$\Rightarrow \left(k+1\right)^2-4\times\left(k+4\right)\times\left(1\right)=0$

$\Rightarrow k^2+2k+1-4k-16=0$ [1]

(Using $\left(a+b\right)^2=a^2+2ab+b^2$)

$\Rightarrow k^2-2k-15=0$

Splitting the middle term,

$\Rightarrow k^2-5k+3k-15=0$

$\Rightarrow k(k-5)+3(k-5)=0$

$\Rightarrow \left(k+3\right)\left(k-5\right)=0$

$\Rightarrow k=-3$ or $k=5$ [1]

Substitute, $k=5$ in

$(k+4)x^2+(k+1)x+1=0$

$\Rightarrow 9x^2+6x+1=0$

$\Rightarrow \left(3x+1\right)^2=0$

$\Rightarrow x=\frac{-1}{3},\frac{-1}{3}$ [1]

Substitute, $k=-3$ in

$(k+4)x^2+(k+1)x+1=0$

$x^2-2x+1=0$

$\Rightarrow \left(x-1\right)^2=0$

$\Rightarrow x=1,1$

Hence, the equal root of the given quadratic equation is either 1 or $\frac{-1}{3}$. [1]

Value Based

PREVIOUS YEARS'

EXAMINATION QUESTIONS

4 Marks Question

1. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250km/hour than the usual speed. Find the usual speed of the plane. What value is depicted in this question?

[TERM 2, 2016]

Solutions

1. Let the usual speed of the plane be x km/h.

Let the time taken by the plane to reach the destination be .

Using $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$,

So, $t_A = \frac{1500}{x}$ [1]

The increased speed is $(x+250)\,km/hr$.

So, $t_B = \frac{1500}{x+250}$

$\therefore t_A - t_B = 30\,\text{min}$ [1]

It is given in the question that the plane got delayed by half an hour.

$\Rightarrow \frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$

$\Rightarrow \frac{1500(x+250-x)}{x(x+250)} = \frac{1}{2}$

$\Rightarrow x^2 + 250x = 750000$

$\Rightarrow x^2 + 250x - 750000 = 0$ [1]

Splitting the middle term,

$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow x(x+1000) - 750(x+1000) = 0$

$\Rightarrow (x-750)(x+1000) = 0$

$\Rightarrow x = 750$ or $x = -1000$

Speed cannot be in negative, $x = 750$

Therefore, the usual speed of the plane is 750km/hr.

The promptness shown by the pilot is appreciable. The pilot is a caring person who has set an example of a trust worthy citizen. [1]

Smart Notes ....

CHAPTER 5

Arithmetic Progression

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Terms of AP	1 mark	1 mark	2 marks	1, 2 marks	1 mark	1, 2, 4 marks
Sum of AP	2, 4 marks	2, 4 marks	3, 3, 4 marks	3, 4 marks	2, 3 marks	3 marks
Word Problem on AP						

[TOPIC 1] Arithmetic Progression

Summary

Sequence and Series

Sequence: A sequence is an arrangement of number in a definite order, according to a definite rule.

Terms: Various numbers occurring in a sequence are called terms or element.

Consider the following lists of number:

3, 6, 9, 12, ...

4, 8, 12, 16, ...

–3, –2, –1, 0, ...

In all the list above, we observe that each successive terms are obtained by adding a fixed number to the preceding terms. Such list of numbers is said to form on **Arithmetic Progression (AP).**

Arithmetic Progression: An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the **common difference (d)** of the A.P.

Common difference can be positive, negative or zero.

Let us denote first term of A.P. by a or t, second term by a_2 or t_2 and nth term by a_n or t_n & the common difference by d. Then the A.P. becomes

a_1, a_2, a_3 .... a_n

where $a_2 - a_1 = d$

or $a_2 = a_1 + d$

similarly $a_3 = a_2 + d$

- $\therefore$ ***In general,*** $a_n - a_{n-1} = d$

  or $a_n = a_{n-1} + d$

Thus a, $a + d$, $a + 2d$, ...

forms an A.P. whose first term is 'a' & common difference is 'd'

This is called general form of an A.P.

Finite A.P. : An A.P. containing finite number of terms is called finite A.P.

e.g. 147, 149, 151 .... 163.

Infinite A.P. : An A.P. containing infinite terms is called infinite A.P.

e.g. 6, 9, 12, 15 ....

nth Term of an A.P.

Let a_1, a_2, a_3 ..... be an A.P., with first term as a, and common difference as d.

First term is $= a$ (i)

Second term $(a_2) = a + d$ (ii)

$= a + (2-1)d$

Third term $(a_3) = a_2 + d$ (iii)

$= a + d + d$ [from (i)]

$= a + 2d$

or $= a + (3-1)d$

Fourth term $a_4 = a_3 + d$

or $= a + 2d + d$ [from (iii)]

$= a + 3d$

$= a + (4-1)d$

$\therefore$ nth term $a_n = a + (n-1)d$

- ***The nth term of the A.P. with first term a & common difference d is given by*** $a_n = a + (n-1)d$

a_n is also called as general term of an A.P.

If there are P terms in the A.P. then a_p represents the last term which can also be denoted by l.

TO FIND nth TERM FROM THE END OF AN A.P.

Consider the following A.P. a, $a + d$, $a + 2d$,...$(l - 2d)$, $(l - d)$, l

where ℓ is the last term

last term $l = l - (1-1)d$

2nd last term $l - d = l - (2-1)d$

3rd last term $l - 2d = l - (3-1)d$

....

....

- ***nth term from the end =*** $l - (n-1)d$

CONDITION FOR TERMS TO BE IN A.P.

If three numbers $a, b, c,$ in order are in A.P. Then,

$b - a$ = common difference = $c - b$

$\Rightarrow b - a = c - b$

$\Rightarrow 2b = a + c$

- ***a, b, c are in A.P. iff***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. In an AP, if $d = -2$, $n = 5$ and $a_n = 0$ then the value of a is

(*a*) 10 (*b*) 5

(*c*) –8 (*d*) 8

[TERM 2, 2011]

2. The common difference of the A.P.

$$\frac{1}{p}, \frac{(1-p)}{p}, \frac{(1-2p)}{p} \dots \dots \dots \text{ is:}$$

(*a*) p (*b*) $-p$

(*c*) –1 (*d*) 1

[TERM 2, 2013]

3. If k, $2k-1$ and $2k+1$ are three consecutive terms of an A.P., the value of k is

(*a*) 2 (*b*) 3

(*c*) –3 (*d*) 5

[TERM 2, 2014]

4. The first three terms of an AP respectively are $3y-1, 3y+5$ and $5y+1$. Then y equals:

(*a*) –3 (*b*) 4

(*c*) 5 (*d*) 2

[TERM 2, 2014]

5. For what value of k will $k+9, 2k-1, 2k+7$ are the consecutive terms of an A.P.?

[TERM 2, 2016]

6. Find the $9th$ term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.

[TERM 2, 2016]

7. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

[TERM 2, 2017]

8. In an AP, if the common difference (d) = –4, and the seventh term (a_7) is 4, then find the first term.

[DELHI 2018]

2 Marks Questions

9. Find whether –150 is a term of the AP 17, 12, 7, 2, ...?

[TERM 2, 2011]

10. How many two-digit numbers are divisible by 3?

[TERM 2, 2012]

11. How many three digit natural numbers are divisible by 7?

[TERM 2, 2013]

12. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

[TERM 2, 2014]

13. Find the middle term of the A.P. 6, 13, 20, ..., 216.

[TERM 2, 2015]

14. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of an A.P. is three times its 11^{th} term.

[TERM 2, 2016]

15. Which term of the progression

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots\dots \text{ is the first negative term?}$$

[TERM 2, 2017]

3 Marks Questions

16. Find the value of the middle term of the following AP: –6, –2, 2, ..., 58.

[TERM 2, 2011]

17. Determine the AP whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30.

[TERM 2, 2011]

18. If the seventh term of an AP is $\frac{1}{9}$ and its ninth term is $\frac{1}{9}$, find its 63^{rd} term.

[TERM 2, 2014]

19. Which term of A.P. 3, 15, 27, 39,...will be 132 more than its 54^{th} term.

[TERM 2, 2017]

4 Marks Questions

20. Show that $a_1, a_2, a_3, \dots\dots, a_n$ form A.P. where a_n is defined as $a_n = 9 - 5n$.

[TERM 2, 2017]

Solutions

1. Given that,

$d = -2$, $n = 5$ and $a_n = 0$

We know that

$a_n = a + (n-1)\,d$ [½]

Substituting the given values in above equation.

$\Rightarrow 0 = a + (5-1)(-2)$

$\Rightarrow 0 = a + (4)(-2)$

$\Rightarrow 0 = a - 8$

$\Rightarrow a = 8$ [½]

Hence, the correct option is (d).

2. The given A.P. is

$\frac{1}{p}, \frac{(1-p)}{p}, \frac{(1-2p)}{p} \ldots \ldots \ldots$

And,

First term $(a_1) = \frac{1}{p}$

Second term $(a_2) = \frac{(1-p)}{p}$ [½]

Common difference $(d) = a_2 - a_1$

$= \frac{(1-p)}{p} - \frac{1}{p}$

$= \frac{1-p-1}{p}$

$\Rightarrow d = -1$

Thus the common difference of the given A.P. is "–1".

Hence the correct option is (*c*). [½]

3. Let a, b and c are in A.P. then $b - a = c - b$

Here the three consecutive terms of an A.P. are k, 2k –1 and $2k + 1$

$\Rightarrow 2k - 1 - k = 2k + 1 - (2k-1)$

$\Rightarrow 2k - 1 - k = 2k + 1 - 2k + 1$

$\Rightarrow k - 1 = 2$

$\Rightarrow k = 2 + 1 = 3$

The correct answer is (b). [1]

4. If a, b and c are in AP,

$b - a = c - b$

$2b = a + c$

$2(3y+5) = 3y - 1 + 5y + 1$

$6y + 10 = 8y$

$10 = 8y - 6y$

$2y = 10$

$y = 5$

Hence the correct option is (c). [1]

5. Arithmetic mean, $2b = a + c$

$a = k + 9, b = 2k - 1, c = 2k + 7$

By putting the values of a, b and c in equation

$2(2k-1) = k + 9 + 2k + 7$

$4k - 2 = 3k + 16$

$4k = 3k + 18$

$k = 18$

Hence, for $k = 18$; $k+9, 2k-1, 2k+7$ will be the consecutive terms of an A.P. [1]

6. The last term of the AP from the end, $l = 185$

Common difference, $d = 9 - 5 = 4$

To find the 9th term from the end, rearrange the A.P. 185, 181,.......,13, 9, 5

The first term of this A.P. is 185 and the common difference is –4.

The nth term of the AP is given by $a_n = a + (n-1)\,d$. [½]

So, the 9th term of the A.P. is

= 185 + (9 – 1) (–4)

= 185 – 8 × 4

= 185 – 32

= 153

Hence, the 9th term from the end is 153. [½]

7. Let be the first term d be the common difference of Arithmetic progression.

As,

$a_n = a + (n-1)d$

$\therefore a_{21} = a + (21-1)\,d = a + 20d$

$a_7 = a + (7-1)d = a + 6d$ [½]

Given $a_{21} - a_7 = 84$

$\Rightarrow (a + 20d) - (a + 6d) = 84$

$\Rightarrow a + 20d - a - 6d = 84$

$\Rightarrow 20d - 6d = 84$

$\Rightarrow 14d = 84$

Dividing both sides by 14

$\Rightarrow \frac{14d}{14} = \frac{84}{14}$

$\Rightarrow d = 6$

Therefore, the common difference of A.P. is 6 [½]

8. The nth term = $a + (n-1)d$

Where a = first term, $n = 7$, $d = -4$

Putting these values,

a + (7 – 1) (–4) = 4

a + 6(–4) = 4

⇒ a = 4 + 24

⇒ a = 28.

So the first term is 28. [1]

9. Given, the AP 17, 12, 7, 2, ...

We have, $a = 17$, $d = 12 - 17 = -5$

Let, $a_n = -150$

But, $a_n = a + (n-1)d$ [1]

Substitute the value of a_n in the above equation.

$-150 = 17 + (n-1)(-5)$

$-150 = 17 - 5n + 5$

$-150 = 22 - 5n$

$-150 - 22 = -5n$

$-172 = -5n$

$\Rightarrow n = \frac{172}{5}$

But "n" cannot be a fractions. "n" is always a whole number.

Therefore, –150 is not a term of the given AP. [1]

10. Numbers divisible by 3 are multiples of 3.

3, 6, 9, 12, 15…

The smallest two-digit number divisible by 3 is 12. And the largest 2-digit number divisible by 3 is 99.

So, the series of the 2-digit multiples of 3 starts with 12 and ends with 99. The difference between the numbers is 3.

Therefore, the A.P. is 12, 15, 18, 21, 24 .90, 93, 96, 99. [1]

In the sequence, the first term, $a = 12$. The last term, $l = 99$. The common difference, $d = 3$. The n^{th} term is $a_n = 99$.

Now, $a_n = a_1 + (n-1)d$

$\Rightarrow 99 = 12 + (n-1)3$

$\Rightarrow 99 = 12 + 3n - 3$

$\Rightarrow 99 = 3n + 9$

$\Rightarrow 3n = 90$

$\Rightarrow n = 30$

Therefore, there are 30 two-digit numbers divisible by 3. [1]

11. Following are the three digit natural numbers divisible by 7:

105,112,119,126........,........,...994

The given series is A.P.

First term (a) = 105

Common diff (d) = 7

n^{th} term (a_n) = 994 [1]

It is known that the term of an A.P. is given by,

$a_n = a + (n-1)d$

Substituting $a = 105$, $d = 7$ and $a_n = 994$

$994 = 105 + (n-1)7$

$\Rightarrow 994 = 105 + 7n - 7$

$\Rightarrow 994 = 98 + 7n$

$\Rightarrow 7n = 994 - 98$

$\Rightarrow 7n = 896$

$\Rightarrow n = 128$

Thus 128 three digit natural numbers are divisible by 7. [1]

12. Numbers that are divisible by both 2 and 5 will be are multiples of 10.

The natural numbers between 101 and 999 which are divisible by both 2 and 5 will be : 110, 120, 130, ..., 990.

This forms an A.P. with first term as 110, common difference 10 and last term as 990.

n^{th} term of an A.P. $a_n = a+(n-1)d$ [1]

Substitute the value of a, d and a_n

$\Rightarrow 990 = 110+(n-1)10$

$\Rightarrow 990 - 110 = 10n-10$

$\Rightarrow 880 = 10n-10$

$\Rightarrow 10n = 890$

$\Rightarrow n = 89$

Therefore, 89 natural numbers lie between 101 and 999 which are divisible by 2 and 5. [1]

13. 6, 13, 20, ..., 216.

In the given sequence $a = 6$, $a_n = 216$ and $d = 7$

We know that $a_n = a+(n-1)d$

Substitute $a = 6$, $a_n = 216$ and $d = 7$ in the formula to find the number of terms we get,

$216 = 6+(n-1)7$

$\Rightarrow 216 = 6+7n-7$

$\Rightarrow 216 = 7n-1$

$\Rightarrow 217 = 7n$

$\Rightarrow n = \frac{217}{7} = 31$ [1]

Therefore the number of terms in the given sequence is 31 which is odd.

Therefore the middle term will be $\left(\frac{n+1}{2}\right)th$ term of the A.P.

$\Rightarrow \frac{31+1}{2} = 16th$ term

We know that nth term in an A.P. is calculated by the formula $a_n = a+(n-1)d$

$\Rightarrow 16th$ term $= 6+(16-1)7$

$= 6 + 105$

Now here $n = 16$

Hence, the middle term is 111. [1]

14. Given, 4^{th} term is zero.

So, $a + 3d = 0$

$\Rightarrow a = -3d$

n^{th} Term of an A.P. is given by,

$a_n = a+(n-1)d$ [1]

$a_{11} = a+10d$

Substitute the value of a,

$a_{11} = -3d+10d = 7d$

Similarly, $a_{25} = a+24d$

$a_{25} = -3d+24d = 21d$

Or, $a_{25} = 3\times 7d$

$a_{25} = 3\times a_{11}$

Hence proved. [1]

15. Given,

First term $a = 20$

Common difference

$d = 19\frac{1}{4}-20 = \frac{77-80}{4} = -\frac{3}{4}$

Let n^{th} term of AP be the first negative term

$\Rightarrow a_n < 0$

$\Rightarrow a_n < 0$ [1]

$\Rightarrow 20+(n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow 20+\left(\frac{3}{4}-\frac{3n}{4}\right) < 0$

$\Rightarrow 20+\frac{3}{4}-\frac{3n}{4} < 0$

$\Rightarrow \frac{83}{4}-\frac{3n}{4} < 0$

$\Rightarrow 83-3n < 0$

$\Rightarrow 3n > 83$

$\Rightarrow n > 27\frac{2}{3}$

$\Rightarrow n \geq 28$

Therefore, 28^{th} term is the first negative term of the A.P. [1]

16. Clearly, –6, –2, 2, ..., 58 is an AP with the first term and common difference $d = -2 - (-6) = -2 + 6 = 4$. Let there be n terms in the given AP. Then,

$a_n = 58$

$a_n = a + (n-1)d$

$a + (n-1)d = 58$

$(-6) + (n-1)4 = 58$ [1]

$-6 + 4n - 4 = 58$

$4n - 10 = 58$

$4n = 58 + 10$

$\Rightarrow 4n = 68$

$\Rightarrow n = \frac{68}{4} = 17$

Here, n is odd, so, the middle te rm is $\left(\frac{n+1}{2}\right)^{th}$ term [1]

i.e. $\left(\frac{17+1}{2}\right)^{th} = \left(\frac{18}{2}\right)^{th} \Rightarrow 9^{th}$ term is the middle term and is given by,

$a_9 = a + (9-1)d$

$= -6 + 8\,(4)$

$= -6 + 32$

$= 26$

The value of the middle term of the given AP is 26. [1]

17. Given that, fourth term $a_4 = 18$ and the difference of the ninth term from the fifteenth term is 30, $a_{15} - a_9 = 30$ [1]

Clearly,

$a_4 = 18$

$\Rightarrow a + 3d = 18$ ....(i)

$a_{15} - a_9 = 30$

$a + 14d - (a + 8d) = 30$

$a + 14d - a - 8d = 30$

$\Rightarrow 6d = 30$

$\Rightarrow d = \frac{30}{6} = 5$ [1]

Substitute the value of d in equation (i)

$a + 3(5) = 18$

$a = 18 - 15 = 3$

$\therefore$ First term $a_1 = 3$

Second term $a_2 = a_1 + d = 3 + 5 = 8$

Third term $a_3 = a_1 + 2d = 3 + 2(5) = 13$

Therefore, the AP is 3, 8, 13, ... [1]

18. Let a be the first term and d be the common difference of the given A.P.

$a_7 = \frac{1}{9}$

$a_9 = \frac{1}{7}$

$a_7 = a + (7-1)\,d = \frac{1}{9}$

$a + 6d = \frac{1}{9}$ .....(1) [1]

$a_9 = a + (9-1)\,d = \frac{1}{7}$

$a + 8d = \frac{1}{7}$.....(2)

Subtracting (1) from (2)

$2d = \frac{2}{63}$

$\Rightarrow d = \frac{1}{63}$

Put $d = \frac{1}{63}$ in the equation (1)

$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9}$ [1]

$\Rightarrow a = \frac{1}{63}$

$a_{63} = a + (63-1)\,d$

$= \frac{1}{63} + 62\left(\frac{1}{63}\right)$

$= \frac{63}{63} = 1$

Hence, $a_{63} = 1$ [1]

19. Here, $a = 3$ and $d = 15 - 3 = 12$.

Now, we first find the 54[th] term of the given A.P.

As we know that n^{th} term is given by $a_n = a + (n-1)d$ where $n = 54$ [1]

$\Rightarrow a_{54} = 3 + (54-1)12$

$\Rightarrow a_{54} = 3 + 53 \times 12$

$\Rightarrow a_{54} = 3 + 636$

$\Rightarrow a_{54} = 639$ [1]

Let's say a_n is the term which is 132 more than 54[th] term. So, according to the Question,

$a_n = a_{54} + 132$

$\Rightarrow a + (n-1)d = 639 + 132$

$\Rightarrow 3 + (n-1)12 = 771$

$\Rightarrow 3 + 12n - 12 = 771$

$\Rightarrow 12n - 9 = 771$

$\Rightarrow 12n = 771 + 9$

$\Rightarrow 12n = 780$

$\Rightarrow n = \frac{780}{12}$

$\Rightarrow n = 65$

Therefore, 65[th] term will be 132 more than 54[th] term. [1]

20. Given $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 4$

$a_2 = 9 - 5(2) = -1$

$a_3 = 9 - 5(3) = -6$ [1]

Difference between, a_2 and a_1

$= a_2 - a_1$

$= -1 - 4 = -5$ [1]

Difference between a_3 and a_2

Also, $a_n = 9 - 5n$ can be written as

$a_n = 4 + 5 - 5n$

$\Rightarrow a_n = 4 + (1-n)(5)$

$\Rightarrow a_n = 4 + (n-1)(-5)$ which is the standard form of the n^{th} term of an A.P. [1]

Also, as the difference is same between these terms, hence, they form an A.P.

Hence proved. [1]

[TOPIC 2] Sum of n Terms of an A.P.

Summary

Sum of n Terms of an A.P.

Let a be the first term and d be the common difference of an A.P. l is the last term where $l = a + (n - 1)\ d$.

Sum of first n terms of the given A.P. is given by

$$S_n = a + (a + d) + (a + 2d) + ...(l - 2d) + (l - d) + l \quad .(i)$$

Writing in reverse order

$$S_n = l + (l - d) + (l - 2d) + ...(a + 2d) + (a + d) + a \quad .(ii)$$

Adding (i) and (ii) we get

$$2S_n = \underbrace{(a + l) + (a + l) + (a + l) + ... + (a + l)}_{n \text{ times}}$$

$$2S_n = n(a + l)$$

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}[a + a + (n - 1)\ d]$$

$[\because\ l = a + (n - 1)d]$

$$S_n = \frac{n}{2}[2a + (n - 1)\ d]$$

$$S_n = \frac{n}{2}(a + a_n)$$

where $a_n = a + (n - 1)d$

Selection of Terms in A.P.

Some times certain number of terms in A.P. are required. The following ways of selecting terms are convenient.

Number of terms	Terms	common difference
3	$a - d,\ a,\ a + d$	d
4	$a - 3d,\ a - d,\ a + d,\ a + 3d$	$2d$
5	$a - 2d,\ a - d,\ a,\ a + d,\ a + 2d$	d
6	$a - 5d,\ a - 3d,\ a - d,\ a + d,\ a + 3d,\ a + 5d$	$2d$

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

▶ 1 Mark Question

1. If the n^{th} term of an A.P. is $(2n + 1)$, then the sum of its first three terms is

(*a*) $6n + 3$ (*b*) 15

(*c*) 12 (*d*) 21

[TERM 2, 2012]

▶ 2 Marks Questions

2. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

[TERM 2, 2014]

3. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of first terms.

[TERM 2, 2015]

4. How many terms of the A.P. 18, 16, 14 ... be taken so that their sum is zero?

[TERM 2, 2016]

5. How many terms of A.P 27, 24, 21, ... should be taken so that their sum is zero (0)?

[TERM 2, 2017]

6. Find the sum of first 8 multiple of 3.

[DELHI 2018]

▶ 3 Marks Questions

7. Find the sum of all multiples of 7 lying between 500 and 900.

[TERM 2, 2012]

8. Find the number of terms of the A.P. $18,\ 15\frac{1}{2}, 13,\ ....,\ -49\frac{1}{2}$ and find the sum of all its terms.

[TERM 2, 2013]

9. The sum of the 5^{th} and the 9^{th} terms of an AP is 30. If its 8^{th} term is three times its 25^{th} term, find the AP.

[TERM 2, 2014]

10. If S_n denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

[TERM 2, 2015]

11. The 14^{th} term of an AP is twice its 8^{th} term. If its 6^{th} term is –8, then find the sum of its first 20 terms.

[TERM 2, 2015]

12. If the ratio of the sum of first n terms of two A.P's is $(7n + 1) : (4n + 27)$ find the ratio of their m^{th} terms.

[TERM 2, 2015]

13. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.

[TERM 2, 2016]

14. If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first mn terms.

[TERM 2, 2017]

15. Find the sum of terms of the series

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots\ldots$$

[TERM 2, 2017]

16. The sum of n term of an A.P is $3n^2 + 5n$. Find the A.P and its 15^{th} term.

[TERM 2, 2017]

4 Marks Questions

17. Find the common difference of an A.P. whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

[TERM 2, 2012]

18. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

[TERM 2, 2014]

19. Find the 60^{th} term of the AP 8, 10, 12, ..., if it has a total of terms and hence find the sum of its last 10 terms.

[TERM 2, 2015]

20. If the ratio of the sum of the first n terms of two A.Ps is $(7n + 1) : (4n + 27)$, then find the ratio of their 9^{th} terms.

[TERM 2, 2017]

21. The ratio of the sums of first m and first n terms of an A. P. is $m^2 : n^2$.

Show that the ratio of its m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

[TERM 2, 2017]

22. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

[DELHI 2018]

Solutions

1. We have,

$a_n = (2n+1)$

$\Rightarrow a_1 = 2\times1+1 = 3$

So, the given sequence is an A.P. with first term $a = a_1 = 3$.

And the second term, $a_2 = 2\times2+1 = 5$.

So, the common difference,

$d = a_2 - a_1 = 5 - 3 = 2$ [½]

Therefore, the sum of first 3 terms of the A.P. is given by

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$= \frac{3}{2}\{6+(3-1)2\}$$

$$= \frac{3}{2}(6+4)$$

$$= \frac{3}{2}(10) = 15$$

Hence the correct option is (b). [½]

2. Let a be the first term and d be the common difference.

Given

$a = 5$

$a_n = 45$

$S_n = 400$

We know $a_n = a+(n-1)\ d$

$45 = 5+(n-1)d$]

$40 = (n-1)d$ ....(1)

And $S_n = \frac{n}{2}(a+a_n)$ [1]

$$400 = \frac{n}{2}(5+45)$$

$$\frac{n}{2} = \frac{400}{50}$$

$n = 2\times8 = 16$

On substituting $n = 16$ in (1)

$40 = (16 - 1)\ d$

$40 = (15)\ d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Thus, the common difference is $\frac{8}{3}$. [1]

3. $S_5 + S_7 = 167$ (Given)

$S_{10} = 235$ (Given)

Sum of n terms is,

$$S_n = \frac{n}{2}\{2a + (n-1)\,d\}$$

Where,

n = Number of terms

a = First term

b = Common difference

$$S_5 = \frac{5}{2}\{2a + (5-1)\,d\} \text{ and } S_7 = \frac{7}{2}\{2a + (7-1)\,d\}$$

[1]

$$\frac{5}{2}\{2a + (5-1)\,d\} + \frac{7}{2}\{2a + (7-1)\,d\} = 167$$

On simplifying

$5a + 10d + 7a + 21d = 167$

$12a + 31d = 167$ .....(i)

For the sum of first ten terms,

$$\frac{10}{2}\{2a + (10-1)\,d\} = 235$$

$10a + 45d = 235$

$2a + 9d = 47$ ....(ii)

Multiply equation (ii) by 6 and subtract equation (i) from it

$$6(2a + 9d) - (12a + 31d) = 6(47) - 167$$

$12a + 54d - 12a - 31d = 282 - 167$

$23d = 115$

$$d = \frac{115}{23} = 5$$

Substitute the value of d in equation (ii), we get

$2a + 9(5) = 47$

$2a + 45 = 47$

$2a = 47 - 45 = 2$

$$a = \frac{2}{2} = 1$$

So the values of a and b are $d = 5a$

Hence, the AP is 1, 6, 11, 16, 21, 26, ... [1]

4. The first term of the AP is 18

Common difference = 16 – 18 = –2

Let the sum of the first x terms of the AP be 0.

So, the sum of the first x terms is given by-

$$\frac{x}{2}[2 \times 18 + (x-1)(-2)] = 0$$

$$\Rightarrow \frac{x}{2}[36 + (-2x + 2)] = 0$$

$$\Rightarrow \frac{x}{2}[36 - 2x + 2] = 0$$ [1]

$$\Rightarrow \frac{x}{2}[38 - 2x] = 0$$

$$\Rightarrow x[19 - x] = 0$$

So, we get,

$x = 0$ or $19 - x = 0$

Ignoring $x = 0$ we get,

$x = 19$

Hence, the sum of first 19 terms of the AP is 0. [1]

5. Here $a = 27$ and $d = 24 - 27 = -3$,

According to the Question , the sum should be zero i.e. $S_n = 0$

Applying the formula for sum of n terms of an A.P,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$\Rightarrow 0 = \frac{n}{2}(2 \times 27 + (n-1)(-3))$$

$$\Rightarrow 0 = \frac{n}{2}\{54 + (-3n + 3)\}$$ [1]

$$\frac{n}{2} = 0 \quad or \quad 54 + (-3n + 3) = 0$$

$n = 0$ or $57 - 3n = 0$

Ignoring $n = 0$, we get;

$57 - 3n = 0$

$\Rightarrow 3n = 57$

$\Rightarrow n = 19$

Therefore, 19 terms of this A.P. should be taken so that their sum is zero (0). [1]

6. The first 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24.

It form an AP with the first term (a) = 3, the difference (d) = 3 and the last term (l) = 24.

The sum of as an AP:

$S_n = \frac{n}{2}(a+l)$ [1]

$= \frac{8}{2}(3+24)$

$= 4 \times 27$

$\Rightarrow S_n = 108$

So the sum of the first 8 multiples of 3 is equal to 108. [1]

7. Between 500 and 900, the first multiple of 7 is $a = 504$ and the last multiple of 7 is $l = 896$ and the common difference is $d = 7$

Hence, the A.P will be

504, 511, 518, ..., 896 [1]

The number of terms in this A.P is given by;

$a_n = a+(n-1)d$

$\Rightarrow 896 = 504+(n-1)7$

$\Rightarrow (n-1)7 = 392$

$\Rightarrow n-1 = 56$

$\Rightarrow n = 57$ [1]

Therefore, the sum of all the multiples of 7 lying between 500 and 900 is given by

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow S_{57} = \frac{57}{2}[2 \times 504+(57-1)7]$

$\Rightarrow S_{57} = \frac{57}{2}[1008+56 \times 7]$

$\Rightarrow S_{57} = \frac{57}{2}[1008+392]$

$\Rightarrow S_{57} = \frac{57}{2} \times 1400$

$\Rightarrow S_{57} = 57 \times 700$

$\Rightarrow S_{57} = 39900$

Therefore, the sum of all multiples of 7 lying between 500 and 900 is 39900. [1]

8. The given A.P. is:

$18,\ 15\frac{1}{2}, 13,\ ....,\ -49\frac{1}{2}$

And,

First term (a_1) = 18

Second term $(a_2) = 15\frac{1}{2}$

Common difference (d) = $a_2 - a_1$

$= 15\frac{1}{2} - 18$

$= \frac{31-36}{2}$

$\Rightarrow d = -\frac{5}{2}$ [1]

Let the A.P. has n terms.

$a_n = -49\frac{1}{2}$

Also, $a_n = a + (n - 1) d$

Thus,

$-49\frac{1}{2} = a + (n - 1) d$

$\frac{-99}{2} = 18+(n - 1)\frac{-5}{2}$

$\frac{-99}{2} = 18 + \left(\frac{-5n}{2}+\frac{5}{2}\right)$

$-99 = 41 - 5n$

$5n = 140$

$n = 28$ [1]

$\therefore$ The number of terms in the given A.P. is 28

Sum of n terms (S_n) :

$S_n = \frac{n}{2}[2a + (n - 1) d]$

$S_{28} = \frac{28}{2}\left[2 \times 18 + (28 - 1) \frac{-5}{2}\right]$

$S_{28} = 14\left[\frac{72-135}{2}\right]$

$S_{28} = 7 \times -63$

$S_{28} = -441$

Thus the sum of all the terms of the A.P. is –441. [1]

9. 5^{th} term of an A.P. = $a + 4d$

9^{th} term of an A.P. = $a + 8d$

Sum of 5^{th} and 9^{th} term of an A.P. = 30

$\Rightarrow a+4d+a+8d=30$

Add the like terms

$\Rightarrow 2a+12d=30$

$\Rightarrow 2(a+6d)=30$

$\Rightarrow a+6d=\frac{30}{2}=15$

$\Rightarrow a+6d=15$ ....(1) [1]

25^{th} term of an A.P. = $a + 24d$

8^{th} term of an A.P. $= a + 7d$

Given that the 25^{th} term is three times its 8^{th} term

$\Rightarrow a+24d=3(a+7d)$

$\Rightarrow a+24d=3a+21d$

$\Rightarrow 3a-a=24d-21d$

$\Rightarrow 2a=3d$ ....(2)

Equation (1) can be written as

$a+2(3d)=15$ ....(3) [1]

Substitute the value of 3d from equation (2) in the above equation (3).

Therefore,

$a+2(2a)=15$

$a + 4a = 15$

$5a = 15$

$a = 3$

Substitute the value of a in equation (2)

$2\times 3=3d$

$d = 2$

Hence, first term of the A.P. is 3 and the common difference is 2 therefore, the A.P. is 3, 5, 7, 9, ... [1]

10. To prove: $S_{12}=3(S_8-S_4)$

Proof: The sum of n terms of an AP is given by the formula,

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

Where 'n' denotes the number of terms in an AP, 'a' denotes the first term and 'd' denotes the common difference of AP.

S_8 denotes the sum of first 8 terms of an AP.

$S_8=\frac{8}{2}[2a+(8-1)d]$ [1]

$\Rightarrow S_8=4[2a+7d]=8a+28d$ ....(i)

S_4 denotes the sum of first 4 terms of an AP.

$S_4=\frac{4}{2}[2a+(4-1)d]$

$\Rightarrow S_4=2[2a+3d]=4a+6d$ ....(ii)

Now, from equations (i) and (ii);

$3(S_8-S_4)=3\{8a+28d-(4a+6d)\}$

$3(S_8-S_4)=3\{8a+28d-(4a+6d)\}$ [1]

$=3(4a+22d)$

$=6(2a+11d)$

$=\frac{12}{2}[2a+(12-1)d]$

$= S_{12}$

Hence proved. [1]

11. $14th$ term of the AP = 2 × $8th$ term of the AP (Given)

Also, $a_n=a+(n-1)d$

Given that $a_{14} = 2 \times a_8$

$\Rightarrow a+(14-1)d=2\{a+(8-1)d\}$

$\Rightarrow a+13d=2a+14d$

$a + d = 0$ ....(a)

Also,

$a_6 = -8$ (Given)

$a+(6-1)d=-8$

$a + 5d = -8$ ....(b) [1]

Subtract equation (a) from (b)

$a+5d-(a+d)=-8-0$

$\Rightarrow a+5d-a-d=-8-0$

$\Rightarrow 4d=-8$

$\Rightarrow d=\frac{-8}{4}=-2$

Substitute the value of d in equation (a)

$a - 2 = 0$

$\Rightarrow a = 2$ [1]

So the values of a and d are 2 and –2 respectively.

Sum of first n term is given by;

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{20} = \frac{20}{2}\{2 \times 2 + (20-1)(-2)\}$$

$$S_{20} = 10(4 - 38)$$

$S_{20} = -340$ [1]

12. Sum of first n terms of an A.P is,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

According to the Question,

$$\frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{(7n+1)}{(4n+27)}$$

$$\Rightarrow \frac{(2a + (n-1)d)}{(2a' + (n-1)d')} = \frac{(7n+1)}{(4n+27)} \quad ....(1)$$ [1]

term of an A.P. is $a_n = a + (n-1)d$

So the ratio of the m^{th} terms is

$$a_m : a'_m = a + (m-1)d : a' + (m-1)d'$$

Multiply both sides by 2

$$a_m : a'_m = 2a + 2[(m-1)d] : 2a' + 2[(m-1)d']$$

$$= 2a + (2m-2)d : 2a' + (2m-2)d'$$ [1]

Replace n by $(2m - 1)$ in equation (1) we get;

$$\frac{(2a + (2m-2)d)}{(2a' + (2m-2)d')} = \frac{(7(2m-1)+1)}{(4(2m-1)+27)}$$

$$\frac{2(a + (m-1)d)}{2(a' + (m-1)d')} = \frac{(14m-6)}{(8m+23)}$$

$$\frac{a_m}{a'_m} = \frac{(14m-6)}{(8m+23)}$$

Hence, ratio of the m^{th} terms is $\frac{(14m-6)}{(8m+23)}$ [1]

13. Let us suppose that the first term and the common difference of the given AP is 'a' and 'd' respectively.

We are given the sum of first 7 terms, $S_7 = 49$

We know that

$$S = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \frac{7}{2}(2a + 6d) = 49$$

$$\Rightarrow \frac{7}{2} \times 2(a + 3d) = 49$$

$\Rightarrow a + 3d = 7 \quad ....(1)$ [1]

Sum of the first 17 terms, $S_{17} = 289$

$$\frac{17}{2}(2a + 16d) = 289$$

$$\Rightarrow \frac{17}{2} \times 2(a + 8d) = 289$$

$$\Rightarrow a + 8d = \frac{289}{17} = 17$$

$\Rightarrow a + 8d = 17 \quad ....(2)$ [1]

Subtracting (2) from (1),

$5d = 10$

$\Rightarrow d = 2$

Substituting the value of d in (1), we get,

$\Rightarrow a + 3(2) = 7$

$\Rightarrow a = 7 - 6 = 1$

Now sum of first n terms of the A.P. will be

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + 2(n-1)]$$

$$= n(1 + n - 1) = n^2$$

Hence, the sum of the first n terms of the AP is n^2. [1]

14. We know that n^{th} term of an A.P is given by

$$a_n = a + (n-1)d$$

Where, a is first term and d is the common difference of the given A.P.

So, n^{th} term $= a + (n-1)d = \frac{1}{m}$

Or, $a = \frac{1}{m} - (n-1)d \quad ....(i)$ [1]

And, m^{th} term $= a+(m-1)d = \frac{1}{n}$

Or, $a = \frac{1}{n} - (m-1)d$ ....(ii)

Equating (i) and (ii), we'll get

$\Rightarrow \frac{1}{m} - (n-1)d = \frac{1}{n} - (m-1)d$

$\Rightarrow \frac{1}{m} - \frac{1}{n} = (n-1)d - (m-1)d$

$\Rightarrow \frac{n-m}{mn} = nd - d - md + d$

$\Rightarrow \frac{n-m}{mn} = (n-m)d$

$\Rightarrow d = \frac{1}{mn}$ ....(iii)

Putting the value of d in equation (i), we'll get

$a = \frac{1}{m} - (n-1)\left(\frac{1}{mn}\right)$

$\Rightarrow a = \frac{1}{m} - \frac{n}{mn} + \frac{1}{mn}$

$\Rightarrow a = \frac{1}{m} - \frac{1}{m} + \frac{1}{mn}$

$\Rightarrow a = \frac{1}{mn}$ ....(iv)

Now, the mn^{th} term will be given by

mn^{th} term $= a + (mn-1)d$

Putting the value of a andd in the above equation.

$\Rightarrow mn^{th}$ term $= \frac{1}{mn} + (mn-1)\left(\frac{1}{mn}\right)$

$\Rightarrow mn^{th}$ term $= \frac{1}{mn} + \frac{mn}{mn} - \frac{1}{mn}$ [1]

$\Rightarrow mn^{th} = a_{mn}$ = term = 1 ....(v)

We know that the sum of n terms of A.P is given by;

$S_n = \frac{n}{2}(a + a_n)$

Therefore, sum of first mn terms is given by;

$S_{mn} = \frac{mn}{2}(a + a_{mn})$

Put values of a and a_{mn} from equation (iv) and (v) we get;

$S_{mn} = \frac{mn}{2}\left(\frac{1}{mn} + 1\right)$

$\Rightarrow S_{mn} = \frac{1}{2} + \frac{mn}{2}$

$\Rightarrow S_{mn} = \frac{1}{2}(1 + mn)$

Hence, the sum of first mn terms is

$S_{mn} = \frac{1}{2}(1 + mn)$ [1]

15. Breaking the given series in two series, we'll get

$\Rightarrow (4+4+4....) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n}....\right)$

$\Rightarrow (4+4+4....) - \frac{1}{n}(1+2+3+....)$

We Know that sum of an A.P is given by the formula

$S_n = \frac{n}{2}(2a + (n-1)d)$

For the sequence $(4+4+4....)$

$a = 4,\ d = 0$

$\therefore S_n = \frac{n}{2}(2\times 4 + (n-1)0)$

$\Rightarrow S_n = \frac{n}{2}(8) = 4n$ [1]

For the sequence $\frac{1}{n}(1+2+3.....)$

$a = 1,\ d = 1$

$\Rightarrow S_n = \frac{1}{n} \times \frac{n}{2}(2\times 1 + (n-1)1)$

$\Rightarrow S_n = \frac{1}{2}(2 + n - 1)$

$\Rightarrow S_n = \frac{1}{2}(n+1)$ [1]

Now, combining sum of both sequences with respect to $(4+4+4....)-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}....\right)$, we get;

$\Rightarrow 4n-\frac{1}{2}(n+1)$

$\Rightarrow \frac{8n-n-1}{2}$

$\Rightarrow \frac{7n-1}{2}$

Hence, the sum of the n^{th} term of the series is $\frac{7n-1}{2}$. [1]

16. $S_n = 3n^2 + 5n$

$S_1 = 3(1)^2 + 5(1) = 8$

As we know, $S_1 = a_1 = a = 8$ [1]

$S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$

So, $a_2 = S_2 - S_1 = 22 - 8 = 14$

$S_3 = 3(3)^2 + 5(3) = 27 + 15 = 42$

So, $a_3 = S_3 - S_2 = 42 - 22 = 20$

So, the A.P is 8, 14, 20, ... with common difference $d = a_2 - a_1 = 14 - 8 = 6$ [1]

Now, we will find the 15th term of this A.P.

$a_n = a + (n-1)d$

$\Rightarrow a_{15} = 8 + (15-1) \times 6$

$\Rightarrow a_{15} = 8 + 14 \times 6$

$\Rightarrow a_{15} = 92$

So, 15th term of the A.P is 92. [1]

17. Let the common difference of the A.P be *d*. First term is 5.

Therefore, the A.P can be written as

$5, 5+d, 5+2d, 5+3d, 5+4d, 5+5d, 5+6d, 5+7d$

[1]

The above sequence represents the first 8 terms of the A.P.

Now, the sum of the first 4 terms of the A.P

$= 5 + 5 + d + 5 + 2d + 5 + 3d = 20 + 6d$ [1]

And, the sum of the next four terms of the A.P

$= 5 + 4d + 5 + 5d + 5 + 6d + 5 + 7d = 20 + 22d$

Now, it is given that

Sum of the first 4 terms

$= \frac{1}{2} \times$ sum of the next four terms [1]

$\Rightarrow 20 + 6d = \frac{1}{2} \times (20 + 22d)$

$\Rightarrow 20 + 6d = 10 + 11d$

$\Rightarrow 11d - 6d = 20 - 10$

$\Rightarrow 5d = 10$

$\Rightarrow d = 2$

Therefore, the common difference of the A.P is 2. [1]

18. Let '*a*' and '*d*' be the first term and the common difference of an A.P. respectively.

The n^{th} term of an A.P is given by,

$a_n = a + (n-1)d$

And the sum of *n* terms of an A.P,

$S_n = \frac{n}{2}[2a + (n-1)d]$ [1]

So, sum of first 10 terms of an A.P is,

$S_{10} = \frac{10}{2}[2a + (10-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2a + 9d]$

$\Rightarrow 210 = \frac{10}{2}[2a + 9d]$

$\Rightarrow 210 = 5[2a + 9d]$ [1]

$\Rightarrow 42 = [2a + 9d]$ ....(1)

Now, the 15^{th} term from the last $= (50-15+1)^{th} = 36^{th}$ term from the beginning

$a_{36} = a + 35d$ [1]

Thus, sum of the last 15 terms

$= \frac{15}{2}[2a_{36} + (15-1)d]$

$\Rightarrow \frac{15}{2}[2(a + 35d) + (15-1)d]$

$\Rightarrow \frac{15}{2}[2(a + 35d) + 14d]$

$\Rightarrow \frac{15}{2}[2(a+35d+7d)]$

$\Rightarrow 15[a+35d+7d]$

$\Rightarrow 15[a+42d]$

$\Rightarrow 2565 = 15[a+42d]$

$\Rightarrow 171 = [a+42d]$ ....(2)

Solving (1) and (2) , we get,

$a = 3,\ d = 4$ [1]

Hence, the required A.P is, 3, 7, 11, 15, ... 199

19. The given AP is 8, 10, 12, ...,

First term, $a = 8$

Common difference, $d = 10 - 8 = 2$

$a_n = a+(n-1)d$

$a_{60} = 8+(60-1)2$ [1]

$a_{60} = 8+59\times 2$

$a_{60} = 126$

$S_n = \frac{n}{2}\{2a+(n-1)d\}$ [1]

Sum of first 60 terms

$S_{60} = \frac{60}{2}\{2\times 8+(60-1)2\}$

$S_{60} = 30(16+59\times 2)$

$S_{60} = 4020$ [1]

Sum of first 50 terms

$S_{50} = \frac{50}{2}\{2\times 8+(50-1)2\}$

$S_{50} = 25(16+49\times 2)$

$S_{50} = 2850$

Sum of last 10 terms is $S_{60} - S_{50}$

$= 4020 - 2850 = 1170$ [1]

Hence, the sum of the last 10 terms is 1170.

20. Let a_1, a_2 be first terms of the two A.Ps.

Let d_1, d_2 be common differences of the two A.Ps.

Sum of the n terms are

$S_n = \frac{n}{2}\{2a_1+(n-1)d_1\}$ and $S_n' = \frac{n}{2}\{2a_2+(n-1)d_2\}$

$\therefore \frac{S_n}{S_n'} = \frac{\frac{n}{2}\{2a_1+(n-1)d_1\}}{\frac{n}{2}\{2a_2+(n-1)d_2\}} = \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}$ [1]

Given,

$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$

$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}$ [1]

Divide both numerator and denominator on the LHS by 2.

As, $a_{19} = a + 8d$

Comparing with $\left\{a+\frac{(n-1)}{2}d\right\}$ we get,

$a+\frac{(n-1)}{2}d = a+8d$

$\Rightarrow \frac{(n-1)}{2} = 8$

After solving we get,

$\Rightarrow n = 17$ [1]

Therefore,

$\frac{a_9}{a_9'} = \frac{\{a_1+8d_1\}}{\{a_2+8d_2\}} = \frac{7(17)+1}{4(17)+27} = \frac{120}{95} = \frac{24}{19}$

Therefore the ratio of the 9^{th} terms of the two A.Ps is $\frac{24}{19}$. [1]

21. We know that sum of first n terms of an A.P is given by the formula

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

Similarly sum of first m terms of the same A.P. will be

$S_m = \frac{m}{2}\{2a+(m-1)d\}$

According to the Question

$\frac{S_m}{S_n} = \frac{m^2}{n^2}$ [1]

$\Rightarrow \frac{S_m}{S_n} = \frac{\frac{m}{2}(2a+(m-1)d)}{\frac{n}{2}(2a+(n-1)d)} = \frac{m^2}{n^2}$

$$\Rightarrow \frac{m\{2a+(m-1)d\}}{n\{2a+(n-1)d\}} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$ [1]

$$\Rightarrow n\times(2a+(m-1)d) = m\times(2a+(n-1)d)$$

$$\Rightarrow 2na+n(m-1)d = 2ma+m(n-1)d$$

$$\Rightarrow 2an+mnd-nd = 2am+mnd-md$$

$$\Rightarrow 2an-nd = 2am-md$$

$$\Rightarrow md-nd = 2am-2an$$

$$\Rightarrow (m-n)d = 2a(m-n)$$

$$\Rightarrow d = 2a$$

We know that n^{th} term of an A.P is given by

$$a_n = a+(n-1)d \quad \text{....(i)}$$

Similarly term of the A.P will be given by

$$a_m = a+(m-1)d \quad \text{....(ii)}$$

Ratio of m^{th} to the n^{th} term will be,

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$$ [1]

Put $d = 2a$

$$\Rightarrow \frac{a_m}{a_n} = \frac{a+(m-1)2a}{a+(n-1)2a}$$

$$\Rightarrow \frac{a_m}{a_n} = \frac{a+2am-2a}{a+2an-2a}$$

$$\Rightarrow \frac{a_m}{a_n} = \frac{a+2am-2a}{a+2an-2a}$$

$$\Rightarrow \frac{a_m}{a_n} = \frac{2am-a}{2an-a} = \frac{a(2m-1)}{a(2n-1)}$$

$$\Rightarrow \frac{a_m}{a_n} = \frac{2m-1}{2n-1}$$

So, the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$

Hence proved. [1]

22. Assume that the four consecutive numbers are $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

$(a-3d)+(a-d)+(a+d)+(a+3d) = 32$

$\Rightarrow 4a = 32$

$\Rightarrow a = 8$ [1]

Now the numbers are $(8-3d)$, $(8-d)$, $(8+d)$ and $(8+3d)$.

According to the Question

$$\frac{(8-3d)(8+3d)}{(8-d)(8+d)} = \frac{7}{15}$$ [1]

$$\Rightarrow \frac{(8^2-9d^2)}{(8^2-d^2)} = \frac{7}{15}$$

$$\Rightarrow \frac{(64-9d^2)}{(64-d^2)} = \frac{7}{15}$$

$$\Rightarrow 960-135d^2 = 448-7d^2$$

$$\Rightarrow 128d^2 = 512$$ [1]

Divide both sides by 128.

$$\Rightarrow d^2 = 4$$

$$\Rightarrow d = \pm 2$$

As the terms are consecutive hence the difference cannot be negative

Hence $d = 2$

And the four consecutive terms are:

$(8-3\times 2)$, $(8-2)$, $(8+2)$ and $(8+3\times 2)$.

Or 2, 6, 10, 14. [1]

Value Based

PREVIOUS YEARS' EXAMINATION QUESTIONS

4 Marks Questions

1. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this Question?

[TERM 2, 2014]

2. Ramkali required Rs 2500 after 12 weeks to send her daughter to school. She saved Rs 100 in the first week and increased her weekly saving by Rs 20 every week. Find whether she will be able to send her daughter to school after 12 weeks.

What value is generated in the above situation?

[TERM 2, 2015]

Solutions

1. It is given that number of trees planted by each class is double the class in which they are studying and each class has two sections.

Number of trees planted by class 1 $= 2 \times 2 = 4$

Number of trees planted by class 2 $= 2 \times 4 = 8$ [1]

Number of trees planted by class 3 $= 2 \times 6 = 12$ and so on.

Thus it forms an A.P. as 4, 8, 12, ... 48

Number of terms = total classes in school

$= n = 12$ [1]

Sum of n terms of an A.P is given by;

$$S_n = \frac{n}{2}(a + l)$$ [1]

So, sum of 12 terms of an A.P. = $S_{12} = \frac{12}{2}(4 + 48)$

$\Rightarrow S_{12} = 6(4 + 48) = 24 + 288$

$\Rightarrow S_{12} = 312$

Hence, 312 trees were planted by the students. [1]

2. Initial saving for first week is Rs100 and increases every week by Rs20, this forms an A.P., where the first term is $a = 100$ and difference, $d = 20$,

Now to find if required total saving is Rs2500 after 12 weeks, we need to find the sum of this AP for 12 terms. So, $n = 12$ [1½]

Using $S_n = \frac{n}{2}(2a + (n-1)d)$

$$S_{12} = \frac{12(2 \times 100 + (12-1)20)}{2}$$ [½]

$S_{12} = 6(200 + (11)20)$

$S_{12} = 6(200 + 220)$

$S_{12} = 6(420) = 2520$ [1]

Hence we can say that after 12 weeks of saving Ramkali will generate Rs 2520, which is more than 2500.

So she will be able to send her daughter to school after 12 weeks. [1]

Smart Notes ....

CHAPTER 6

Coordinate Geometry

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Distance Formula	1, 3 marks	1, 3 marks	1 mark	2 marks	2 marks	2, 3 marks
Section Formula	2 marks	2 marks		3 marks	2 marks	2 marks
Area of Triangle			2, 3 marks		3 ,4 marks	
Collinear of Points			4 marks	4 marks		
Midpoint Formula				2 marks		

[TOPIC 1] Distance between two Points and Section Formula

Summary

Coordinates of a Point

Location of the position of a point on a plane requires a pair of co-ordinate axes. The distance of a point from the x-axis is called its y-coordinate, or ordinate. The distance of a point from the y-axis is called its x-co-ordinate or abscissa. The co-ordinates of a point on the x-axis are of the form $(x, 0)$ and of a point on the y-axis are of the form $(0, y)$.

DISTANCE FORMULA

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is called the distance formula.

- ***In particular, the distance of a point P(x, y) from the origin O(0, 0) is given by***

$$OP = \sqrt{x^2 + y^2}$$

COLLINEAR POINTS

Three points A, B, C are said to be collinear if they lie on the same straight line.

Test For Collinearity of ThreePoints

In order to show that three given points A, B, C are collinear, we find distances AB, BC and AC. If the sum of any two of these distances is equal to the third distance, then the given points are collinear.

- *In a triangle , sum of any two sides is greater than the third side.*
- *Any point on x-axis is of the form (x, 0).*
- *Any point on y-axis is of the form (0, y).*
- *Circumcentre of a triangle is equidistant from its three vertices.*

Section Formula

Section formula : The coordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n are given by

$$x = \frac{mx_2 + nx_1}{m+n},\ y = \frac{my_2 + ny_1}{m+n}$$

Midpoint formula : The coordinates of the midpoint M of a line segment AB with end points $A(x_1, y_1)$ and $B(x_2, y_2)$ are $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

- *Diagonals of a parallelogram bisect each other.*

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The point P which divides the line segment joining the points $A(2, -5)$ and $B(5, 2)$ in the ratio 2 : 3 lies in the quadrant

   (*a*) I (*b*) II

   (*c*) III (*d*) IV

   [TERM 1, 2011]

2. The mid-point of segment AB is the point P(0, 4). If the coordinates of B are $(-2,3)$ then the coordinates of A are

   (*a*) $(2,5)$ (*b*) $(-2,-5)$

   (c) $(2,9)$ (*d*) $(-2,11)$

   [TERM 1, 2011]

3. The distance of the point $(-3,4)$ from the x-axis is

   (*a*) 3 (*b*) –3

   (*c*) 4 (d) 5

   [TERM 1, 2012]

4. In Figure , $P(5,-3)$ and $Q(3,y)$ are the points of trisection of the line segment joining $A(7,-2)$ and $B(1,-5)$. Then y equals?

(*a*) 2 (*b*) 4

(*c*) –4 (d) $-\frac{5}{2}$

[TERM 1, 2012]

5. ABCD is a rectangle whose three vertices are B (4, 0), C (4, 3) and D (0, 3). The length of one of its diagonals is

(*a*) 5 (*b*) 4

(*c*) 3 (*d*) 25

[TERM 1, 2014]

6. If the distance between the points (4,k) and (1,0) is 5,then what can be the possible values of *k*?

[TERM 1, 2014]

2 Marks Questions

7. If the distances of P(x,y)from A(5,1)and B(–1,5)are equal, then prove that 3x = 2y

[TERM 1, 2017]

8. Find that value(s) of x for which the distance between the points P(x, 4) and Q(9, 10) is 10 units.

[TERM 1, 2011]

9. Find the value of k, if the point P (2, 4) is equidistant from the points A(5, k)and(k, 7)

[TERM 1, 2012]

10. IfA(5, 2),B(2, –2) and C(–2, t)and are the vertices of a right angled triangle with $\angle B = 90^\circ$, then find the value of t.

[TERM 1, 2015]

11. Find the ratio in which the point $P\left(\frac{3}{4},\frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2},\frac{3}{2}\right)$ and B(2, –5).

[TERM 1, 2015]

12. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B. Find the value of p.

[TERM 1, 2015]

13. Find the ratio in which *y*-axis divides the line segment joining the points A(5, –6) and B(–1, –4). Also find the coordinates of the point of division.

[TERM 1, 2015]

14. The *x*-coordinate of a point P is twice its *y*-coordinate. If P is equidistant from Q(2, –5) and R(–3,6), find the coordinates of P.

[TERM 1, 2016]

15. Let P and Q be the points of trisection of the line segment joining the points A (2, -2) and B (-7, 4) such that P is nearer to A. Find the coordinates of P and Q.

[TERM 1, 2016]

16. Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

[TERM 1, 2016]

17. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, –5)is the mid-point of PQ, then find the coordinates of P and Q.

[TERM 1, 2017]

3 Marks Questions

18. If two vertices of an equilateral triangle are (3,0) and (6,0), find the third vertex.

[TERM 1, 2011]

19. Find the coordinates of a point P, which lies on the line segment joining the points A (–2, –2) and B(2, –4) such that AP = $\frac{3}{7}$ AB

[TERM 1, 2014]

20. Find the ratio in which the y-axis divides the line segment joining the points (–4, –6) and (10, 12) Also find the coordinates of the point of division.

[TERM 1, 2013]

21. If the point A(0, 2) is equidistant from the points B(3, p) and C(p, 5), find p. Also find the length of AB.

[TERM 1, 2014]

22. If the point P(*k* – 1, 2) is equidistant from the points A(3, *k*) and B(*k*, 5), find the values of *k*.

[TERM 1, 2014]

23. Find the ratio in which the line segment joining the points A(3, – 3) and B(– 2, 7) is divided by *x*-axis. Also find the coordinates of the point of division.

[TERM 1, 2015]

24. If the points A(–2, 1),B(a,b) and C(4, –1)are collinear and a – b =1, find the values of a and b.

[TERM 1, 2014]

25. If the point P (x, y) is equidistant from the points A (a + b, a – b) and B (a – b, a + b) Prove that bx = ay

[TERM 1, 2016]

26. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, -2) and Q(3, 7) ? Also find the value of y

[TERM 1, 2017]

4 Marks Question

27. Find the ratio in which the point $P(x, 2)$ divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$. Also find the value of x.

[TERM 1, 2014]

Solutions

1. Given that, Coordinate of point $A(2, -5)$ and $B(5, 2)$.

Let (x, y) be the coordinate of the point P, which divides the line segment AB in the ratio 2 : 3.

Coordinate of P are given by

$$=\left(\frac{2\times5+3\times2}{2+3},\frac{2\times2+3(-5)}{2+3}\right)$$ [½]

$$=\left(\frac{10+6}{5},\frac{4-15}{5}\right)$$

$$=\left(\frac{16}{5},\frac{-11}{5}\right)$$

Clearly, from the coordinate of P, x is positive and y is negative.

Therefore, P lies in IV quadrant.

Hence, the correct option is (d). [½]

2. Let the coordinates of A be (x, y)

According to the question, P is the midpoint of the line segment AB.

Coordinates of the midpoint of the line segment AB = Coordinate of the point P

$$\left(\frac{x-2}{2},\frac{y+3}{2}\right)=(0,4)$$ [½]

Equating the coordinate on both the sides.

$$\Rightarrow \frac{x-2}{2}=0 \text{ and } \frac{y+3}{2}=4$$

$\Rightarrow x - 2 = 0$ and $y + 3 = 8$

$\Rightarrow x = 2$ and $y = 5$

Therefore, the coordinate of point A is (2, 5)

Hence, the correct option is (a). [½]

3. We know that, y-coordinate or ordinate of a point is the distance from the x-axis.

Therefore, the distance of the point (–3, 4) from the x-axis is 4 units.

Option (c) is correct. [1]

4. P and Q are the points of trisection of AB,

$\therefore$ $AP = PQ = QB$

Thus, Q divide AB internally in the ratio 2 : 1.

Let $A(7, -2) \equiv (x_1, y_1)$ and $B(1, -5) \equiv (x_2, y_2)$.

Applying the section formula at point Q (3, y) where m : n ≡ 2 : 1, we have

$$y=\frac{my_2+ny_1}{m+n}$$ [½]

$$\Rightarrow y=\frac{2\times(-5)+1\times(-2)}{2+1}$$

$$\Rightarrow y=\frac{-10-2}{3}$$

$$\Rightarrow y=\frac{-12}{3}$$

$\Rightarrow y = -4$

Thus, the y-coordinate of point Q is –4. [½]

$\therefore$ Option (c) is correct.

5. For the rectangle $ABCD$, the diagonals are AC and BD.

Distance between two points

$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$ [½]

Let $B = (x_1, y_1) = (4, 0)$ and $D = (x_2, y_2) = (0, 3)$

Length of $BD = \sqrt{(0-4)^2+(3-0)^2}$

Length of $BD = \sqrt{(4)^2+(3)^2}$

Length of $BD = \sqrt{16+9} = \sqrt{25} = 5$

The correct answer is (a). [½]

6. We know that distance d between two points is given by

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$ [½]

It is given that the distance between the two points is 5

Putting the values

$$\Rightarrow 5=\sqrt{(4-1)^2+(k-0)^2}$$

On squaring both sides,

$\Rightarrow 25 = 9 + k^2$

$\Rightarrow 25 - 9 = k^2$

$\Rightarrow 16 = k^2$

$\Rightarrow k = \pm 4$

Hence, the possible values of k are 4 and –4. [½]

7. Given, $PA = PB$

Using distance formula,

$$\Rightarrow \sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2} \quad [1]$$

Squaring both sides

$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 - 10y + 25$

$\Rightarrow -10x - 2y = 2x - 10y$

Dividing both sides by 4

$\Rightarrow 3x = 2y$

Hence proved. [1]

8. Given that, the distance between the points $P(x, 4)$ and $Q(9, 10)$ is 10 units.

Using the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between the point P and Q is given by:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given, $PQ = 10$

$$10 = \sqrt{(9-x)^2 + (10-4)^2}$$

$$10 = \sqrt{(9-x)^2 + (6)^2}$$

$$10 = \sqrt{(9-x)^2 + 36} \quad [1]$$

On squaring both the sides.

$$(10)^2 = \left(\sqrt{(9-x)^2 + 36}\right)^2$$

$\Rightarrow 100 = (9-x)^2 + 36$

$\Rightarrow 100 = 81 + x^2 - 18x + 36$

$\Rightarrow 100 = x^2 - 18x - 117$

$\Rightarrow x^2 - 18x + 117 - 100 = 0$

$\Rightarrow x^2 - 18x + 17 = 0$

$\Rightarrow x(x-17) - 1(x-17) = 0$

$\Rightarrow (x-17)(x-1) = 10$

$\Rightarrow (x-17) = 0$ or $(x-1) = 0$

$\Rightarrow x = 17$ or $x = 1$

Therefore, the coordinates of the point P could be (17, 4) or (1, 4). [1]

9. Since P is equidistant from points A and B,

$\therefore \; PA = PB$

$$\Rightarrow \sqrt{(5-2)^2 + (k-4)^2} = \sqrt{(k-2)^2 + (7-4)^2}$$

$$\Rightarrow \sqrt{9 + (k-4)^2} = \sqrt{(k-2)^2 + 9} \quad [1]$$

On squaring both sides.

$\Rightarrow 9 + (k-4)^2 = (k-2)^2 + 9$

$\Rightarrow (k-4)^2 = (k-2)^2$

$\Rightarrow k^2 + 16 - 8k = k^2 + 4 - 4k$

$\Rightarrow 16 - 8k = 4 - 4k$

$\Rightarrow 4k = 12$

$\Rightarrow k = 3$

Therefore, the value of k is 3. [1]

10. Given $A(5, 2)$, $B(2, -2)$ and $C(-2, t)$ as the vertices of a right angled triangle with with $\angle B = 90°$

Distance between the two points is given by the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\Rightarrow \text{Distance } AB = \sqrt{(2-5)^2 + (-2-2)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25}$$

$\Rightarrow AB = 5$

Now,

$$\text{Distance } BC = \sqrt{(-2-2)^2 + (t+2)^2}$$

$$= \sqrt{16 + t^2 + 4 + 4t}$$

$$\Rightarrow BC = \sqrt{t^2 + 4t + 20}$$

$$\text{Distance } CA = \sqrt{(5+2)^2 + (2-t)^2} \quad [1]$$

$$= \sqrt{49 + 4 + t^2 - 4t}$$

$$\Rightarrow CA = \sqrt{t^2 - 4t + 53}$$

Using Pythagoras theorem we get,

$(AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow \; t^2 - 4t + 53 = 25 + t^2 + 4t + 20$

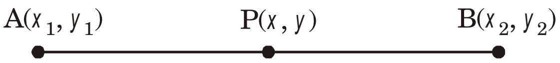
$\Rightarrow \; -8t = 45 - 53$

$\Rightarrow \; -8t = -8$

$\Rightarrow \; t = 1$

Hence, the value of t is 1. [1]

11. Let P divides the line segment joining the points A and B in the ratio $m : n$.



Then the coordinates of point P is given by the section formula,

$$P=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

Where (x_1, y_1) and (x_2, y_2) are the coordinates of the points joined to make the line segment.

Here $P=\left(\frac{3}{4},\frac{5}{12}\right)$. Let $(x_1, y_1)=\left(\frac{1}{2},\frac{3}{2}\right)$ and $(x_2, y_2) = (2, -5)$

Using the section formula we get,

$$\left(\frac{3}{4},\frac{5}{12}\right)=\left(\frac{2m+\frac{1}{2}n}{m+n},\frac{-5m+\frac{3}{2}n}{m+n}\right)$$ [1]

Comparing the x and y coordinates of both the sides.

$$\Rightarrow \frac{3}{4}=\frac{2m+\frac{1}{2}n}{m+n} \text{ and } \frac{5}{12}=\frac{-5m+\frac{3}{2}n}{m+n}$$

Using $\frac{3}{4}=\frac{2m+\frac{1}{2}n}{m+n}$ we get,

$$3(m+n)=4\left(2m+\frac{1}{2}n\right)$$

$\Rightarrow 3m + 3n = 8m + 2n$

$\Rightarrow 3n - 2n = 8m - 3m$

$\Rightarrow n = 5m$

Hence, P divides the given line segment in the ratio 1 : 5. [1]

12. $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$ (Given)

The coordinate of $A(4, 7)$ and $B(p, 3)$ so, $x_2 = p$, $x_1 = 4$, $y_2 = 3$ and $y_1 = 7$

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$AB=\sqrt{(p-4)^2+(3-7)^2}$$

$$AB=\sqrt{p^2-8p+16+16}$$

On squaring both sides.

$AB^2 = p^2 - 8p + 32$

Similarly,

$$BC=\sqrt{(p-7)^2+(3-3)^2}$$

$$BC=\sqrt{p^2+49-14p}$$

On squaring both the sides.

$BC^2 = p^2 + 49 - 14p$

And $AC=\sqrt{(4-7)^2+(7-3)^2}$

$$AC=\sqrt{9+16}$$

On squaring both the sides.

$AC^2 = 25$ [1]

ABC is a right angled triangle.

By Pythagoras theorem

$AB^2 + BC = AC^2$

Putting the value of AB, BC and AC in Pythagoras theorem

$p^2 - 8p + 32 + p^2 + 49 - 14p = 25$

$\Rightarrow 2p^2 - 22p + 81 = 25$

$\Rightarrow 2p^2 - 22p + 56 = 0$

$\Rightarrow p^2 - 11p + 28 = 0$

$\Rightarrow p^2 - 7p - 4p + 28 = 0$

$\Rightarrow p(p - 7) - 4(p - 7) = 0$

$\Rightarrow (p - 4)(p - 7) = 0$

$\Rightarrow p = 4, 7$

Here, if $p = 7$ point B and C will coincide.

A, B and C are given vertices of a triangle, therefore, $p \neq 7$

Hence, $p = 4$ [1]

13. Let $(0, a)$ be a point on the y-axis dividing the line segment AB in the ratio $k : 1$.

We make use of section formula to get

$$(0,a)=\left(\frac{-k+5}{k+1},\frac{-4k-6}{k+1}\right)$$

Comparing the x and y coordinate on both the sides.

$$\Rightarrow \frac{-k+5}{k+1}=0,\ \frac{-4k-6}{k+1}=a$$

Solving each one independently,

$$\Rightarrow \frac{-k+5}{k+1}=0$$

$\Rightarrow -k + 5 = 0$

$\Rightarrow k = 5$ [1]

and

$$\Rightarrow \frac{-4k-6}{k+1} = a$$

Substituting $k = 5$

$$\Rightarrow \frac{-4 \times 5 - 6}{5+1} = a$$

$$\Rightarrow a = \frac{-26}{6}$$

$$\Rightarrow a = \frac{-13}{3}$$

Thus, the y-axis divide the line segment in the ratio 5 : 1.

Hence, the coordinates of point of division are $\left(0, \frac{-13}{3}\right)$. [1]

14. Let the y-coordinate of the point be a.

Then according to the question, x-coordinate will be $2a$.

So, the coordinates of the point P are$(2a, a)$.

Since, the point P $(2a, a)$ is equidistant from Q(2,-5) and R (-3,6).

So we can use distance formula such that,

$$\sqrt{(2a-2)^2 + (a-(-5))^2} = \sqrt{(2a-(-3))^2 + (a-6)^2}$$

$$\Rightarrow \sqrt{(2a-2)^2 + (a+5)^2} = \sqrt{(2a+3)^2 + (a-6)^2}$$

$$\Rightarrow \sqrt{4a^2 + 4 - 8a + a^2 + 25 + 10a} = \sqrt{4a^2 + 9 + 12a + a^2 + 36 - 12a}$$

$$\Rightarrow \sqrt{5a^2 + 2a + 29} = \sqrt{5a^2 + 45}$$ [1]

Squaring both sides,

$\Rightarrow 5a^2 + 2a + 29 = 5a^2 + 45$

$\Rightarrow 5a^2 + 2a - 5a^2 = 45 - 29$

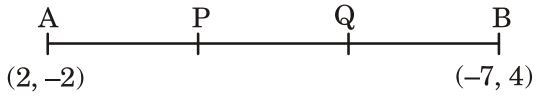
$\Rightarrow 2a = 16$

$\Rightarrow a = 8$

Since $a = 8$, then x-coordinate is $2a$ i.e. $2 \times 8 = 16$.

Hence, the coordinates of the point P are (16, 8). [1]

15. P 1and Q are the points of trisection. Hence, $AP = PQ = QB$



Thus, P divides AB internally in the ratio 1 : 2 and Q divides AB internally in the ratio 2 : 1.

Using section formula, coordinates of P are,

$$P = \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

$$P = \left(\frac{-3}{3}, 0\right) = (-1, 0)$$

$P = (-1, 0)$ [1]

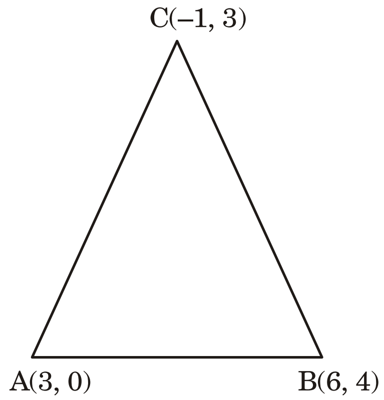
Similarly, coordinates of Q are,

$$Q = \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right) = \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$$

$$Q = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

$Q = (-4, 2)$ [1]

16. Let the coordinates for point A (3, 0), coordinates for point B (6, 4) and the coordinates for point C (-1, 3).



Using Distance formula,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(6-3)^2 + (4-0)^2}$$

$$AB = \sqrt{25} = 5$$

Similarly,

$$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{50}$$

$$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{25} = 5$$

So, two sides are equal. Hence, $AC = AB$. [1]

Now, apply the Pythagoras theorem on the given triangle. If $AB^2 + AC^2 = BC^2$ then it is right angled isosceles triangle because it already has two equal sides.

So, $BC^2 = (\sqrt{50})^2 = 50$ and AB 2 = 5^2 = 25,
$AC^2 = 5^2 = 25$
$AB^2 + AC^2 = 25 + 25 = 50 = BC^2$
Hence, the given triangle is right angled isosceles triangle. [1]

17. Given,
$P(0, y)$
$Q(x, 0)$
Midpoint of $PQ = (2, -5)$
Using midpoint formula,

$$\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$$

$$\Rightarrow \left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$$ [1]

On comparing the x and y coordinates of both the sides.

$$\Rightarrow \frac{x}{2} = 2 \text{ and } \frac{y}{2} = -5$$

$\Rightarrow x = 4, y = -10$

Therefore, coordinates of P and Q are (0, –10) and (4, 0) respectively. [1]

18. Let the equilateral triangle be ABC and its vertices be $A(3, 0)$, $B(6, 0)$ and $C(x, y)$
Now as we know that length of each sides of an equilateral triangle are same,
$AB = BC = AC$

$\Rightarrow AB^2 = BC^2 = AC^2$

Now let's take, $AB^2 = BC^2$
$\Rightarrow (6-3)^2 + (0-0)^2 = (x-6)^2 + (y-0)^2$
$\Rightarrow (3)^2 = 36 + x^2 - 12x + y^2$
$\Rightarrow (x^2 - 12x + y^2 + 27) = 0$ (1) [1]
Now, $AB^2 = AC^2$
$\Rightarrow (6-3)^2 + (0-0)^2 = (x-3)^2 + (y-0)^2$
$\Rightarrow (3)^2 = 9 + x^2 - 6x + y^2$
$\Rightarrow x^2 - 6x + y^2 = 0$ (2)
Subtract equation (2) from equation (1),
$\Rightarrow (x^2 - 12x + y^2 + 27) - (x^2 - 6x + y^2) = 0$
$\Rightarrow -6x + 27 = 0$
$\Rightarrow -6x = -27$

$$\Rightarrow x = \frac{9}{2}$$ [1]

Substituting this value in equation (2)

$$\Rightarrow \left(\frac{9}{2}\right)^2 - 6 \times \left(\frac{9}{2}\right) + y^2 = 0$$

$$\Rightarrow \frac{81}{4} - 27 + y^2 = 0$$

$$\Rightarrow y^2 = \frac{108-81}{4}$$

$$\Rightarrow y^2 = \frac{27}{4}$$

$$\Rightarrow y = \pm\frac{3\sqrt{3}}{2}$$

Hence the third vertex could be $C\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$ or $C\left(\frac{9}{2}, -\frac{3\sqrt{3}}{2}\right)$ [1]

19. Let the coordinates of point P be (x, y)
Now, it is given that,

$$AP = \frac{3}{7} AB$$

$$\Rightarrow AP = \frac{3}{7}(AP + PB)$$

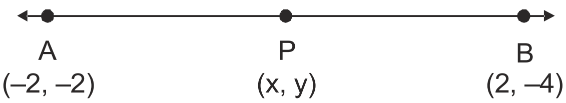
$$\Rightarrow 7AP = 3(AP + PB)$$

$$\Rightarrow 7AP - 3AP = 3PB$$

$$\Rightarrow 4AP = 3PB$$

$$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$$ [1]

$\therefore$ The point P divides the line segment AB in the ratio 3 : 4.



$m = 3$, $n = 4$, $x_1 = -2$, $x_2 = 2$, $y_1 = -2$, $y_2 = -4$
Therefore, using section formula,

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$(x, y) = \left(\frac{3(2)+4(-2)}{3+4}, \frac{3(-4)+4(-2)}{3+4}\right)$$

Comparing x and y coordinates on both the sides.

$$\Rightarrow x = \frac{3\times 2+4\times(-2)}{7}$$

$$\Rightarrow x = \frac{6-8}{7}$$ [1]

$$\Rightarrow x = \frac{-2}{7}$$

And,

$$\Rightarrow y = \frac{3\times(-4)+4\times(-2)}{7}$$

$$\Rightarrow y = \frac{-12-8}{7}$$

$$\Rightarrow y = \frac{-20}{7}$$

Therefore, the coordinates of $P(x, y) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

[1]

20. Let us assume a point such that that the line joining the points (–4, –6) and (10, 12) in the ratio $k : 1$

Let this point on the y axis be $(0, y)$

Now using the section formula

$$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(0, y) = \left(\frac{10k-4}{k+1}, \frac{12k+(-6)}{k+1}\right)$$ [1]

On comparing the x coordinate of both the sides.

$$\Rightarrow \frac{10k+(-4)}{k+1} = 0 \Rightarrow 10k-4 = 0 \Rightarrow k = \frac{4}{10} = \frac{2}{5}$$

and, $y = \frac{12k-6}{k+1}$ [1]

Substituting the value of k,

$$y = \frac{12\times\frac{2}{5}-6}{\frac{2}{5}+1}$$

$$\Rightarrow y = -\frac{6}{7}$$

Hence, the y axis is dividing the line in the ratio

2 : 5 at point $\left(0, -\frac{6}{7}\right)$ [1]

21. The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

It is given that A is equidistant from B and C.

$\therefore \quad AB = AC$ [1]

Squaring both the sides

$AB^2 = AC^2$

$(3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$9 + p^2 + 4 - 4p = p^2 + 9$

$4 - 4p = 0$

$4p = 4$

$p = 1$ [1]

Thus, the value of p is 1.

Length of AB

$= \sqrt{(3-0)^2+(p-2)^2}$

Substituting the value of p.

$= \sqrt{(3-0)^2+(1-2)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ units [1]

22. Distance between two points

$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance between $A(3, k)$ and $P(k-1, 2)$

$= \sqrt{(k-1-3)^2+(2-k)^2}$ [1]

$AP = \sqrt{(k-4)^2+(2-k)^2}$

Apply the identity $(A-B)^2 = A^2 - 2AB + B^2$

$AP = \sqrt{k^2-8k+16+4-4k+k^2}$

$AP = \sqrt{2k^2-12k+20}$

Distance between $B(k, 5)$ and $P(k - 1, 2)$

$= \sqrt{(k-1-k)^2+(2-5)^2}$

$BP = \sqrt{(-1)^2+(-3)^2}$

$BP = \sqrt{1+9}$

$BP = \sqrt{10}$ [1]

Since P is equidistant from A and B, $AP = BP$

$\sqrt{2k^2-12k+20} = \sqrt{10}$

Squaring both sides, we get

$2k^2 - 12k + 20 = 10$

Write the equation in the form $ax^2 + bx + c = 0$

$2k^2 - 12k + 10 = 0$

$k^2 - 6k + 5 = 0$

Split the middle term

$k^2 - 5k - k + 5 = 0$

$k(k - 5) - 1(k - 5) = 0$

$(k - 5)(k - 1) = 0$

Set each factor to zero

$k = 5$ or $k = 1$

The values of k are 1, 5. [1]

23. Let the x-axis divide the line segment AB at $K(x, 0)$ in the ratio $m : 1$

Let the coordinates of A be (x_1, y_1) and that of B be (x_2, y_2)

By section formula coordinates of K are

$$\left(\frac{mx_2 + x_1}{m+1}, \frac{my_2 + y_1}{m+1}\right) \quad ....(i)$$

$$(x, 0) = \left(\frac{mx_2 + x_1}{m+1}, \frac{my_2 + y_1}{m+1}\right) \quad [1]$$

Comparing the y coordinates on both the sides.

$$\frac{my_2 + y_1}{m+1} = 0$$

Substitute the values of y_1 and y_2

$$\frac{m \cdot 7 - 3}{m+1} = 0$$

$7m - 3 = 0$

$$m = \frac{3}{7}$$

Therefore the x-axis divides the line segment AB in the ratio $\frac{3}{7} : 1 = 3 : 7$ [1]

Substituting the value of m in (i),

$$\text{Coordinates of } K = \left(\frac{\frac{3}{7}\cdot(-2) + 3}{\frac{3}{7} + 1}, 0\right)$$

$$\text{Coordinates of } K = \left(\frac{-\frac{6}{7} + 3}{\frac{3}{7} + 1}, 0\right)$$

$$\text{Coordinates of } K = \left(\frac{\frac{21-6}{7}}{\frac{3+7}{7}}, 0\right)$$

$$\text{Coordinates of } K = \left(\frac{\frac{15}{7}}{\frac{10}{7}}, 0\right)$$

$$\text{Coordinates of } K = \left(\frac{3}{2}, 0\right) \quad [1]$$

24. The given points are $A(-2, 1)$, $B(a, b)$ and $C(4, -1)$

Since the given points are collinear, the area of the triangle ABC is zero.

Area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$$

Here $x_1 = -2$, $y_1 = 1$, $x_2 = a$, $y_2 = b$ and $x_3 = 4$, $y_3 = -1$ [1]

$$\frac{1}{2}\left[-2(b+1) + a(-1-1) + 4(1-b)\right] = 0$$

$-2b - 2 - 2a + 4 - 4b = 0$

$2a + 6b = 2$

Divide above equation by 2,

$a + 3b = 1$ ....(1)

Given:

$a - b = 1$ ....(2) [1]

Subtracting equation (1) from (2)

$4b = 0$

$b = 0$

Subtracting $b = 0$ in (2).

$a - 0 = 1$

$a = 1$

Thus, the values of a and b are1 and 0 respectively. [1]

25. According to the question, point P is equidistant from the points A and point B.

Hence, $PA = PB$

Using the distance formula, distance

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\sqrt{(x-(a+b))^2 + (y-(b-a))^2} = \sqrt{(x-(a-b))^2 + (y-(a+b))^2} \quad [1]$$

$$x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a) =$$
$$x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$$

$$-2x(a+b)-2y(b-a)=-2x(a-b)-2y(a+b)$$ [1]

$$-2x(a+b)+2x(a-b)=-2y(a+b)+2y(b-a)$$

$$2x(-a-b+a-b)=2y(-a-b+b-a)$$

$$x(-2b)=y(-2a)$$

$$-2bx=-2ay$$

$$bx=ay$$

Hence proved. [1]

26. Section Formula $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

Let the given point divide line segment in ratio $x:1$

Using section formula and point $\left(\frac{24}{11},y\right)$

$$\left(\frac{24}{11},y\right)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$ [1]

Comparing the x and y coordinate.

$$\frac{24}{11}=\frac{3x+2}{x+1},\ y=\frac{7x-2}{x+1}$$

For $\frac{24}{11}=\frac{3x+2}{x+1}$

$24(x+1)=33x+22$

$\Rightarrow 24x+24=33x+22$

$\Rightarrow 2=9x$ [1]

Now, for $y=\frac{7x-2}{x+1}$

$$y(x+1)=7x-2$$

$$\Rightarrow xy+y=7x-2$$

Put $x=\frac{2}{9}$

$$\Rightarrow \frac{2}{9}y+y=7\times\frac{2}{9}-2$$

$$\Rightarrow \frac{2}{9}y+y=\frac{14}{9}-2$$

$$\Rightarrow \frac{11}{9}y=\frac{-4}{9}$$

$$\Rightarrow y=\frac{-4}{11}$$

Therefore, the point $\left(\frac{24}{11},\frac{-4}{11}\right)$ divides the line PQ in ratio 2 : 9. [1]

27. Let us assume the point P divides the line segment joining the points A (12, 5) and B (4, –3) in the ratio $r:1$

Using section formula $\left(\frac{mx_2+x_1}{m+1},\frac{my_2+y_1}{m+1}\right)$

Then the coordinates of P are, $\left(\frac{4r+12}{r+1},\frac{-3r+5}{r+1}\right)$

[1]

Given, the coordinates of P are $(x, 2)$

$$\therefore (x,2)=\left(\frac{4r+12}{r+1},\frac{-3r+5}{r+1}\right)$$

$$\therefore \frac{4r+12}{r+1}=x \text{ and } \frac{-3r+5}{r+1}=2$$ [1]

Now, $\frac{-3r+5}{r+1}=2$

$$\Rightarrow -3r+5=2(r+1)$$

$$\Rightarrow 5r=3$$

$$\Rightarrow r=\frac{3}{5}$$

Substitute $r=\frac{3}{5}$ in $\frac{4r+12}{r+1}=x$, we get

$$\Rightarrow x=\frac{4\left(\frac{3}{5}\right)+12}{\frac{3}{5}+1}$$ [1]

$$\Rightarrow x=\frac{\frac{12}{5}+12}{\frac{3}{5}+1}$$

$$\Rightarrow x=\frac{\frac{72}{5}}{\frac{8}{5}}$$

$$\Rightarrow x=\frac{72}{8}$$

$$\Rightarrow x=9$$

Hence, the value of x is 9 and the point $P(x, 2)$ divides the line segment joining the points A (12, 5) and B (4, –3) in the ratio $\frac{3}{5}:1=3:5$ [1]

[TOPIC 2] Centroid and Area of Triangle

Summary

Centroid of a Triangle

The coordinates of the centroid of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by $\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$.

Area of a Triangle

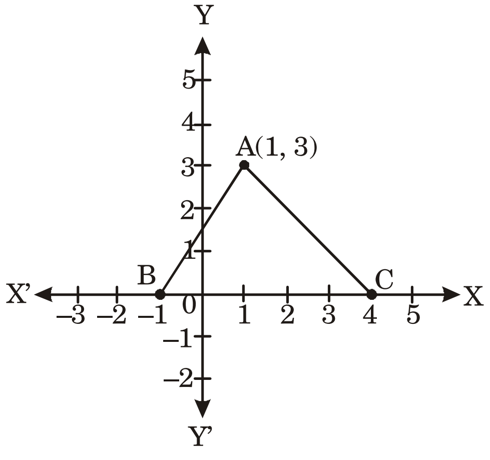
The area of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by area

$$(\Delta ABC) = \left|\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\right|.$$

PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 2

1 Mark Questions

1. In Fig, find the area of triangle ABC (in sq. units) is:



(a) 15

(b) 10

(c) 7.5

(d) 2.5

[TERM 1, 2013]

2. If the points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear, then the value of x is:

(a) –63 (b) 63

(c) 60 (d) –60

[TERM 1, 2014]

2 Marks Question

3. Find the relation between x and y if the points A(x, y),B(–5, 7) and $C(-4,5)$ are collinear.

[TERM 1, 2015]

3 Marks Questions

4. If (3,3), (6,y), (x,7) and (5,6) are the vertices of a parallelogram taken in order, find the values of x and y

[TERM 1, 2011]

5. Find the value of k, if the points P(5,4), Q(7,k) and R(9,-2) are collinear.

[TERM 1, 2011]

6. Find the area of the quadrilateral ABCD whose vertices are A(–3, –1), B(–2, –4), C(4, –1) and D(3, 4).

[TERM 1, 2012]

7. If the points A(x, y), B(3, 6) and C(-3, 4) are collinear, show that x – 3y + 15 = 0.

[TERM 1, 2012]

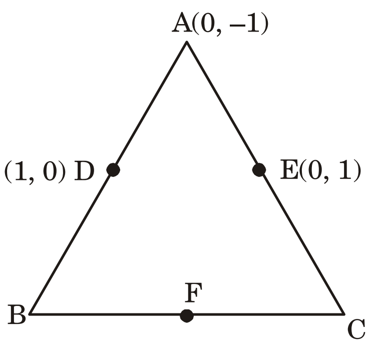
8. Prove that the points(7, 10),(–2, 5)and(3, -4) are the vertices of an isosceles right triangle.

[TERM 1, 2013]

9. Find the area of the triangle ABC with (1, –4) and mid-points of sides through A being (2, –1) and.(0, –1)

[TERM 1, 2015]

10. In the given figure, ABC is a triangle coordinates of whose vertex A are (0, –1)D and E respectively are the mid-points of the sides AB and AC and their coordinates are(1, 0)and (0, 1) respectively. If F is the mid-point of BC, find the areas of ΔABC and ΔDEF.



[TERM 1, 2016]

11. Show that ΔABC ,whereA(–2, 0), B(2, 0), C(0, 2) and ΔPQR whereP (–4, 0),Q (4, 0) R(4, 0)are similar triangles.

[TERM 1, 2017]

12. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, –2) . If the third vertex is $\left(\frac{7}{2}, y\right)$,find the value of y.

[TERM 1, 2017]

13. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides. $A(x_1, y_1)$

[DELHI, 2018]

4 Marks Questions

14. The three vertices of a parallelogram $ABCD$ are $A(3, -4)$, $B(-1, -3)$ and $C(-6, 2)$. Find the coordinates of vertex D and find the area of parallelogram $ABCD$.

[TERM 1, 2013]

15. If $A(-3, 5)$, $B(-2, -7)$, $C(1, -8)$ and $D(6, 3)$ are the vertices of a quadrilateral $ABCD$, find its area.

[TERM 1, 2014]

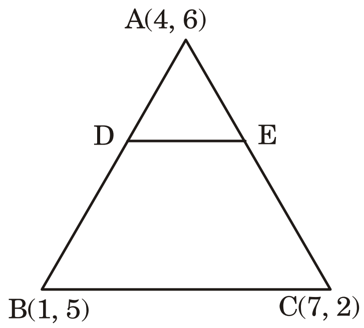
16. If $A(-4, 8)$, $B(-3, -4)$, $C(0, -5)$ and $D(5, 6)$ are the vertices of a quadrilateral $ABCD$, find its area.

[TERM 1, 2015]

17. Find the values of so that the area of the triangle with vertices (1, –1), (–4, 2k) and (–k, –5) is 24 square units.

[TERM 1, 2015]

18. In fig., the vertices of ΔABC are $A(4, 6)$, $B(1, 5)$, $C(7, 2)$. A line segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of and compare it with area of ΔABC.



[TERM 1, 2016]

19. If the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear, then find the value of k.

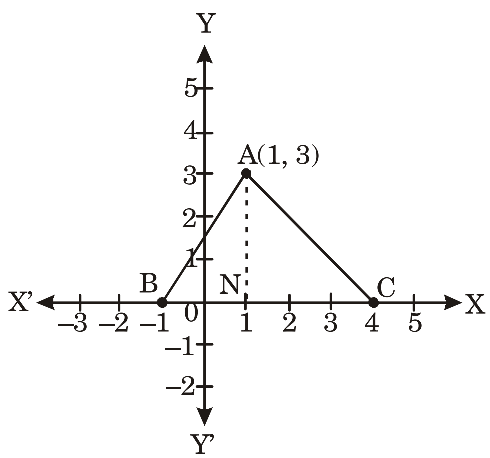
[TERM 1, 2017]

20. If a ≠ b ≠ 0, prove that the points (a, a^2), (b, b^2), (0, 0) will not be collinear.

[TERM 1, 2017]

Solutions

1.



[½]

Construction: Draw AN ⊥ BC.

Here, BC = 4 – (–1) = 5 units and AN = 3 units.

In ΔABC,

The base is BC and the height is AN.

Now,

$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times \text{ BC } \times \text{ AN}$$

$$= \frac{1}{2} \times 5 \times 3$$

= 7.5 Sq. Units.

Thus the area of the given triangle is 7.5 sq units.

Hence the option "c" is the correct answer. [½]

2. It is given that the three points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear.

Area of $\Delta ABC = 0$ (Points are collinear)

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

[½]

Here, $x_1 = x$, $y_1 = 2$, $x_3 = -3$, $y_2 = -4$ and $x_3 = 7$, $y_3 = -5$

$x[-4 - (-5)] - 3(-5 - 2) + 7[2 - (-4)] = 0$

$x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0$

$x - 3 \times (-7) + 7 \times 6 = 0$

$x + 21 + 42 = 0$

$x + 63 = 0$

$x = -63$

Hence, the correct option is (a). [½]

3. If A, B, C are collinear then area of $\Delta ABC = 0$

Area of the triangle with coordinates (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by-

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$ [1]

Now, if A, B, C are collinear, then

$\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}\left|x(7-5) - 5(5-y) - 4(y-7)\right| = 0$$

$$\Rightarrow \frac{1}{2}\left|2x - 25 + 5y - 4y + 28\right| = 0$$

$$\Rightarrow \frac{1}{2}\left|2x + y + 3\right| = 0$$

$\Rightarrow 2x + y + 3 = 0$

$\Rightarrow 2x + y = -3$

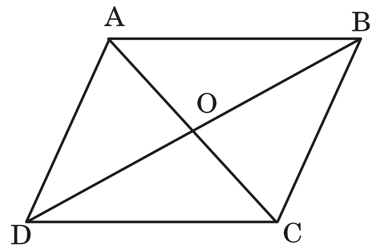
Hence, the relation between x and y is $2x + y = 3$.

[1]

4. Let A, B, C and D be the vertices of the parallelogram.

Therefore, the coordinates of the vertices are $A(3, 3)$, $B(6, y)$, $C(x, 7)$ and $D(5, 6)$.

[1]



We know that the diagonals of a parallelogram bisect each other. Therefore, the coordinates of the midpoint of AC are the same as the coordinates of the midpoint of BD, i.e.

$$\left(\frac{3+x}{2}, \frac{3+7}{2}\right) = \left(\frac{6+5}{2}, \frac{y+6}{2}\right)$$

$$\Rightarrow \left(\frac{3+x}{2}, \frac{10}{2}\right) = \left(\frac{11}{2}, \frac{y+6}{2}\right)$$ [1]

Comparing the x and y coordinates on both the sides.

$$\frac{3+x}{2} = \frac{11}{2} \text{ and } \frac{10}{2} = \frac{y+6}{2}$$

$\Rightarrow 3 + x = 11$ and $y + 6 = 10$

$\Rightarrow x = 8$ and $y = 4$

Therefore, the values of x and y are 8 and 4 respectively. [1]

5. Since the points $P(5, 4)$, $Q(7, k)$ and $R(9, -2)$ are collinear.

Then area of $\Delta PQR = 0$ [1]

Area of the triangle with coordinates (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by-

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$$ [1]

$$\Rightarrow \frac{1}{2}\left|5(k-(-2)) + 7(-2-4) + 9(4-k)\right| = 0$$

$$\Rightarrow \frac{1}{2}\left|5(k+2) - 42 + 9(4-k)\right| = 0$$

$\Rightarrow |5k + 10 - 42 + 36 - 9k| = 0$

$\Rightarrow |4k + 4| = 0$

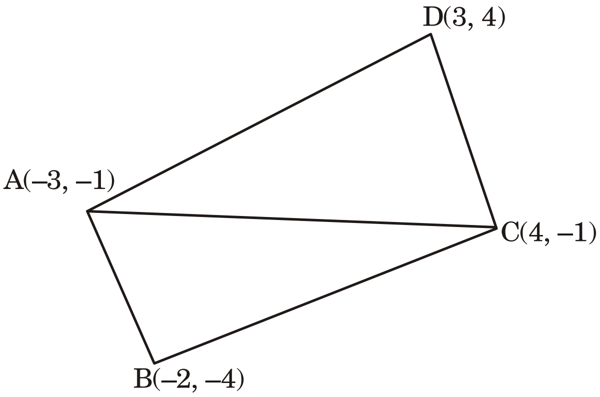
$\Rightarrow -4k + 4 = 0$

$\Rightarrow -4k = -4$

$\Rightarrow k = 1$

Hence the value of $k = 1$. [1]

6.



The vertices of quadrilateral are A (–3, –1), B(–2, –4), C(4, –1) and D(3, 4).

Let us join AC to split the quadrilateral $ABCD$ into two triangles ABC and ADC.

Therefore,

Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔADC. [1]

Area of a triangle is given by

$$\text{Area of } \Delta = \frac{1}{2}\left[\begin{array}{l} x_1(y_2 - y_3) + x_2(y_3 - y_1) + \\ x_3(y_1 - y_2) \end{array}\right]$$

For ΔABC, $(x_1, y_1) = (-3, -1)$, $(x_2, y_2) = (-2, -4)$ and $(x_3, y_3) = (4, -1)$

Substituting the values, we have

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2}\left[\begin{array}{l} x_1(y_2 - y_3) + x_2(y_3 - y_1) + \\ x_3(y_1 - y_2) \end{array}\right]$$

$$= \frac{1}{2}\left[-3(-4+1) - 2(-1+1) + 4(-1+4)\right]$$

$$= \frac{1}{2}[9 - 0 + 12]$$

$$= \frac{1}{2} \times 21$$

$$= \frac{21}{2}$$ [1]

Similarly, For ΔADC, $(x_1, y_1) \equiv (-3, -1)$, $(x_2, y_2) \equiv (3, 4)$ and $(x_3, y_3) = (4, -1)$

Substituting the values, we have

$$\text{Area of } \Delta\text{ADC} = \frac{1}{2}\left[\begin{array}{l} x_1(y_2 - y_3) + x_2(y_3 - y_1) + \\ x_3(y_1 - y_2) \end{array}\right]$$

$$= \frac{1}{2}\left[-3(4+1) + 3(-1+1) + 4(-1-4)\right]$$

$$= \frac{1}{2}\left[-3(4+1) + 3(-1+1) + 4(-1-4)\right]$$

$$= \frac{1}{2} \times -35$$

$$= -\frac{35}{2}$$

However, area can't be negative,

$$\text{Area of } \Delta\text{ADC} = \frac{1}{2}|-35|$$

$$= \frac{1}{2} \times 35$$

$$= \frac{35}{2}$$

Therefore, Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔADC.

$$= \frac{21}{2} + \frac{35}{2}$$

$$= \frac{56}{2}$$

= 28 square units

Therefore, Area of quadrilateral $ABCD$ is 28 sq. units. [1]

7. If the points $A(x, y)$, $B(3, 6)$ and $C(-3, 4)$ are collinear, then they all are points lying on a straight line.

Hence, the area of the triangle formed with these three points A, B and C will be zero.

For ΔABC,

$$(x_1, y_1) \equiv (x, y), (x_2, y_2) \equiv (3, 6) \text{ and } (x_3, y_3) \equiv (-3, 4)$$ [1]

Substituting the values, we have

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2}\left[\begin{array}{l} x_1(y_2 - y_3) + x_2(y_3 - y_1) + \\ x_3(y_1 - y_2) \end{array}\right] = 0$$

$$\frac{1}{2}\left[x(6-4) + 3(4-y) - 3(y-6)\right] = 0$$

$$\frac{1}{2}[6x - 4x + 12 - 3y - 3y + 18] = 0$$ [1]

$$\frac{1}{2} \times 2[x - 3y + 15] = 0$$

$x - 3y + 15 = 10$

Hence, proved. [1]

8. Let the points be $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$

Using distance formula,

$$AB = \sqrt{(7+2)^2 + (10-5)^2} = \sqrt{81+25} = \sqrt{106}$$

$$BC = \sqrt{(-2-3)^2 + (5+4)^2} = \sqrt{25+81} = \sqrt{106}$$

$$CA = \sqrt{(7-3)^2 + (10+4)^2} = \sqrt{16+196} = \sqrt{212}$$ [1]

Here,

$AB = BC$

$\Rightarrow$ ΔABC is an isosceles triangle. [1]

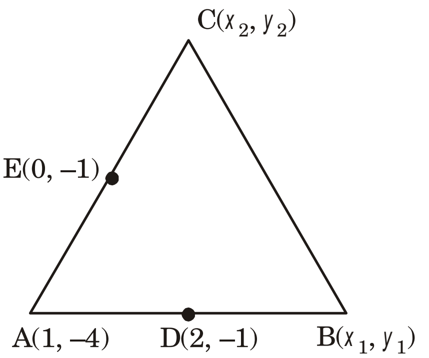
Also,

$AB^2 + BC^2 = 106 + 106 = 212 = AC^2$

$\therefore$ ΔABC is also a right angled triangle.

Hence, The points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right angled triangle. [1]

9.



Given coordinates of point and mid-points of sides through A being (2, –1) and (0, –1).

Let co-ordinate of the point B and C be (x_1, y_1) and (x_2, y_2) respectively.

Also, let point D and E be the mid points of sides AB and AC respectively.

The mid-point formula is given as:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ [1]

Since D is the mid-point of AB we get,

$$2 = \frac{1 + x_1}{2} \text{ and } -1 = \frac{-4 + y_1}{2}$$

$$\Rightarrow 4 = 1 + x_1 \text{ and } -2 = -4 + y_1$$

$$\Rightarrow x_1 = 3 \text{ and } y_1 = 2$$

The coordinates of point B are (3, 2).

Now since E is the mid-point of AC we get,

$$0 = \frac{1 + x_2}{2} \text{ and } -1 = \frac{-4 + y_2}{2}$$

$$\Rightarrow 0 = 1 + x_2 \text{ and } -2 = -4 + y_2$$

$$\Rightarrow x_2 = -1 \text{ and } y_2 = 2$$ [1]

The coordinates of point C are (–1, 2).

Area of the triangle ABC having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula,

$$\text{Area} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$\Rightarrow \text{Area of the } \Delta ABC = \frac{1}{2}\begin{bmatrix} 1(2-2) + 3(2+4) \\ -1(-4-2) \end{bmatrix}$$

$$= \frac{1}{2}\left[0 + 18 - 1(-6)\right]$$

$$= \frac{1}{2}(18 + 6)$$

$$= \frac{1}{2} \times 24 = 12$$

Hence, the area of the triangle ABC is 12 unit^2. [1]

10. Let the coordinates of B and C be (x_2, y_2) and (x_3, y_3).

Since, D is the mid-point of AB. So,

$$(1, 0) = \left(\frac{x_2 + 0}{2}, \frac{y_2 - 1}{2}\right)$$

$$\Rightarrow 1 = \frac{x_2 + 0}{2} \text{ and } 0 = \frac{y_2 - 1}{2}$$

$$\Rightarrow 1 = \frac{x_2}{2} \text{ and } 0 = \frac{y_2 - 1}{2}$$

$$\Rightarrow x_2 = 2 \text{ and } y_2 = 1$$ [1]

Thus, the coordinates of B are (2, 1).

Similarly, E is the mid-point of AC. So,

$$(0, 1) = \left(\frac{x_3 + 0}{2}, \frac{y_3 - 1}{2}\right)$$

$$\Rightarrow 0 = \frac{x_3 + 0}{2} \text{ and } 1 = \frac{y_3 - 1}{2}$$

$$\Rightarrow 0 = \frac{x_3}{2} \text{ and } 1 = \frac{y_3 - 1}{2}$$

$$\Rightarrow x_3 = 0 \text{ and } y_3 = 3$$

Thus, the coordinates of C are (0, 3).

We know that F is the mid-point of BC. So, its coordinates are

$$\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$ [1]

Now we know that the area of the triangle is given by-

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

So, area of ΔABC is

$$= \frac{1}{2}\left|0(1-3) + 2(3+1) + 0(-1-1)\right|$$

$$= \frac{1}{2} \times 8 = 4 \text{ square unit}$$

Area of the is ΔDEF

$$= \frac{1}{2}\left|1(1-2) + 0(2-0) + 1(0-1)\right|$$

$$= \frac{1}{2} \times |-2| = \frac{1}{2} \times 2 = 1 \text{ square unit}$$

Hence, the area of ΔABC is 4 and the area of ΔDEF is 1 square unit. [1]

11. To prove that ΔABC and ΔPQR are similar triangles we need to find prove that the ratios of the lengths of their corresponding sides are equal.

We know that distance d between two points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ [1]

In ΔABC

$$AB = \sqrt{(2-(-2))^2 + (0-0)^2} = 4$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2} = 2\sqrt{2}$$

$$CA = \sqrt{(0-(-2))^2 + (2-0)^2} = 2\sqrt{2}$$

In ΔPQR

$$PQ = \sqrt{(4-(-4))^2 + (0-0)^2} = 8$$

$$QR = \sqrt{(0-4)^2 + (4-0)^2} = 4\sqrt{2}$$

$$PR = \sqrt{(0+4)^2 + (4-0)^2} = 4\sqrt{2}$$ [1]

Now, the ratios of the corresponding sides will be

$$\Rightarrow \frac{AB}{PQ} = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow \frac{BC}{QR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{CA}{PR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

We can see that the ratios of the corresponding sides of the triangles ABC and PQR are same.

Therefore, ΔABC and ΔPQR are similar triangles.

Hence proved. [1]

12. We know that area of triangle can be calculated using the formula

$$A = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$ [1]

Here, $x_1 = 2, y_1 = 1, x_2 = 3, y_2 = -2, x_3 = \frac{7}{2}, y_3 = y$

$$\therefore 5 = \frac{1}{2}\left|2(-2-y) + 3(y-1) + \frac{7}{2}(1-(-2))\right|$$

$$\Rightarrow 5 = \frac{1}{2}\left|-4 - 2y + 3y - 3 + \frac{21}{2}\right|$$ [1]

$$\Rightarrow 10 = \left|\frac{-14+21}{2} + y\right|$$

$$\Rightarrow 10 = \left|\frac{7}{2} + y\right|$$

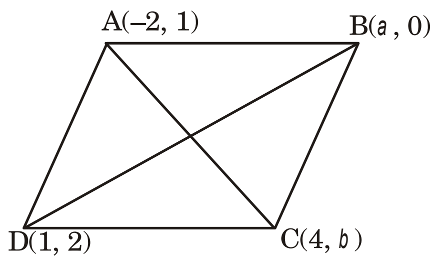
$$\Rightarrow 10 = \frac{7}{2} + y \text{ or } -10 = \frac{7}{2} + y$$

$$\Rightarrow 10 - \frac{7}{2} = y \text{ or } -10 - \frac{7}{2} = y$$

$$y = \frac{13}{2} \text{ or } y = \frac{-27}{2}$$

Hence, the value of y can be $\frac{13}{2}$ or $\frac{-27}{2}$. [1]

13. Consider the parallelogram $ABCD$:



The diagonals of a parallelogram bisect each other.

So coordinates of midpoint of AC = Coordinates of midpoint of BD

Coordinates of midpoint of a line segment joining the points $A(x_1, y_1)$

and $B(x_2, y_2) = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$

Therefore, $\left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$

$\left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2}{2}\right)$

$\left(1, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, 1\right)$ [1]

Comparing x and y coordinates of both the sides.

$\Rightarrow \frac{a+1}{2} = 1$ and $\frac{1+b}{2} = 1$

$\Rightarrow a+1 = 2$ and $1+b = 2$

$\Rightarrow a = 2-1$ and $b = 2-1$

$\Rightarrow$ a=1 and b=1

So the coordinates are $A(-2, 1)$, $B(1, 0)$, $C(4, 1)$, $D(1, 2)$

Length of any side having coordinates (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So the length of side $AB = \sqrt{(1-(-2))^2 + (0-1)^2}$

Length of side AB $= \sqrt{(1+2)^2 + (-1)^2}$

Length of side $AB = \sqrt{3^2 + 1}$

Length of side $AB = \sqrt{9+1}$

Length of side $AB = \sqrt{10}$ [1]

Since $ABCD$ is a parallelogram, opposite sides are equal.

AB = CD=$\sqrt{10}$

Length of side $BC = \sqrt{(4-1)^2 + (1-0)^2}$

Length of side $BC = \sqrt{3^2 + 1^2}$

Length of side $BC = \sqrt{9+1}$

Length of side $BC = \sqrt{10}$

Since $ABCD$ is a parallelogram, opposite sides are equal.

BC = AD=$\sqrt{10}$

Therefore, the length of the sides of the parallelogram are

$AB = BC = CD = DA = \sqrt{10}$ units [1]

14. The three vertices are $A(3, -4)$, $B(-1, -3)$ and $C(-6, 2)$.

Let the coordinates of the vertex D be (x, y). [1]

In a parallelogram, the diagonals bisect each other.

Hence, Mid point of AC = Mid point of BD

$\Rightarrow \left(\frac{3+(-6)}{2}, \frac{-4+2}{2}\right) = \left(\frac{-1+x_2}{2}, \frac{-3+y_2}{2}\right)$

$\Rightarrow \left(\frac{-3}{2}, \frac{-2}{2}\right) = \left(\frac{-1+x_2}{2}, \frac{-3+y_2}{2}\right)$

$\Rightarrow -3 = -1 + x_2, -2 = -3 + y_2$

$\Rightarrow x = -2, y = 1.$ [1]

Hence, the coordinates of the fourth vertex D is $(-2, 1)$.

Now, for the area of $ABCD$ = area of triangle ABC + area of triangle ACD

= 2 × area of triangle ABC

The area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ [1]

Now substituting the values,

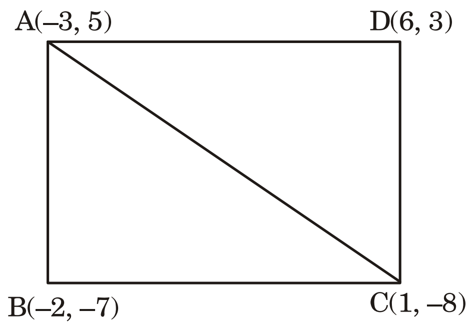
Area

$= \frac{1}{2}\left|\left[3(-3-2) + (-1)(2-(-4)) + (-6)(-4-(-3))\right]\right|$

$= \frac{1}{2}\left|\left[-15 - 6 + 6\right]\right| = \frac{|-15|}{2}$ square units

Hence area of parallelogram $2 \times \frac{15}{2} = 15$ square units. [1]

15.



Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD

Area of

$\Delta ABC = \frac{1}{2} \times |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

[1]

Area of

$$\Delta ABC = \frac{1}{2} \times |-3(-7-(-8)) + (-2)(-8-5) + 1(5-(-7))|$$

Area of

$$\Delta ABC = \frac{1}{2} \times |-3(-7+8) - 2(-8-5) + 1(5+7)|$$

Area of $\Delta ABC = \frac{1}{2} \times |-3(1) - 2(-13) + 1(12)|$

Area of $\Delta ABC = \frac{1}{2} \times |-3 + 26 + 12|$

Area of $\Delta ABC = \frac{1}{2} \times (35)$

Area of ΔABC = 17.5 square units [1]

Area of

$$\Delta ADC = \frac{1}{2} \times |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Area of

$$\Delta ADC = \frac{1}{2} \times |-3(3-(-8)) + 6(-8-5) + 1(5-3)|$$

Area of $\Delta ADC = \frac{1}{2} \times |-3(3+8) + 6(-8-5) + 1(5-3)|$

Area of $\Delta ADC = \frac{1}{2} \times |-3(11) + 6(-13) + 1(2)|$

Area of $\Delta ADC = \frac{1}{2} \times |-33 - 78 + 2|$ [1]

Area of $\Delta ADC = \frac{1}{2} \times |-109|$

Area of $\Delta ADC = \frac{1}{2} \times 109$

Area of ΔADC = 54.5 square units

Therefore, Area of quadrilateral $ABCD$ = 17.5 + 54.5 = 72 square units [1]

16. In this quadrilateral $ABCD$, we have 2 triangles i.e. ΔABC and ΔACD

Area of a triangle

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left\{\begin{matrix}-4(-4-(-5)) - 3(-5-8) \\ +0(8-(-4))\end{matrix}\right\}$$ [1]

Now,

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}\left\{\begin{matrix}-4(-4-(-5)) - 3(-5-8) \\ +0(8-(-4))\end{matrix}\right\}$$

$$= \frac{1}{2}\{-4(-4+5) - 3(-13) + 0\}$$

$$= \frac{1}{2}\{-4 + 39\} = \frac{35}{2} = 17.5$$

Area of ΔABC = 17.5 units^2 [1]

Now,

$$\text{Area of } \Delta ACD = \frac{1}{2}\{-4(-5-6) + 0(6-8) + 5(8-(-5))\}$$

$$= \frac{1}{2}\{-4(-11) + 0 + 5(8+5)\}$$

$$= \frac{1}{2}\{44 + 5(13)\} = \frac{1}{2}\{44 + 65\}$$

$$= \frac{109}{2} = 54.5$$ [1]

Area of ΔACD = 54.5 units^2

Now area of quadrilateral $ABCD$ will be sum of area of ΔABC and ΔACD.

Therefore Area of quadrilateral $ABCD$ = 17.5 + 54.5 = 72 sq. units

Hence, answer is 72 sq. units. [1]

17. Area of a triangle with coordinates (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by-

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$ [1]

Putting in the coordinates,

$$x_1 = 1, x_2 = -4, x_3 = -k, y_1 = -1, y_2 = 2k, y_3 = -5$$

$$\Rightarrow \frac{1}{2}[1(2k+5) - 4(-5+1) - k(-1-2k)] = 24$$ [1]

$$\Rightarrow \frac{1}{2}[2k + 5 + 16 + k + 2k^2] = 24$$

$$\Rightarrow \frac{1}{2}[2k^2 + 3k + 21] = 24$$

$$\Rightarrow 2k^2 + 3k + 21 = 48$$

$$\Rightarrow 2k^2 + 3k - 27 = 0$$ [1]

$$\Rightarrow 2k^2 - 6k + 9k - 27 = 0$$

$$\Rightarrow 2k(k-3) + 9(k-3) = 0$$

$$\Rightarrow (k-3)(2k+9) = 0$$

$$\Rightarrow k - 3 = 0 \text{ and } 2k + 9 = 0$$

$$\Rightarrow k = 3 \text{ and } k = \frac{-9}{2}$$ [1]

18. Given $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

$\frac{AD}{AB} = \frac{1}{3}$ and $\frac{AE}{AC} = \frac{1}{3}$

$\frac{\text{AD}}{\text{AB}} = \frac{1}{3}$

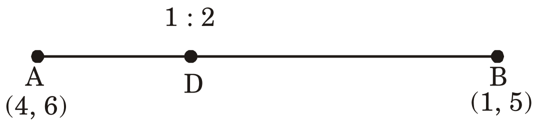
$\Rightarrow 3\text{AD}=\text{AD}+\text{DB}$

$\Rightarrow 3\text{AD}-\text{AD}=\text{DB}$

$\Rightarrow 2\text{AD}=\text{BD}$

$\Rightarrow \frac{\text{AD}}{\text{BD}} = \frac{1}{2}$

Thus point D divides line AB in ratio 1 : 2



Using section formula, Coordinates of D

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Putting values in the given above formula

$m_1 = 1, m_2 = 2$

$x_1 = 4, x_2 = 1$

$y_1 = 6, y_2 = 5$

$$\text{Point } D = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1\times1+2\times4}{1+2}, \frac{1\times5+2\times6}{1+2}\right)$$

$$= \left(\frac{1+8}{3}, \frac{5+12}{3}\right)$$

$$\text{Point D}=\left(\frac{9}{3}, \frac{17}{3}\right)$$ [1]

$\frac{\text{AE}}{\text{AC}} = \frac{1}{3}$

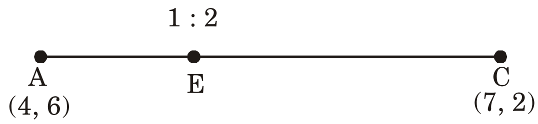
$\Rightarrow 3\text{AE} = \text{AE} + \text{CE}$

$\Rightarrow 3\text{AE} - \text{AE} = \text{CE}$

$\Rightarrow 2\text{AE} = \text{CE}$

$\Rightarrow \frac{\text{AE}}{\text{CE}} = \frac{1}{2}$

Thus point E divides line AC in ratio 1 : 2



Using section formula, Coordinates of E

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Putting values in the given above formula

$m_1 = 1, m_2 = 2$

$x_1 = 4, x_2 = 7$

$y_1 = 6, y_2 = 2$

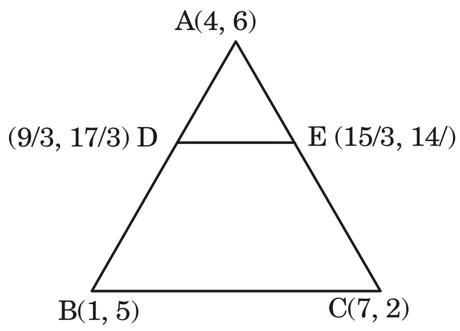
$$\text{Point } E = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1\times7+2\times4}{1+2}, \frac{1\times2+2\times6}{1+2}\right)$$

$$= \left(\frac{7+8}{3}, \frac{2+12}{3}\right)$$

$$= \left(\frac{15}{3}, \frac{14}{3}\right)$$ [1]

Now, Finding area of ΔABC and ΔADE



Area of $\Delta ABC =$

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Given,

$x_1 = 4, x_2 = 1$

$x_3 = 7, y_1 = 6$

$y_2 = 5, y_3 = 2$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$$

$$= \frac{1}{2}[4(3)+1(-4)+7(1)]$$

$$= \frac{1}{2}[12-4+7]$$

$$= \frac{15}{2} \text{sq. units}$$ [1]

Area of ΔADE =

$$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

Given,

$$x_1 = 4, x_2 = \frac{9}{3}$$

$$x_3 = \frac{15}{3}, y_1 = 6$$

$$y_2 = \frac{17}{3}, y_3 = \frac{14}{3}$$

$$= \frac{1}{2}\left[4\left(\frac{17}{3}-\frac{14}{3}\right)+\frac{9}{3}\left(\frac{14}{3}-6\right)+\frac{15}{3}\left(6-\frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4(1)+3\left(\frac{-4}{3}\right)+5\left(\frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\times\frac{5}{3} = \frac{5}{6} \text{ sq units}$$

Therefore,

$$\frac{\text{ar}(\Delta \text{ADE})}{\text{ar}(\Delta \text{ABC})} = \frac{\frac{5}{6}}{\frac{15}{2}} = \frac{5}{6}\times\frac{2}{15} = \frac{1}{9}$$

Hence, ratio is 1:9. [1]

19. Given points are collinear so area of triangle formed must be zero.

$$\Rightarrow \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)| = 0$$ [1]

Given;

$$\Rightarrow x_1 = k+1, y_1 = 2k;$$

$$\Rightarrow x_2 = 3k, y_2 = 2k+3;$$

$$\Rightarrow x_3 = 5k-1, y_3 = 5k;$$

$$\Rightarrow \frac{1}{2}\left|\begin{matrix}(k+1)\{(2k+3)-(5k)\}+(3k)\{5k-2k\} \\ +(5k-1)\{(2k)-(2k+3)\}\end{matrix}\right| = 0$$ [1]

$$\Rightarrow (k+1)\{-3k+3\}+(3k)\{3k\}+(5k-1)\{-3\} = 0$$

$$\Rightarrow -3k^2+3k-3k+3+9k^2-15k+3 = 0$$

$$\Rightarrow 6k^2-15k+6 = 0$$

Dividing the above equation by 3

$$\Rightarrow 2k^2-5k+2 = 0$$

$$\Rightarrow 2k^2-4k-k+2 = 0$$

$$\Rightarrow 2k(k-2)-1(k-2) = 0$$ [1]

$$\Rightarrow (k-2)(2k-1) = 0$$

$$\Rightarrow k-2 = 0 \text{ and } 2k-1 = 0$$

$$\Rightarrow k = 2 \text{ and } k = \frac{1}{2}$$

Therefore, the values of $k = \frac{1}{2}$ and 2 [1]

20. We know that three points are collinear if the area of the triangle formed by them is zero.

The area of triangle can be calculated using the formula

$$A = \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$ [1]

Here, $x_1 = a, y_1 = a^2, x_2 = b, y_2 = b^2, x_3 = 0, y_3 = 0$

$$A = \frac{1}{2}\left|a\left(b^2-0\right)+b\left(0-a^2\right)+0\left(a^2-b^2\right)\right|$$ [1]

$$A = \frac{1}{2}\left|ab^2-a^2b\right|$$

According to the question $a \neq b \neq 0$

$$\therefore A = \frac{1}{2}\left|ab^2-a^2b\right| \neq 0$$ [1]

Hence, the points $(a, a^2), (b, b^2), (0, 0)$ are non collinear.

Hence proved. [1]

Smart Notes ....

CHAPTER 7

Triangles

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Area of Triangle	1 mark	1 mark				4 marks
Question based on Proving properties of Triangle	3, 4 marks	3, 4 marks				

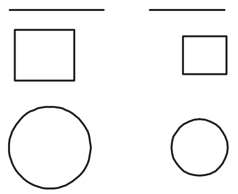
Summary

Similar Triangles

Similar figures: Geometric figures which have the same shape but different sizes are known as similar figures.

Illustrations:

1. Any two line-segments are similar
2. Any two squares are similar
3. Any two circles are similar



Two congruent figures are always similar but two similar figures need not be congruent.

Similar polygons: Two polygons of the same number of sides are said to be similar if

(i) their corresponding angles are equal (i.e., they are equiangular) and

(ii) their corresponding sides are in the same ratio (or proportion)

Similar triangles: Since triangles are also polygons, the same conditions of similarity are applicable to them.

Two triangles are said to be similar if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or proportion).

BASIC-PROPORTIONALITY THEOREM (Thales theorem)

Theorem 1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Theorem 2 : (Converse of BPT theorem) If a line divides any two sides of a triangle in the same ratio, prove that it is parallel to the third side.

Criteria for Similarity of Two Triangles

Two triangles are said to be similar if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportional).

Thus, two triangles ABC and $A'B'C'$ are similar if

(i) $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C$ and

(ii) $\dfrac{AB}{A'B'} = \dfrac{BC}{B'C'} = \dfrac{CA}{C'A'}$

In this section, we shall make use of the theorems discussed in earlier sections to derive some criteria for similar triangles which in turn will imply that either of the above two conditions can be used to define the similarity of two triangles.

CHARACTERISTIC PROPERTY 1 (AAA SIMILARITY)

Theorem 3: If in two triangles, the corresponding angles are equal, then the triangles are similar.

CHARACTERISTIC PROPERTY 2 (SSS SIMILARITY)

Theorem 4: If the corresponding sides of two triangles are proportional, then they are similar.

CHARACTERISTIC PROPERTY 3 (SAS SIMILARITY)

Theorem 5: If one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional than the two triangles are similar.

Areas of Similar Triangles

Theorem 6:The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Pythagoras Theorem

Theorem 7:In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Theorem 8: (Converse of Pythagoras theorem) In a triangle if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

- *The mid-point of the hypotenuse of a right triangle is equidistant from the vertices.*

PREVIOUS YEARS' EXAMINATION QUESTIONS

1 Mark Questions

1. $\Delta DEF \sim \Delta ABC$; If $DE : AB = 2 : 3$ and ar(ΔDEF) is equal to 44 square units, then area(ΔABC) in square units is

   (*a*) 99 (*b*) 120

   (*c*) $\frac{176}{9}$ (*d*) 66

   [TERM 2, 2012]

2. The famous mathematician who gave an important truth called "Basic proportionality theorem" belongs to:

   (*a*) China

   (*b*) India

   (*c*) Babylonia

   (*d*) Greece

   [TERM 2, 2013]

3. Find the length of the diagonal of a square whose each side is 8 cm.

   [TERM 2, 2014]

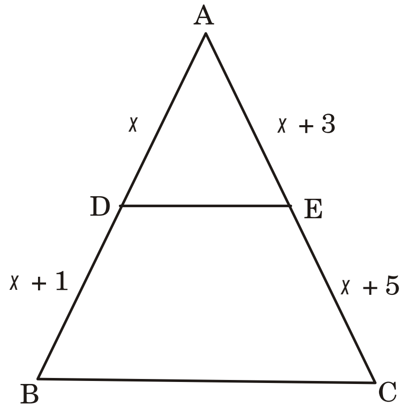
4. In a , right angled at *B*, $AC - AB = 2$ cm , $BC = 8$ cm, find the value of *AC*.

   [TERM 2, 2015]

5. In ΔABC, *D* and *E* are point on side *AB* and *AC* resp. such that DE | | BC . If *AE* = 2 cm, *AD* = 3 cm and *BD* = 4.5 cm then find *CE*.

   [TERM 2, 2016]

6. In a $\Delta ABC, DE \parallel BC$, then find the value of *x*.



   [TERM 2, 2017]

7. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then $\frac{ar\Delta ABC}{ar\Delta PQR}$.

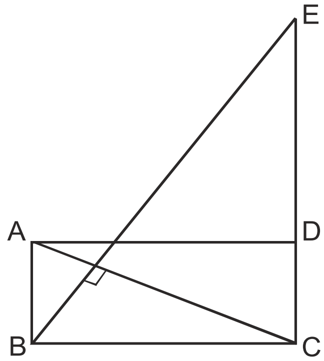
   [DELHI, 2018]

2 Marks Questions

8. In ΔABC $AB = AC$, and *D* is a point on side *AC* such that $BC^2 = AC.CD$. Prove that $BD = BC$.

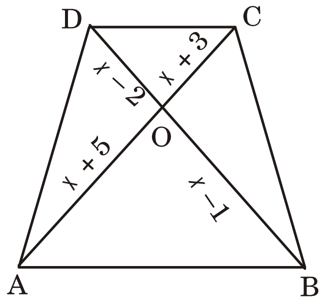
   [TERM 2, 2011]

9. In given figure *ABCD* is a rectangle, in which BC = 2AB. *A* point *E* lies on *CD* produced such that $CE = 2BC$. Find $AC : BE$



   [TERM 2, 2011]

10. In the given figure, if AB | | DC, find the value of *x*.

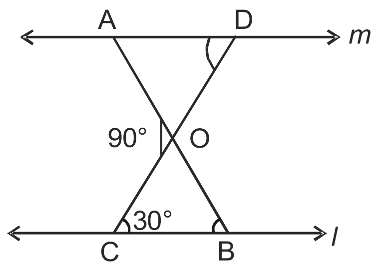


   [TERM 2, 2012]

11. In ΔABC, $\angle A = 90°$ $AN \perp BC$, BC = 13 cm and AC = 5 Cm. Find the ratio of ar(ΔNAC) : ar(ΔABC)

   [TERM 2, 2013]

12. In the figure, $\Delta OAD \sim \Delta OBC$, If $\angle AOC = 90°$ and $\angle OBC = 30°$, find $\angle ODA$ and $\angle COB$



   [TERM 2, 2014]

13. In Figure 4, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54, then find the lengths of sides AB and AC.

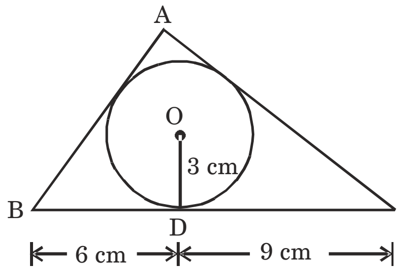
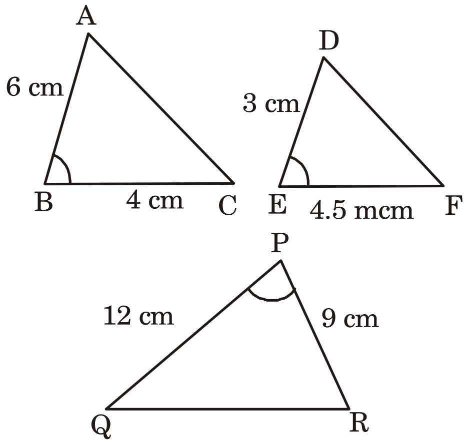


Figure 4

[TERM 2, 2015]

14. State which of the two triangles given in the figure are similar. Also state the similarity criterion used.



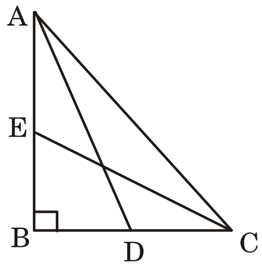
[TERM 2, 2015]

3 Marks Questions

15. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that: $AP \times PC = BP \times DP$.

[TERM 2, 2011]

16. ΔABC is right angled at B. AD and CE are the two medians drawn from A and C, respectively. If $AC = 5$ cm, $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



[TERM 2, 2011]

17. D, E, F are respectively the mid-points of the sides AB, BC and CA of ΔABC. Find the ratios of area of ΔDEF and ΔABC.

[TERM 2, 2012]

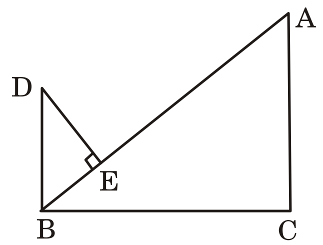
18. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals

[TERM 2, 2012]

19. Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. If $AB = 2CD$, find the ratio of the area of triangles AOB and COD.

[TERM 2, 2012]

20. In the given figure, $DB \perp BC$; $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$

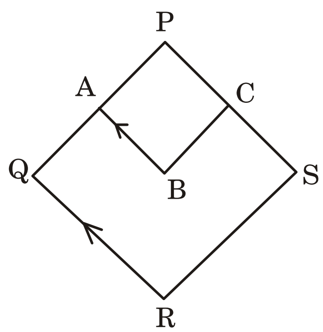


[TERM 2, 2013]

21. In an equilateral triangle ABC, AD is an altitude drawn from A on the side BC. Prove that $\frac{3}{4}AB^2 = AD^2$

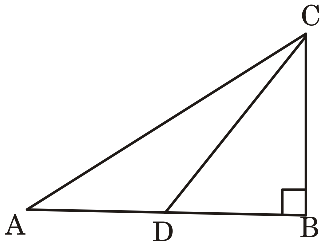
[TERM 2, 2013]

22. In the figure $AB \parallel QR$ and $BC \parallel RS$. Prove that $\frac{PA}{PQ} = \frac{PC}{PS}$



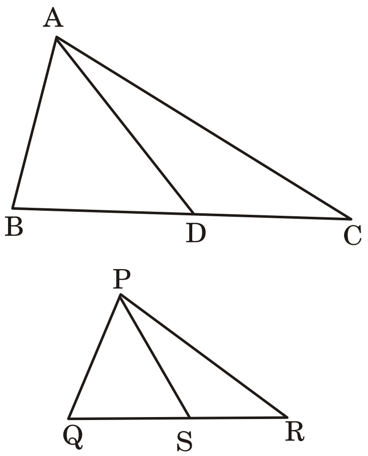
[TERM 2, 2014]

23. In the figure if $CD = 17$m, $BD = 8$m and $AD = 4$m, then find the value of AC.



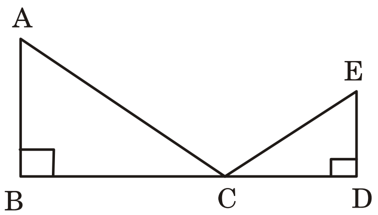
[TERM 2, 2014]

24. In the given figure $\triangle ABD \sim \triangle PQS$ when AD and PS are medians. Prove that $\triangle ABC \sim \triangle PQR$.



[TERM 2, 2015]

25. In the given figure $\angle B = \angle D = 90^\circ$. If AB = 12 cm, AC = 13 cm, CE = 5 cm and ED = 4 cm, find the length of BD.



[TERM 2, 2015]

26. State and prove Pythagoras theorem.

[TERM 2, 2017]

27. If the area of two similar triangles are equal, prove that they are congruent.

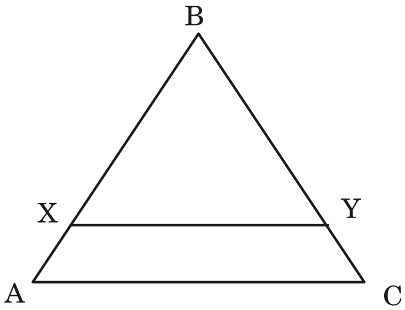
[DELHI, 2018]

28. Prove that the area of equilateral triangle described on one side of the square is equal to half of the area of an equilateral triangle described on one of its diagonal.

[DELHI, 2018]

4 Marks Questions

29. In the given figure, in ΔABC, $XY / / AC$ and XY divides the ΔABC into two regions such that ar (Δ BXY)=) 2ar($ACYX$). Determine $\frac{AX}{AB}$.

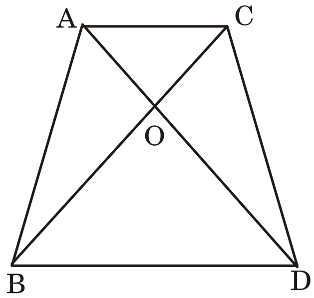


[TERM 2, 2011]

30. Two poles of height p and q metres are standing vertically on a level ground, a metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{pq}{p+q}$ metres.

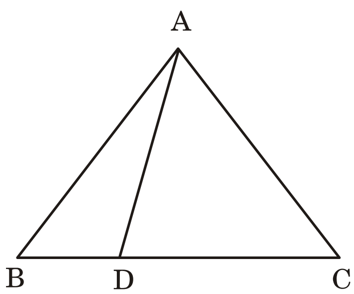
[TERM 2, 2011]

31. In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$



[TERM 2, 2012]

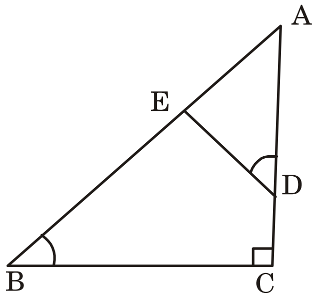
32. In the given figure ΔABC is an equilateral. D is a point on BC such that $BD = \frac{1}{3}BC$ Prove that $9AD^2 = 7AB^2$



[TERM 2, 2013]

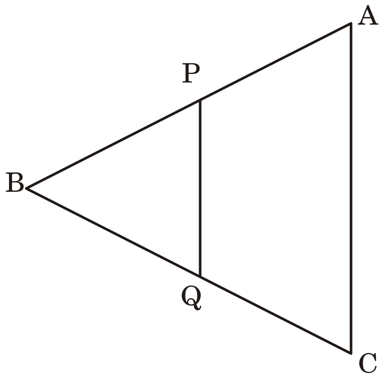
33. In ΔABC, If $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.

Also if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 5.4$ cm, then find DE.



[TERM 2, 2015]

34. In this given ΔABC, $PQ \parallel AC$ and it divides the Δ into two equal parts in area. Then find the ratio of $\frac{AP}{AB}$.



[TERM 2, 2017]

35. Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

[DELHI, 2018]

Solutions

1. We know that,

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{DE^2}{AB^2} \quad \text{...(i)}$$

We have,

$ar(\Delta DEF) = 44$ square units, and $\frac{DE}{AB} = \frac{2}{3}$ [½]

Substituting above values in equation (i),

$$\frac{44}{\Delta ABC} = \frac{2^2}{3^2}$$

$$\Rightarrow \frac{44}{\Delta ABC} = \frac{4}{9}$$

$$\Rightarrow \Delta ABC = \frac{44 \times 9}{4}$$

$$\Rightarrow \Delta ABC = 11 \times 9 = 99$$

The area(ΔABC) in square units is 99.

Hence, the correct option is (a). [½]

2. Famous Greek mathematician Thales (600 B.C.) gave an important truth concerning equiangular triangles. Which is called Basic proportionality Theorem.

Hence the correct option is (d). [1]

3. Let the length of each side of square be $a = 8\,\text{cm}$, and suppose the length of the diagonal be ,

Now using Pythagoras theorem,

$$\Rightarrow d^2 = 2a^2$$

$$\Rightarrow d = \sqrt{2}a$$

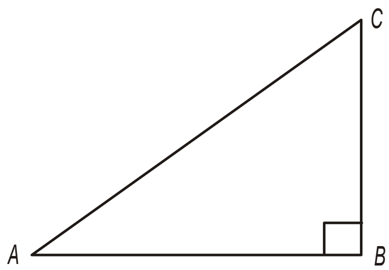
$$\Rightarrow d = \sqrt{2} \times 8 = 8\sqrt{2}$$

Hence length of the diagonal is $8\sqrt{2}\text{cm}$. [1]

4. It is given that ΔABC is right angled at B, Also, $AC - AB = 2$ cm and $BC = 8$ cm.

$AC - AB = 2$ cm This implies

$AB = AC - 2$ cm ...(i)



[½]

Using the Pythagoras theorem we get,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = (AC - 2)^2 + (BC)^2$$

Using (i)

$$\Rightarrow (AC)^2 = (AC)^2 + 4 - 4AC + (8)^2$$

$$\Rightarrow (AC)^2 = (AC)^2 + 4 - 4AC + 64$$

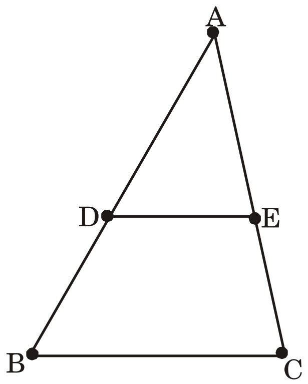
$$\Rightarrow 0 = -4AC + 68$$

$$\Rightarrow 4AC = 68$$

$$\Rightarrow AC = 17$$

Hence, the value of AC is 17 cm. [½]

5. In ΔABC, we have DE | | BC.



[½]

By basic proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{3}{4.5} = \frac{AE}{EC}$$

$$\frac{3}{4.5} = \frac{2}{EC}$$

$$EC = \frac{2 \times 4.5}{3}$$

$$EC = \frac{9}{3}$$

$$EC = CE = 3$$

Thus $CE = 3\text{cm}$ [½]

6. Given:

$\Delta ABC, DE \parallel BC, AD = x, DB = x+1,$
$AE = x+3$, and $EC = x+5$

Now, according to Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{x+1} = \frac{x+3}{x+5}$$

$$\Rightarrow x(x+5) = (x+3)(x+1)$$

$$\Rightarrow x^2 + 5x = x(x+1) + 3(x+1)$$ [½]

On simplifying the expression, we get,

$$\Rightarrow x^2 + 5x = x^2 + x + 3x + 3$$

Taking all the terms to the left side,

$$\Rightarrow x^2 - x^2 + 5x - 4x - 3 = 0$$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

Hence, the value of x is 3. [½]

7. According to the theorem "If two triangles are similar then the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides."

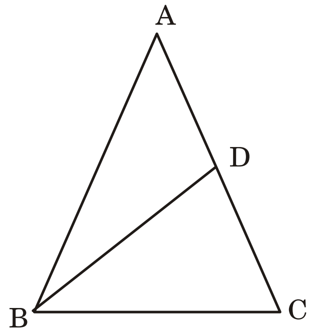
$$\frac{ar\Delta ABC}{ar\Delta PQR} = \left(\frac{AB}{PQ}\right)^2$$

$$= \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{9}$$

$$\therefore \frac{ar\Delta ABC}{ar\Delta PQR} = \frac{1}{9}$$ [1]

8. Consider ΔABC where $AB = AC$ and D is a point on side AC.



It is given that

$$\Rightarrow BC \times BC = AC \times CD$$

Rearranging the terms,

$$\Rightarrow \frac{AC}{BC} = \frac{BC}{CD}$$ [1]

Which means that ΔABC is similar to ΔBDC.

$$\Rightarrow \frac{AC}{BC} = \frac{AB}{BD} \quad ...(i)$$

(Corresponding sides of similar triangles are proportional)

As it is given that $AB = AC$,

Therefore, from (i) $BD = BC$.

Hence proved. [1]

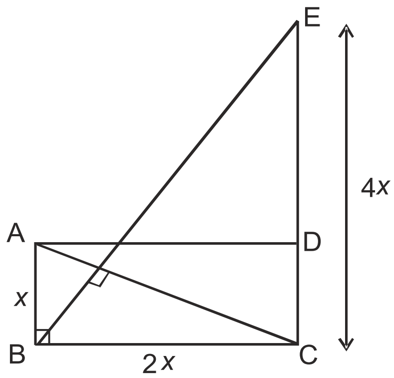
9. ABCD is a rectangle and each angle of rectangle is 90°.

Let length of $AB = x$ ...(i)

It is given that $BC = 2AB$,

$\Rightarrow BC = 2x$ (Using(i)) ...(ii)

$CE = 2BC = 2(2x) = 4x$ (Using (ii))



Apply Pythagoras Theorem in right angled ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = x^2 + (2x)^2$$

$$\Rightarrow AC^2 = 5x^2$$

$$\Rightarrow AC = \sqrt{5}x$$

(Neglecting the negative value of square root as length cannot be in negative) ...(iii) [1]

Apply Pythagoras Theorem in right angled ΔBCE,

$$BE^2 = BC^2 + CE^2$$

$$\Rightarrow BE^2 = (2x)^2 + (4x)^2$$

$$\Rightarrow BE^2 = 20x^2$$

$\Rightarrow BE = \sqrt{20}x = 2\sqrt{5}x$ (Neglecting the negative value of square root as length cannot be in negative) ...(iv)

From (iii) and (iv),

$$AC : BE = \sqrt{5}x : 2\sqrt{5}x$$

Hence, $AC : BE = 1 : 2$. [1]

10. In ΔOAB and ΔOCD,

$\angle AOB = \angle COD$ (vertically opposite angles)

$\angle OBA = \angle ODC$ (alternate interior angles)

Hence, $\Delta OAB \sim \Delta OCD$

(AAA similarity criterion)

Since, triangles OAB and OCD are similar,

Therefore, $\frac{OA}{OC} = \frac{OB}{OD}$ [1]

Substituting values from the given figure, we get,

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

Solving the equation,

$$\Rightarrow (x+5)(x-2) = (x-1)(x+3)$$

$$\Rightarrow x^2 + 5x - 2x - 10 = x^2 + 3x - x - 3$$

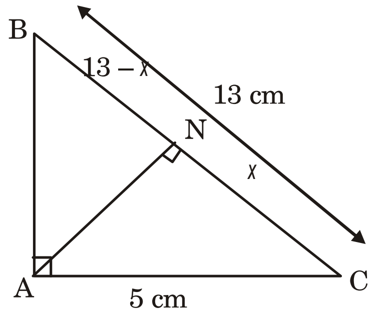
Cancelling the same terms on both sides and solving, we get,

$$\Rightarrow 3x - 10 = 2x - 3$$

$$\Rightarrow x = 7$$

Hence, the value of x is 7. [1]

11.



In right ΔBAC,

$$AB^2 + AC^2 = BC^2$$

$$AB^2 = 13^2 - 5^2 = 144$$

$AB = 12$ cm [½]

Now, in right ΔANC Using Pythagoras theorem,

$$AN^2 + NC^2 = AC^2$$

$l^2 + x^2 = 25$ ...(i)

Now, in right ΔANB, Using Pythagoras theorem

$$AN^2 + BN^2 = AB^2$$

$$l^2 + (13 - x)^2 = (12)^2$$

$l^2 + 169 + x^2 - 26x = 144$ ...(ii) [½]

By substituting the value of l^2 from eqn(i) in eqn(ii),we get.

$$25 - x^2 + 169 + x^2 - 26x = 144$$

$$x = \frac{25}{13}$$

Put the value of x in eqn (i)

$$l^2 + \left(\frac{25}{13}\right)^2 = 25$$

$l = \frac{60}{13}$ [½]

Now, area of triangle is given by

$\frac{1}{2} \times \text{base} \times \text{height}$

$$\frac{area\Delta ANC}{area\Delta ABC} = \frac{\frac{1}{2} \times \frac{25}{13} \times \frac{60}{13}}{\frac{1}{2} \times 5 \times 12}$$

$$= \frac{25}{169}$$

Hence

$ar(\Delta NAC): ar(\Delta ABC) = 25:169$ [½]

12. If $\angle AOC = 90°$,

then $\angle COB = 90°$ since AOB is a straight line and sum of angles on a straight lines are 180

In ΔOBC, $\angle OBC = 30°$, $\angle COB = 90°$,

By using angle sum property of a triangle,

$\angle OCB = 180° - (\angle COB + \angle OBC)$ [½]

$= 180° - (90° + 30°)$

$\angle OCB = 180° - 120°$

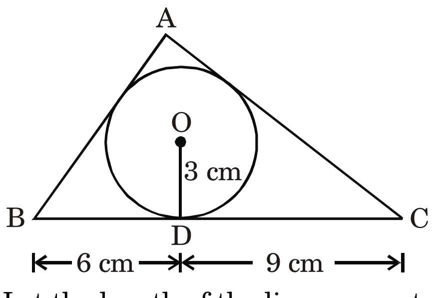
$\angle OCB = 60°$

Now since, $\Delta OAD \sim \Delta OBC$

$\angle OCB = \angle ODA = 60°$

Hence $\angle OCB = 60°$ and $\angle COB = 90°$ [½]

13. Let us suppose that the circle touches the sides AB and AC of the triangle of the triangle at point F and E, respectively.



Let the length of the line segment AF be x cm.

According to the theorem, that the lengths of tangents drawn form an external point to circle are equal.

In ΔABC,

$CE = CD = 9\text{cm}$

(Tangents of the circle from point C)

$BF = BD = 6\text{cm}$

(Tangents of the circle from point B)

$AB = AF + FB = (x+6)\text{cm}$

$BC = BD + DC = 6 + 9 = 15\text{cm}$

$CA = CE + EA = (9 + x)\text{cm}$

Area of $\Delta OBC = \frac{1}{2} \times BC \times OD$

$= \frac{1}{2} \times 15 \times 3 = \frac{45}{2}\text{cm}^2$ [½]

Area of $\Delta OCA = \frac{1}{2} \times AC \times OE$

$= \frac{1}{2} \times (x+9) \times 3 = \frac{3}{2}(x+9)\text{cm}^2$ [½]

Area of $\Delta OAB = \frac{1}{2} \times AB \times OF$

$= \frac{1}{2} \times (x+6) \times 3 = \frac{3}{2}(x+6)\text{cm}^2$

Area of $\Delta ABC =$ Area of $\Delta OBC +$ Area of $\Delta OCA +$ Area of ΔOAB

$\Rightarrow 54 = \frac{45}{2} + \frac{3}{2}(x+9) + \frac{3}{2}(x+6)$ [½]

Multiply the above equation by 2,

$\Rightarrow 108 = 45 + 3x + 27 + 3x + 18$

$\Rightarrow 18 = 6x$

Divide above equation by 6,

$\Rightarrow x = 3$

So,

$AB = x + 6 = 3 + 6 = 9\text{cm}$

$AC = 9 + x = 9 + 3 = 12\text{cm}$

Hence, the lengths of sides AB = 9cm and AC = 12cm. [½]

14. In ΔABC and ΔFED we have,

$\angle ABC = \angle FED$

$\frac{AB}{FE} = \frac{6}{4.5} = \frac{60}{45} = \frac{4}{3}$

And, $\frac{BC}{ED} = \frac{4}{3}$

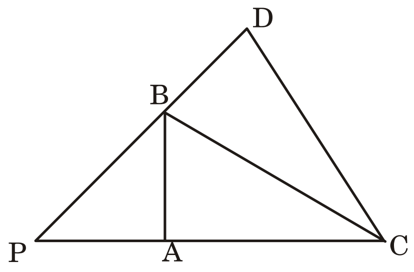
$\Rightarrow \frac{AB}{FE} = \frac{BC}{ED}$ [1]

$\Rightarrow \Delta ABC \sim \Delta FED$ by SAS Rule

Since one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. [1]

15. ΔABC and ΔDBC are right angled triangles at A and D respectively.

ΔDPC is a right angled triangle at D. Represent the given information as a diagram,



Using Pythagoras Theorem,

$CP^2 = CD^2 + DP^2$

$\Rightarrow CD^2 = CP^2 - DP^2$

Now, $CP = CA + AP$ and $DP = DB + BP$

$\Rightarrow CD^2 = (CA + AP)^2 - (DB + BP)^2$

Use $(a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow CD^2 = CA^2 + 2CA.AP + AP^2 - (DB^2 + 2DB.BP + BP^2)$ [1]

$\Rightarrow CD^2 = CA^2 + AP^2 + 2 CA.AP - DB^2 - BP^2 - 2 DB.BP$

$\Rightarrow CD^2 + DB^2 = CA^2 - (BP^2 - AP^2) + 2 CA.AP - 2 DB.BP$

$\Rightarrow CB^2 = CA^2 - AB^2 + 2 CA.AP - 2 DB.BP$ [1]

$\Rightarrow 2 AB^2 = 2 CA.AP - 2 DB.BP$

$(CB^2 = AB^2 + AC^2 \Rightarrow CB^2 - AC^2 = AB^2)$

$\Rightarrow AB^2 = (PC - AP)AP - (DP - BP)BP$

$\Rightarrow AB^2 = AP.PC - AP^2 - DP.BP + BP^2$

$\Rightarrow AB^2 = AP.PC - DP.BP + AB^2$

$\Rightarrow AP \times PC = DP \times BP$

Hence proved. [1]

16. Applying Pythagoras Theorem in ΔABC,

$AC^2 = AB^2 + BC^2$ ...(i)

Applying Pythagoras Theorem in ΔABD,

$AD^2 = AB^2 + BD^2$

$BD = \frac{1}{2}BC$ as AD is median.

$AD^2 = AB^2 + \left(\frac{1}{2}BC\right)^2$

$AD^2 = AB^2 + \frac{1}{4}BC^2$ ...(ii) [1]

Subtract (ii) from (i),

$AC^2 - AD^2 = AB^2 + BC^2 - \left(AB^2 + \frac{1}{4}BC^2\right)$

Substitute AC 5 cm, $AD = \frac{3\sqrt{5}}{2}$ cm

$\Rightarrow 5^2 - \left(\frac{3\sqrt{5}}{2}\right)^2 = \frac{3}{4}BC^2$

$\Rightarrow 25 - \frac{45}{4} = \frac{3}{4}BC^2$

$\Rightarrow \frac{55}{4} = \frac{3}{4}BC^2$

$\Rightarrow BC^2 = \frac{55}{3}$.....(iii)

Substitute the above value in equation (i), we get

$5^2 = AB^2 + \left(\frac{55}{3}\right)$

$\Rightarrow AB^2 = 25 - \frac{55}{3} = \frac{20}{3}$ ...(iv)

Consider ΔEBC,

$CE^2 = BC^2 + BE^2$

$\Rightarrow CE^2 = BC^2 + \left(\frac{1}{2}AB\right)^2$ [1]

(CE is median, so $BE = \frac{1}{2}(AB)$)

$\Rightarrow CE^2 = BC^2 + \frac{1}{4}AB^2$

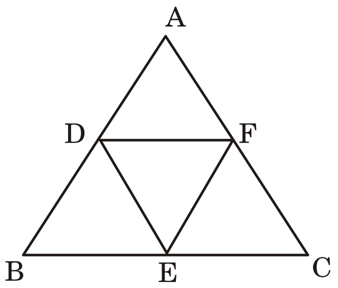
$$\Rightarrow CE^2 = \frac{55}{3} + \frac{1}{4}\left(\frac{20}{3}\right)$$

(Using (iii) and (iv))

$$\Rightarrow CE^2 = \frac{55}{3} + \left(\frac{5}{3}\right) = 20$$

$$\Rightarrow CE = \sqrt{20} = 2\sqrt{5}cm \cdot$$ [1]

17. According to the given information, the following figure can be drawn:



According to the mid-point theorem, the line joining the midpoints of two sides of a triangle is parallel to third side and half of it. [1]

Therefore, $AB \mid\mid EF$, $BC \mid\mid DF$ and $AC \mid\mid DE$, and or

$$\frac{EF}{AB} = \frac{1}{2}$$

$$\frac{DF}{BC} = \frac{1}{2}$$

$$\frac{DE}{AC} = \frac{1}{2}$$ [1]

We know that if in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

Hence, $\Delta ABC \sim \Delta DFE$ (by SSS congruency rule)

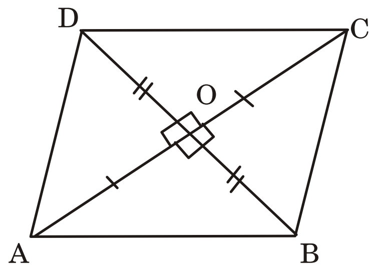
Again, according to the theorem, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \frac{ar(\Delta DFE)}{ar(\Delta ABC)} = \frac{EF^2}{AB^2} = \left[\frac{(EF)^2}{(2EF)^2}\right]$$

$$= \frac{1^2}{2^2} = \frac{1}{4}$$

Hence, the ratio of the areas of two triangles will be 1 : 4 [1]

18. Consider a rhombus $ABCD$ as shown in the figure:



We know that the diagonals of a rhombus bisect each other at right angles. Therefore;

$$AO = CO = \frac{1}{2} AC \quad ...(i)$$

$$BO = DO = \frac{1}{2} BD \quad ...(ii)$$

Also, $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$ ...(iii)

Now, consider the right triangle AOB.

$$AB^2 = AO^2 + BO^2$$

$$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$$

{from equation (i) and (ii)}

$$\Rightarrow AB^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2 \quad ...(iv)$$ [1]

Similarly,

$$BC^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2 \quad (v)$$

And, $CD^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2$ ...(vi)

Again $AD^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2$ ...(vii) [1]

Add equations (iv) , (v) , (vi) and (vii), we get;

$$AB^2 + BC^2 + CD^2 + AD^2$$

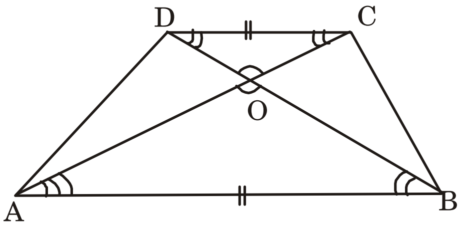
$$= \frac{1}{4}AC^2 + \frac{1}{4}AC^2 + \frac{1}{4}AC^2 + \frac{1}{4}AC^2 +$$

$$\frac{1}{4}BD^2 + \frac{1}{4}BD^2 + \frac{1}{4}BD^2 + \frac{1}{4}BD^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence proved. [1]

19. Considering the given situation, following diagram can be drawn:



Given that $AB \| DC$. AC and BD are diagonals that intersect each other at O.

Therefore, $\angle AOB = \angle COD$ (vertically opposite angles)

Also, $\angle OAB = \angle OCD$ (alternate angles) [1]

Similarly, $\angle OBA = \angle ODC$ (alternate angles)

Hence, $\Delta AOB \sim \Delta COD$ (by AAA similarity criteria)

Also, we know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides [1]

So, $\dfrac{ar\Delta AOB}{ar\Delta COD} = \dfrac{(AB)^2}{(CD)^2}$

Or, $\dfrac{ar\Delta AOB}{ar\Delta COD} = \dfrac{(2CD)^2}{(CD)^2}$

$= \dfrac{(2)^2}{(1)^2} = \dfrac{4}{1}$

Therefore, $ar\Delta AOB : ar\Delta COD = 4 : 1$ [1]

20. $BD \| AC$

In ΔBED and ΔABC,

$\angle BED = \angle ACB$ $\left(each\ 90^o\right)$ [1]

$\angle DBA = \angle BAC$ (Alternate interior angles)

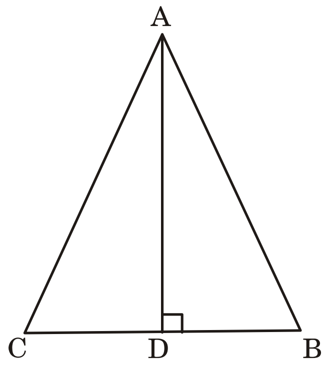
$\Rightarrow \Delta BED \sim \Delta ACB$ (By AAA rule) [1]

So, $\dfrac{BE}{AC} = \dfrac{DE}{BC}$ (Ratio of corresponding sides of similar traingles)

Or, $\dfrac{BE}{DE} = \dfrac{AC}{BC}$

Hence Proved. [1]

21.



In the ΔABC, AD is the Altitude.

$AD^2 + DB^2 = AB^2$ [1]

$DB = \dfrac{1}{2}BC$ (Perpendicular to base bisects the side in equilateral triangle)

$AD^2 + \left(\dfrac{1}{2}BC\right)^2 = AB^2$

$\Rightarrow AD^2 + \dfrac{BC^2}{4} = AB^2$

$\Rightarrow 4AD^2 + BC^2 = 4AB^2$

$\Rightarrow 4AD^2 = 4AB^2 - BC^2$ [1]

$\Rightarrow 4AD^2 = 4AB^2 - AB^2$ ($BC = AB$ in equilateral triangle)

$\Rightarrow 4AD^2 = 3AB^2$

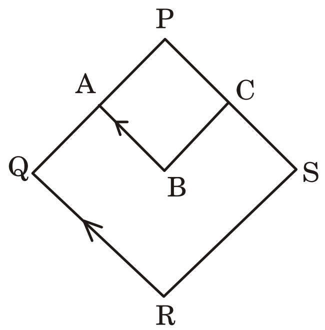
$\Rightarrow AD^2 = \dfrac{3}{4} \times AB^2$

Hence Proved. [1]

22. To prove: $\dfrac{PA}{PQ} = \dfrac{PC}{PS}$

Given: $AB \| QR$ and $BC \| RS$

Proof:



Since $AB \| QR$,

$\angle PAB = \angle PQR$ [Corresponding angles are equal]

$\angle PBA = \angle PRQ$ [Corresponding angles are equal]

Hence in ΔPAB and ΔPQR,

$\angle APB = \angle QPR$ [Common angle]

$\angle PAB = \angle PQR$ and $\angle PBA = \angle PRQ$

$\Delta PAB \sim \Delta PQR$ since all the angles are equal.

Similarly $BC \| RS$ [1]

$\angle PCB = \angle PSR$ [Corresponding angles are equal]

$\angle PBC = \angle PRS$ [Corresponding angles are equal]

Hence in ΔPCB and ΔPSR,

$\angle CPB = \angle SPR$ [common angle]

$\angle PCB = \angle PSR$ and $\angle PBC = \angle PRS$

$\Delta PCB \sim \Delta PSR$ since all the angles are equal.

Now $\Delta PCB \sim \Delta PSR$ and $\Delta PAB \sim \Delta PQR$, [1]

Therefore their corresponding sides are proportional,

$$\frac{PC}{PS} = \frac{PB}{PR} \quad ...(i)$$

$$\frac{PA}{PQ} = \frac{PB}{PR} \quad ...(ii)$$

Hence from (i) and (ii),

$$\frac{PA}{PQ} = \frac{PC}{PS}$$

Hence proved. [1]

23. In right triangle CBD using Pythagoras theorem,

$CD^2 = BD^2 + BC^2$

$17^2 = 8^2 + BC^2$

$BC^2 = 17^2 - 8^2 = 289 - 64$

$BC^2 = 225$

$BC = 15$ m [1]

Now since $AD = 4$m and $BD = 8$m,

$AB = AD + BD$

$AB = 12$ m [1]

Now in triangle ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 15^2 = 144 + 225$

$AC^2 = 369$

$AC = 3\sqrt{41}$ m [1]

Hence required answer is $AC = 3\sqrt{41}$ m.

24. Given: $\Delta ABD \sim \Delta PQS$

To prove: $\Delta ABC \sim \Delta PQR$

Proof:

Since $\Delta ABD \sim \Delta PQS$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ (Corresponding sides of similar triangles are proportional) ...(i) [1]

Also, $\angle BAD = \angle QPS, \angle B = \angle Q, \angle ADB = \angle PSQ$ (Corresponding angles of similar triangles are equal). ...(ii)

Now it is given that AD and PS are medians.

Therefore $BD = \frac{BC}{2}$ and QS=$\frac{QR}{2}$

From (i) we get,

$$\frac{AB}{PQ} = \frac{\frac{BC}{2}}{\frac{QR}{2}}$$

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$ ...(iii) [1]

Now in ΔABC and ΔPQR we have,

$\frac{AB}{PQ} = \frac{BC}{QR}$ (From (iii))

Also $\angle B = \angle Q$ (From (ii))

$\Rightarrow \Delta ABC \sim \Delta PQR$ by SAS criterion.

Hence proved. [1]

25. In ΔABC, we have $AB = 12$ cm, $AC = 13$ cm and $\angle B = 90^\circ$

Using the Pythagoras theorem we have,

$(AC)^2 = (AB)^2 + (BC)^2$ [1]

$\Rightarrow (13)^2 = (12)^2 + (BC)^2$

$\Rightarrow 169 = 144 + (BC)^2$

$\Rightarrow (BC)^2 = 169 - 144 = 25$

$\Rightarrow BC = 5$ cm

Now in ΔEDC, we have $CE = 5$ cm and $ED = 4$ cm and $\angle D = 90^\circ$ [1]

Using the Pythagoras theorem we have,

$(CE)^2 = (CD)^2 + (ED)^2$

$\Rightarrow (5)^2 = (CD)^2 + (4)^2$

$\Rightarrow 25 = (CD)^2 + 16$

$\Rightarrow (CD)^2 = 25 - 16 = 9$

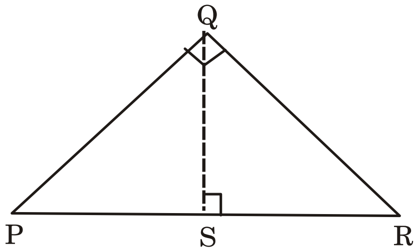
$\Rightarrow CD = 3$ cm

$BD = BC + CD$

$\Rightarrow BD = 5 \text{ cm} + 3 \text{ cm} = 8 \text{ cm}$

Hence, the length BD is 8 cm. [1]

26. Pythagoras theorem: In a right triangle, the square of the hypotenuse is equal to sum of the squares of the other two sides.



Given: ΔPQR is a right triangle, right angled at Q.

To prove: $PR^2 = PQ^2 + QR^2$

Construction: Draw $QS \perp PR$

Proof: In ΔPQS and ΔPQR,

$\angle PSQ = \angle PQR = 90^\circ$

$\angle QPS = \angle QPR$ (common) [1]

(by AAA similarity criterion)

$\Rightarrow \frac{PS}{PQ} = \frac{PQ}{PR}$

$\Rightarrow PS \times PR = PQ^2$ ...(i)

Now, In ΔQSR and ΔPQR,

$\angle QSR = \angle PQR = 90^\circ$

$\angle QRS = \angle QRP$ (common)

$\therefore \Delta QSR - \Delta PQR$ (by AA similarity criterion)

$\Rightarrow \frac{RS}{QR} = \frac{QR}{PR}$

$\Rightarrow RS \times PR = QR^2$ ...(ii) [1]

Adding (i) and (ii)

$\Rightarrow PQ^2 + QR^2 = PS \times PR + RS \times PR$

$\Rightarrow PQ^2 + QR^2 = PR(PS + RS)$

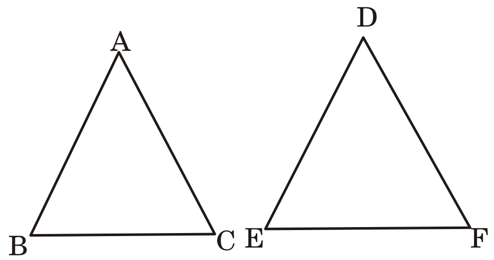
$\Rightarrow PQ^2 + QR^2 = PR.PR$

$\Rightarrow PQ^2 + QR^2 = PR^2$

$\Rightarrow PR^2 = PQ^2 + QR^2$

Hence proved [1]

27. Let the two triangles be ΔABC and ΔDEF



If two triangles are similar than the ratio of areas is equal to square of ratio of its corresponding sides

$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DF}\right)^2$ [1]

As the areas are same

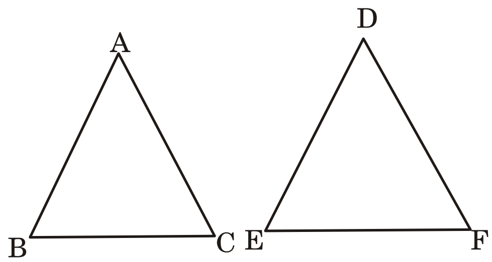
$\therefore \left(\frac{BC}{EF}\right)^2 = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DF}\right)^2 = 1$

$\Rightarrow \frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} = 1$

or

$BC = EF, AB = DE$ and $AC = DF$.

Now in ΔABC and ΔDEF



[1]

$AB = DE$

$BC = EF$

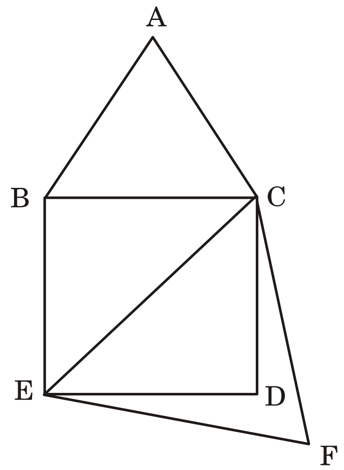
$CA = FD$

Hence by SSS congruency

$\Delta ABC \cong \Delta DEF$

Hence proved. [1]

28. The diagram will be:



Let the length of the side of the square be a.

Diagonal of the square EC = $\sqrt{2}a$ [1]

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2$

Area of $\Delta ECF = \frac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2$ [1]

$= \frac{\sqrt{3}}{4} \times 2a^2$

$= \frac{\sqrt{3}}{2} \times a^2$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} \times a^2\right)$

Which is equal to half of area of ΔABC.

Hence proved. [1]

29. ar (ΔBXY)=2ar($ACYX$).

ar (Δ BXY) = 2ar(Δ $ABC - \Delta BXY$)

3ar(Δ BXY)=2 ar(Δ ABC)

$\Rightarrow \frac{ar(\Delta BXY)}{ar(\Delta ABC)} = \frac{2}{3}$ ...(i) [1]

Consider ΔBXY and Δ ABC,

$\angle BXY = \angle BAC$ (Corresponding angles as $XY \mid\mid AC$)

$\angle BYX = \angle BCA$ (Corresponding angles as $XY \mid\mid AC$)

$\therefore \Delta BYX \cong \Delta BCA$ (AA similarity criterion)

$\Rightarrow \frac{ar(\Delta BXY)}{ar(\Delta ABC)} = \left(\frac{BX}{AB}\right)^2$ ...(ii) [1]

From (i) and (ii),

$\left(\frac{BX}{AB}\right)^2 = \frac{2}{3}$

Now, $BX = AB - AX$

$\left(\frac{AB - AX}{AB}\right)^2 = \frac{2}{3}$

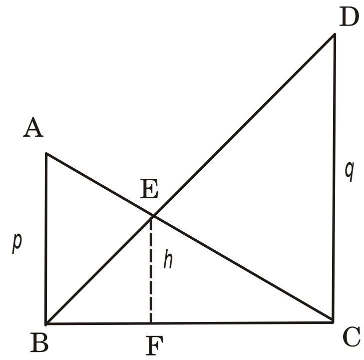
$\Rightarrow \left(1 - \frac{AX}{AB}\right)^2 = \frac{2}{3}$ [1]

Taking square root,

$1 - \frac{AX}{AB} = \sqrt{\frac{2}{3}}$

$\Rightarrow \frac{AX}{AB} = 1 - \sqrt{\frac{2}{3}}$ [1]

30. Let height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole be h.



[1]

Consider ΔABC and ΔEFC

$\angle ABC = \angle EFC = 90^\circ$

$ACB = \angle ECF$

$\Rightarrow \Delta ABC \cong \Delta EFC$ (AAA similarity criterion)

$\Rightarrow \frac{AB}{EF} = \frac{BC}{FC}$

$\Rightarrow \frac{p}{h} = \frac{a}{FC}$

$\Rightarrow FC = \frac{ah}{p}$ ...(i) [1]

Consider ΔDCB and ΔEFB

$\angle DCB = \angle EFB = 90^\circ$

$\angle DBC = \angle EBF$

$\Rightarrow \Delta DCB \cong \Delta EFB$ (AA similarity criterion)

$$\Rightarrow \frac{DC}{EF} = \frac{BC}{FB}$$

$$\Rightarrow \frac{q}{h} = \frac{a}{FB}$$

$$\Rightarrow FB = \frac{ah}{q} \quad ...(ii) \quad [1]$$

Add equations (i) and (ii),

$$\Rightarrow FC + FB = \frac{ah}{p} + \frac{ah}{q}$$

$$\Rightarrow a = ah\left(\frac{1}{p} + \frac{1}{q}\right)$$

$$\Rightarrow h = \frac{1}{\frac{1}{p} + \frac{1}{q}} = \frac{1}{\frac{q+p}{pq}}$$

$$\Rightarrow h = \frac{pq}{p+q}$$

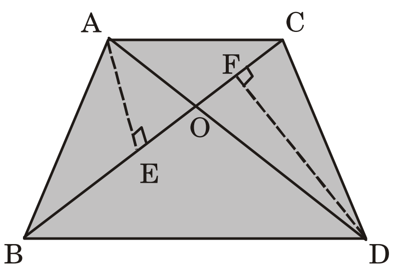
Hence proved. [1]

31. Given: BC is the common base of ΔABC and (ΔDBC).

To Prove: $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$

Draw,

$AE \perp BC$ and $DF \perp BC$ in ΔABC and ΔDBC respectively.



Hence, $ar(\Delta ABC) = \dfrac{1}{2} \times BC \times AE$ ...(i)

And, $ar(\Delta DBC) = \dfrac{1}{2} \times BC \times DF$ ...(ii) [1]

Taking ratio of equation (i) and equation (ii), we get,

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AE}{DF}$$

But we need to prove that,

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Hence, we need to prove that,

$$\frac{AO}{DO} = \frac{AE}{DF} \quad [1]$$

In ΔAOE and ΔDOF,

$\angle AEO = \angle DFO = 90^\circ$

$\angle AOE = \angle DOF$ (Vertically opposite angles)

Hence, by AA similarity criterion

$\Delta AOE = \Delta DOF$

We know that, if two triangles are similar, their corresponding sides are in the same ratio

Therefore, $\dfrac{AE}{DF} = \dfrac{AO}{DO}$ ...(iii) [1]

Now, we have,

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AE}{DF} \quad ...(iv)$$

On comparing equation (iii) and equation (iv), we get,

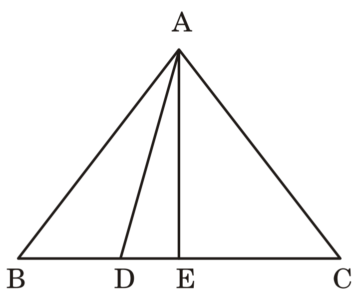
$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Hence proved. [1]

32. It is given that in an equilateral triangle ABC, the side BC is trisected at D such that $BD = \dfrac{1}{3}BC$.

To prove: $9AD^2 = 7AB^2$

Construction: Draw $AE \perp BC$.



[1]

In ΔABE and ΔACE .

$AB = AC$

$AE = AE$ $\because$ Common sides are equal.

$\angle AEB = \angle AEC = 90°$

$\therefore \Delta AEB \cong \Delta AEC$

Also, $BE = EC$ By C.P.C.T

$$BE = EC = \frac{BC}{2}$$

In a right angled triangle ADE.

$$AD^2 = AE^2 + DE^2 \quad ......(i)$$

In a right angled triangle ABE.

$$AB^2 = AE^2 + BE^2 \quad ......(ii) \quad [1]$$

Subtract equation (ii) from equation (i) we get,

$$\Rightarrow AD^2 - AB^2 = DE^2 - BE^2$$

$$\Rightarrow AD^2 - AB^2 = (BE - BD)^2 - BE^2$$

$$\Rightarrow AD^2 - AB^2 = \left(\frac{BC}{2} - \frac{BC}{3}\right)^2 - \left(\frac{BC}{2}\right)^2$$

$$\Rightarrow AD^2 - AB^2 = \left(\frac{(3BC - 2BC)}{6}\right)^2 - \left(\frac{BC}{2}\right)^2 \quad [1]$$

$$\Rightarrow AD^2 - AB^2 = \frac{BC^2}{36} - \frac{BC^2}{4}$$

$$\Rightarrow AD^2 = \frac{(36AB^2 + AB^2 - 9AB^2)}{36}$$

$$\Rightarrow AD^2 = \frac{(28AB^2)}{36}$$

$$\Rightarrow AD^2 = \frac{(7AB^2)}{9}$$

$$\Rightarrow 9AD^2 = 7AB^2$$

Hence Proved. [1]

33. In ΔADE and ΔABC

Given ,

$\angle ADE = \angle B$ (Given)

$\angle A = \angle A$ (Common)

$\angle C = \angle E$ (each 90°) [1]

Hence, $\Delta ADE \sim \Delta ABC$ by R.H.S criterion.

Now $\Delta ADE \sim \Delta ABC$

So,

$$\frac{AC}{AE} = \frac{AB}{AD} = \frac{BC}{DE}$$

$AB = BE + AE$

$\Rightarrow AB = 4.2 + 7.2 = 11.4\text{cm}$ [1]

Now,

$$\frac{AC}{AE} = \frac{AB}{AD} = \frac{BC}{DE}$$

$$\Rightarrow \frac{AC}{7.2} = \frac{11.4}{7.6} = \frac{5.4}{DE} \quad [1]$$

Now,

$$\Rightarrow \frac{11.4}{7.6} = \frac{5.4}{DE}$$

$$\Rightarrow 11.4 \times DE = 5.4 \times 7.6$$

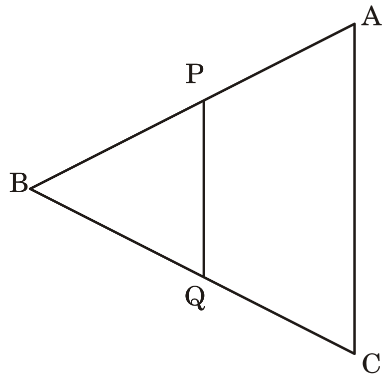
$$\Rightarrow DE = \frac{7.6 \times 5.4}{11.4}$$

Hence $DE = 3.6$ cm . [1]

34. Now, Given: In

ΔABC, $PQ \parallel AC$ and $\Delta PBC \sim \Delta ABC$

and $ar(PBC) = ar(APQC)$



To find: ratio of $\frac{AP}{AB}$

Solution: $ar(PBC) = ar(APQC)$ ...(i)

As $\Delta PBC \sim \Delta ABC$ [1]

So,

$$\frac{ar(PBC)}{ar(ABC)} = \left(\frac{PB}{AB}\right)^2 \times \left(\frac{BQ}{BC}\right)^2 = \left(\frac{PQ}{AC}\right)^2$$

Taking $\frac{ar(PBC)}{ar(\Delta ABC)} = \left(\frac{PB}{AB}\right)^2$

$$\frac{ar(\Delta PBC)}{ar\Delta PBC + ar(APQC)} = \left(\frac{PB}{AB}\right)^2$$ [1]

$$\Rightarrow \frac{ar(\Delta PBC)}{2ar(\Delta PBC)} = \left(\frac{PB}{AB}\right)^2$$ ... from (ii)

$$\Rightarrow \frac{1}{2} = \left(\frac{PB}{AB}\right)^2$$

Taking square root of both the sides, we get

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{PB}{AB}$$ [1]

$$\Rightarrow 1 - \frac{1}{\sqrt{2}} = 1 - \frac{PB}{AB}$$

$$\Rightarrow \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{AB - PB}{AB}$$

$$\Rightarrow \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{AP}{AB}$$

After rationalization of the denominator, we get

$$\Rightarrow \frac{AP}{AB} = \frac{2-\sqrt{2}}{2}$$

So, the ratio of is

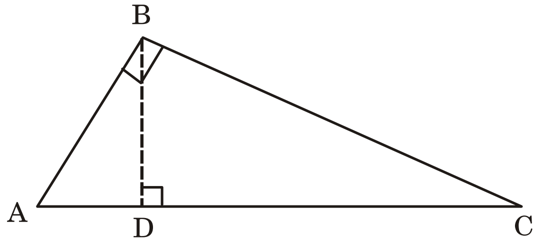
$$\Rightarrow \frac{AP}{AB} \text{is} \frac{2-\sqrt{2}}{2}.$$ [1]

35. Consider a right triangle ABC right angled at B. Draw $BD \perp AC$

Consider $\Delta\ ADB$ and $\Delta\ ABC$,

$\angle ADB = \angle ABC = 90^\circ$

$\angle BAD = \angle CAB$ (Common)



[1]

$\Delta\ ADB \sim \Delta\ ABC$ (Using AA similarity criterion)

$\frac{AD}{AB} = \frac{AB}{AC}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow AB^2 = AD \times AC$ .... (i) [1]

Consider ΔBDC and ΔABC,

$\angle BDC = \angle ABC = 90^\circ$

$\angle BCD = \angle ACB$ (Common)

$\Delta\ BDC \sim \Delta\ ABC$ (Using AA similarity criterion)

$\frac{DC}{BC} = \frac{BC}{AC}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow BC^2 = CD \times AC$ ...(ii) [1]

Adding (i) and (ii), we get

$$AB^2 + BC^2 = (AD \times AC) + (CD \times AC)$$

$$\Rightarrow AB^2 + BC^2 = AC \times (AD + CD)$$

$$\Rightarrow AB^2 + BC^2 = AC^2$$ [1]

Hence proved.

Smart Notes ....

Smart Notes ....

CHAPTER 8

Circles

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Tangent to Circle			2 marks	1 marks	1,2,2,4 marks	1,1 marks
Question based on Proving Properties of Tangent	3 marks	3 marks	2,3, 4 marks	2,2,4, 4 marks	4 marks	2,4,4 marks

Summary

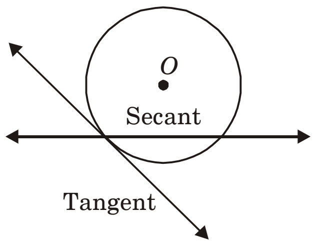
Circles

DEFINITIONS

Secant: A line, which intersects a circle in two distinct points, is called a secant.

Tangent: A line meeting a circle only in one point is called a tangent to the circle at that point.

The point at which the tangent line meets the circle is called the point of contact.



Length of tangent: The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

- ***There is no tangent passing through a point lying inside the circle.***
- ***There is one and only one tangent passing through a point lying on a circle.***
- ***There are exactly two tangents through a point lying outside a circle.***

Theorem 1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

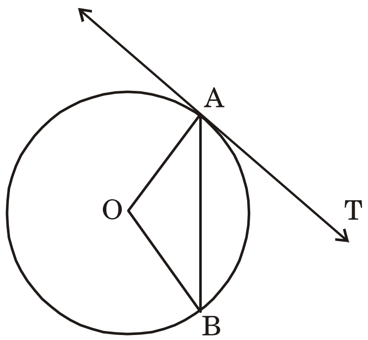
Theorem 2 : The lengths of tangents drawn from an external point to a circle are equal.

- ***The centre lies on the bisector of the angle between the two tangents.***

PREVIOUS YEARS' EXAMINATION QUESTIONS

1 Mark Questions

1. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 110°$, then $\angle BAT$ is equal to



(*a*) 100° (*b*) 40°
(*c*) 50° (*d*) 90°

[TERM 2, 2011]

2. The radii of two circles are 4 cm and 3 cm respectively. The diameter of the circle having area equal to the sum of the areas of the two circles (in cm) is

(*a*) 5
(*b*) 7
(c) 10
(*d*) 14

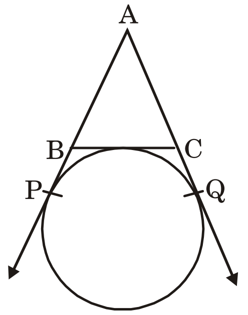
[TERM 2, 2011]

3. From a point Q, 13 cm away from the centre of a circle, the length of tangent PQ to the circle is 12 cm. The radius of the circle (in cm) is

(*a*) 25
(*b*) $\sqrt{313}$
(*c*) 5
(d) 1

[TERM 2, 2012]

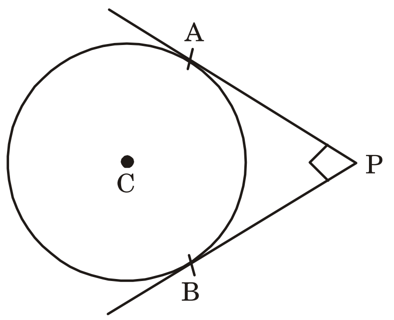
4. In Figure 1, AP, AQ and BC are tangents to the circle. If AB = 5, BC = 4 and AC = 6 cm, then the length of AP (in cm) is



(*a*) 7.5 (*b*) 15
(*c*) 10 (*d*) 9

[TERM 2, 2012]

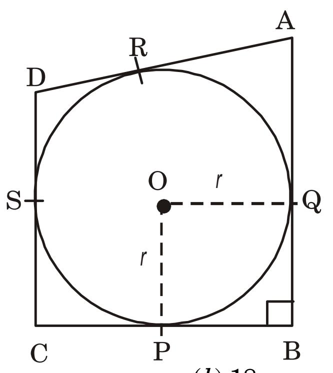
5. In Fig. PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then the length of each tangent is:



(*a*) 3 cm (*b*) 4 cm
(*c*) 5 cm (*d*) 6 cm

[TERM 2, 2013]

6. In Fig.2, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively, If AB = 29 cm, AD = 23 cm, $\angle B = 90°$ and DS = 5 cm, then the radius of the circle (in cm.) is:



(*a*) 11 (*b*) 18
(*c*) 6 (*d*) 15

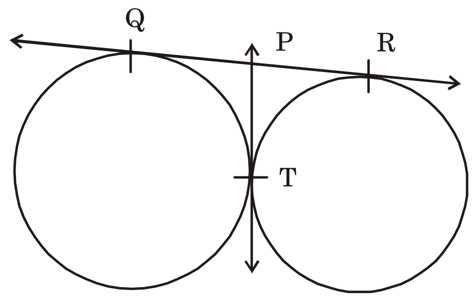
[TERM 2, 2013]

7. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. The value of $\angle APB$ is

(a) 30° (b) 45°
(c) 60° (d) 90°

[TERM 2, 2014]

8. In figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If QP = 3.8, then the length of QR (in cm) is :



(*a*) 3.8 (*b*) 7.6
(*c*) 5.7 (*d*) 1.9

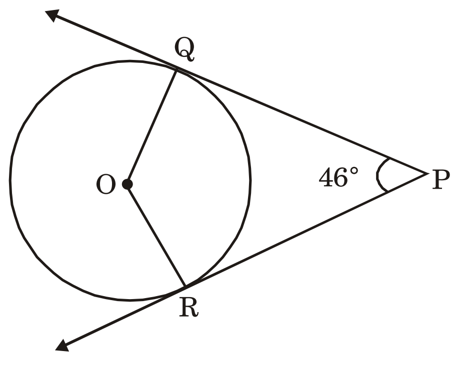
[TERM 2, 2014]

9. In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is

(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1

[TERM 2, 2014]

10. In figure, PQ and PR are two tangents to a circle with centre O. If $\angle PQR = 46°$, then $\angle QOR$ equals:



(a) 67°
(b) 134°
(c) 44°
(d) 46°

[TERM 2, 2014]

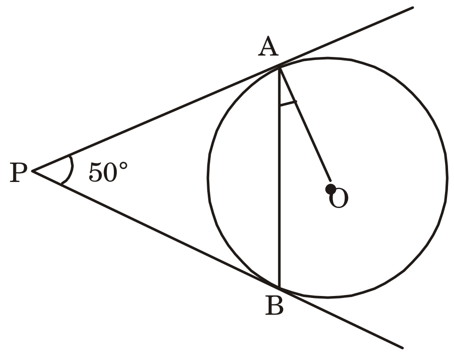
11. A chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is

(*a*) $5\sqrt{2}$ (*b*) $10\sqrt{2}$

(*c*) $\frac{5}{\sqrt{2}}$ (*d*) $10\sqrt{3}$

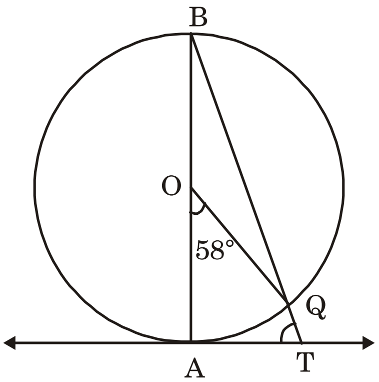
[TERM 2, 2014]

12. In given Fig., *PA* and *PB* are tangents to the circle with centre O such that $\angle APB = 50^\circ$. Write the measure of $\angle OAB$.



[TERM 2, 2015]

13. In Fig., AB is the diameter of a circle with centre *O* and *AT* is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



[TERM 2, 2015]

14. From an external point *P*, tangents *PA* and *PB* are drawn to a circle with centre *O*. If $\angle PAB = 50^\circ$, then find $\angle AOB$.

[TERM 2, 2016]

15. If the angle between two tangents drawn from an external point *P* to a circle of radius a and centre *O*, is 60°, then find the length of *OP*.

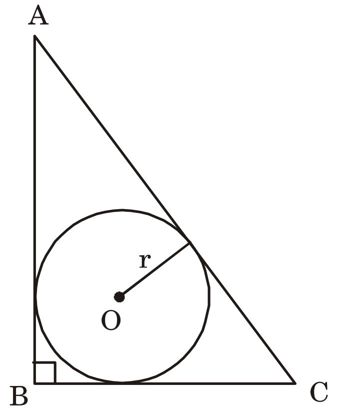
[TERM 2, 2017]

2 Marks Questions

16. Two concentric circles are of radii 7 cm and *r* cm respectively, where $r > 7$. A chord of the larger circle of length 48 cm, touches the smaller circle. Find the value of *r*.

[TERM 2, 2011]

17. In Figure, a right triangle *ABC*, circumscribes a circle of radius r. If *AB* and *BC* are of lengths 8 cm and 6 cm respectively, find the value of *r*.

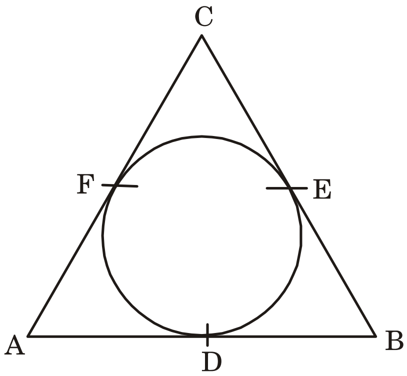


[TERM 2, 2012]

18. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

[TERM 2, 2012]

19. In Fig., a circle inscribed in triangle *ABC* touches its sides *AB*, *BC* and *AC* at points *D*, *E* and *F* respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of *AD*, *BE* and *CF*.

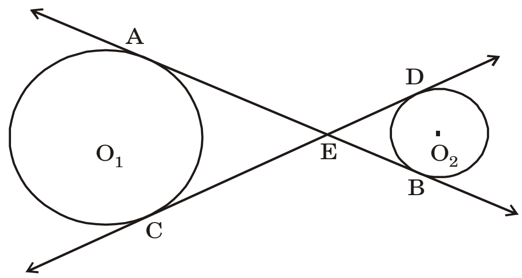


[TERM 2, 2013]

20. Prove that the parallelogram circumscribing a circle is a rhombus.

[TERM 2, 2013]

21. In Figure, common tangents *AB* and *CD* to the two circles with centres O_1 and O_2 intersect at *E*. Prove that *AB* = *CD*.



[TERM 2, 2014]

22. Prove that the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

[TERM 2, 2014]

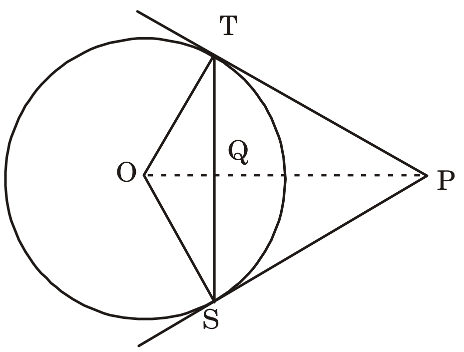
23. The incircle of an isosceles triangle ABC, in which $AB = AC$, touches the sides BC, CA and AB at D, E and F respectively. Prove that $BD = DC$.

[TERM 2, 2014]

24. If from an external point P of a circle with center O, two tangents PQ and PR are drawn such that $\angle QPR = 120°$, prove that $2\,PQ = PO$

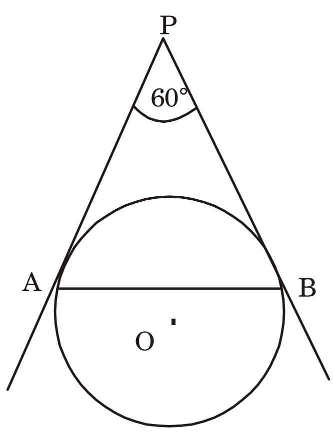
[TERM 2, 2014]

25. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If $OP = 2\,r$, show that $\angle OTS = \angle OST = 30°$



[TERM 2, 2016]

26. In Fig., AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60°$. Find the length of chord AB

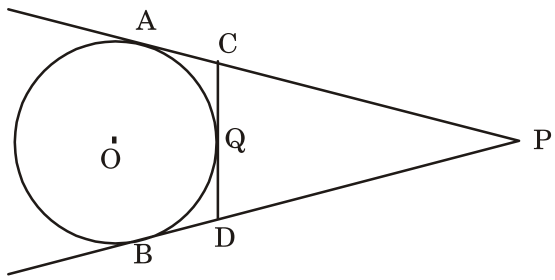


[TERM 2, 2016]

27. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

[TERM 2, 2017]

28. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.



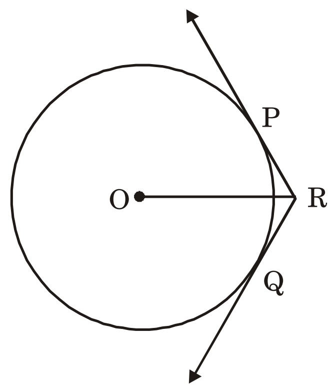
[TERM 2, 2017]

3 Marks Questions

29. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

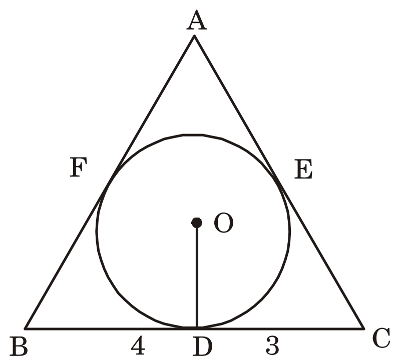
[TERM 2, 2015]

30. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that $OR = PR + RQ$.



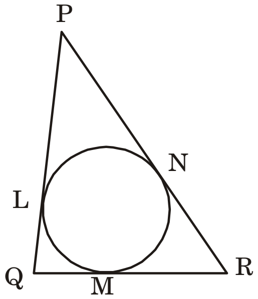
[TERM 2, 2015]

31. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21$ cm^2, then find the lengths of sides AB and AC.



[TERM 2, 2011]

32. In Figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths QM, RN and PL.



[TERM 2, 2012]

33. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

[TERM 2, 2017]

4 Marks Questions

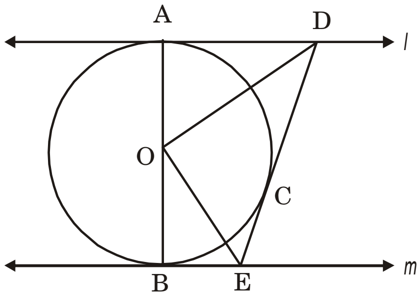
34. Prove that the lengths of tangents drawn from an external point to a circle are equal.

[TERM 2, 2012]

35. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

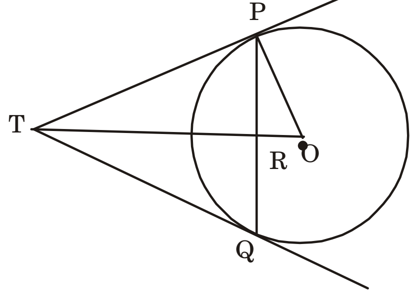
[TERM 2, 2013]

36. In fig., l and m are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line l at D and m at E. Prove that $\Delta DOE = 90°$



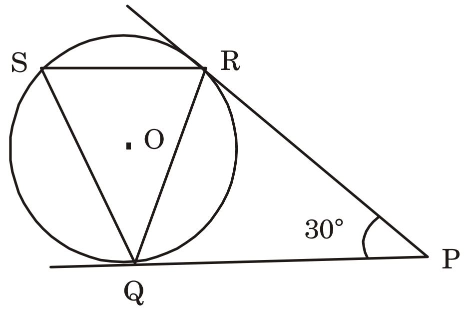
[TERM 2, 2013]

37. In Figure 4, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



[TERM 2, 2014]

38. In Fig., tangents PQ and PR are drawn from an external point P to a circle with center O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.

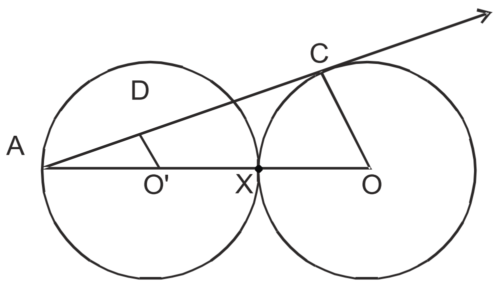


[TERM 2, 2015]

39. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

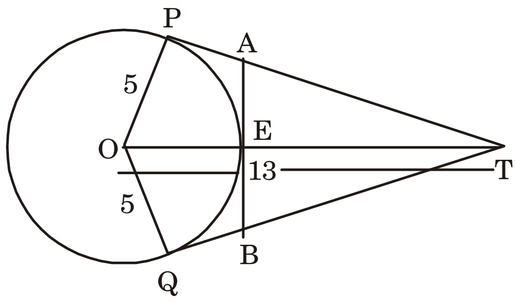
[TERM 2, 2015]

40. In the given figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. $O'D$ is perpendicular to AC. Find the value of $\frac{DO'}{CO}$



[TERM 2, 2016]

41. In Fig., O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



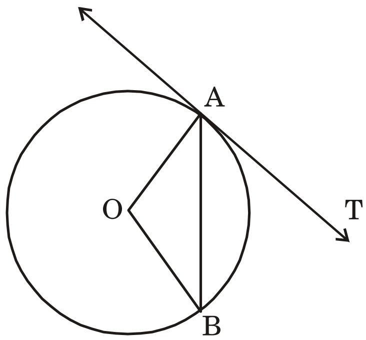
[TERM 2, 2016]

42. Prove that the lengths of tangents drawn from an external point to a circle are equal.

[CBSE 2017]

Solutions

1. Consider the given diagram



Here, in ΔAOB, $\angle AOB = 100°$, $OA = OB$ = Radius of the circle

$\Rightarrow$ ΔAOB is an isosceles triangle.

$\therefore$ $\angle OBA = \angle OAB = x$

In ΔAOB apply angle sum property.

$\therefore$ $\angle AOB + \angle OBA + \angle OAB = 180°$

$\Rightarrow$ $100° + x + x = 180°$

$\Rightarrow$ $100° + 2x = 180°$

$\Rightarrow$ $2x = 180° - 100°$

$\Rightarrow$ $2x = 80°$

$\Rightarrow$ $x = 40°$

$\therefore$ $\angle OBA = \angle OAB = 40°$ [½]

Clearly $\angle OAT = 90°$ (as tangents are perpendicular to the radius at the point of contact.)

From the diagram,

$\angle OAT = \angle OAB + \angle BAT$

$\Rightarrow$ $90° = 40° + \angle BAT$

$\Rightarrow$ $\angle BAT = 90° - 40° = 50°$

Hence, the correct option is (c). [½]

2. Let r_1 and r_2 denote the radii of two circles and R be the radius of the bigger circle.

$\therefore$ r_1 = 4 cm and r_1 = 3 cm

According to the question,

Sum of the areas of two smaller circles = Area of the bigger circle

$\pi r_1^2 + \pi r_2^2 = \pi R^2$ [½]

Substituting the values of r_1 and r_2 in the above equation.

$\Rightarrow$ $\pi(4)^2 + \pi(3)^2 = \pi R^2$

$\Rightarrow$ $(4)^2 + (3)^2 = R^2$

$\Rightarrow$ $16 + 9 = R^2$

$\Rightarrow$ $R^2 = 6 + 9$

$\Rightarrow$ $R^2 = 25$

$\Rightarrow$ $R = \sqrt{25}$

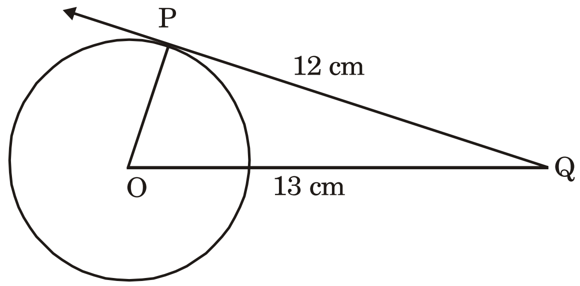
$\Rightarrow$ $R = 5$

Diameter = 2 × Radius

Therefore, the required diameter of the circle is

Hence, the correct option is (c). [½]

3.



We know that, tangent at a point is perpendicular to the radius through that point. Therefore, OP is perpendicular to PQ. [½]

In right triangle OPQ, Using pythagoras theorem, we have

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow$ $(13)^2 = OP^2 + (12)^2$

$\Rightarrow$ $OP^2 = 169 - 144$

$\Rightarrow$ $OP^2 = 25$

$\Rightarrow$ $OP = 5$

Therefore, the length of the radius of the circle, OP is 5 cm.

The correct option is (c). [½]

4. Let us suppose that the tangent BC touches the circle at point O.

Now, we know that, tangents drawn from an exterior point to a circle are equal in length.

$\therefore$ $BP = BO$ [From external point B]

$CQ = CO$ [From external point C]

$AP = AQ$ [From external point A]

Now, In ΔABC, [½]

$AB + BC + AC$ = 5 cm + 4 cm + 6 cm

$\Rightarrow$ $AB + (BO + CO) + AC$ = 15 cm

$\Rightarrow$ $(AB + BP) + (CQ + AC)$ = 15 cm

[$\because$ $BP = BO$ and $CQ = CO$]

$\Rightarrow$ $AP + AQ$ = 15 cm

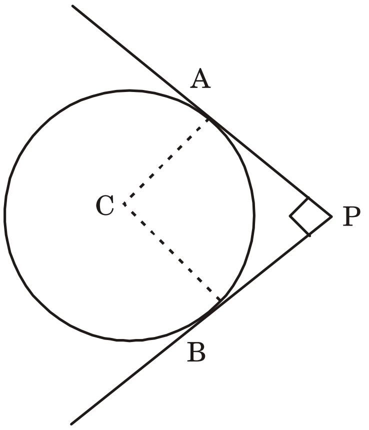
$\Rightarrow$ $2AP$ = 15 cm [$\because$ $AP = AQ$]

$\Rightarrow$ AP = 7.5 cm

Therefore, length of the tangent AP is 7.5 cm.

The correct answer is (a). [½]

5.



It is given that $AP \perp PB$ and radius of the circle is 4 cm

Construction: Join CA and CB.

It is known fact that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ $AC \perp AP$ and $BC \perp PB$

$\Rightarrow$ $\angle CAP = \angle CBP = 90°$

Also,

In quadrilateral $ACBP$,

$\angle CAP + \angle APB + \angle PBC + \angle BCA = 360°$ [½]

(Angle sum property)

$\Rightarrow$ $90° + 90° + 90° + \angle BCA = 360°$

$\Rightarrow 270° + \angle BCA = 360°$

$\Rightarrow$ $\angle BCA = 360° - 270°$

$\Rightarrow$ $\angle BCA = 90°$

And also,

$AC = CB$ (Radius of the circle)

Here all sides of $APBC$ are equal and all angles are 90°

$\therefore$ $APBC$ is a square.

$\therefore$ $AC = CB = BP = PA = 4$ cm

Thus the length of each tangent is "4 cm"

Hence the correct answer is (b). [½]

6. Here it is given that AB, BC, CD and AD are tangents to the circle with centre O and touch the circle at Q, P, S and R respectively.

And also,

$BA = 29$ cm, $AD = 23$ cm, $\angle B = 90°$ and $DS = 5$ cm

We know that the lengths of the tangents drawn from an external point to a circle are equal.

$DS = DR = 5$ cm

$AR = AD - DR = 23 - 5 = 18$ cm ($AD = 23$ cm)

$\Rightarrow$ $AQ = AR = 18$ cm

$\therefore$ $QB = AB - AQ = 29 - 18 = 11$ cm (AB = 29 cm)

And,

$QB = BP = 11$ cm

$\angle PBQ = 90°$ [Given]

We know that, angle between the tangent and the radius at the point of contact is a right angle.

Thus,

$\angle OPB = 90°$ and $\angle OQB = 90°$ [½]

Now

In quadrilateral $OPBQ$,

$\angle PBQ + \angle OPB + \angle OQB + \angle POQ = 360°$

[Angle sum property of a quadrilateral]

$\Rightarrow$ $90° + 90° + 90° = \angle POQ = 360°$

$\Rightarrow$ $270° + \angle POQ = 360°$

$\Rightarrow$ $\angle POQ = 360° - 270°$

$\Rightarrow$ $\angle POQ = 90°$

Here all sides of $OPBQ$ are equal and all angles are

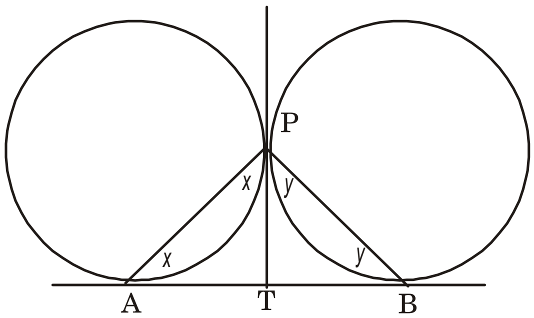
$\therefore$ $OPBQ$ is a square.

$\therefore$ $OQ = QB = BP = PO = r = 11$ cm

Thus the radius of the circle is "11 cm"

Hence the correct answer is (a). [½]

7. The diagram is represented as follows:



In ΔTAP

TA = TP (Tangents from an external point are equal) [½]

So $\angle TAP = \angle TPA = x$ (Corresponding angles are equal)

In ΔTBP

TB = TP (Tangents from an external point are equal)

So $\angle TBP = \angle TPB = y$ (Corresponding angles are equal)

In ΔPAB, $\angle PAB + \angle PBA + \angle APB$ (Sum of angles of a triangle)

$\Rightarrow\ x + y + x + y = 180°$

$\Rightarrow\ 2x + 2y = 180°$

$\Rightarrow\ 2(x + y) = 180°$

$\Rightarrow x + y = \dfrac{180°}{2}$

Therefore, $\angle APB = x + y = 90°$

The correct answer is (d). [½]

8. It is known that the length of the tangents drawn from an external point to a circle is equal.

$QP = PT = 3.8$ cm ....(i)

$PR = PT = 3.8$ cm ....(ii)

From equations (i) and (ii), we get:

$QP = PR = 3.8$ cm

Now,

$QR = QP + PR$

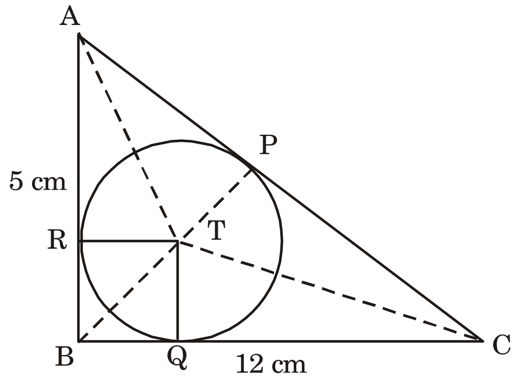
$= 3.8$ cm $+ 3.8$ cm

$= 7.6$ cm

Hence, the correct option is (b). [1]

9. Consider the triangle *ABC*, right angled at *B* where AB = 5 cm and BC = 12 cm.

T is the centre of the circle and *TP*, *TR* and *TQ* are the radii of the inscribed circle.



ABC is a right angle triangle with an inscribed circle centered at *T*.

Let *r* be the radius of the circle, so $TQ = TR = TP$

AB, *BC* and *AC* are tangents to the circle at *R*, *Q* and *P*.

Using Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow\ AC^2 = 5^2 + 12^2$

$\Rightarrow\ AC^2 = 25 + 144$

$\Rightarrow\ AC^2 = 169$

$\Rightarrow \text{AC} = \sqrt{169} = 13$ cm

Area of $\Delta\text{ABC} = \dfrac{1}{2} \times \text{AB} \times \text{BC}$

$\Rightarrow$ Area of $\Delta\text{ABC} = \dfrac{1}{2} \times 5 \times 12 = 30\,\text{cm}^2$

Also, Area of ΔABC = Area ΔTAB + Area ΔTBC + Area ΔTCA

$\Rightarrow 30 = \dfrac{1}{2} \times \text{TR} \times \text{AB} + \dfrac{1}{2} \times \text{TQ} \times \text{BC} + \dfrac{1}{2} \times \text{TP} \times \text{AC}$

$\Rightarrow 30 = \dfrac{1}{2} \times \text{r} \times \text{AB} + \dfrac{1}{2} \times \text{r} \times \text{BC} + \dfrac{1}{2} \times \text{r} \times \text{AC}$

(*TP*, *TR* and *TQ* are the radii of the circle)

$\Rightarrow 30 = \dfrac{1}{2} \times \text{r}\left(\text{AB} + \text{BC} + \text{AC}\right)$ [½]

Substitute the value for the length of the sides

$\Rightarrow 30 = \dfrac{1}{2} \times \text{r}\left(5 + 12 + 13\right)$

$\Rightarrow 30 = \dfrac{1}{2} \times \text{r} \times 30$

$\Rightarrow r = \dfrac{30 \times 2}{30} = 2\,\text{cm}$

The correct answer is (c). [½]

10. Given: $\angle QPR = 46°$

PQ and *PR* are tangents.

Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

So, we have $OQ \perp PQ$ and $OR \perp RP$.

$\angle OQP = \angle ORP = 90°$

So, in quadrilateral *PQOR*, we have

$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360°$

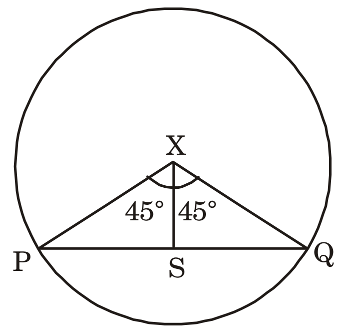
(Angle sum property)

$\Rightarrow\ 90° + 46° + 90° + \angle ROQ = 360°$

$\angle ROQ = 360° - 226° = 134°$

Hence, the correct option is (b). [1]

11. The figure represents the circle of radius 10 cm with the chord that subtends right angle at the centre.



Consider ΔPXS

$$\cos 45° = \frac{XS}{PX}$$

$$\Rightarrow \cos 45° = \frac{XS}{10}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{XS}{10}$$

$$\Rightarrow XS = \frac{10}{\sqrt{2}} = 5\sqrt{2}\,\text{cm}$$

Thus the correct answer is (a). [1]

12. Here PA and PB are the two tangents.

We know that the tangents drawn from the external point are equal.

$\therefore \quad PA = PB$

The ΔPAB is an isosceles triangle with $PA = PB$

Therefore $\angle PAB = \Delta PBA$

...(i) (Angles opposite to equal sides)

In ΔPAB,

$$\angle APB + \angle PAB + \angle PBA = 180°$$

$$\Rightarrow 50° + 2\angle PAB = 180° \quad \text{(Using (i))}$$

$$2\angle PAB = 180° - 50°$$

$$\Rightarrow \angle PAB = \frac{130°}{2}$$

$$\Rightarrow \angle PAB = 65° \quad ...\text{(ii)}$$ [½]

Now we know that the radius is perpendicular to the tangent at the point of contact.

$$\therefore \angle OAB + \angle PAB = 90°$$

$$\Rightarrow \angle OAB + 65° = 90° \quad \text{(Using (ii))}$$

$$\Rightarrow \angle OAB = 90° - 65°$$

$$\Rightarrow \angle OAB = 25°$$

Hence $\angle OAB = 25°$ [½]

13. It is given that $\angle AOQ = 58°$.

Therefore $\angle ABQ = \frac{1}{2} \times \angle AOQ$ (Since angle subtended by an arc at the centre of the circle is twice the angle subtended by it at any point on the remaining part of the circle).

$$\Rightarrow \angle ABQ = \frac{1}{2} \times 58°$$

$$\Rightarrow \angle ABQ = 29°$$

Now we know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Therefore $\angle BAT = 90°$ [½]

In ΔABT,

$\angle ABT + \angle BAT + \angle BTA = 90°$ (Since the sum of all the angles in a triangle is 180°)

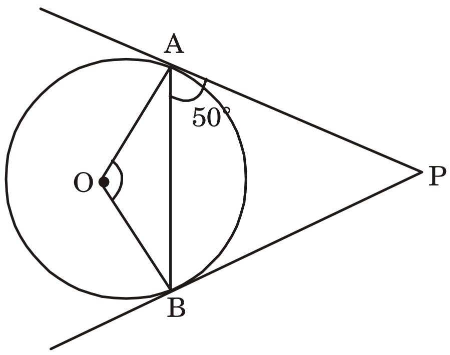
$$\Rightarrow 29° + 90° + \angle BTA = 180°$$

$$\Rightarrow \angle BTA = 180° - 119°$$

$$\Rightarrow \angle BTA = 61°$$

Hence, $\angle ATQ = 61°$ [½]

14.



It is given that PA and PB are tangents to the given circle.

$\therefore \quad \angle PAO = 90°$ (Radius is perpendicular to the tangent at the point of contact.)

Also, $\angle PAB = 50°$

$\therefore \quad \angle OAB = \angle PAO - \angle PAB = 90° - 50° = 40°$

In ΔOAB, $OB = OA$ (Radii of the circle)

$\therefore \quad \angle OAB = \angle OBA = 40°$ (Angles opposite to equal sides are equal.)

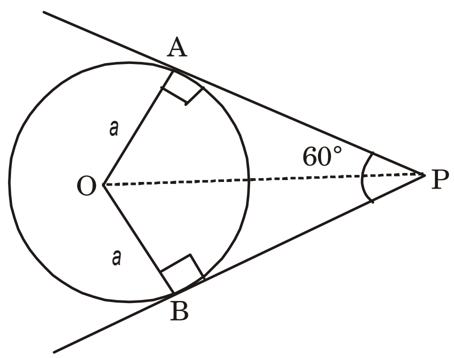
Now, using the angle sum property of triangles,

$$\angle AOB + \angle OAB + \angle OBA = 180°$$

$$\angle AOB = 180° - 40° - 40° = 100°$$

Hence, $\angle AOB = 100°$ [1]

15.



Along with center O, PA and PB are two tangents drawn to the circle

$\angle APB = 60°$

In ΔOPB and ΔOPA

The radii of the circle is equal ($OB = OA = a$)

$\angle OBP = \angle OAP = 90°$ .... At point of contact tangents are perpendicular to radius

$BP = PA$ ....From an external point to the circle lengths of tangents drawn are equal

So, $\Delta OPB \cong \Delta OPA$ (SAS congruence)

$\therefore \quad \angle OPB = \angle OPA = 30°$ (By CPCT)

In ΔOPB

$$\sin 30° = \frac{OB}{OP}$$

$$\Rightarrow \frac{1}{2} = \frac{a}{OP}$$

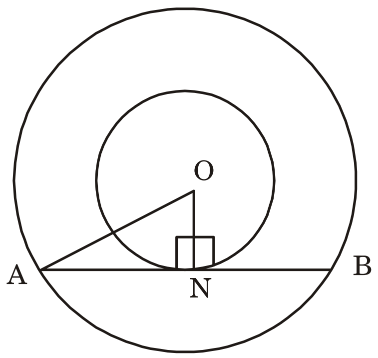
$\Rightarrow \quad OP = 2a$

Therefore, the length of $OP = 2a$ [1]

16. Given that the radius of the smaller circle is 7 cm and the radius of the bigger circle is r cm.

Let AB, be the chord of the bigger circle which touches the smaller circle. Here, $AB = 48$ cm

Join OA and drop a perpendicular from O on chord AB meeting at point N.



We know that, perpendicular from the centre of the circle on a chord bisects the chord.

$\therefore$ ON $\cong AB$ and

$$AN = NB = \frac{AB}{2} = \frac{48}{2} = 24\,\text{cm} \quad [1]$$

Now, in ΔONA, $ON = 7$ cm, $OA = r$ cm and $AN = 24$ cm

Applying Pythagoras theorem in ΔONA,

$OA^2 = ON^2 + NA^2$

$r^2 = (7)^2 + (24)^2$

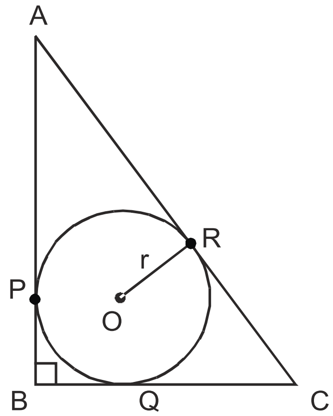
$r^2 = 49 + 576$

$r^2 = 625$

$r = \sqrt{625} = 25$

Therefore, the radius (r) of the bigger circle is 25 cm. [1]

17. Let us suppose that the circle touches the triangle ABC, on sides AB, BC and AC at P, Q and R respectively.



Now, we know that, tangents drawn from an external point to a circle are equal in length.

$\therefore \quad AP = AR$, $BP = BQ$ and $CQ = CR$. $BPOQ$ is a square as every angle measures 90°

We have, $AR = AP$

$\Rightarrow \quad AR = AB - BP$

$= (8 - r)$

Similarly, $CR = CQ$

$\Rightarrow \quad CR = CB - BQ$ [1]

Now, $AC = AR + CR = (8 - r + 6 - r) = (14 - 2r)$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (14 - 2\,r)^2 = 8^2 + 6^2$

$\Rightarrow \quad (14 - 2\,r)^2 = 10^2$

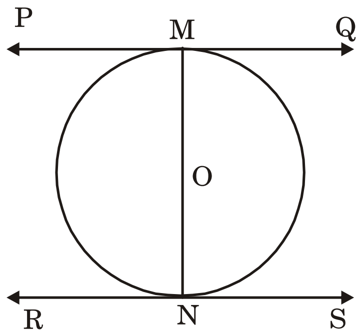
$\Rightarrow \quad (14 - 2\,r) = 10$

$\Rightarrow \quad 2\,r = 4$

$\Rightarrow \quad r = 2$

Therefore, the radius of the circle is 2 cm. [1]

18. Let MN be a diameter of a given circle, PQ and RS be the tangents drawn to the circle at points M and N respectively.



[1]

Since, we know that the tangent at a a point to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $MN \perp PQ$ and $MN \perp RS$

$\Rightarrow$ Ð$PMN = 90°$ and $\angle MNS = 90°$

$\Rightarrow$ $\angle PMN = \angle MNS$

Now, since $\angle PMN$ and $\angle MNS$ are pair of alternate angles for the pair of lines PQ and RS.

$\therefore$ $PQ \mid\mid RS$

Hence proved. [1]

19. Lengths of sides of the triangle are given as If $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm

Assume $AD = AF = p$ cm, $BD = BE = q$ and $CE = CF = r$ cm (Tangents drawn from an external point to the circle are equal) [1]

$2\,(p + q + r) = AB + BC + AC = 30$ cm

$\Rightarrow$ $(p + q + r) = 15$ cm

$\Rightarrow$ $AB = AD + DB = p + q = 12$ cm

$\therefore$ $r = CF = 15 - 12 = 3$ cm

$\Rightarrow$ $AC = AF + FC = p + r = 10$ cm

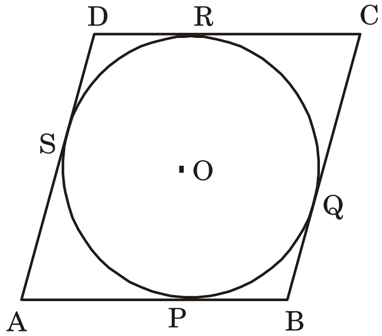
$\therefore$ $q = BE = 15 - 10 = 5$ cm

$\Rightarrow$ $BC = BE + EC = q + r = 8$ cm

$\therefore$ $p = AD = 15 - 8 = 7$ cm

Thus $AD = 7$ cm, $BE = 5$ cm and $CF = 3$ cm [1]

20. Let $ABCD$ is the parallelogram circumscribing a circle with centre O.



To Prove: $ABCD$ is a rhombus.

In parallelogram $ABCD$,

$AS = AP$ (tangents drawn to a circle from an exterior point)

$BQ = BP$ (tangents drawn to a circle from an exterior point)

$CQ = CR$ (tangents drawn to a circle from an exterior point)

$DS = DR$ (tangents drawn to a circle from an exterior point) [1]

Thus,

$AS + BQ + CQ + DS = AP + BP + CR + DR$

$(AS + DS) + (BQ + CQ) = (AP + BP) + (CR + DR)$

$\therefore$ $AD + BC = AB + CD$

$\because$ $AD = BC$ and $AB = CD$

(Opposite sides of parallelogram)

$\therefore$ $2BC = 2AB$

$\Rightarrow$ $BC = AB$

$\therefore$ $AB = BC = DC = AD$

$\therefore$ $ABCD$ is a rhombus.

Hence proved. [1]

21. Tangents from an external point to a circle is equal in length.

$\Rightarrow$ $EA = EC$ and $EB = ED$

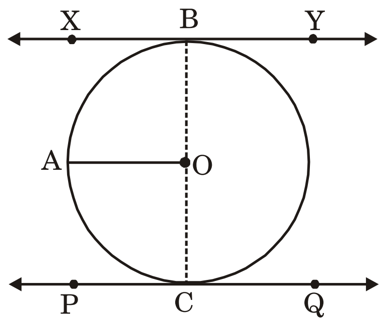
Add the two equations

$\Rightarrow$ $EA + EB = EC + ED$

$\Rightarrow$ $AB = CD$

Hence proved. [2]

22.



Let XBY and PCQ be two parallel tangents to a circle with centre O

Construction: Join OB and OC.

Now, $XB \mid\mid AO$ [1]

$\angle XBO + \angle AOB = 180°$ (sum of adjacent interior angles is 180°)

Now, $\angle XBO = 90°$ (A tangent to a circle is perpendicular to the radius through the point of contact)

$90° + \angle AOB = 180°$ (Co-interior angles)

$\angle AOB = 180° - 90° = 180°$

Similarly, $\angle AOC = 90°$

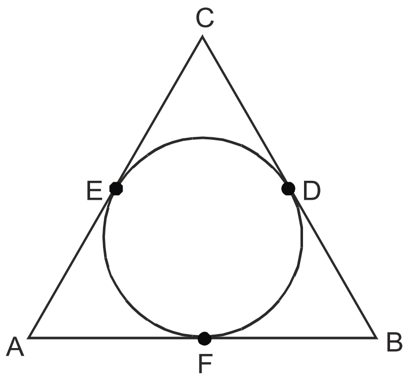
$\angle AOB + \angle AOC = 90° + 90° = 180°$

Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

Hence proved. [1]

23. Consider the isosceles triangle ABC where $AB = AC$



Tangents from an external point on the circle are equal in length.

$\Rightarrow BD = BF$ ...(i) [1]

Also, $CD = CE$ ... (ii)

Since $AB = AC$ and $AF = AE$

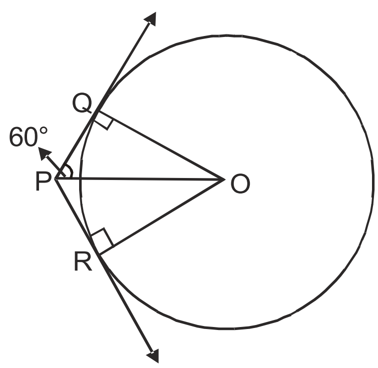
$\Rightarrow BF = CE$ ...(iii)

Using equations (i), (ii) and (iii)

$BD = DC$

Hence proved. [1]

24. Let us draw the circle with extent point P and two tangents PQ and PR.



We know that the radius is perpendicular to the tangent at the point of contact.

$\angle OQP = 90°$ [1]

We also know that the tangents drawn to a circle from an external point are equally inclined joining the centre to that point.

$\angle QPO = 60°$

Now, in ΔQPO

$$\cos 60° = \frac{PQ}{PO}$$

$$\frac{1}{2} = \frac{PQ}{PO}$$

$2PQ = PO$

Hence proved. [1]

25. Given $OP = 2\,r$, $\angle OTP = 90°$ (radius drawn at the point of contact is perpendicular to the tangent)

Now, In ΔOTP, $\sin \angle OPT = \dfrac{OT}{OP} = \dfrac{r}{2r} = \dfrac{1}{2}$

$\Rightarrow \sin \angle OPT = \sin 30°$

$\Rightarrow \angle OPT = 30°$

Sum of angles of a triangle is equal to 180°

Therefore, $\angle OPT + \angle TOP + \angle TPO = 180°$

$90° + 30° + \angle TOP = 180°$

$120° + \angle TOP = 180°$

$\angle TOP = 60°$ [1]

So, ΔOTP is a right angled triangle

Similarly, ΔOSP is a right angled triangle and $\angle SOP = 60°$

Hence, $\angle TOS = \angle TOP + \angle SOP = 60° + 60° = 120°$

In ΔOTS, $\angle TOS + \angle OTS + \Delta OST = 60°$

$\angle OTS + \angle OST + 120° = 180°$

$\angle OTS + \angle OST = 60°$

$\therefore$ $OT = OS$ (Radii of the same circle)

$\therefore$ $\angle OTS = \angle OST = 30°$

Hence proved. [1]

26. Two tangents AP and BP are drawn to the circle with centre O from an external point P.

Tangents drawn from an external point to a circle are equal in length, so $PA = PB$

Now, in ΔPAB, sides PA and PB are of the same length i.e. ΔPAB is an isosceles triangle such that

$PA = PB$ amd $\angle PAB = \angle PBA = x$ (Let suppose)

Given that $\angle APB = 60°$, we can find $\angle PAB$ and $\angle PBA$.

We know that the sum of angles of a triangle is 180°. [1]

In ΔPAB,

$\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow$ $x + x + 60° = 180°$

$\Rightarrow$ $2x = 120°$

$\Rightarrow$ $x = 60°$

Thus, $\angle PAB = \angle PBA = 60°$

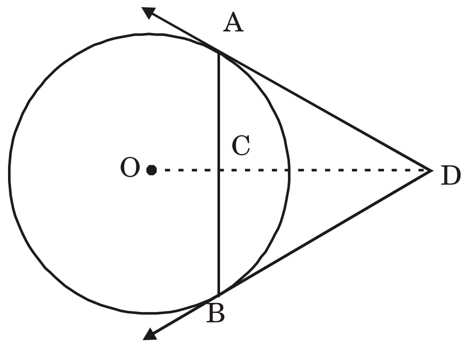
From this we conclude that is an equilateral triangle with $AP = BP = AB$

Now we know that $AP = 5$ cm

$\therefore$ $AB = AP = 5$ cm [1]

Hence, the length of the chord AB is 5 cm.

27.



Let AB be chord of circle with centre O.

Let AD and BD be the tangents at A and B.

OD meets AB at C.

To Prove $\angle DAC = \angle DBC$

Line segment joining the centre to external point bisects the angle between two tangents.

$\angle DAC = \angle DBC$ ....(i) [1]

In ΔDCA and ΔDCB

$DA = DB$ [Tangents from an external points are equal]

$\angle ADC = \angle BDC$ [From (i)]

$DC = DC$ [Common]

$\Delta DCA \cong \Delta DCB$ [By SAS]

$\Rightarrow$ $\angle DAC = \angle DBC$ [By C.P.C.T]

Hence proved. [1]

28. Given: PA and PB are tangents to the circle from an external point P.

CD is a tangent touching the circle at Q

$PA = 12$ cm

To find: $PC + PD$ [1]

We know that the lengths of tangents to a circle from same point are equal.

So,

Similarly, $CA = CQ = 3$ cm

And $DB = DQ = 3$ cm

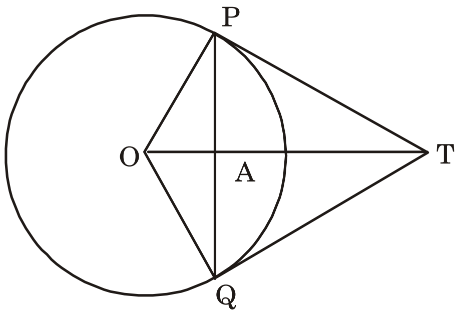
Now, $PC = PA - CA = 12 - 3 = 9$ cm

And $PD = PB - DB = 12 - 3 = 9$ cm

$\therefore$ $PC + PD = 9 + 9 = 18$ cm

Hence, $PC + PD = 18$ cm [1]

29.



Given: TP and TQ are the tangents drawn to the circle from the point T outside the circle of centre O.

To Prove: OT is the right bisector of line segment PQ.

Proof: In ΔPOT and ΔQOT

$PT = QT$ (Lengths of the tangents drawn from an external point to a circle)

$OP = OQ$ (Radii of the circle)

$OT = OT$ (Common side)

$\Rightarrow$ $\Delta POT \cong \Delta QOT$ (By SSS Rule)

$\Rightarrow$ $\angle PTO = \angle QTO$ (By CPCT Rule) ...(i) [1]

Suppose OT intersect PQ at A.

Now in ΔPTA and ΔQTA

$PT = QT$ (Lengths of the tangents drawn from an external point to a circle)

$\angle PTO = \angle QTO$ (Using (i))

$TA = TA$ (Common Side)

$\Rightarrow$ $\Delta PTA \cong \Delta QTA$ (By SAS Rule)

$\Rightarrow$ $\angle PAT = \angle QAT$ (By CPCT Rule)

And $PA = QA$ (By CPCT Rule) ...(ii) [1]

Now $\angle PAT + \angle QAT = 180°$ (Linear pair)

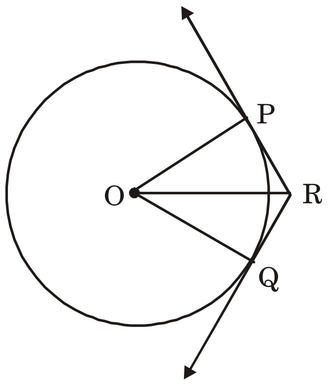
$\Rightarrow$ $2\angle PAT = 180°$ ...(iii)

$\Rightarrow$ $\angle PAT = 90°$

From (ii) and (iii) it is proved that OT is the right bisector of line segment PQ.

Hence proved. [1]

30. On joining OP and OQ, we get ΔOPR and ΔOQR



In ΔOPR and ΔOQR

$\angle OPR = \angle OQR = 90°$ (Tangent is perpendicular to circle at point of contact)

$OP = OQ$ (Radii of same circle)

OR is the common side.

By right hand side congruency

$\Delta OPR \cong \Delta OQR$

PR = RQ ....(i) [1]

Also,

$\angle ORQ + \angle ORP = \angle PRQ$

$\angle PRQ = 120°$ (Given)

$\angle ORP + \angle ORQ = 120°$

$\angle ORP + \angle ORP = 120°$

$\angle ORP = 60°$

Also,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$ [1]

In ΔOPR

$$\cos 60° = \frac{PR}{OR}$$

$$\frac{1}{2} = \frac{PR}{OR}$$

$OR = 2PR$

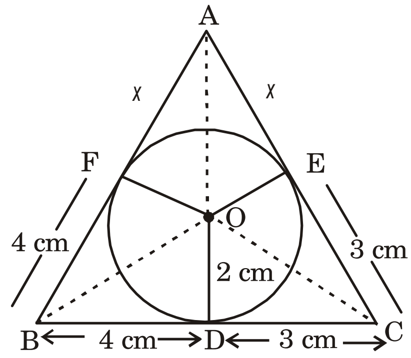
$OR = PR + PR$

$OR = PR + RQ$ (From (i) $PR = RQ$)

Hence proved. [1]

31. It is given that radius of the circle is 2cm, area of $\Delta ABC = 21$ cm^2 where $BD = 4$cm and $CD = 3$cm.

Join OA, OB, and OC.



Since length of the tangents drawn from an external points are equal,

$\therefore$ $CD = CE = 3$ cm and $BD = BF = 4$ cm

And let $AF = AE = x$ cm

Now the other two sides are,

$AB = (4 + x)$ cm and AC = $(3 + x)$ cm [1]

Now since area of triangle ABC = area of ($\Delta AOB + \Delta BOC + \Delta AOC$)

Area of ΔAOB =

$$\frac{1}{2} \times base \times height = \frac{1}{2} \times (4 + x) \times 2 = (4 + x)\text{cm}^2$$

Area of ΔAOC =

$$\frac{1}{2} \times base \times height = \frac{1}{2} \times (3 + x) \times 2 = (3 + x)\text{cm}^2$$

Area of ΔBOC =

$$\frac{1}{2} \times base \times height = \frac{1}{2} \times (4 + 3) \times 2 = 7\text{cm}^2$$

And as area of $\Delta BOC = 21$ cm^2 [1]

Therefore

$\Rightarrow$ $21 = (4 + x) + (3 + x) + 7$

$\Rightarrow$ $21 = 14 + 2x$

$\Rightarrow$ $2x = 7$

$\Rightarrow$ $x = 3.5$

Hence the side $AB = 4 + x = 4 + 3.5 = 7.5$ cm

$AC = 3 + x = 3 + 3.5 = 6.5$ cm

Hence the length of AB is 7.5 cm and the length of AC is 6.5 cm. [1]

32. We know that, tangents drawn to a circle from an external point are equal in length.

$\therefore$ $PL = PN$, $QL = QM$ and $RM = RN$

Now, let us suppose $PL = x = PN$

$\Rightarrow$ $QL = 10 - x = QM$

Now, since $PN = x$

$\Rightarrow$ $RN = 12 - x = RM$ [1]

Now, $QR = 8$ cm

$\Rightarrow QM + MR = 8$

$\Rightarrow 10 - x + 12 - x = 8$

$\Rightarrow 22 - 2x = 8$

$\Rightarrow 2x = 14$

$\Rightarrow x = 7$ [1]

Therefore, $PL = x = 7$ cm

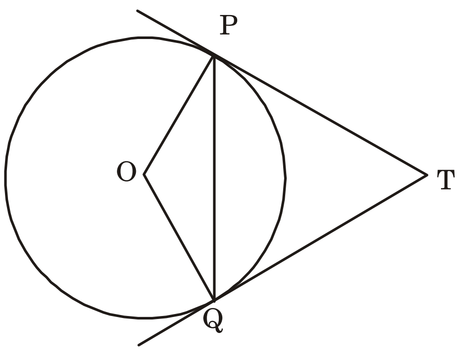
$QM = 10 - x = 10 - 7 = 3$ cm, and

$RN = 12 - x = 12 - 7 = 5$ cm

The length of QM, PL and RN are 3 cm, 7 cm and 5 cm [1]

33. Given: Two tangents TP and TQ are drawn to a circle with centre O from an external point T.

To prove: $\angle PTQ = 2\angle OPQ$



Proof: $TP = TQ$ (Tangents drawn from an external point to a circle are equal in length) [1]

$\therefore$ In ΔTPQ

$\angle TPQ = \angle TQP$ (Angles opposite to equal sides of a triangle)

$\angle PTQ + \angle TPQ + \angle TQP = 180°$

$\angle PTQ + \angle TPQ + \angle TPQ = 180°$

$\angle PTQ + 2\angle TPQ = 180°$

$$\angle TPQ = \frac{180° - \angle PTQ}{2} \quad \text{....(i)} \quad [1]$$

Also, $\angle TPO = 90°$ (Tangent at any point of a circle is perpendicular to the radius)

$\angle OPQ + \angle TPQ = 90°$

$$\angle OPQ + \frac{180° - \angle PTQ}{2} = 90° \quad \text{(Using (i))}$$

$2\angle OPQ + 180° - \angle PTQ = 180°$

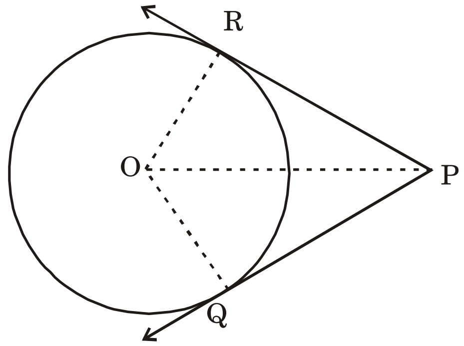
$2\angle OPQ = \angle PTQ$

Hence proved. [1]

34. Given: PQ and PR are two tangents from an external point P to a circle with centre O.

To prove: Lengths of the tangents PQ and PR are equal i.e., $PR = PQ$

Construction: Join OP, OQ and OR.



[1]

Proof: In order to prove that $PR = PQ$, we shall first prove that $\Delta ORP \cong \Delta OQP$.

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ $OR \perp PR$ and $OQ \perp PQ$.

$\Rightarrow \angle ORP = \angle OQP = 90°$ ....(i) [1]

Now, in right triangles ORP and OQP, we have

$OR = OQ$ [Radii of the circle]

$\angle ORP = \angle OQP$ [From (i)]

And, $OP = OP$ [Common] [1]

So, by RHS-criterion of congruence, we get

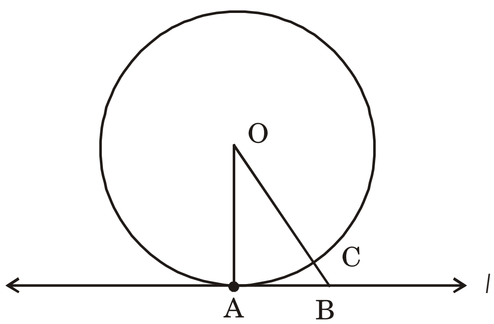
$\Delta ORP \cong \Delta OQP$

$\Rightarrow PR = PQ$ [CPCT]

Hence proved. [1]

35. In a circle $C(O, r)$ and a tangent l touches the circle at point A.

Construction: Consider a point B on l, other than A, on the tangent l. Join OB. Let OB intersect the circle in C.



[1]

To prove: $OA \perp l$

Proof: Among all line segments joining the point O to a point on l, the perpendicular is shortest to l.

$OA = OC$ (Radius of the circle)

Now, $OB = OC + BC$

$\therefore \quad OB > OC$

$\Rightarrow \quad OB > OA$

$\Rightarrow \quad OA > OB$ [1]

As B is any point on the tangent l, OA is shorter than any other line segment joining O to any point on l.

Therefore, $OA \perp l$ as smallest line is perpendicular to radius. [1]

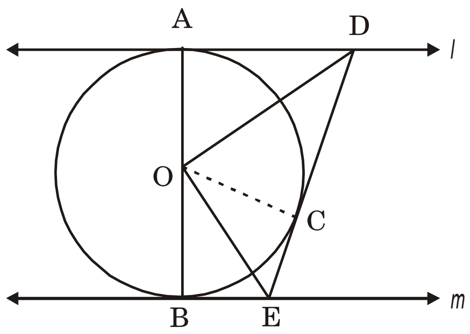
So, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Hence proved. [1]

36. l and m are two tangents to the circle with centre O parallel to each other and touching the circle at A and B respectively. DE is another tangent at the point C, which intersects l at D and m at E.

To prove: $\angle DOE = 90°$

Construction: Join OC.



[1]

Proof:

Taking ΔODA and ΔODC,

$OA = OC$ (Radii of the same circle)

$AD = DC$

(Length of tangents drawn from an external point to the circle are equal in length)

$DO = OD$ (Common side) [1]

$\Delta ODA \cong \Delta ODC$ (SSS congruence property)

$\therefore \quad \angle DOA = \angle COD$ ....(i) (C.P.C.T.)

Similarly, $\Delta OEB \cong \Delta OEC$

Therefore, $\angle EOB = \angle COE$ ....(i)

And AOB is a diameter of the circle which is a straight line and the angle on a straight line is a straight angle (180°).

$\therefore \quad \angle DOA + \angle COD + \angle COE + \angle EOB = 180°$ [1]

From (i) and (ii),

$2\angle COD + 2\angle COE = 180°$

$\Rightarrow \angle COD + \angle COE = 90°$

$\Rightarrow \angle DOE = 90°$

Hence proved. [1]

37. It is given that $OP = 10$ cm and $PQ = 16$ cm

Since OR is a perpendicular bisector of the chord so $PR = RQ = 8$ cm

In ΔORP, apply Pythagoras theorem [1]

$OP^2 = OR^2 + RP^2$

$OR^2 = OP^2 - RP^2$

$OR = \sqrt{10^2 - 8^2}$

$OR = \sqrt{100 - 64}$

$OR = \sqrt{36} = 6$ cm [1]

Apply Pythagoras theorem in right angled ΔPRT

$PT^2 = PR^2 + RT^2$

Apply Pythagoras theorem in right angled ΔOPT

$OT^2 = OP^2 + PT^2$

Substitute the value of PT^2 in the above equation

$OT^2 = OP^2 + PR^2 + RT^2$

$(OR + RT)^2 = OP^2 + PR^2 + RT^2$

Use $(a + b)^2 = a^2 + 2ab + b^2$

$OR^2 + 2.OR\,.\,RT + RT^2 = OP^2 + PR^2 + RT^2$ [1]

$OR^2 + 2.OR\,.\,RT = OP^2 + PR^2$

$6^2 + 2\,.\,6\,.\,RT = 10^2 + 8^2$

$36 + 12RT = 100 + 64$

$12RT = 100 + 64 - 36$

$12RT = 128$

$RT = \frac{128}{12} = 10.67$ cm

Apply Pythagoras theorem in right angled ΔPRT

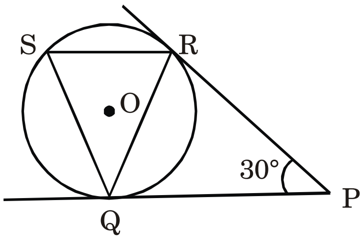
$PT^2 = PR^2 + RT^2$

$PT^2 = 82 + (10.67)^2$

$PT^2 = 64 + 113.849$

$PT = \sqrt{177.849} = 13.336$ cm

Length of the tangent PT is 13.36 cm. [1]

38. 

As it is given that PR and PQ are tangents drawn from point P to the same circle.

Therefore we can say that,

$PR = PQ$ Since, tangents drawn from an external point to a circle are equal in length. [1]

Now since $PR = PQ$, this implies,

$\angle PRQ = \angle PQR$ [Angles opposite to equal sides in a triangle are equal] ...(i)

Now using angle sum property of triangle, in ΔPQR

$\angle PQR + \angle PRQ + \angle RPQ = 180°$

Using equation (i) and as given $\angle RPQ = 30°$ [1]

$2\angle PQR + 30° = 180°$

$2\angle PQR = 150°$

$\angle PQR = 75°$

Now by using alternate segment theorem,

$\angle RQP = \angle RSQ = 75°$ ...(ii), [1]

And since RS | | PQ,

Therefore by using alternate angles property,

$\angle RQP = \angle SRQ = 75°$ ...(iii)

Now using angle sum property of triangle, in ΔSQR

$\angle SRQ + \angle RSQ + \angle SQR = 180°$

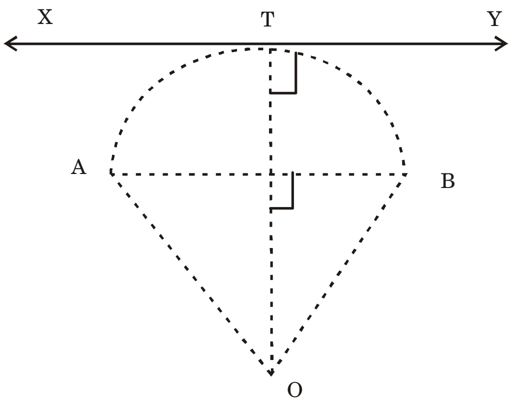
Using equation (ii) and (iii),

$75° + 75° + \angle SQR = 180°$

$150° + \angle SQR = 180°$

$\angle SQR = 30°$

Hence the required answer is $\angle RQS = 30°$ [1]

39.  [1]

Let XY be the tangent at the mid-point of the arc A. [1]

T be the point of contact.

AB be the chord.

D is the mid-point of AB.

Construction: Join OA, OB, OT where O is the center of the circle. [1]

$\angle OTY = 90°$

$\angle ODB = 90°$

Hence, the corresponding angles are equal.

AB is parallel to XY.

Hence proved. [1]

40. AO' = O'X = XO = OC

(Since the two circles are equal)

So, $OA = AO' + O'X + XO$

Hence, $OA = 3O'A$ [1]

In $\Delta AO'D$ and ΔAOC,

$\angle DAO' = \angle CAO$ (Common angle)

$O'D$ is perpendicular to AC [1]

$\angle ADO' = \angle ACO$ (The tangents drawn at any point of a circle is perpendicular to the radius through the point of contact)

$\angle ADO' - \angle ACO$ (By AA test of similarity)

$$\frac{DO'}{CO} = \frac{O'A}{OA}$$

$$= \frac{O'A}{3O'A} = \frac{1}{3}$$ [1]

Hence, $\frac{DO'}{CO} = \frac{1}{3}$ [1]

41. From the given figure,

$TP = TQ$ Since, two tangents drawn from an external point is equal.

Also,

$\angle TQO = \angle TPO = 90°$ Since, tangent drawn to a circle is at right angle to the radius.

In ΔTOQ,

$QT^2 + OQ^2 = OT^2$

$\Rightarrow QT^2 = 13^2 - 5^2 = 144$

$\Rightarrow$ $QT = 12$ cm [1]

So,

$OT - OE = ET$

$\Rightarrow$ $13 - 5 = 8$ cm

$OE \perp AB$ Since, tangent drawn to a circle is at right angle to the radius. [1]

Let $BQ = x$ cm

$\therefore$ $QB = EB = x$ cm Since, two tangents drawn from an external point is equal.

And, $\angle OEB = 90°$ Since, tangent drawn to a circle is at right angle to the radius.

Consider ΔBET,

$BE^2 + TE^2 = TB^2$

$\Rightarrow$ $x^2 + 8^2 = (12 - x)^2$ [1]

$\Rightarrow$ $x^2 + 64 = x^2 - 24x + 144$

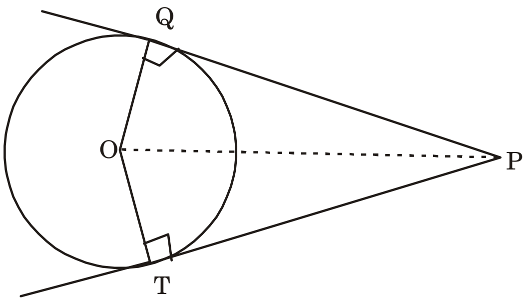
$\Rightarrow$ $24x = 80$

$$\Rightarrow x = \frac{80}{24} = \frac{10}{3}$$

So, $AB = 2x = 2 \times \frac{10}{3}$

$$AB = \frac{20}{3} cm$$ [1]

42. Let us consider the following diagram.



[1]

Given: A circle having two tangents PQ and PT, drawn from an external point P.

To prove: $PQ = PT$

Construction: Join PO, OQ and OT.

Proof: In ΔPOQ and ΔPOT [1]

$OQ = OT$ [Radii of the circle]

$\angle PQO = \angle PTO = 90°$ $\therefore$ The tangents drawn at any point of a circle is perpendicular to the radius through the point of contact.

$OP = OP$ [Common] [1]

$\therefore$ $\Delta POQ \cong \Delta POT$ [RHS congruence criterion]

Also, $PQ = PT$ $\because$ Corresponding parts of the congruent triangles are equal.

Hence proved. [1]

Smart Notes ....

CHAPTER 9

Constructions

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Construction of Triangle	3 marks	3 marks	4 marks	4 marks		
Tangent to Circle					4 marks	4 marks
Division of Line Segment			2 marks			

[TOPIC 1] Construction of a Line Segment

Summary

Division of a Line Segment in a given Ratio

In this chapter we shall study some constructions by using the knowledge of the earlier constructions done in previous class.

Solved Examples (Three Marks Each)

Illustration 1

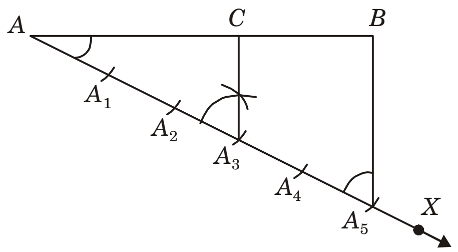
Question:

To divide a line segment in a given ratio 3 : 2.

Solution:

Given a line segment AB, we want to divide it in the ratio 3 : 2.

Steps of construction:



1. Draw any ray AX, making an acute angle with AB.
2. Locate 5 points A_1, A_2, A_3, A_4 and A_5 on AX so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
3. Join BA_5.
4. Through the point A_3, draw *a* line parallel to A_5B (by making an angle equal to $\angle AA_5B$) intersecting AB at the point C

Then, $AC : CB = 3 : 2$

Justification:

Since A_3C is parallel to A_5B, therefore,

$$\frac{AA_3}{A_3A_5} = \frac{AC}{CB} \text{ (By Basic proportionality theorem)}$$

By construction,

$$\frac{AA_3}{A_3A_5} = \frac{3}{2}.$$

Therefore,

$$\frac{AC}{CB} = \frac{3}{2}.$$

This shows that C divides AB in the ratio 3 : 2

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

2 Marks Question

1. Draw a line segment of length 8cm and divide it internally in the ratio 4 : 5.

[TERM 2, 2017]

Solutions

1. The steps to divide a line segment of length 8cm in the ratio of 4 : 5 are as follows:

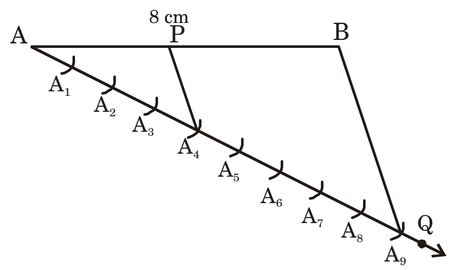
Step 1: Draw a line segment AB of 8cm and draw a ray from A making an acute angle with line segment AB. [½]

Step 2: Make 9 points, $A_1, A_2, A_3, A_4, ..., A_9$ on AQ such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 .... A_8A_9$

Step 3: Join BA_9. [½]

Step 4: Through the point , draw a line parallel to BA_9 by making an angle equal to $\angle AA_9B$ at A_4 intersecting AB at point P.

P is the point dividing line segment AB in the ratio of 4 : 5.



[1]

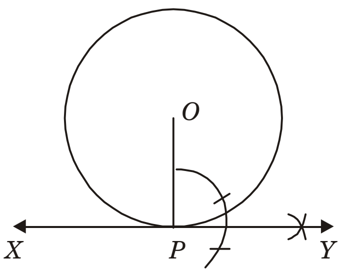
[TOPIC 2] Construction of a Tangent to a Circle from a Point Outside it.

Summary

Construction of Tangents From a Point Outside the Circle

WHEN CENTRE IS GIVEN

When point of tangency is on the circle.



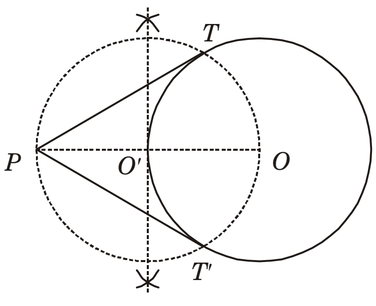
Given : A circle with centre O.

Required : To draw a tangent from point P on the circle.

Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of given radius.
2. Take a point P on the circle.
3. Join OP.
4. Construct $\angle OPX = 90°$.
5. Produce XP to Y to get XPY as the required tangent.

When point of tangency is outside the circle



Given : A circle with centre O.

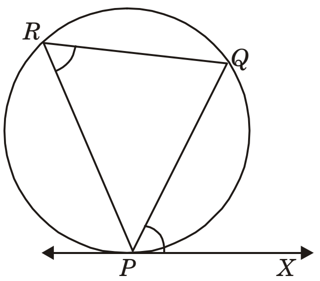
Required : To draw a tangent from an external point i.e., P.

Steps of construction:

1. Join the centre O of the circle to the given external point i.e., P.
2. Draw $\perp$ bisector of OP, intersecting OP at O'.
3. Taking O' as centre and $OO' = PO'$ as radius, draw a circle to intersect the given circle at T and T'.
4. Join PT and PT' to get the required tangents as PT and PT'.

WHEN CENTRE IS NOT GIVEN

When point of tangency is on the circle.



Given : A circle and a point P on it.

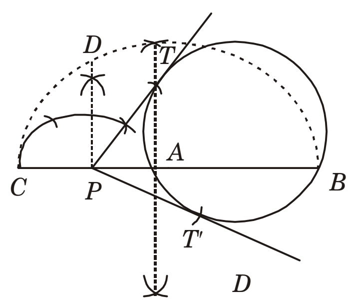
Required : To draw a tangent at P without using centre of the circle.

Steps of construction:

1. Draw any chord PQ of the circle through P as in figure.
2. Take any point R on the major arc PQ and join PR and QR.
3. Construct $\angle QPX$ equal to $\angle PRQ$.

   Then PX is the required tangent at P to the circle.

When point of tangency is outside the circle.



Given : A circle and a point P outside it.

Required : To draw a tangent from point P without using the centre.

Steps of construction:

1. Let P be the external point from where the tangents are to be drawn to the given circle.
2. Through P draw a secant PAB to intersect the circle at A and B.
3. Produce AP to a point C such that $AP = PC$.
4. Draw a semi-circle with BC as diameter.
5. Draw $PD \perp CB$, intersecting the semi-circle at D.
6. With P as centre and PD as radius draw arcs to intersect the given circle at T and T'.
7. Joint PT and PT'. PT and PT' are the required tangents.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

3 Marks Question

1. Draw a right triangle ABC in which $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

[TERM 2, 2014]

4 Marks Questions

2. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of to each other.

[TERM 2, 2016]

3. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length.

[TERM 2, 2016]

Solutions

1. Follow the given steps to construct the figure.

Step 1:Draw a line AB = 6 cm segment from point B, draw a ray making an angle of 90° with AB. Now with B as center and radius 8 cm draw an arc cutting the ray at point C. Join AC, to form ΔABC. Thus, ΔABC is created. [½]

Step 2: Bisect BC and name the midpoint of BC as E. So, the center of circle is E. [½]

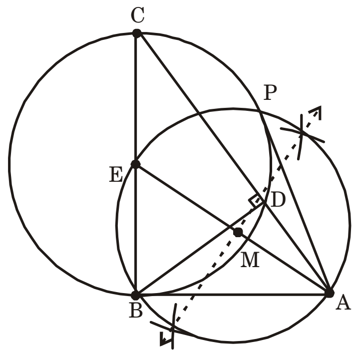
Step 3: Join points A and E. Bisect AE and name the midpoint of AE is M. [½]

Step 4: With M as centre and ME as radius, draw a circle. [½]

Step 5: Let it intersect given circle at B and P.

Step 6: Join AP and AB.

Here, AB and AP are the required tangents to the circle from A.



[1]

2. Steps of construction:

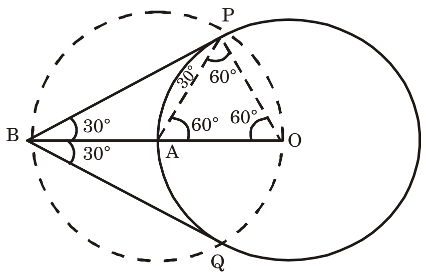
(i) Take a point O on the plane of the paper and draw a circle of radius

(ii) Produce OA to B such that OA = AB = 4 cm [½]

(iii) Draw a circle with center at A and radius AB.

(iv) Suppose it cuts the circle drawn in step (i) at P and Q. [½]

(v) Join BP and BQ to get the required tangents.



[1]

Justification:

In ΔOAP, OA = OP = 4 cm

(radii of the same circle)

Also, AP = 4 cm

(radii of the circle with centre A)

So, ΔOAP isequilateral. [½]

∠PAO = 60° And therefore, ∠BAP = 120°

In ΔBAP, we have BA = AP and ∠BAP = 120° [½]

$\angle ABP = \angle APB = 30^o$

Similarly we can get

∠ABQ = 30°

Hence, ∠PBQ = 60° [1]

3. The steps to draw tangents on the given circle are:

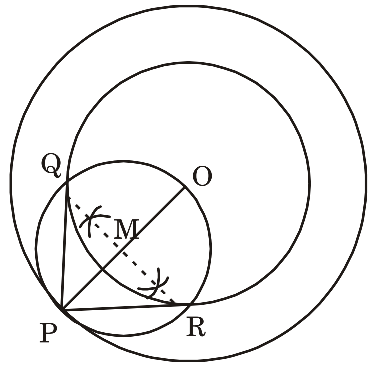
Step 1: Draw a circle with radius 3 cm and centre O. [½]

Step 2: Draw another circle of radius and centre O. Locate a point P on this circle and join OP. [½]

Step 3: Bisect OP and let M be the midpoint of OP. [½]

Step 4: Now taking M as centre and MO as radius, draw a circle. Let it intersect the circles at points Q and R as given in the diagram. [½]

Step 5: Join PQ and PR which is the required tangents. [½]



[1]

After construction, it can be observed that PQ and PR each measure 4cm. [½]

[TOPIC 3] Construction of a triangle Similar to a given Triangle

Summary

Some Constructions of Triangles

1. **Rules of Congruency of Two Triangles**
   (i) **SAS :** Two triangles are congruent, if any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle.
   (ii) **SSS :** Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.
   (iii) **ASA :** Two triangles are congruent if any two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle.
   (iv) **RHS :** Two right triangles are congruent if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.
2. **Uniqueness of a Triangle**
   A triangle is unique if
   (i) two sides and the included angle is given
   (ii) three sides and angle is given
   (iii) two angles and the included side is given and,
   (iv) in a right triangle, hypotenuse and one side is given.

Note : At least three parts of a triangle have to be given for constructing it but not all combinations of three parts are sufficient for the purpose.

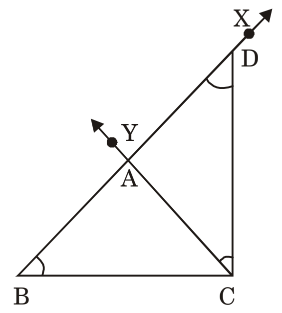
Basic Constructions of Triangles:

Statement 1 : To construct a triangle, given its base, a base angle and sum of other two sides.

Given : Base BC, a base angle, say ∠B and the sum AB + AC of the other two sides of a triangle ΔABC

Required : To construct a ΔABC.

Steps of construction



1. Draw the base BC and at the point B make an angle, say XBC equal to the given angle.
2. Cut a line segment BD equal to AB + AC from the ray BX.
3. Join DC and make an angle DCY equal to ∠BDC.
4. Let CY intersect BX at A (see fig.)

Then, ABC is the required triangle.

Note : The construction of the triangle is not possible if the sum AB + AC ≤ BC.

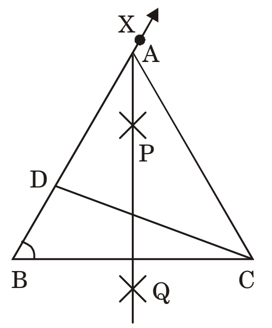
Statement 2 : To construct a triangle given its base, a base angle and the difference of the other two sides.

Given : The base BC, a base angle, say Z B and the difference of other two sides AB – AC or AC – AB.

Require : Construct the triangle ABC.

Case (i) : Let AB > AC that is AB – AC is given.

Steps of Construction :

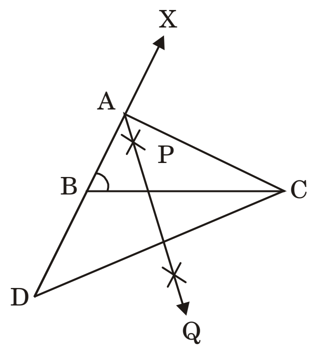


1. Draw the base BC and at point B make an angle say XBC equal to the given angle.
2. Cut the line segment BD equal to AB – AC from ray BX.

3. Join DC and draw the perpendicular bisector, say PQ of DC.
4. Let it intersect BX at a point A. Join AC (see fig.) I hen ABC is the required triangle.

Case (ii) : Let AB < AC that is AC – AB is given.

Steps of Construction :



1. Draw the base BC and at B make an angle XBC equal to the given angle.
2. Cut the line segment BD equal to AC – AB from the line BX extended on opposite side of line segment BC.
3. Join DC and draw the perpendicular bisector, say PQ of DC.
4. Let PQ interseel BX at A. Join AC (see fig.) Then, ABC is the required triangle.

Statement 3 : To construct a triangle, given its perimeter and its two base angles.

Given : The base angles, say ∠B and ∠C and BC + CA + AB.

Required : Construct the triangle ABC.

Steps of Construction :

1. Draw a line segment, say XY equal to BC + CA – FAB.
2. Make angle LXY equal to ∠B and MYX equal to ∠C.
3. Bisect ∠LXY and ∠MYX. Let these bisectors intersect at a point A. (see fig. (i))

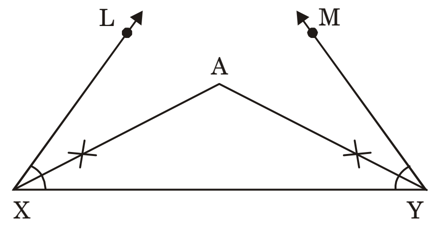


Fig (i)

4. Draw perpendicular bisectors PQ of AX and RS of AY.
5. Let PQ intersect XY at B and RS intersect XY at C. join AB and AC. (see fig. (ii))

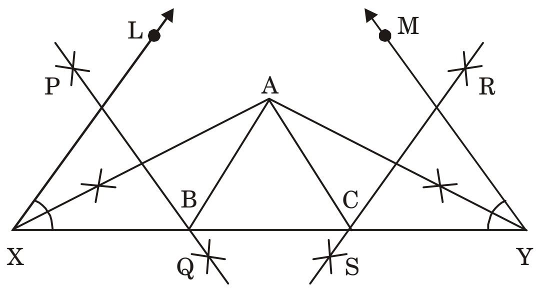


Fig (ii)

Then ABC is the required triangle.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 3

3 Marks Questions

1. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC.

[TERM 2, 2011]

2. Draw a triangle ABC with BC = 7 cm, ∠B = 45° and ∠C = 60°. Then construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of ΔABC.

[TERM 2, 2012]

3. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of first triangle.

[TERM 2, 2013]

4. Construct a triangle with sides 5 cm, 5.5 cm and 6.5 cm. Now construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

[TERM 2, 2014]

4 Marks Questions

5. Construct a triangle ABC with, BC = 7 cm, $\angle B = 60°$ and $AB = 6$. Construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

[TERM 2, 2015]

6. Construct a ΔABC in which AB = 6cm, ΔA = 30° and ΔB = 60°. Construct another $\Delta AB'C'$ similar to ΔABC with base AB' = 8 cm.

[TERM 2, 2015]

7. Construct a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are times the corresponding sides of the ΔABC.

[TERM 2, 2017]

8. Construct an isosceles triangle with base 8cm and altitude 4 cm.

Construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the isosceles triangle.

[TERM 2, 2017]

Solutions

1. Steps of construction for triangle ABC:

(1) Draw a line segment BC = 6 cm

(2) From point B, draw a ray making an angle of 60° with BC.

(3) Now with B as center and radius 5 cm draw an arc cutting the ray at point A.

(4) Join AC, to form ΔABC. [1]

Now steps of construction for similar triangle,

(1) Draw a ray BX making an acute angle opposite to vertex A.

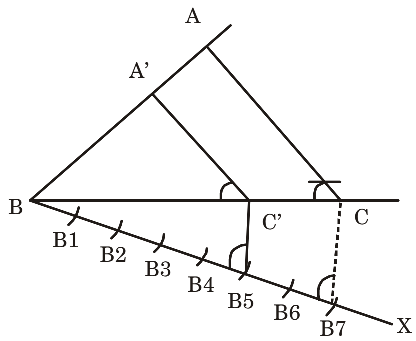
(2) Mark 7 points $B_1, B_2, B_3, B_4, B_5, B_6, B_7$ at equal distance from each other on BX such that

$$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$$

(3) Join the point C and B_7 and draw B_5C' parallel to B_7C.

(4) Draw $C'A'$ parallel to CA. [1]

Hence $A'B'C$ is the required triangle as shown below.



[1]

2. In order to construct the given triangle, follow the following steps:

Step 1 : Draw BC = 7 cm. [½]

Step 2 : At B, construct $\angle B = 45°$ and at C, construct $\angle C = 60°$. They intersect each other at A. Thus, ΔABC is constructed.

Step 3: Construct an acute angle $\angle CBZ$ at B on opposite side of vertex A of ΔABC [½]

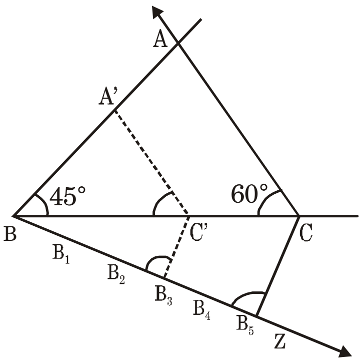
Step 4: Along BZ, mark off 5 points B_1, B_2, B_3, B_4, B_5 such that

$$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5.$$

Step 5: Join B_5C [½]

Step 6: Since we have to construct a triangle each of whose sides is $\frac{3}{5}$ of the corresponding sides of ΔABC. So, take three parts out of five equal parts on BZ i.e., from B_3, draw $B_3C' \mid\mid B_5C$, meeting BC BC at C'. [½]

Step 7: From C', draw $C'A' \mid\mid CA$, meeting BA at A'. Thus, $A'BC'$ is the required triangle, each of whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC as shown below [½]



[½]

3. Following are the steps of constructing a required triangle.

Step1: Draw a line segment AB = 4 cm and then draw an arc of radius 5 cm considering A as a center.

Step 2: Draw an arc of radius 6 cm considering B as center. [½]

Step 3: Name the point where both the arcs from step 1 and step 2 intersect as C.

Step 4: Join BC and AC. ΔABC is formed.

Step 5: Draw a ray AY which forms an acute angle with AB on the opposite side of the vertex C [½]

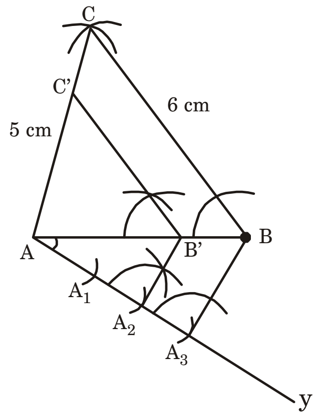
Step 6: Locate three points A_1, A_2, and A_3 on lines AY such that $AA1 = A_1A_2 = A_2A_3$. [½]

Step 7: Join B and A_3.

Step 8: Draw a line segment from point A_2 and parallel to BA_3 which intersects AB at point B'.

Step 9: Draw a line segment from B' and parallel to BC which intersects AC at the point C'. [½]

Thus, required triangle $\Delta AB'C'$ whose sides are $\frac{2}{3}$ times the corresponding sides of first triangle (ΔABC) has been constructed.



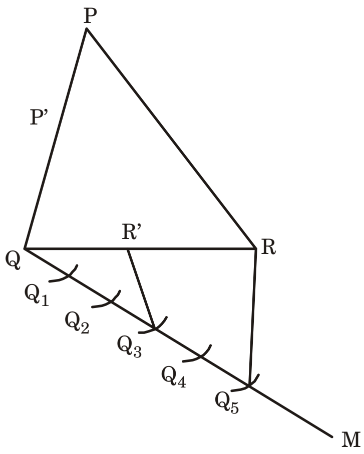
[1]

4. Step 1: Construct a triangle PQR such that QR = 5.5 cm. Considering Q as center, mark an arc of length 5 cm and considering R as center, mark an arc of length 6.5 cm. The point of intersection of arcs is called P. [½]

Step 2: Draw a line QM such that it makes an acute angle with QR. [½]

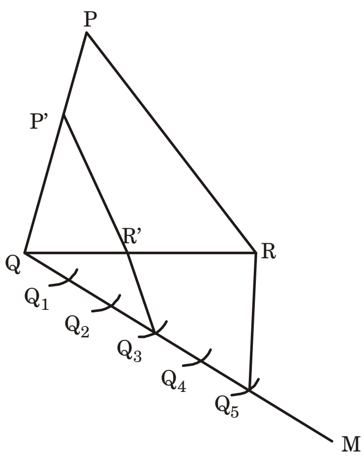
Step 3: Draw 5 points Q_1, Q_2, Q_3, Q_4 and Q_5 on equal distance from each other on QM and then join the fifth mark to point R. [½]

Step 4: Draw a line Q_3R' parallel to Q_5R from Q_3 [½]



[½]

Step 5: Draw a line $R'P'$ parallel to RP



[½]

Step 6: **P'QR'** is the required triangle.

5. Steps of construction for triangle ABC:

(1) Draw a line segment BC = 7 cm [½]

(2) From point B, draw a ray making an angle of 60° with BC. [½]

(3) Now with B as center and radius 6 cm draw an arc cutting the ray at point A. [½]

(4) Join AC, to form ΔABC. [½]

Now steps of construction for similar triangle,

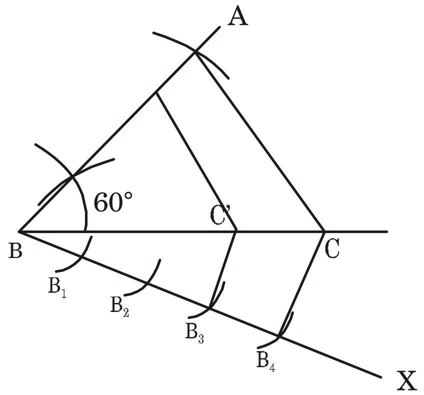
(1) Draw a ray BX making an acute angle opposite to vertex A. [½]

(2) Mark 4 points B_1, B_2, B_3, B_4 at equal distance from each other on BX.

(3) Join the point C and B_4 and draw parallel to B_4C'. [½]

(4) Draw C'A' parallel to CA.

Hence A'BC' is the required triangle.



[1]

6. The ratio of the sides and

Hence, the ratio of to AB' is $AB = \frac{8}{6} = \frac{4}{3}$. [½]

Steps of Construction:

Step 1: Draw the line segment AB = 6 cm and make an angle of 30° with A as centre and make another angle of 60° with B as centre. Mark the point of intersection as C. [1]

Step 2: With A as centre, make any acute angle BAX. [½]

Step 3: Now cut the 4 arcs A_1, A_2, A_3, A_4 of any radius on line AX such that

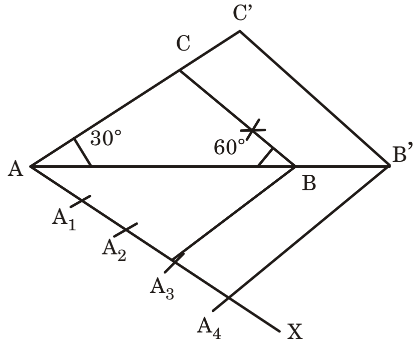
$AA_1 = AA_2 = A_2A_3 = A_3A_4$.

Step 4: Join point B to the third arc A_3. [½]

Step 5: Draw a line from A_4 which is parallel to A_3B cutting AB at point B'.

Step 6: Now draw a line from B' parallel to BC which cuts AC at C'. [½]

Now we have the required $\Delta ABC'C'$.



[1]

7. Given below are the steps of construction

(1) Start with drawing BC = 7 cm

(2) At point C, construct [½]

$\angle BCY = 180^\circ - (105^\circ + 45^\circ) = 30^\circ$ and at B,

construct $\angle CBX = 45°$

(3) The point of intersection of CY and BX gives A. So, is obtained. [½]

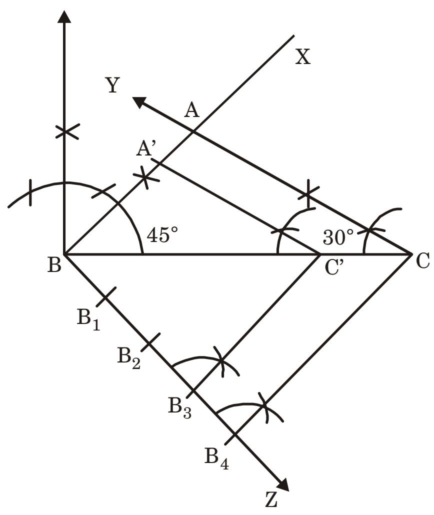
(4) Draw ray BZ making an acute angle with BC on opposite side to vertex A. [½]

(5) Identify 4 points B_1, B_2, B_3 and B_4 on BZ such that $BB_1=B_1B_2=B_2B_3=B_3B_4$ [½]

(6) Join the point B_4C and draw line through B_3 parallel to B_4C to intersect BC at C [½]

(7) Draw line through C' parallel to line CA to intersect BA at A'.

Now we have the required ΔA'BC'. [½]



[1]

8. The steps of constructions are as below:

   Step 1: Draw a line segment BC = 8 cm

   Step 2: Draw a perpendicular bisector of BC as PQ, intersecting BC at D. [½]

   Step 3: Taking D as centre, draw an arc of 4cm, cutting PQ at A. [½]

   Step 4: Join AB and AC. [½]

   Step 5: Draw a ray BX making an acute angle with line segment BC. [½]

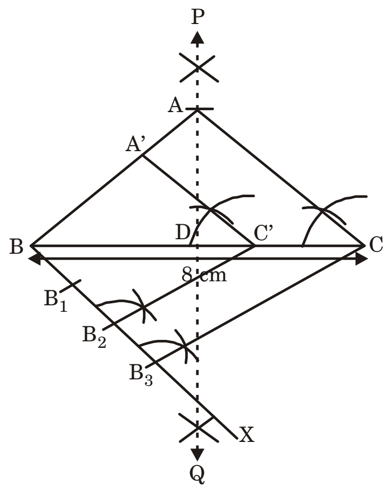
   Step 6: Make 3 points, B_1, B_2, B_3, on BC such that $BB_1 = B_1B_2 = B_2B_3$ and so on.

   Step 7: Join CB_3. [½]

   Step 8: Through the point , draw a line parallel to meeting BC at C'.

   Step 9: Through the point , draw a line parallel to AC meeting AB at A'[½]

Triangle A'BC' is the required triangle as shown below. [½]



[1]

Smart Notes ....

CHAPTER 10

Introduction to Trigonometry

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Question based on Trignometric Values	1 mark	1 mark				4 marks
Question based on Trignometric Identities	1,3,4 marks	1,3,4 marks				

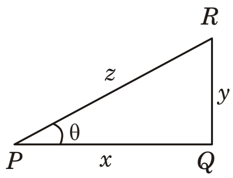
[TOPIC 1] Trigonometric Ratios

Summary

Trigonometry

TRIGONOMETRY: It is that branch of mathematics, which deals with the measurement of angles and the problems related with angles.

TRIGONOMETRIC RATIOS (T-RATIOS)



Let $\angle RPQ = \theta$ be the given angle of a right-angled ΔPQR.

In right-angled ΔPQR, let base = $PQ = x$ units, Perpendicular = $QR = y$ units and hypotenuse = $PR = z$ units.

Trigonometric ratios for θ are defined as below:

(i) sine $\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{y}{z}$, and is written as sin θ.

(ii) cosine $\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{x}{z}$, and is written as cos θ.

(iii) tangent $\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{y}{x}$, and is written as tan θ.

(iv) cosecant $\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{z}{y}$, and is written as cosec θ.

(v) secant $\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{z}{x}$, and is written as sec θ.

(vi) cotangent $\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{x}{y}$, and is written as cot θ.

RECIPROCAL RELATION

We have

(i) $\text{cosec}\ \theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

- ***The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.***

POWER OF T-RATIOS

We write $(\sin\theta)^2 = \sin^2\theta$; $(\sin\theta)^3 = \sin^3\theta$; $(\cos\theta)^3 = \cos^3\theta$; etc.

QUOTIENT RELATION OF T-RATIOS

Theorem 1: For any acute angle θ, prove that

(i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(ii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

SQUARE RELATION

Theorem 2: For any acute angle θ, prove that

(i) $\sin^2\theta + \cos^2\theta = 1$;

(ii) $1 + \tan^2\theta = \sec^2\theta$;

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$.

- ***The value of sin θ increases from 0 to 1 as the angle θ increases from 0º to 90°.***
- ***The value of cos θ decreases from 1 to 0 as the angle θ increases from 0º to 90°.***

VALUES OF ALL THE TRIGONOMETRIC RATIOS OF 0°, 30°, 45°, 60° AND 90°.

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n. d.
cosecθ	n. d.	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	n. d.
cot θ	n. d.	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

If A and B are two complementary acute angles, i.e., $A + B = 90^{\circ}$, then we have

sin A = sin (90° – B) = cos B

cos A = cos (90° – B) = sin B

tan A = tan (90° – B) = cot B

cosec A = cosec (90° – B) = sec B

sec A = sec (90° – B) = cosec B

cot A =cot (90° – B) = tan B

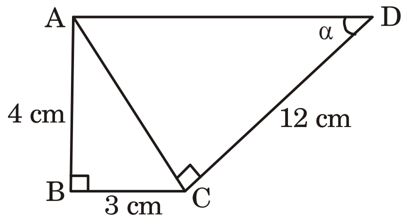
PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. In figure below, the value of sec x is



(a) $\frac{13}{12}$ (b) $\frac{5}{12}$

(c) $\frac{12}{5}$ (d) $\frac{12}{13}$

[TERM 2, 2011]

2. In figure, AB = 4cm and BC = 3cm, then cot θ equals:

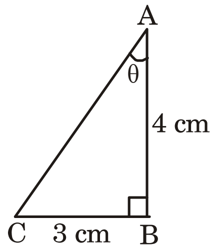


Figure II

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$

(c) $\frac{4}{3}$ (d) $\frac{3}{5}$

[TERM 2, 2011]

3. The maximum value of $\frac{1}{\cos ec\theta}$ is

(a) 1

(b) –1

(c) 0

(d) Can't be determined

[TERM 2, 2011]

4. $3 \sin^2 20° - 2 \tan^2 45° + 3 \sin^2 70°$ is equal to:

(a) 0 (b) 1

(c) 2 (d) –1

[TERM 2, 2012]

5. Given that $\sin\theta = \frac{a}{b}$, then tan θ is equal to:

(a) $\frac{b}{\sqrt{a^2+b^2}}$ (b) $\frac{b}{\sqrt{b^2-a^2}}$

(c) $\frac{a}{\sqrt{a^2-b^2}}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

[TERM 2, 2012]

6. The value of $\sin^2 30° - \cos^2 30° + \tan^2 45°$ is:

(a) $\frac{-1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) 0

[TERM 2, 2013]

7. The value of cot 10° cot 15° cot 75° cot 80° is equal to:

(a) 0 (b) 2

(c) 1 (d) cannot be determined

[TERM 2, 2013]

8. If θ=45°, then find the value of $2\text{cosec}^2\theta + 3\sec^2\theta$.

[TERM 2, 2014]

9. Find the value of cos θ + sec θ, when it is given that $\cos\theta = \frac{1}{2}$

[TERM 2, 2014]

10. If 24 cot A = 7, find the value of sin A.

[TERM 2, 2015]

11. Express cosec 48° + tan 88° in terms of t-ratio of angle 00 and 45°

[TERM 2, 2016]

12. If $\tan 2A = \cot(A + 60°)$, find the value of A, where $2A$ is an acute angle.

[TERM 2, 2017]

13. If $\sin\alpha = \frac{1}{2}$, then find the value of $3 \sin a - 4 \sin^3 a$.

[TERM 2, 2017]

14. Given $\sqrt{3}\tan 5\theta = 1$, find the value of θ.

[TERM 2, 2015]

2 Marks Questions

15. If A, B and C are interior angles of ΔABC, then show that

$$\tan\left(\frac{\angle A + \angle B}{2}\right) = \cot\frac{\angle C}{2}$$

[TERM 2, 2011]

16. If $\sqrt{3}sin\theta - cos\theta = 0$ and 0° < θ < 90°, find the value of θ.

[TERM 2, 2012]

17. Take A = 60 and B = 30. Write the value of cosA, cosB and cos(A + B). Is cos(A + B) = cosA + cosB?

[TERM 2, 2016]

18. Find cosec 30° & cos 60° geometrically.

[TERM 2, 2017]

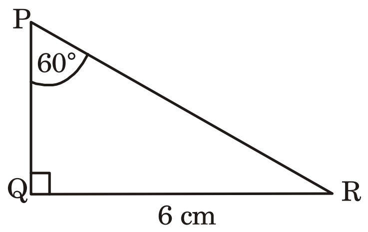
19. sin (A + B) = 1 & sin (A – B) $\frac{1}{2}$,

$0 \le A + B = 90^\circ$ & $A > B$, then find *A* & *B*.

[TERM 2, 2017]

3 Marks Questions

20. In fig, ΔPQR is, right angled at Q, QR = 6 cm, $\angle QPR = 60°$. Find the length of PQ and PR.



[TERM 2, 2011]

21. If $\sin\theta = \frac{3}{5}$, evaluate $\frac{\text{cosec}\,\theta - \cot\theta}{2\cot\theta}$

[TERM 2, 2014]

22. If 3 tan *A* = 4 then prove that:

(i) $\sqrt{\frac{\sec A - \text{Cosec}A}{\sec A + \text{Cosec}A}} = \frac{1}{\sqrt{3}}$

(ii) $\sqrt{\frac{1-\sin A}{1+\text{Cos}A}} = \frac{1}{2\sqrt{2}}$

[TERM 2, 2016]

4 Mark Questions

23. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$,

$0 < A + B \le 90°, A > B$, find A and B.

[TERM 2, 2011]

24. Evaluate:

$$\frac{\cot(90° - \theta)\sin(90° - \theta)}{sin\theta} + \frac{(\cot 40°)}{(\tan 50°)}$$
$$-\left(\cos^2 20° + \cos^2 70°\right)$$

[TERM 2, 2012]

25. Evaluate:

$$\tan^2 30^\circ.\sin 30^\circ + \cos 60^\circ.\sin^2 90^\circ.$$
$$\tan^2 60^\circ - 2\tan 45^\circ.\cos^2 0^\circ.\sin 90^\circ$$

[TERM 2, 2015]

26. Find Trigonometric ratios of 30° & 45° in all values of T.R.

[TERM 2, 2017]

Solutions

1. ΔABC is a right angled triangle, therefore, using Pythagoras Theorem,

$AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow AC = \sqrt{4^2 + 3^2}$

$\Rightarrow AC = \sqrt{16+9} = 5cm$ ....(i)

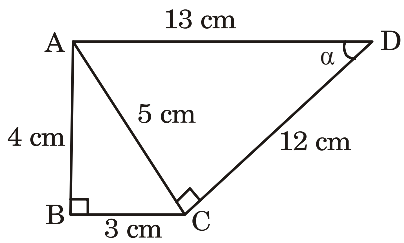
Similarly, ΔACD is a right angled triangle, therefore, using Pythagoras Theorem,

$AD = \sqrt{AC^2 + CD^2}$

$\Rightarrow AD = \sqrt{5^2 + 12^2}$ (Using value of *AC* from (i))

$\Rightarrow AD = \sqrt{25+144} = 13cm$ **[½]**

Using values of *AC* and *AD*, we have



$\cos\alpha = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{CD}}{\text{AD}} = \frac{12}{13}$

Now, $\sec\alpha = \frac{1}{\cos\alpha} = \frac{13}{12}$

Hence, correct option is (a). **[½]**

2. It is given that $AB = 4$cm and $BC = 3$cm

$\tan\theta = \frac{\text{Side opposite to }\angle\theta}{\text{Side adjacent to }\angle\theta} = \frac{BC}{AC} = \frac{3}{4}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

Hence, the correct option is (c). **[1]**

3. We know

$$\frac{1}{cosec\theta} = \sin\theta$$

The maximum value of sin θ is 1 and the minimum value of sin θ is –1. Hence, the maximum value of $\frac{1}{cosec\theta}$ is 1.

The correct option is (a). **[1]**

4. $3\sin^2 20° - 2\tan^2 45° + 3\sin^2 70°$ (Given)

$\Rightarrow 3(\sin^2 20° + \sin^2 70°) - 2\tan^2 45°$ (1)

We know that, sin 70° = sin(90 – 20)° = cos 20° and tan 45° = 1

Substituting these values in equation (1), we get,

$\Rightarrow 3(\sin^2 20° + \cos^2 20°) - 2$

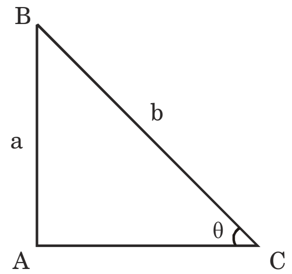
Also, by an identity $\sin^2 20° + \cos^2 20° = 1$

$\Rightarrow 3 - 2 = 1$

Hence, the correct option is (b). **[1]**

5. We have,

$$sin\theta = \frac{a}{b}$$



$sin\theta = \frac{AB}{BC} = \frac{a}{b}$ (Given)

By Pythagoras theorem, we know,

$BC^2 = AB^2 + AC^2$

$\Rightarrow b^2 = a^2 + AC^2$

$\Rightarrow AC^2 = b^2 - a^2$

$\Rightarrow AC = \sqrt{b^2 - a^2}$ [½]

Using trigonometric ratios,

$$\tan\theta = \frac{AB}{AC}$$

$$\tan\theta = \frac{a}{\sqrt{b^2 - a^2}}$$

Hence, the correct option is (d). [½]

6. $\sin^2 30^o = \left(\frac{1}{2}\right)^2$

$\cos^2 30^o = \left(\frac{\sqrt{3}}{2}\right)^2$

$\tan^2 45^0 = (1)^2$ [½]

Putting the values in the given question

$\Rightarrow \sin^2 30^o - \cos^2 30^o + \tan^2 45^0$

$= \frac{1}{4} - \frac{3}{4} + 1$

$= 1 - \frac{2}{4}$

$= \frac{4-2}{4}$

$= \frac{2}{4} = \frac{1}{2}$

Hence the correct option is (c). [½]

7. $\cot 10^o \cot 15^o \cot 75^o \cot 80^o$

$= \cot(90-80)^o \cot 80^o \cot(90-75)^o \cot 75^o$

$= \tan 80^o . \cot 80^o . \tan 75^o . \cot 75^o$

= 1 × 1

= 1

Hence the correct option is (c). **[1]**

8. If θ = 45° then,

$\Rightarrow 2\text{cosec}^2\theta + 3\sec^2\theta = 2\text{cosec}^2 45° + 3\sec^2 45°$

$\Rightarrow 2\left(\sqrt{2}\right)^2 + 3\left(\sqrt{2}\right)^2 = 2\times 2 + 3\times 2$

$\Rightarrow 4 + 6$

$\Rightarrow 10$

Hence the value of $2\text{cosec}^2\theta + 3\sec^2\theta = 10$ **[1]**

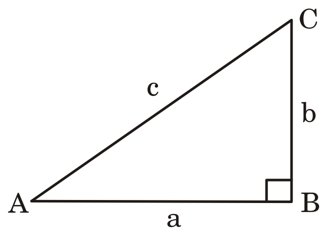
9. Since, $\cos\theta = \frac{1}{2}$

$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{2}} = 2$

Hence,

$\cos\theta + \sec\theta = \frac{1}{2} + 2 = \frac{1+4}{2} = \mathbf{\frac{5}{2}}$ **[1]**

10.



24 cot A = 7

$\Rightarrow \cot A = \frac{7}{24}$

$\cot A = \frac{Base}{Perpendicular}$

$\Rightarrow a = 7$ and $b = 24$

Using the Pythagoras theorem we get,

$c^2 = a^2 + b^2$

$\Rightarrow c^2 = (7)^2 + (24)^2$

$\Rightarrow c^2 = 49 + 576 = 625$ [½]

$\Rightarrow$ c = 25

We know that $\sin A = \frac{Perpendicular}{hypotenuse}$

$\Rightarrow \sin A = \frac{24}{25}$ [½]

11. $\cos ec48^o + \tan 88^o$

Using the identity

$\cos ec\theta = \sec(90 - \theta)$ and $\tan\theta = \cot(90 - \theta)$,

we get

$\cos ec48^o + \tan 88^o = \sec(90 - 48) + \cot(90 - 88)$

Thus $\cos ec48^o + \tan 88^o = \sec 42^o + \tan 2^o$ **[1]**

12. Given $\tan 2A = \cot(A + 60^\circ)$

$\Rightarrow \tan 2A = \cot\left(A + 60^\circ\right)$

As we know tan2A = cot(90° – 2A)

$\Rightarrow \cot\left(90^\circ - 2A\right) = \cot\left(A + 60^\circ\right)$ [½]

On equating the angles

$\Rightarrow 90^\circ - 2A = A + 60^\circ$

$\Rightarrow 3A = 30^\circ$

Divide the above equation by 3,

$\Rightarrow A = 10^\circ$

Hence, the value of A is 10° [½]

13. It is given that $\sin\alpha = \frac{1}{2}$,

So, substituting value of sin *a*,

$3\sin\alpha - 4\sin^3\alpha = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - 4\left(\frac{1}{8}\right)$

$= \frac{3}{2} - \frac{1}{2}$

= 1 **[1]**

14. $\sqrt{3}\tan 5\theta = 1$ (Given)

$\Rightarrow \tan 5\theta = \frac{1}{\sqrt{3}}$

We know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow 5\theta = 30^\circ$

$\Rightarrow \theta = \frac{30^\circ}{5} = 6^\circ$

Hence, the value of θ is 6°. **[1]**

15. The sum of all the interior angles of a triangle is 180°.

So,

$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + \angle B = 180^\circ - \angle C$

Divide both the sides by 2,

$\frac{\angle A + \angle B}{2} = 90^\circ - \frac{\angle C}{2}$ ....(i) **[1]**

Applying tan to both the sides of the equation

$\tan\left(\frac{\angle A + \angle B}{2}\right) = \tan\left(90^\circ - \frac{\angle C}{2}\right)$

Also

tan(90° – θ) = cotθ°

$\Rightarrow \tan\left(\frac{\angle A + \angle B}{2}\right) = \cot\frac{\angle C}{2}$

Hence proved. **[1]**

16. We have,

$\sqrt{3}sin\theta - cos\theta = 0$

Dividing above equation by 2, we get,

$\left(\frac{\sqrt{3}}{2}\right)\sin\theta - \left(\frac{1}{2}\right)\cos\theta = 0$ (1) [½]

Since, $\cos 30^\circ = \frac{\sqrt{3}}{2}$ and $\sin 30^\circ = \frac{1}{2}$, using this in equation 1, we get,

$\cos(30^\circ)\sin(\theta)-\sin(30^\circ)\cos(\theta)=0$ (2) [½]

We know that, $\sin(A-B)=\sin A\cos B-\cos A\sin B$, using this in equation 2

$\Rightarrow \sin(\theta-30°)=\sin(n\pi)$

$\Rightarrow \theta-30° = n\pi$

$\Rightarrow \theta = n\pi+30°$

$\Rightarrow \theta = n\pi+\frac{\pi}{6}$

The value of θ is $n\pi+\frac{\pi}{6}$ **[1]**

17. Here, It is given that $A = 60$ and $B = 30$

$\therefore \cos A = \cos 60^\circ = \frac{1}{2}$

and $\cos B = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos A+\cos B = \cos 60^\circ + \cos 30^\circ = \frac{1}{2}+\frac{\sqrt{3}}{2}$

$= \frac{1+\sqrt{3}}{2}$ (i) **[1]**

And

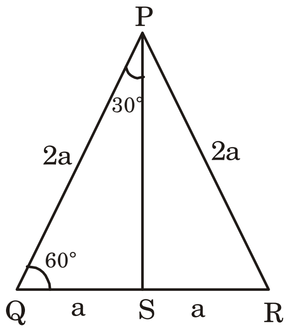
$\cos(A+B) = \cos(60+30) = \cos 90 = 0$ (ii)

From (i) and (ii)

$\cos A = \frac{1}{2}, \cos B = \frac{\sqrt{3}}{2}, \cos(A+B)=0$

and $\cos(A+B) \neq \cos A+\cos B$ **[1]**

18.



Make an equilateral triangle PQR

$\angle P = \angle Q = \angle R = 60^\circ$

Draw a perpendicular PS from P to the side QR

As, a perpendicular bisector divides the equilateral triangle into two congruent triangles,

So, $\triangle PQS \cong \triangle PRS$ (by RHS congruency criterion)

$\therefore QS = RS$ (CPCT)

Here length of each side is 2a

$\Rightarrow QS = RS = a$ **[1]**

In, ΔPQS

$\operatorname{cosec}30^\circ = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{PQ}{QS}$

$\Rightarrow \operatorname{cosec}30^\circ = \frac{2a}{a} = 2$

And, $\cos 60^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{QS}{PQ}$

$\Rightarrow \cos 60^\circ = \frac{a}{2a} = \frac{1}{2}$

Hence, cosec 30° = 2 and $\cos 60^\circ = \frac{1}{2}$. **[1]**

19. Given Sin $(A + B) = 1$

$\Rightarrow \sin(A+B) = \sin 90^\circ \quad (\because \sin 90° = 1)$

$\Rightarrow A+B = 90°$ ......(*i*)

Also $\sin(A-B) = \frac{1}{2}$

$\Rightarrow \sin(A-B) = \sin 30^\circ \quad \left(\because \sin 30° = \frac{1}{2}\right)$ **[1]**

$\Rightarrow A-B = 30°$ ......(*ii*)

Adding Eqn. (i) and (ii)

$\Rightarrow 2A = 120°$

Divide above equation by 2,

$\Rightarrow A = 60°$

Substituting value of A in Eqn. (i), we have

$\Rightarrow B = 30°$

Hence, $A = 60°$ and B = 30°. **[1]**

20. In ΔPQR,

The value of $\sin 60° = \frac{\sqrt{3}}{2}$,

So,

$\Rightarrow \frac{QR}{PR} = \frac{\sqrt{3}}{2}$ **[1]**

Substitute $QR = 6$ cm, we get

$\Rightarrow \frac{6}{PR} = \frac{\sqrt{3}}{2}$

$\Rightarrow PR = \frac{12}{\sqrt{3}}$ cm

$= \frac{12\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = 4\sqrt{3}$ cm **[1]**

In right angled ΔPQR,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow \left(4\sqrt{3}\right)^2 = PQ^2 + 6^2$

$\Rightarrow 48 = PQ^2 + 36$

$\Rightarrow PQ^2 = 12$

$\Rightarrow PQ = \sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}\text{cm}$

Hence, $PQ = 2\sqrt{3}cm$ and $PR = 4\sqrt{3}cm$. **[1]**

21. $\sin\theta = \frac{3}{5}$ (Given)

$\cos^2\theta = 1 - \sin^2\theta$ **[1]**

$\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos\theta = \frac{4}{5}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{5}{3}$ **[1]**

Now substituting these values in the given equation,

$$\frac{\text{cosec}\,\theta - \cot\theta}{2\cot\theta} = \frac{\frac{5}{3} - \frac{4}{3}}{2 \cdot \frac{4}{3}}$$

$$\Rightarrow \frac{\frac{5}{3} - \frac{4}{3}}{2 \cdot \frac{4}{3}} = \frac{1}{3} \cdot \frac{3}{8} = \frac{1}{8}$$

Hence, $\frac{\text{cosec}\,\theta - \cot\theta}{2\cot\theta} = \frac{1}{8}$ **[1]**

22. $3\tan A = 4$

$\Rightarrow \tan A = \frac{4}{3}$ and $\cot A = \frac{1}{\tan A}$

$\Rightarrow \cot A = \frac{3}{4}$ ....(i)

$\Rightarrow \tan^2 A = \frac{16}{9}$

And also,

$\sec^2 A = 1 + \tan^2 A$

$\sec^2 A = 1 + \frac{16}{9} = \frac{25}{9}$ **[1]**

Taking square root,

$\sec A = \frac{5}{3}$

And,

$\text{cosec}^2 A = 1 + \cot^2 A$

$\text{cosec}^2 A = 1 + \frac{9}{16} = \frac{25}{16}$ (Using(i))

Taking square root,

$\text{cosec} A = \frac{5}{4}$

(i) Taking LHS

$$\sqrt{\frac{\sec A - \text{Cosec} A}{\sec A + \text{Cosec} A}}$$

$$= \sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}}$$

$$= \sqrt{\frac{\frac{5}{12}}{\frac{35}{12}}}$$

$$= \sqrt{\frac{5}{35}} = \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}}$$ **[1]**

Hence, LHS=RHS

(ii) $\sqrt{\frac{1 - \sin A}{1 + \cos A}} = \frac{1}{2\sqrt{2}}$

$\sin A = \frac{1}{\text{cosec} A} = \frac{4}{5}$

And,

$\cos A = \frac{1}{\sec A} = \frac{3}{5}$

Taking LHS,

$$\sqrt{\frac{1 - \sin A}{1 + \cos A}}$$

$$= \sqrt{\frac{1 - \frac{4}{5}}{1 + \frac{3}{5}}}$$

$$= \sqrt{\frac{\frac{1}{5}}{\frac{8}{5}}}$$

$$= \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

Hence, LHS=RHS. **[1]**

23. It is given that $\tan(A+B)=\sqrt{3}$

We know that $\tan 60^\circ = \sqrt{3}$

$\Rightarrow A+B=60^\circ$ ....(I) **[1]**

$\tan(A-B)=\dfrac{1}{\sqrt{3}}$

We know that $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\Rightarrow A-B=30^\circ$ ....(II) **[1]**

Add equations (I) and (II),

$\Rightarrow A+B+A-B=60^\circ+30^\circ$

$\Rightarrow 2A=90^\circ$ **[1]**

Dividing both sides by 2,

$\Rightarrow A=45^\circ$

Substituting the value of A in (I).

$\Rightarrow 45^\circ+B=60^\circ$

$\Rightarrow B=60^\circ-45^\circ=15^\circ$

Hence, $A=45^\circ$ and B = 15°. **[1]**

24. We have,

$$\frac{\cot(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin\theta}+\frac{(\cot 40^\circ)}{(\tan 50^\circ)}-\begin{pmatrix}\cos^2 20^\circ\\+\cos^2 70^\circ\end{pmatrix}$$

We know,

$\sin(90-\theta)=\cos\theta, \cos(90-\theta)=\sin\theta,$
$\tan(90-\theta)=\cot\theta$

$$\Rightarrow \frac{\cos(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin(90^\circ-\theta)\sin\theta}+\frac{(\cot 40^\circ)}{(\tan 50^\circ)}-\left(\cos^2 20^\circ+\cos^2 70^\circ\right)$$ **[1]**

$$\Rightarrow \frac{\cos(90^\circ-\theta)}{\sin\theta}+\frac{(\cot 40^\circ)}{(\tan 50^\circ)}-\left(\cos^2 20^\circ+\cos^2 70^\circ\right)$$

$$\Rightarrow \frac{\sin\theta}{\sin\theta}+\frac{\tan(90^\circ-40^\circ)}{\tan 50^\circ}-\left(\cos^2 20^\circ+\cos^2 70^\circ\right)$$ **[1]**

$$\Rightarrow 1+\frac{\tan 50^\circ}{\tan 50^\circ}-\left(\cos^2 20^\circ+\left(1-\sin^2 70^\circ\right)\right)$$

$$\Rightarrow 1+1-\left(\cos^2 20^\circ+\left(1-\sin^2 70^\circ\right)\right)$$ **[1]**

$$\Rightarrow 2-\left(\cos^2 20^\circ+1-\sin^2 70^\circ\right)$$

$$\Rightarrow 2-\left(\cos^2 20^\circ+1-\sin^2(90^\circ-20^\circ)\right)$$

$$\Rightarrow 2-\left(\cos^2 20^\circ+1-\cos^2 20^\circ)\right)$$

$\Rightarrow 2-1$

$\Rightarrow 1$

Hence, the solution is 1. **[1]**

25. $\tan^2 30^\circ.\sin 30^\circ+\cos 60^\circ.\sin^2 90^\circ.$
$\tan^2 60^\circ-2\tan 45^\circ.\cos^2 0^\circ.\sin 90^\circ$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2\times\frac{1}{2}+\frac{1}{2}\times(1)^2\times\left(\sqrt{3}\right)^2-2\times1\times(1)^2\times1$$ **[2]**

$$\Rightarrow \frac{1}{3}\times\frac{1}{2}+\frac{1}{2}\times1\times3-2$$ **[1]**

$$\Rightarrow \frac{1}{6}+\frac{3}{2}-2$$

$$\Rightarrow -\frac{1}{3}$$ **[1]**

26. Following table shows all trigonometric ratios of $\angle 30^\circ$ & $\angle 45^\circ$ in all values of T.R.

θ	30°	45°	
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	**[1]**
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	**[1]**
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	**[½]**
$\cot\theta$	$\sqrt{3}$	1	**[½]**
$\sec\theta$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**[½]**
$\text{cosec}\,\theta$	2	$\sqrt{2}$	**[½]**

[TOPIC 2] Trigonometric Identities

Summary

Trigonometric Identities

An equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) involved.

Following are the three trigonometric identities which are used to solve the basic trigonometric equations.

(i) $\sin^2 \theta + \cos^2 \theta = 1$;

(ii) $1 + \tan^2 \theta = \sec^2 \theta$;

(iii) $1 + \cot^2 \theta = \text{cose}^2 \theta$.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. $4 \tan^2 A - 4 \sec^2 A$ is equal to:

(*a*) –1 (*b*) –4

(*c*) 0 (*d*) 4

[TERM 2, 2011]

2. $[(\sec A + \tan A)(1 - \sin A)]$ on simplification gives

(*a*) $\tan^2 A$ (*b*) $\sec^2 A$

(*c*) $\cos A$ (*d*) $\sin A$

[TERM 2, 2011]

3. Evaluate: $\sin^2 A + \cos^2 A + \cot^2 A$

[TERM 2, 2015]

4. Find the value of $(\sec^2 \theta - 1) \cdot \cot^2 \theta$

[TERM 2, 2016]

2 Marks Questions

5. If $\sqrt{3} \tan \theta = 3 \sin \theta$, find the value of $\sin^2 \theta - \cos^2 \theta$.

[TERM 2, 2011]

6. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin \theta + \cos \theta$.

[TERM 2, 2012]

7. $\left[\frac{1 - \tan A}{1 - \cot A}\right]^2 = \tan^2 A$; $\angle A$ is acute

[TERM 2, 2014]

8. Prove the following identity: $\frac{\sin^4 \theta + \cos^4 \theta}{1 - 2\sin^2 \theta \cos^2 \theta} = 1$

[TERM 2, 2015]

3 Marks Questions

9. Prove that $(\text{cosec}\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$.

[TERM 2, 2011]

10. Prove that: $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$

[TERM 2, 2012]

11. Prove that: $\frac{(1 + \tan^2 A)\cot A}{\text{cosec}^2 A} = \tan A$

[TERM 2, 2012]

12. If $\tan\theta + \frac{1}{\tan\theta} = \sqrt{2}$, find the value of $\tan^2 \theta + \frac{1}{\tan^2 \theta}$

[TERM 2, 2013]

13. Prove that:

$$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2 \theta - 1}$$

[TERM 2, 2013]

14. Prove that: $(1 + \cos\theta - \text{cosec}\theta).(1 + \tan\theta + \sec\theta)=2$

[TERM 2, 2014]

15. Prove the identity:

$$\frac{1}{\text{cosec}\,\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\text{cosec}\,\theta-\cot\theta}$$

[TERM 2, 2016]

16. Prove that:

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$$

[TERM 2, 2017]

17. If A, B, C are interior angles of a ΔABC, then

Show that: $\sin\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$

[TERM 2, 2017]

4 Marks Questions

18. Prove the following identity:

$$(\sin\theta+\sec\theta)^2+(\cos\theta+\text{cosec}\theta)^2=(1+\sec\theta\,\text{cosec}\theta)^2$$

[TERM 2, 2011]

19. Prove that

$$\frac{1}{\sec\theta-\tan\theta}-\frac{1}{\cos\theta}=\frac{1}{\cos\theta}-\frac{1}{\sec\theta+\tan\theta}$$

[TERM 2, 2011]

20. Prove that: $\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\frac{\cos\theta}{1-\sin\theta}$

[TERM 2, 2012]

21. Prove that: $\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=(\cos\theta+\sin\theta)$

[TERM 2, 2012]

22. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then show that

$\tan\theta = 1\ or\ \frac{1}{2}$

[TERM 2, 2013]

23. Prove the following identity:

$$\left(\frac{1+\tan A}{1+\cot A}\right)^2=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$$

[TERM 2, 2015]

24. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, show that

$\cos\theta-\sin\theta=\sqrt{2}\sin\theta\cdot$

[TERM 2, 2015]

Solutions

1. We know that

$1+tan^2A=sec^2A$ or

$tan^2A-sec^2A=-1$ ....(i)

The given equation is $4\tan^2 A - 4\sec^2 A$

By taking 4 as a common

$=4(tan^2A-sec^2A)$

Using equation (i),

$\Rightarrow 4tan^2A-4sec^2A=4(-1)=-4$

Hence, the correct option is (b). **[1]**

2. Using $\sec A=\frac{1}{\cos A}$ and $\tan A=\frac{\sin A}{\cos A}$ in given expression, we get

$$\left[(\sec A+\tan A)(1-\sin A)\right]=\left[\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)\right]$$

Apply $(a + b)(a - b) = a^2 - b^2$ where $a = 1$ and $b = \sin A$

$\Rightarrow [(\sec A+\tan A)(1-\sin A)]=\frac{1-\sin^2 A}{\cos A}$ ...(i) **[½]**

Also,

$1-sin^2A=cos^2A$ ....(ii)

Substitute equation (ii) in equation (i),

$\Rightarrow [(\sec A+\tan A)(1-\sin A)]=\frac{\cos^2 A}{\cos A}=\cos A$

Hence, the correct option is (c). **[½]**

3. $\sin^2 A+\cos^2 A+\cot^2 A$ (Given)

We know that $\sin^2 A+\cos^2 A=1$

$\Rightarrow \sin^2 A+\cos^2 A+\cot^2 A=1+\cot^2 A$

$\Rightarrow \text{cosec}^2 A\left(\text{Since } 1+\cot^2\text{A}=\text{cosec}^2\text{A}\right)$

Hence, $\sin^2 A+\cos^2 A+\cot^2 A=\text{cosec}^2 A$ **[1]**

4. $\left(\sec^2\theta - 1\right).\ \cot^2\theta$

Using the identity $\sec^2\theta - 1 = \tan^2\theta$, we get

$\left(\sec^2\theta - 1\right).\ \cot^2\theta = \tan^2\theta.\ \cot^2\theta$ **[½]**

$=\frac{\sin^2\theta}{\cos^2\theta}.\ \frac{\cos^2\theta}{\sin^2\theta}=1$

$\left[\text{As } \tan\theta=\frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta=\frac{\cos\theta}{\sin\theta}\right]$

Thus $\left(\sec^2\theta - 1\right).\ \cot^2\theta = 1$ **[½]**

5. $\sqrt{3}\tan\theta = 3\sin\theta$ (Given)

Putting $\tan\theta = \frac{\sin\theta}{\cos\theta}$ in given equation,

$\sqrt{3}\times\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

On solving,

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

Squaring both the sides,

$\cos^2\theta = \frac{1}{3}$ ....(i) **[½]**

Using the Trigonometric identity,

$\sin^2\theta = 1-\cos^2\theta$ ....(ii)

On Substitute the value of $\cos^2\theta$ from equation (ii) in (i),

$\sin^2\theta = 1-\frac{1}{3} = \frac{2}{3}$ ....(iii) **[½]**

On putting the value of $\sin^2\theta$ and $\cos^2\theta$

$\sin^2\theta - \cos^2\theta = \frac{2}{3}-\frac{1}{3} = \frac{1}{3}$

Hence, the value of $\sin^2\theta - \cos^2\theta$ is $\frac{1}{3}$. **[1]**

6. It is given that,

$\sin\theta - \cos\theta = \frac{1}{2}$

On squaring both the sides,

$(\sin\theta - \cos\theta)^2 = \frac{1}{4}$ **[½]**

$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{1}{4}$

Since, $\sin^2\theta + \cos^2\theta = 1$

$1-2\sin\theta\cos\theta = \frac{1}{4}$

$2\sin\theta\cos\theta = 1-\frac{1}{4} = \frac{3}{4}$

$\sin\theta\cos\theta = \frac{3}{8}$ ....(i) **[½]**

$\sin\theta + \cos\theta$ (Given)

$\Rightarrow \sin\theta + \cos\theta = \sqrt{(\sin\theta-\cos\theta)^2 + 4\sin\theta\cos\theta}$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{1-2\sin\theta\cos\theta + 4\times\frac{3}{8}}$

(Using equation 1)

$\Rightarrow \sin\theta + \cos\theta = \sqrt{1-2\times\frac{3}{8}+4\times\frac{3}{8}}$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{1-\frac{3}{4}+\frac{3}{2}}$

$\Rightarrow \sin\theta + \cos\theta = \frac{\sqrt{7}}{2}$ **[1]**

7. To prove $\left[\frac{1-\tan A}{1-\cot A}\right]^2 = \tan^2 A$,

Using the left hand side of the equation,

$\left[\frac{1-\tan A}{1-\cot A}\right]^2$

Since, $\tan A = \frac{\sin A}{\cos A}$ **[½]**

$\Rightarrow \left[\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right]^2$

$\Rightarrow \left[\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right]^2$ **[½]**

$\Rightarrow \left[\frac{\cos A - \sin A}{\sin A - \cos A}\cdot\frac{\sin A}{\cos A}\right]^2$

$\Rightarrow \left[-\frac{\sin A}{\cos A}\right]^2 = \tan^2 A$

Since LHS = RHS,

Hence proved, $\left[\frac{1-\tan A}{1-\cot A}\right]^2 = \tan^2 A$ **[1]**

8. $\frac{\sin^4\theta + \cos^4\theta}{1-2\sin^2\theta\cos^2\theta} = 1$

Taking the LHS and using the identity $\sin^2 A + \cos^2 A = 1$ we have,

$\frac{\sin^4\theta + \cos^4\theta}{1-2\sin^2\theta\cos^2\theta} =$

$\frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta + \cos^2\theta - \sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta}$ **[1]**

$= \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta(1-\cos^2\theta) + \cos^2\theta(1-\sin^2\theta)}$

$\left(\begin{array}{l}\text{Since } 1-\cos^2\theta = \sin^2\theta \\ \text{and } 1-\sin^2\theta = \cos^2\theta\end{array}\right)$

$= \frac{\sin^4\theta + \cos^4\theta}{\sin^4\theta + \cos^4\theta}$

= 1

Hence proved. **[1]**

9. Proceeding with the left hand side of the given equation and using $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$,

$$(cosec\theta - cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$\Rightarrow (cosec\theta - cot\theta)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$ **[1]**

Apply $\sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow (cosec\theta - cot\theta)^2 = \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$$

$$\Rightarrow (cosec\theta - cot\theta)^2 = \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

(As $a^2 - b^2 = (a-b)(a+b)$) **[1]**

$\Rightarrow (cosec\theta - cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$, Which is same as the Right hand side.

Hence proved. **[1]**

10. Using the Left hand side of the equation

$$\Rightarrow \frac{1+secA}{secA}$$

Putting $\sec A = \frac{1}{\cos A}$

$$\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$ **[1]**

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}}$$

$= \cos A + 1$ **[1]**

Using the Right hand side of the equation

$$\Rightarrow \frac{\sin^2 A}{1-cosA}$$

We know sin2 A = $1 - \cos^2 A$

$$\Rightarrow \frac{1-\cos^2 A}{1-cosA}$$

$$\frac{(\cos A+1)(1-\cos A)}{1-\cos A}$$

$$\Rightarrow 1+\cos A$$

Hence, proved. **[1]**

11. Using the left hand side of the equation

$$\Rightarrow \frac{(1+\tan^2 A)\cot A}{\cos ec^2 A}$$

We know $1 + \tan^2 A = \sec^2 A$ **[1]**

$$\frac{(\sec^2 A)\cot A}{\cos ec^2 A}$$

$$\frac{\frac{1}{\cos^2 A}(\cot A)}{\frac{1}{\sin^2 A}}$$ **[1]**

$$= \frac{\sin^2 A}{\cos^2 A}(\cot A) = \tan^2 A \times \cot A$$

$$= \tan^2 A \times \frac{1}{\tan A}$$

$$= \tan^2 A \times \frac{1}{\tan A} = \text{ tanA}$$

Hence, proved. **[1]**

12. $\tan\theta + \frac{1}{\tan\theta} = \sqrt{2}$ (Given)

Squaring both sides we get,

$$\left(\tan\theta + \frac{1}{\tan\theta}\right)^2 = 2$$ **[1]**

$$\tan^2\theta + \cot^2\theta + 2\tan\theta\frac{1}{\tan\theta} = 2$$

$$\left(\therefore \frac{1}{\tan\theta} = \cot\theta\right)$$ **[1]**

$\tan^2\theta + \cot^2\theta + 2 = 2$

$\tan^2\theta + \cot^2\theta = 0$

The value of $\tan^2\theta + \cot^2\theta = 0$ **[1]**

13. Using the left hand side of the equation,

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$ **[½]**

$$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{(\sin^2\theta - \cos^2\theta)}$$

[2]

$$= \frac{1+1}{\sin^2\theta - (1-\sin^2\theta)} \qquad (\therefore \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{2}{2\sin^2\theta - 1}$$

Hence Proved. **[½]**

14. To prove:

$(1+\cot\theta-\operatorname{cosec}\theta).(1+\tan\theta+\sec\theta)=2$

Solving the left hand side of the equation

$(1+\cot\theta-\operatorname{cosec}\theta).(1+\tan\theta+\sec\theta)$

$$\Rightarrow\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right).\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$\Rightarrow\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right).\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$ **[1]**

$$\Rightarrow\left(\frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cdot\cos\theta}\right)$$

$$\Rightarrow\left(\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cdot\cos\theta-1}{\sin\theta\cdot\cos\theta}\right)$$ **[1]**

$$\Rightarrow\left(\frac{1+2\sin\theta\cdot\cos\theta-1}{\sin\theta\cdot\cos\theta}\right)$$

$$\Rightarrow\left(\frac{2\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}\right)$$

$\Rightarrow 2$

Since LHS = RHS, **[1]**

Hence proved.

15. Taking LHS,

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}$$

Using the identity,

$\operatorname{cosec}^2\theta-\cot^2\theta=1$

Putting $1=\operatorname{cosec}^2\theta-\cot^2\theta$

$$\frac{\operatorname{cosec}^2\theta-\cot^2\theta}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}$$

$$\frac{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)}{(\operatorname{cosec}\theta+\cot\theta)}-\operatorname{cosec}\theta$$

$(\operatorname{cosec}\theta-\cot\theta)-\operatorname{cosec}\theta$

$-\cot\theta$ **[1]**

Taking RHS,

$$\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

$$\operatorname{cosec}\theta-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

Putting, $1=\operatorname{cosec}^2\theta-\cot^2\theta$

$$\operatorname{cosec}\theta-\frac{(\operatorname{cosec}^2\theta-\cot^2\theta)}{\operatorname{cosec}\theta-\cot\theta}$$ **[1]**

$$\operatorname{cosec}\theta-\frac{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)}{\operatorname{cosec}\theta-\cot\theta}$$

$=\operatorname{cosec}\theta-(\operatorname{cosec}\theta+\cot\theta)$

$-\cot\theta$

Thus LHS = RHS. **[1]**

16. Taking LHS,

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$$

Dividing both numerator and denominator by $\cos\theta$, we get

$$=\frac{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}-\frac{1}{\cos\theta}}$$

$$=\frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}$$ **[1]**

$$=\frac{(\tan\theta+\sec\theta)-1}{(\tan\theta-\sec\theta)+1}$$

Multiplying both numerator and denominator by $(\tan\theta-\sec\theta)$

$$=\frac{\{(\tan\theta+\sec\theta)-1\}}{\{(\tan\theta-\sec\theta)+1\}}\frac{(\tan\theta-\sec\theta)}{(\tan\theta-\sec\theta)}$$

$$\frac{(\tan^2\theta-\sec^2\theta)-(\tan\theta-\sec\theta)}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$$ **[1]**

[As $a^2-b^2=(a+b)(a-b)$]

$$=\frac{(-1-\tan\theta+\sec\theta)}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$$

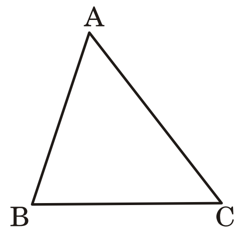
[$\because \tan^2\theta-\sec^2\theta=-1$]

$$=\frac{-1}{\tan\theta-\sec\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}$$

Hence, LHS = RHS, Proved **[1]**

17. In ΔABC,



[1]

Sum of interior angles of triangles = 180° (Angle sum property of triangle)

$A + B + C = 180°$

$\Rightarrow B + C = 180° - A$

Dividing both the sides by 2

$$\Rightarrow \frac{B+C}{2} = \frac{180° - A}{2}$$

$$\Rightarrow \frac{B+C}{2} = 90° - \frac{A}{2} \quad ....(i)$$ **[1]**

Now taking L.H.S of the given expression,

$$\sin\left(\frac{B+C}{2}\right)$$

$$= \sin\left(90° - \frac{A}{2}\right) = \cos\left(\frac{A}{2}\right) \quad \left[\sin\left(90° - \theta\right) = \cos\theta\right]$$

= R.H.S **[1]**

18. We know, $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Substituting in the left hand side,

$$(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = \left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2$$

Using $(a + b)^2 = a^2 + 2ab + b^2$ **[1]**

$$(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = \sin^2\theta + \frac{2\sin\theta}{\cos\theta} + \frac{1}{\cos^2\theta} + \cos^2\theta + \frac{2\cos\theta}{\sin\theta} + \frac{1}{\sin^2\theta}$$

$$(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = 1 + \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta\sin^2\theta} + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

(Also, $\sin^2\theta + \cos^2\theta = 1$) **[1]**

$$\Rightarrow (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = 1 + \frac{1}{\cos^2\theta\sin^2\theta} + 2\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$\Rightarrow (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = 1 + \sec^2\theta\operatorname{cosec}^2\theta + 2\left(\frac{1}{\sin\theta\cos\theta}\right)$$ **[1]**

$$\Rightarrow (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = 1 + \sec^2\theta\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\sec\theta$$

$$\Rightarrow (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1 + \sec\theta\operatorname{cosec}\theta)^2$$

Hence proved. **[1]**

19. Solving left hand side of the equation,

Using, $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} - \frac{1}{\cos\theta}$$

$$\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{\cos\theta}{1 - \sin\theta} - \frac{1}{\cos\theta}$$

$$= \frac{\cos^2\theta - (1 - \sin\theta)}{\cos\theta(1 - \sin\theta)}$$ **[1]**

Apply $\cos^2\theta = 1 - \sin^2\theta$

$$= \frac{1 - \sin^2\theta - (1 - \sin\theta)}{\cos\theta(1 - \sin\theta)} = \frac{\sin\theta - \sin^2\theta}{\cos\theta(1 - \sin\theta)}$$

$$\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{\sin\theta(1 - \sin\theta)}{\cos\theta(1 - \sin\theta)} = \tan\theta \quad ....(i)$$

[1]

Solving right hand side of the equation,

Use $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta} = \frac{1}{\cos\theta} - \frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\cos\theta} - \frac{\cos\theta}{1 + \sin\theta}$$

$$= \frac{1 + \sin\theta - \cos^2\theta}{\cos\theta(1 + \sin\theta)}$$ **[1]**

Apply $\cos^2\theta = 1 - \sin^2\theta$

$$= \frac{1+\sin\theta-\left(1-\sin^2\theta\right)}{\cos\theta\left(1+\sin\theta\right)} = \frac{\sin\theta+\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \tan\theta \quad \text{....(ii)}$$

Using equation (i) and (ii),

$$\frac{1}{\sec\theta-\tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta+\tan\theta} \quad [1]$$

20. Taking LHS of the equation

$$\Rightarrow \frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$\Rightarrow \frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+(\sec^2\theta-\tan^2\theta)} \quad [1]$$

(Using, $\sec^2\theta - \tan^2\theta = 1$)

$$\Rightarrow \frac{\tan\theta+\sec\theta-1}{-(\sec\theta-\tan\theta)+(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}$$

using, $a^2 - b^2 = (a - b)(a + b)$

[1]

Taking ($\sec\theta - \tan\theta$) common from denominator, we get,

$$\Rightarrow \frac{\tan\theta+\sec\theta-1}{(\sec\theta-\tan\theta)(-1+\sec\theta+\tan\theta)}$$

On cancelling the like terms,

$$\Rightarrow \frac{1}{\sec\theta-\tan\theta} \quad [1]$$

$$\Rightarrow \frac{1}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$\Rightarrow \frac{\cos\theta}{1-\sin\theta}$$

This is equal to RHS. [1]

Hence proved.

21. Using the left hand side of the equation,

$$\Rightarrow \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$$

Multiplying by $\cos\theta$ and $\sin\theta$

$$\Rightarrow \frac{\cos^2\theta}{\cos\theta(1-\tan\theta)} + \frac{\sin^2\theta}{\sin\theta(1-\cot\theta)} \quad [1]$$

We know, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\Rightarrow \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta} \quad [1]$$

$$\Rightarrow \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$\Rightarrow \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta} \quad [1]$$

$$\Rightarrow \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta-\sin\theta}$$

$\{\because a^2 - b^2 = (a+b)(a-b)\}$

$\Rightarrow \cos\theta + \sin\theta$ = Right hand side of the equation

Hence, proved. [1]

22. $1 + \sin^2\theta = 3\sin\theta\cos\theta$ (Given)

On dividing both sides by $\cos^2\theta$,

$$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cos\theta}{\cos^2\theta}$$

$\sec^2\theta + \tan^2\theta = 3\tan\theta$ [1]

$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$

$1 + 2\tan^2\theta = 3\tan\theta$

$2\tan^2\theta - 3\tan\theta + 1 = 0$

$2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$ [1]

$2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$(\tan\theta - 1)(2\tan\theta - 1) = 0$ [1]

$\tan\theta - 1 = 0$

$\tan\theta = 1$

Or

$2\tan\theta - 1 = 0$

$2\tan\theta = 1$

$$tan\theta = \frac{1}{2} \quad [1]$$

23. $\left(\frac{1+\tan A}{1+\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

We know

$$\cot A = \frac{1}{\tan A}$$

On using the left hand side of the equation

$$\left(\frac{1+\tan A}{1+\cot A}\right)^2$$

$$\Rightarrow\left(\frac{1+\tan A}{1+\cot A}\right)^2=\left(\frac{1+\tan A}{1+\frac{1}{\tan A}}\right)^2$$ **[1]**

$$\Rightarrow\left(\frac{1+\tan A}{\frac{\tan A+1}{\tan A}}\right)^2$$

$$\Rightarrow\left(\frac{\tan A(1+\tan A)}{\tan A+1}\right)^2$$

$$\Rightarrow(\tan A)^2$$

$$\Rightarrow(\tan^2 A)$$ **[1]**

Also,

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2$$

$$\Rightarrow\left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$ **[1]**

$$\Rightarrow\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$\Rightarrow\left(\frac{-\tan A(1-\tan A)}{1-\tan A}\right)^2$$

$$\Rightarrow(-\tan A)^2$$

$$\Rightarrow(\tan^2 A)$$

Hence proved. **[1]**

24. $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$ (Given)

Squaring both the sides we get,

$$(\cos\theta+\sin\theta)^2=(\sqrt{2}\cos\theta)^2$$ **[1]**

Using $\cos^2\theta+\sin^2\theta=1\Rightarrow\cos^2=1-\sin^2\theta$ we get,

$$\Rightarrow\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta=2\cos^2\theta$$ **[1]**

$$\Rightarrow 1+2\cos\theta\sin\theta=2(1-\sin^2\theta)$$

$$\Rightarrow 1+2\cos\theta\sin\theta=2-2\sin^2\theta$$

On rearranging the terms,

$$\Rightarrow 2-1-2\cos\theta\sin\theta=2\sin^2\theta$$

$$\Rightarrow 1-2\cos\theta\sin\theta=2\sin^2\theta$$ **[1]**

Substituting $1=\sin^2\theta+\cos^2\theta$,

$$\sin^2\theta+\cos^2\theta-2\cos\theta\sin\theta=2\sin^2\theta$$

$$\Rightarrow(\sin\theta-\cos\theta)^2=(\sqrt{2}\sin\theta)^2$$

Taking square root on both the sides,

$$\sin\theta-\cos\theta=\sqrt{2}\sin\theta$$

Hence proved. **[1]**

Smart Notes ....

CHAPTER 11

Some Applications of Trigonometry

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Problem related to Height and Distance	4 marks	4 marks	1,3,4 marks	1,3,4 marks	1,3,4 marks	1,3,4 marks

Summary

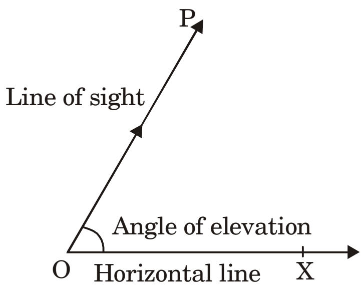
Introduction

LINE OF SIGHT

When an observer looks from a point O at an object P then the line OP is called the line of sight.

ANGLE OF ELEVATION

Assume that from a point O, we look up at an object P, placed above the level of our eye. Then, the angle which the line of sight makes with the horizontal line through O is called the angle of elevation of P, as seen from O.

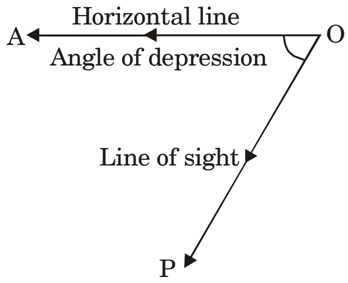


Example: Let OX be a horizontal line on the level ground and let a person at O be looking up towards an object P, say an aeroplane or the top of a tree or the top of a tower, or a flag at the top of a house.

Then, $\angle XOP$ is the angle of elevation of P from O.

ANGLE OF DEPRESSION

Assume that from a point O, we look down at an object P, placed below the level of our eye.



Then, the angle which the line of sight makes with the horizontal line through O is called the angle of depression of P, as seen from O.

PREVIOUS YEARS' EXAMINATION QUESTIONS

1 Mark Questions

1. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 45°. The height of the tower (in metres) is

(*a*) 15

(*b*) 30

(*c*) $30\sqrt{3}$

(*d*) $10\sqrt{3}$

[TERM 2, 2011]

2. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is

(*a*) 45° (*b*) 30°

(c) 60° (*d*) 90°

[TERM 2, 2012]

3. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m.) is:

(*a*) $25\sqrt{3}$ (*b*) $50\sqrt{3}$

(*c*) $75\sqrt{3}$ (d) 150

[TERM 2, 2013]

4. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in meters) is:

(*a*) $\frac{4}{\sqrt{3}}$ (*b*) $4\sqrt{3}$

(*c*) $2\sqrt{2}$ (*d*) 4

[TERM 2, 2014]

5. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower (in metres) is

(*a*) $50\sqrt{3}$ (*b*) $150\sqrt{3}$

(*c*) $150\sqrt{2}$ (*d*) 75

[TERM 2, 2014]

6. The tops of two towers of height *x* and *y*, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

[TERM 2, 2015]

7. In figure 1, a tower *AB* is 20 *m* high and *BC*, its shadow on the ground, is $20\sqrt{3}m$ long. Find the Sun's altitude.

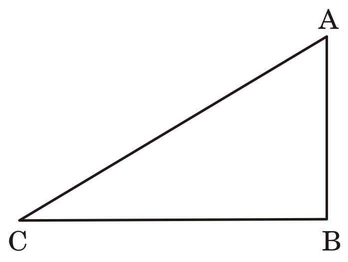


Figure 1

[TERM 2, 2015]

8. In Fig. 1, *AB* is a 6 *m* high pole and *CD* is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point *D* of pole. If $AD = 2.54$ *m*, find the length of the ladder. (Use $\sqrt{3} = 1.732$)

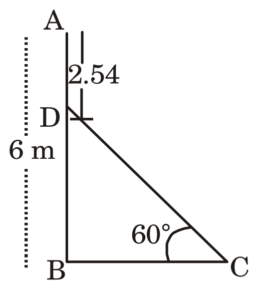


Fig. 1

[TERM 2, 2016]

9. A ladder leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 *m* away from the wall, find the length of the ladder.

[TERM 2, 2016]

10. If a tower 30 *m* high, casts a shadow $10\sqrt{3}m$ long on the ground, then what is the angle of elevation of the sun?

[TERM 2, 2017]

11. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the Sun?

[TERM 2, 2017]

3 Marks Questions

12. From the top of a tower 100 *m* high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars. [Use $\sqrt{3} = 1.73$]

[TERM 2, 2011]

13. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}$ m high cliff are 45° and 60° respectively. Find the height of the tower.

[TERM 2, 2012]

14. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the second pole is 24, find the height of the first pole. $\left[\sqrt{3} = 1.732\right]$

[TERM 2, 2013]

15. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ *m*, find the speed of the aeroplane.

[TERM 2, 2014]

16. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of the two ships as observed from the top of the light house are 60° and 45°. If the height of the light house is 200 m, find the distance between the two ships. $\left[\text{Use} \sqrt{3} = 1.73\right]$

[TERM 2, 2014]

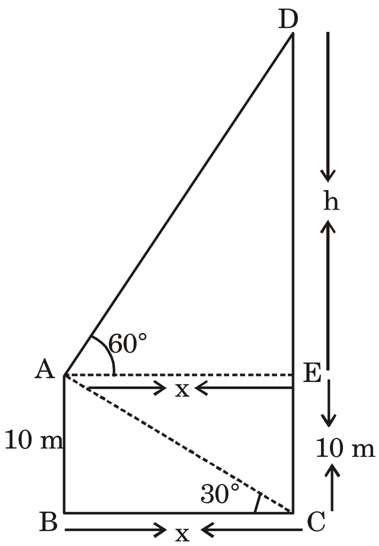
17. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

[TERM 2, 2015]

18. The angle of elevation of an aero plane from a point *A* on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aero plane is flying at a constant height of $1500\sqrt{3}m$, find the speed of the plane in km/hr.

[TERM 2, 2015]

19. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.



[TERM 2, 2016]

20. The angles of depression of the top and bottom of a 50 *m* high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and thehorizontal distance between the tower and the building.

[TERM 2, 2016]

21. On a straight line passing through the foot of a tower, two points *C* and *D* are at distances of 4 *m* and 16 *m* from the foot respectively. If the angles of elevation from *C* and *D* of the top of the tower are complementary, then find the height of the tower.

[TERM 2, 2017]

22. A moving boat is observed from the top of a 150*m* high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

[TERM 2, 2017]

4 Marks Questions

23. Two poles of equal heights are standing opposite to each other on either side of the road, which is 100 *m* wide. From a point between then on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles.

[TERM 2, 2011]

24. The angles of elevation and depression of the top and bottom of a light-house from the top of a 60*m* high building are 30° and 60° respectively. Find

(i) The difference between the heights of the light-house and the building.

(ii) The distance between the light-house and the building.

[TERM 2, 2012]

25. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 *m* high, find the height of the building.

[TERM 2, 2013]

26. From a point D on the ground the angle of elevation of the top of a tower is 45° and that of the top of a flagstaff fixed on the top of the tower is 60°. If the distance between the foot & point D is 120 m, then find the height of the flagstaff.

[Use $\sqrt{3} = 1.73$] [TERM 2, 2014]

27. The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 *m* high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them.

[TERM 2, 2014]

28. From a point *P* on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 *m*, find the height of the tower.

[TERM 2, 2015]

29. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A.

[TERM 2, 2015]

30. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of the tower is 45°. Find the height of the tower PQ and the distance PX.

[TERM 2, 2016]

31. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1.732$]

[TERM 2, 2017]

32. The angle of elevation of a cloud from a point 60m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

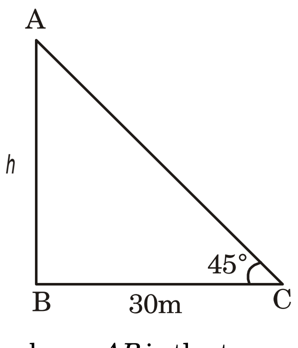
[TERM 2, 2017]

33. As observed from the top of a 100 m high light house from sea level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

[DELHI, 2018]

Solutions

1.



In the figure above, AB is the tower and C is the point 30m away from the foot of the tower.

Let h denote the height of the tower (in metres).

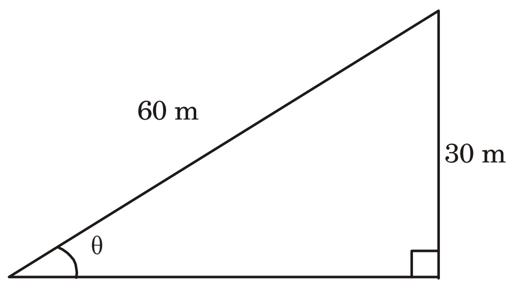
$$\tan 45^o = \frac{AB}{BC} = \frac{h}{30}$$ [½]

$$\Rightarrow 1 = \frac{h}{30}$$

$\Rightarrow h = 30$ [½]

Hence, the correct option is (b).

2.



From the figure,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\Rightarrow \sin\theta = \frac{30}{60}$$ [½]

$$\Rightarrow \sin\theta = \frac{1}{2}$$

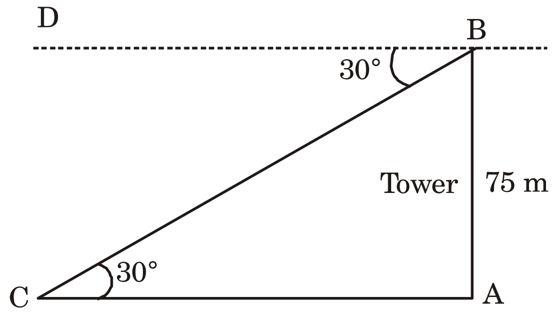
$$\Rightarrow \sin\theta = \sin 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$

The angle of elevation of kite at the ground is 30°.

Option (b) is correct. [½]

3.



Let AB is the Tower of height 75 m and Car is at the point C on the ground.

Now,

According to the question,

$\angle CBD = 30^\circ$

$\angle BCA = 30^\circ$ (alternate opposite angles) [½]

And,

In the ΔABC,

$$Cot\ 30^\circ = \frac{AC}{AB}$$

$$\Rightarrow AC = AB\ Cot\ 30^\circ$$

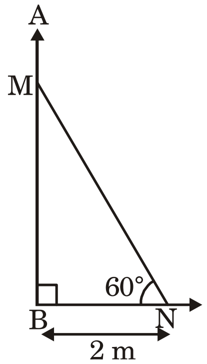
Putting $Cot\ 30^\circ = \sqrt{3}$

$$AC = 75 \times \sqrt{3}$$

$\therefore$ The distance of the car from the base of the tower is $75\sqrt{3}$m [½]

Hence option (c) is correct.

4.



Let the length of the ladder is MN, placed against the wall AB and makes an angle of 60° with the ground.

The foot of the ladder is at N, which is 2 m away from the wall.

$BN = 2\ m$

In right-angled triangle ΔMBN:

$$cos60^\circ = \frac{BN}{MN} = \frac{2}{MN}$$ [½]

Using $\cos 60^\circ = \frac{1}{2}$ we get,

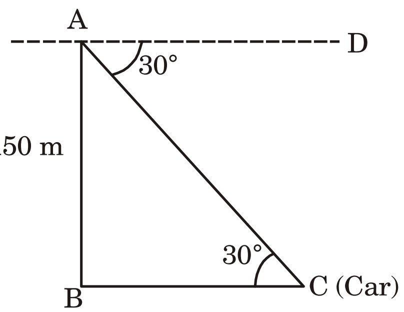
$$\frac{1}{2} = \frac{2}{MN}$$

$$\Rightarrow MN = 4\ m$$

Therefore, the length of the ladder is 4 m. [½]

Hence, the correct option is (d).

5. [½]



Consider the tower AB of height 150 m and $\angle DAC = \angle ACB = 30^\circ$ (alternate angles)

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30^\circ = \frac{AB}{BC}$$

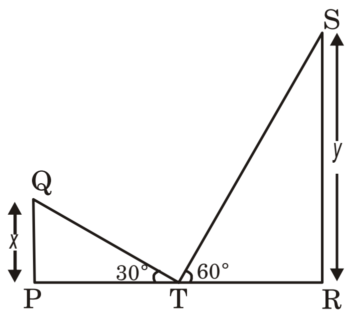
Using $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{BC}$$

$$\Rightarrow BC = 150\sqrt{3}m$$ [½]

Thus the correct answer is (b).

6.



Let T is the centre of the line joining the feet of the two towers PR.

In ΔQPT

$$\tan 30^\circ = \frac{QP}{PT}$$

Using $\tan 30^\circ = \frac{1}{\sqrt{3}}$ we get,

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{PT}$$

$$\Rightarrow PT = \sqrt{3}x$$ [½]

Also,

In ΔSRT

$$\tan 60^\circ = \frac{RS}{TR}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \sqrt{3} = \frac{y}{TR}$$

$\Rightarrow TR = \frac{y}{\sqrt{3}}$

Since it is given that T is the centre of PR this implies that $PT = TR$

$\Rightarrow \sqrt{3}x = \frac{y}{\sqrt{3}}$

$\Rightarrow \frac{x}{y} = \frac{1}{3}$

Hence, the ratio of x and y is 1 : 3. [½]

7. $AB = p = 20\ m$ (Given)

$BC = b = 20\sqrt{3}\ m$ (Given)

Now,

$\tan\theta = \frac{p}{b}$

$\tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 60°$

$\theta = 60°$ [1]

8. Given: $AB = 6\ m$, $AD = 2.54\ m$ and CD is inclined at an angle of 60°

To find: Length of the ladder i.e. CD

Solution: From the figure, we can see that,

$AB = AD + DB = 6\ m$

Since, $AD = 2.54\ m$

So, 2.54 m + $DB = 6\ m$

$DB = 3.46\ m$ [½]

Now in the ΔBCD,

$\frac{BD}{CD} = \sin 60°$

$\Rightarrow \frac{3.46}{CD} = \frac{\sqrt{3}}{2}$

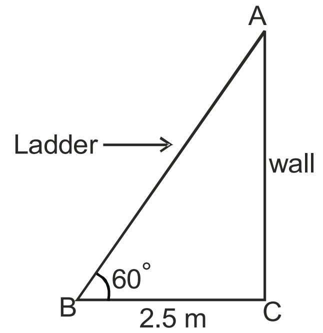
$\Rightarrow \frac{3.46}{CD} = \frac{1.732}{2}$

$\Rightarrow CD = \frac{2 \times 3.46}{1.732}$

$\Rightarrow CD = 3.99\ m$

Hence, the length of the ladder CD is $3.99m \approx 4m$. [½]

9. In the given figure AB is ladder,



$\frac{\text{Base}}{\text{Hypotenuse}} = \cos 60^0$ [½]

Using $\cos 60^\circ = \frac{1}{2}$ we get,

$\frac{BC}{AB} = \frac{1}{2}$

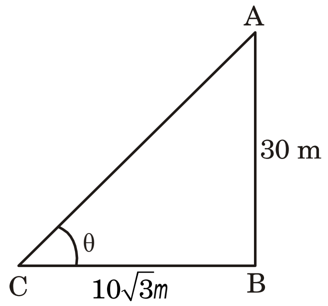
$AB = 2BC$

$AB = 2 \times 2.5$

$AB = 5\ m$

Therefore, the length of the ladder is 5 m. [½]

10.



Given:

AB = 30 m (Height)

BC = $10\sqrt{3}m$ (Length)

Let the angle of elevation be θ

In ΔABC

$\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{30}{10\sqrt{3}}$ [½]

$\Rightarrow \tan\theta = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3}$

$\Rightarrow \tan\theta = \sqrt{3}$

As $\tan 60^\circ = \sqrt{3}$

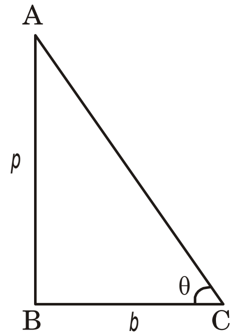
$\theta = 60^\circ$ [½]

Therefore, the angle of elevation of the sun is 60°

11. Let the angle of elevation of the Sun be θ.

Let the height of the tower be p.

And the length of the shadow be b.



It is given than ratio of height to the length of the shadow is $= \sqrt{3} : 1$

$\Rightarrow p : b = \sqrt{3} : 1$ [½]

We know that tan θ is given by perpendicular over base

$$\Rightarrow \tan\theta = \frac{perpendicular}{base} = \frac{AB}{BC}$$

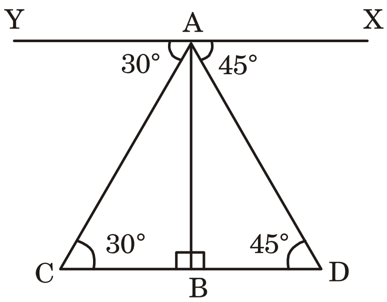
$$\Rightarrow \tan\theta = \frac{p}{b}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\therefore \theta = 60^\circ \left(\because \tan 60^\circ = \sqrt{3}\right)$$ [½]

Hence, the angle of elevation of the sun is 60°.

12. Consider the diagram,



Let MN be the tower of height 100 m. C and D are the position of two cars whose angle of depression from the top of the tower is 30° and 40° respectively.

Clearly, from the diagram $\angle YAC = \angle ACB = 30^o$

and $\angle XAD = \angle ADB = 45^o$

(Alternate interior angles)

In ΔABC,

$$\tan 30^o = \frac{AB}{CB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{CB}$$

$$\Rightarrow CB = 100\sqrt{3}$$

In ΔABD,

$$\tan 45^o = \frac{AB}{BD}$$

$$1 = \frac{100}{BD}$$

$\Rightarrow BD = 100$ [1]

The distance between two cars = CD

$= CB + BD$ (from the diagram)

$= 100\sqrt{3} + 100$

$= 100(\sqrt{3} + 1)$

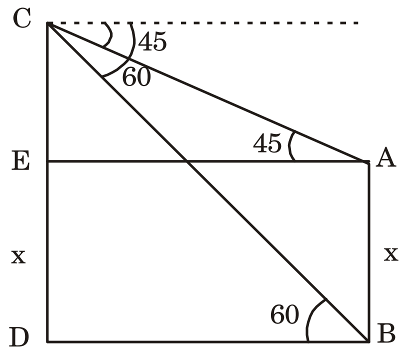
= 100 (1.73 + 1)

= 100 (2.73)

= 273 m

Therefore, the distance between the two cars is 273 m. [1]

13. [1]



Let the height of the tower be AB be x

$\Rightarrow AB = DE = x$

It is given that the height of the cliff $CD = 60\sqrt{3}\ m$

$\Rightarrow CE = 60\sqrt{3} - x$

Now, In right triangle AEC,

$$\tan 45^\circ = \frac{CE}{EA}$$

$$\Rightarrow 1 = \frac{60\sqrt{3} - x}{EA}$$

$\Rightarrow EA = 60\sqrt{3} - x$ (1) [1]

In right triangle CDB

$$\tan 60^\circ = \frac{CD}{DB}$$

$$\Rightarrow \sqrt{3} = \frac{60\sqrt{3}}{DB}$$

$\Rightarrow DB = 60\ m$

$\Rightarrow DB = EA = 60\ m$

Now, from Equation (1), we have

$\Rightarrow EA = 60\sqrt{3} - x$

$\Rightarrow 60 = 60\sqrt{3} - x$

$x = 60\left(\sqrt{3} - 1\right) m$

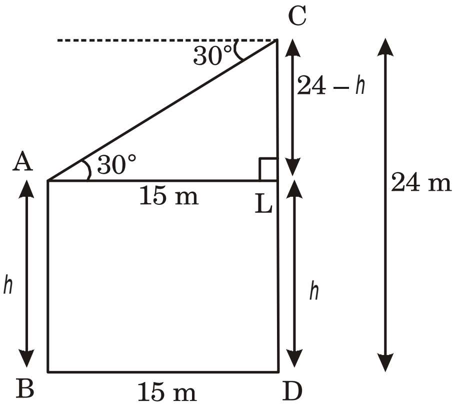
Thus, the height of the tower is $60\left(\sqrt{3} - 1\right) m$ [1]

14. Let two poles AB and CD are apart and $CD = 24$m

According to the question,

Angle of depression of the top of first pole as seen from the top of second pole is 30°.

Using the figure according to the question, [1]



$\angle CAL = 30°$ and BD = 15 m

$\because$ BD = AL

$\therefore$ AL = 15m

Now,

Let the height of the pole AB be h

$\Rightarrow$ LD = h

$\therefore CL = 24 - h$

In the right angled ΔACL,

$$\tan A = \frac{CL}{AL}$$

$$\tan 30° = \frac{24 - h}{15}$$ [1]

Using $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24 - h}{15}$$

$$\frac{15}{\sqrt{3}} = 24 - h$$

$$24 - \frac{15\sqrt{3}}{3} = h$$

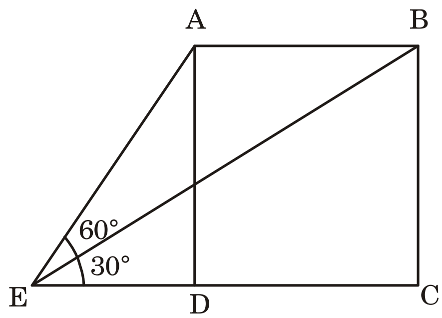
$24 - 5\sqrt{3} = h$

$24 - (5 \times 1.732) = h$

$h = 15.34$

Thus the height of the pole (AB) is 15.34 m [1]

15. [½]



Here $AD = 3000\sqrt{3}$ m

Consider the ΔBCE,

$$\tan 30° = \frac{BC}{EC}$$

$$\tan 30° = \frac{3000\sqrt{3}}{EC}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{EC}$$

$EC = 9000\ m$ [1]

Now consider ΔADE,

$$\tan 60^\circ = \frac{AD}{ED}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{ED}$$

$ED = 3000\ m$

Distance covered by the aeroplane in 30 seconds $= AB = CD = EC - ED$

Distance covered by the aeroplane in 30 seconds $= 9000 - 3000 = 6000\ m$

$$\text{Speed of aeroplane } = \frac{\text{Distance}}{\text{Time}}$$ [½]

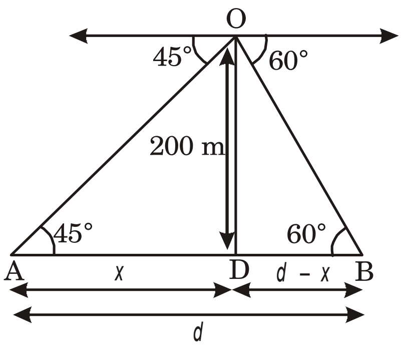
$$\text{Speed of aeroplane } = \frac{6000}{30} = 200\, m/s$$

Hence the speed of the aeroplane is 200 *m/s* [1]

16. Let d be the distance between the two ships.

Suppose the distance of one of the ships from the light house is x meters, then the distance of the other ship from the light house is (d-x) meter.

[1]



In right-angled ΔADO, we have,

$$\tan 45^o = \frac{OD}{AD} = \frac{200}{x}$$

$$1 = \frac{200}{x}$$

$x = 200\ m$

In right-angled ΔBDO, we have

$$\tan 60^o = \frac{OD}{BD} = \frac{200}{d - x}$$

Using $\tan 60^\circ = \sqrt{3}$

$$\sqrt{3} = \frac{200}{d - x}$$

$$d - x = \frac{200}{\sqrt{3}}$$ [1]

Putting $x = 200$

$$\Rightarrow d = \frac{200}{\sqrt{3}} + 200$$

$$d = \frac{200 + 200\sqrt{3}}{\sqrt{3}}$$

$$d = \frac{200\left(1 + \sqrt{3}\right)}{\sqrt{3}}$$

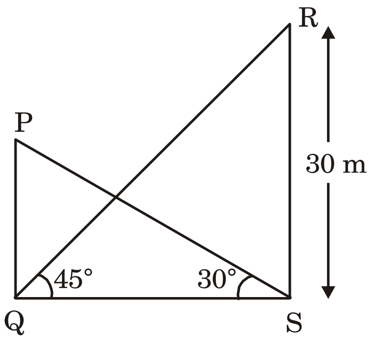
d = 200 × 1.58

d = 316

Thus, the distance between two ships is approximately 316 *m*. [1]

17. Let the height of the building is $PQ = x\ m$.

[½]



In ΔQRS we have,

$$\tan 45^\circ = \frac{RS}{QS}$$

$$\Rightarrow 1 = \frac{30}{QS}$$

$\Rightarrow QS = 30$ m (i) [½]

Now in ΔSPQ we have,

$$\tan 30^\circ = \frac{PQ}{QS}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{PQ}{30} \quad \text{(Using (i))}$$

$$\Rightarrow PQ = \frac{30}{\sqrt{3}}$$ [1]

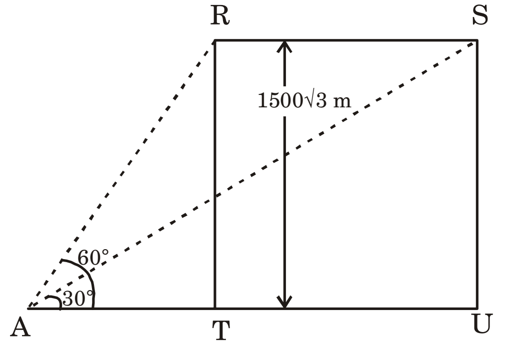
On rationalizing we get,

$$PQ = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow PQ = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\ \text{m}$$

Hence, the height of the building is $10\sqrt{3}$ m . [1]

18. [½]



Let, R and S be the two position of the plane and A be the point of observation. Let, ATU be the horizontal line through A. The angle of elevation of the plane in two position P and Q from a point A are respectively.

$\angle RAT = 60°$, $\angle SAU = 30°$

$RT = 1500\sqrt{3}\ m$ (Given)

In ΔATR,

$$\tan 60° = \frac{RT}{AT}$$

$$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AT}$$ [½]

$AT = 1500\ m$

In ΔASU

$$\tan 30° = \frac{SU}{AU}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AU}$$

$AU = 4500\ m$

Distance travelled by the plane is $4500m - 1500m$ $= 3000\ m$ [1]

$$Speed = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{3000}{15} = 200 \text{ m/sec}$$

$$\Rightarrow 200 \times \frac{18}{5} = 720 \text{ km/hr}$$

Hence the speed of the plane is 720 km/hr. [1]

19. Let CD be the hill and suppose the man is standing on the deck of the ship at point A.

The angle of depression of the base C of the hill CD from point A is 30° and the angle of elevation of the top D of the hill CD is 60°.

So, $\angle EAD = 60°$ and $\angle BCA = 30°$ [½]

In $\Delta AED, \tan 60^o = \frac{DE}{EA}$

Using $\tan 60^\circ = \sqrt{3}$ we get,

$$\sqrt{3} = \frac{h}{x}$$ [½]

Hence, $h = \sqrt{3}x$ (equation I)

In $\Delta ABC, \tan 30^o = \frac{AB}{BC}$

Using $\tan 30^\circ = \frac{1}{\sqrt{3}}$ we get,

$$\frac{1}{\sqrt{3}} = \frac{10}{x}$$ [1]

Hence, $x = 10\sqrt{3}$ (equation II)

Using equations I and II

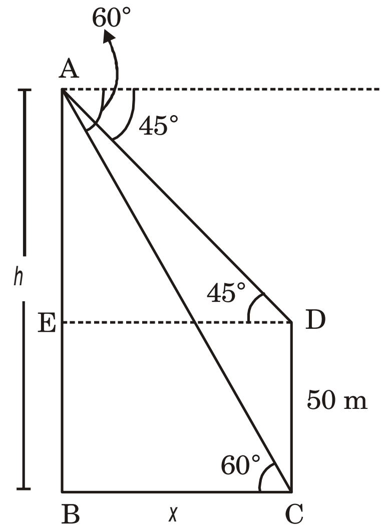
$h = \sqrt{3} \times 10\sqrt{3} = 30$

Hence, $DE = 30\ m$

So, $CD = CE + ED = 10 + 30 = 40\ m$

Thus, the distance of the hill from the ship is and the height of the hill is 40 m. [1]

20. [½]



Let the height of the tower AB be h meters and the horizontal distance between the tower and the building BC be x meters.

From the figure, we can see that,

$AE = AB - EB$

$AE = (h - 50)\ m$

Now, in ΔAED,

$$\tan 45^\circ = \frac{AE}{ED}$$ [½]

Using tan 45° = 1 we get,

$$\Rightarrow 1 = \frac{h-50}{x}$$

$$\Rightarrow x = h - 50 \quad ....(1)$$

In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$\tan 60^\circ = \sqrt{3}$ we get,

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3}x = h \quad ....(2)$$ [½]

Using (1) and (2),

$$x = \sqrt{3}x - 50$$

$$\Rightarrow x\left(\sqrt{3} - 1\right) = 50$$

Solving and putting $\sqrt{3} = 1.73$,

$$\Rightarrow x = \frac{50}{\left(\sqrt{3}-1\right)}$$

$$= \frac{50}{0.73}$$

$$\Rightarrow x = 68.49m$$ [½]

Now substituting the value of x in (1), we get,

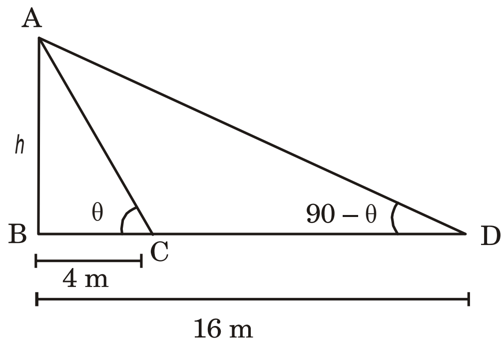
$68.49 = (h - 50)\ m$

$\Rightarrow h = 68.49 + 50$

$\Rightarrow h = 118.49\ m$

Hence, the height of the tower is 118.49 m and the horizontal distance between the tower and the building is 68.449 m. [1]

21. [½]



Let AB be the height of tower

Angle of elevation of top of the tower from point C be θ

Angle of elevation of top of the tower from point D be $90 - \theta$

In ΔABD

$$\tan\left(90^\circ - \theta\right) = \frac{AB}{BD} = \frac{h}{16}$$ [½]

$$\cot\theta = \frac{h}{16} \ ......(i)$$

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{4} \ .....(ii)$$ [½]

Multiply (i) by (ii)

$$\tan\theta \times \cot\theta = \frac{h}{4} \times \frac{h}{16} = \frac{h^2}{64}$$

Since $\tan\theta = \frac{1}{\cot\theta}$ [½]

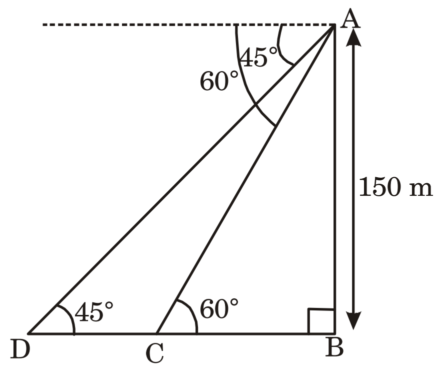
$$\Rightarrow 1 = \frac{h^2}{64}$$

$\Rightarrow h^2 = 64$

$\Rightarrow h = 8\ m$

Therefore, the height of tower is $h = 8\ m$ [1]

22. Let us consider the following diagram.

 [½]

In ΔABC

$$\tan\theta = \frac{p}{b} = \frac{AB}{BC}$$

$$\Rightarrow \tan 60° = \frac{150}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{150}{BC}$$ [½]

$$\Rightarrow BC = \frac{150}{\sqrt{3}}$$

$$= \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 50\sqrt{3}m$$ (i)

In ΔABD

$$\tan\theta = \frac{p}{b} = \frac{AB}{BD}$$

$$\Rightarrow \tan 45° = \frac{150}{BD}$$ [½]

$$\Rightarrow 1 = \frac{150}{BD}$$

$$\Rightarrow BD = 150m$$ (ii)

Also, $BD = BC + CD$

Putting equation (i) and (ii), in the above equation

$$\Rightarrow 150 = 50\sqrt{3} + CD$$

$$\Rightarrow CD = 150 - 50\sqrt{3}a$$

$$\Rightarrow CD = 150 - 50\sqrt{3} = 50\sqrt{3}\left(\sqrt{3}-1\right)$$ [½]

According to the question, time taken to cover the distance from C to D is 2 min.

Converting 2 min in hours

$$2\,\text{min} = \frac{2}{60} = \frac{1}{30}\,\text{hours}$$

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow Speed = \frac{CD}{\frac{1}{30}}$$

$$= \frac{50\sqrt{3}\left(\sqrt{3}-1\right)}{\frac{1}{30}}$$

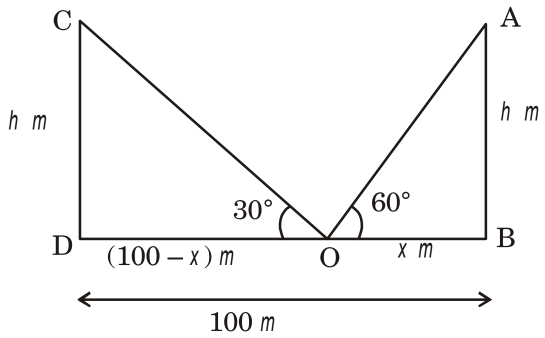
$$= 30 \times \left(50\sqrt{3}\left(\sqrt{3}-1\right)\right)$$

$$= 1500\sqrt{3}\left(\sqrt{3}-1\right)\text{m/h}$$

Hence, the speed of the boat is

$1500\sqrt{3}\left(\sqrt{3}-1\right)$ m/h [1]

23.



[1]

Let AB and CD be the poles of equal heights standing opposite to each other on either sides of the road and distance between them is $BD = 100m$

Let O be the point on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively.

Let $OB = x$ m, therefore $OD = (100 - x)$ m

Now in ΔAOB

$$\tan 60° = \frac{AB}{OB} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x\sqrt{3}$$ (1) [1]

Now in ΔDOC

$$\tan 30° = \frac{CD}{OD} = \frac{h}{100 - x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$ [1]

Using equation (1),

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{100 - x}$$

$$\Rightarrow 100 - x = 3x$$

$\Rightarrow 100 = 4x$

$\Rightarrow x = 25$

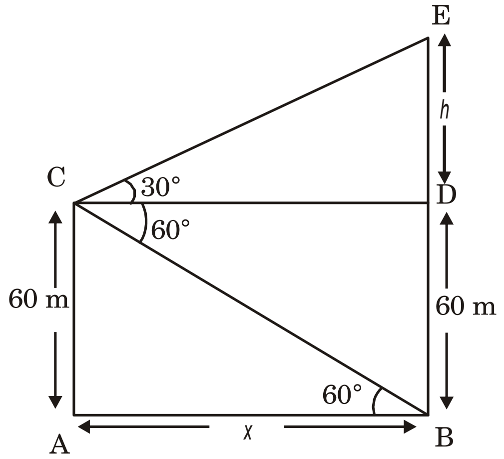
Now using equation (1)

$\Rightarrow h = 25\sqrt{3}$

Hence height of the poles is

$h = 25\sqrt{3}\text{m}$ [1]

24.



[1]

Let AC be the building such that $AC = 60\ m$ and BE be the light house.

Let $AB = CD = x$ be the horizontal distance between the building and light house.

It is given that the angles of elevation and depression of the top and bottom of a light-house from the top of a 60 m high building are 30° and 60° respectively.

$\Rightarrow \angle DCE = 30^\circ$ and $\angle ABC = 60^\circ$ [1]

Let DE be the difference between the heights of the light-house and the building h.

(ii) Now, in right triangle ABC,

$$\tan 60^\circ = \frac{AC}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}}$$

$\Rightarrow x = 20\sqrt{3}$ [1]

Therefore, the distance between the light-house and the building, $x = 20\sqrt{3}m$

(i) In right triangle CDE,

$$\tan 30^\circ = \frac{DE}{CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow h = \frac{x}{\sqrt{3}}$$

$$\Rightarrow h = \frac{20\sqrt{3}}{\sqrt{3}}$$

$\Rightarrow h = 20$ m

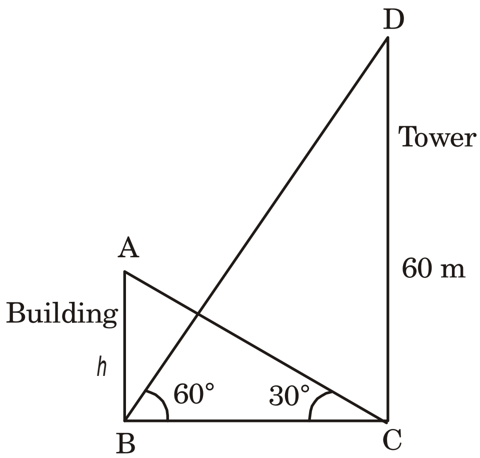
Therefore, the difference between the heights of the light-house and the building is $h = 20\ m$ [1]

25. Assume AB as the building and CD as the tower.

Suppose the height of the building AB as 'h' m.

Given that, $\angle ACB = 30^\circ$, $\angle CBD = 60^\circ$ and $CD = 60$ m.

[1]



ΔBCD is a right angled triangle then,

$cot\ 60^\circ = \dfrac{BC}{CD}$ [1]

$\Rightarrow BC = CD\ cot\ 60^\circ$

$\Rightarrow BC = 60 \times \dfrac{1}{\sqrt{3}}$ [1]

$$= \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 20\sqrt{3}m$$

ΔACB is a right angled triangle then,

$$\tan 30^\circ = \frac{AB}{BC}$$

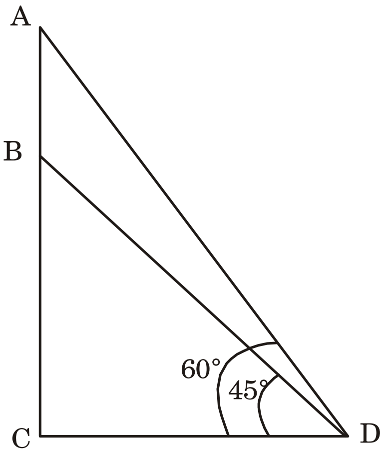
$$\Rightarrow AB = BC \tan 30^\circ$$

$$\Rightarrow h = 20\sqrt{3} \times \frac{1}{\sqrt{3}}$$

$$= 20m$$

$\therefore$ The height of the building is 20 m. [1]

26. Consider BC as the tower and AB as the flagstaff fixed on top of the tower. The distance CD is 120 m.



[1]

In ΔBCD, $\tan 45^\circ = \frac{BC}{CD}$,

$$1 = \frac{BC}{120}$$

$$BC = 120$$

So the height of the tower is 120 m. [1]

Now in ΔACD, $\tan 60^\circ = \frac{AC}{CD}$,

$$\sqrt{3} = \frac{AB+BC}{CD}$$

$$\sqrt{3} = \frac{AB+120}{120}$$ [1]

$$120\sqrt{3} = AB + 120$$

$$AB = 120\sqrt{3} - 120$$

$$AB = 120\left(\sqrt{3} - 1\right)$$

$$AB = 120\,(1.732 - 1)$$

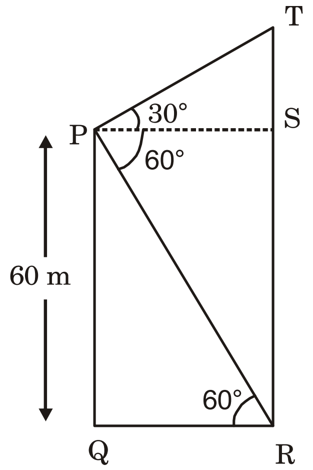
$$AB = 120\,(0.732)$$

$$AB = 87.84 \text{ m}$$

Therefore the height of the flagstaff is 87.84 m.

[1]

27.



[1]

From the given figure we have,

$PQ = 60\ m$, $PS = QR$

In ΔPQR

$$\frac{PQ}{QR} = \tan 60^\circ$$

$$\Rightarrow \frac{60}{QR} = \frac{\sqrt{3}}{1}$$ [1]

$$\Rightarrow QR = \frac{60}{\sqrt{3}}$$

$$= \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 20\sqrt{3}m$$

In ΔTSP

$$\frac{TS}{PS} = \tan 30^\circ$$

$$\Rightarrow \frac{TS}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

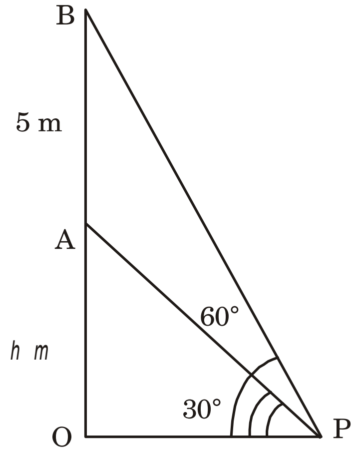
$$\Rightarrow TS = \frac{20\sqrt{3}}{\sqrt{3}} = 20m$$ [1]

Then, the height of the tower, $TR = TS + SR = 20 + 60 = 80\ m$

Hence, difference between the heights of the tower and the building is $80\ m - 60\ m = 20\ m$

And the distance between the tower and the building is $20\sqrt{3}m$. [1]

28.



[1]

Let OA be the tower of the height h meters and AB be the height of flag staff i.e. 5 m.

Now angle of elevation of the top of the tower and the flag staff from point P is 30° and 60° respectively.

$OA = h\ m$, $AB = 5\ m$ and $\angle OPA = 30°$, $\angle OPB = 60°$

In right ΔOAP, $\angle OPA = 30°$,

$\Rightarrow \tan 30° = \frac{h}{OP}$ [1]

Using $\tan 30° = \frac{1}{\sqrt{3}}$ we get,

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{OP}$

$\Rightarrow OP = h\sqrt{3}$ ... (i)

Now in right ΔOBP, $\angle OPB = 60°$,

$\Rightarrow \tan 60° = \frac{h+5}{OP}$

Using $\tan 60° = \sqrt{3}$ we get,

$\Rightarrow \sqrt{3} = \frac{h+5}{OP}$ [1]

Now using equation (i) and substituting the value of OP,

$\Rightarrow \sqrt{3} = \frac{h+5}{h\sqrt{3}}$

$\Rightarrow 3h = h + 5$

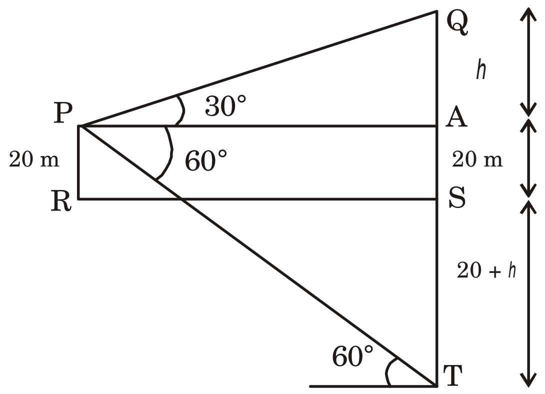
$\Rightarrow 2h = 5$

$\Rightarrow h = \frac{5}{2} = 2.5$

Here we get the value of, $h = 2.5\ m$

Hence the height of the tower is 2.5 m. [1]

29.



[1]

Let the distance of the cloud from A be h meters.

Distance of the reflection of the cloud in the lake will be same as distance of the cloud from the level of water in the lake.

$\Rightarrow TS = 20 + h$

In ΔTAP

$\tan 60° = \frac{AT}{AP}$

$\Rightarrow \sqrt{3} = \frac{40+h}{AP}$

$\Rightarrow AP = \frac{40+h}{\sqrt{3}}$ ....(i) [1]

In ΔAPQ

$\tan 30° = \frac{h}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AP}$

$\Rightarrow AP = h\sqrt{3}$ [1]

Equating both the values of AP we get,

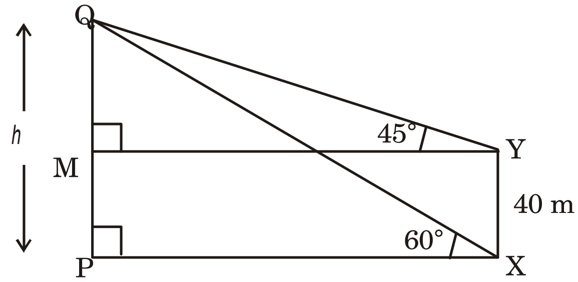
$\frac{40+h}{\sqrt{3}} = \sqrt{3}h$

$\Rightarrow 40 + h = 3h$

$\Rightarrow h = 20\ m$

Distance of the cloud from A is 20 m. [1]

30. According to the question, the angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of the tower is



[1]

$MP = XY = 40\ m$

$QM = h - 40$

In right angled triangle ΔQMY,

Using tan 45° = 1 we get,

$1 = \frac{h-40}{PX}$ $(MY = PX)$

Hence, $PX = h - 40$ (Equation I)

$\tan 60^0 = \frac{QP}{PX}$ [1]

Using $\tan 60^\circ = \sqrt{3}$ we get,

$\sqrt{3} = \frac{QP}{PX}$

$PX = \frac{h}{\sqrt{3}}$ (Equation II)

From equations I and II,

$h - 40 = \frac{h}{\sqrt{3}}$

$\sqrt{3}h - 40\sqrt{3} = h$ [1]

$\sqrt{3}h - h = 40\sqrt{3}$

$1.732\ h - h = 40\ (1.732)$

$h = 94.645\ m$

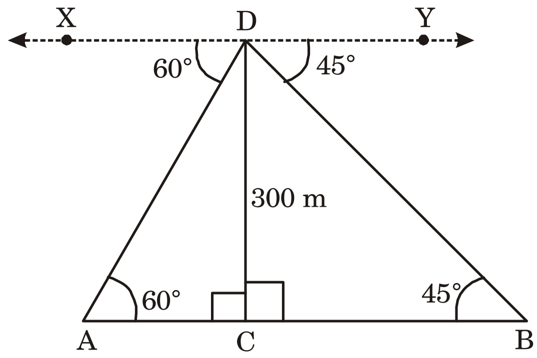
Thus, $PQ = 94.645\ m$

Also, $PX = 94.645 - 40 = 54.645$

(Using Equation I)

Thus, $PX = 54.79\ m$ [1]

31.



[1]

Let height of the aeroplane above the river is CD.

Let A and B be two points on both banks in opposite direction.

Given:

Height of the aeroplane above the river is $CD = 300\ m$ [1]

$\angle ADX = \angle CAD = 60^\circ$ ......(Alternate angle)

$\angle BDY = \angle CBD = 45^\circ$ ......(Alternate angle)

In right ΔBCD

$\tan 45^\circ = \frac{CD}{BC}$

Using tan 45° = 1 we get,

$\Rightarrow 1 = \frac{300}{BC}$

$\Rightarrow BC = 300\ m$

In right ΔACD

$\tan 60^\circ = \frac{CD}{AC}$

Using $\tan 60^\circ = \sqrt{3}$ we get,

$\sqrt{3} = \frac{300}{AC}$

$\Rightarrow AC = \frac{300}{\sqrt{3}} = 100\sqrt{3}m$ [1]

As width of river is AB = BC + AC

$= 300 + 100\sqrt{3}$

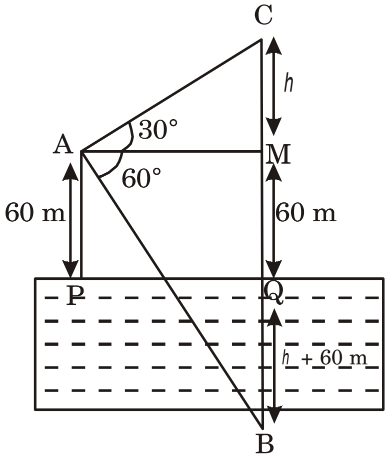
= 300 + 100 × 1.73

= 300 + 173

= 473 m

Therefore, Width of the river is 473 m. [1]

32. The depth of the shadow will be same as the distance of the cloud from the surface of the lake. Let us consider the following diagram.



[1]

Here, $AP = 60\ m$ is the height of the point of observation. $QC = h + 60\ m$ is the height of the cloud from the surface of the lake. $QB = h + 60\ m$ is the depth of the shadow.

In ΔAMC

$$\tan 30° = \frac{CM}{MA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{MA}$$

$$\Rightarrow MA = \sqrt{3}h \quad \text{........ (i)}$$ [1]

In ΔAMB

$$\tan 60° = \frac{BM}{MA}$$

$$\Rightarrow \sqrt{3} = \frac{60 + h + 60}{MA}$$

$$\Rightarrow MA = \frac{120 + h}{\sqrt{3}} \quad \text{........ (ii)}$$ [1]

Equating equation (i) and (ii)

$$\sqrt{3}h = \frac{120 + h}{\sqrt{3}}$$

$\Rightarrow 3h = 120 + h$

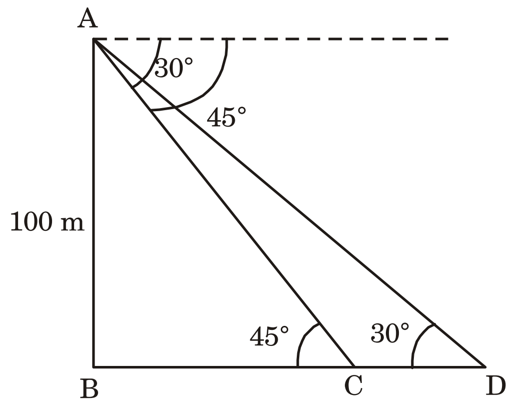
$\Rightarrow 2h = 120$

$\Rightarrow h = 60\ m$

Height of the cloud from the surface of the lake $= QC = h + 60 = 60 + 60 = 120\ m$

Hence, the height of the cloud from the surface of the lake is 120 *m*. [1]

33. [1]



Height of the light house = 100 *m*

Consider ΔABC

$$\tan 45° = \frac{AB}{BC}$$ [1]

Using tan 45° = 1 we get,

$$1 = \frac{100}{BC}$$

BC = 100 *m*

Consider ΔABD

Using $\tan 30° = \frac{1}{\sqrt{3}}$ we get,

$$\frac{1}{\sqrt{3}} = \frac{100}{BD}$$ [1]

$BD = 100\sqrt{3}$ m

Distance between the two ships = $BD - BC$

Distance between the two ships $= 100\sqrt{3} - 100$

Distance between the two ships $= 100\left(\sqrt{3} - 1\right)$

Distance between the two ships = 100 (1.732 – 1)

Distance between the two ships = 100 (0.732)

Distance between the two ships is 73.2 m [1]

Smart Notes ....

Smart Notes ....

CHAPTER 12

Areas Related to Circles

Chapter Analysis of Previous Three Years' Board Exams

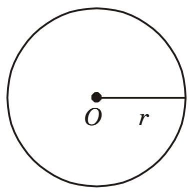
Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Finding Area of Shaded Region	3 marks	3 marks	3,4,4 marks	3,3, 4 marks	3,4 marks	3,3 marks

Summary

Terms Related to Circles

CIRCLE



The set of points which are at a constant distance of units from a fixed point O is called a circle with centre O and radius = r units. The circle is denoted by $C(O, r)$.

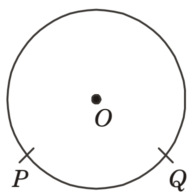
The fixed point O is called the centre and the constant distance r units is called its radius.

CIRCUMFERENCE

The perimeter (or length of boundary) of a circle is called its circumference.

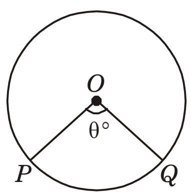
ARC

A continuous piece of a circle is called an arc of the circle.



In the given figure, PQ is an arc of a circle, with centre O, denoted by $\widehat{PQ}$. The remaining part of the circle, shown by the dotted lines, represents $\widehat{QP}$.

CENTRAL ANGLE

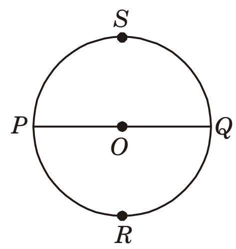


An angle subtended by an arc at the centre of a circle is called its central angle.

In the given figure of a circle with centre O, central angle of $\widehat{PQ}$ = $\angle POQ = \theta°$.

If $\theta° < 180°$ then the arc $\widehat{PQ}$ is called the minor arc and the arc $\widehat{QP}$ is called the major arc.

1.5 SEMICIRCLE

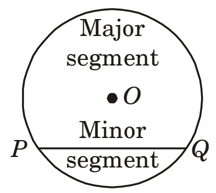


A diameter divides a circle into two equal arcs. Each of these two arcs is called a semicircle.

In the given figure of a circle with centre O, $\widehat{PRQ}$ and $\widehat{QSP}$ are semicircles.

An arc whose length is less than the arc of a semicircle is called a **minor arc.** An arc whose length is more than the arc of a semicircle is called a **major arc**.

SEGMENT

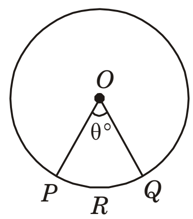


A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.

The segment containing the minor arc is called a minor segment, while the segment containing the major arc is the major segment.

The centre of the circle lies in the major segment.

SECTOR OF A CIRCLE



The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

In the given figure, *OPRQO* is a sector of the circle with centre *O*.

If arc *PQ* is a minor arc then *OPRQO* is called the minor sector of the circle.

The remaining part of the circle is called the major sector of the circle.

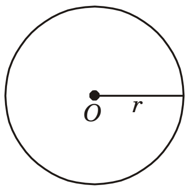
QUADRANT

One-fourth of a circular disc is called a quadrant. The central angle of a quadrant is 90º.

FORMULAE

Circumference and area of a circle

For a circle of radius *r*, we have



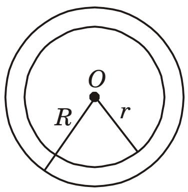
(i) Circumference of the circle = $2\pi r$

(ii) Area of the circle = πr^2

(iii) Area of the semicircle = $\frac{1}{2}\pi r^2$

(iv) Perimeter of the semicircle = $(\pi r + 2r)$

Area of a ring



Let *R* and *r* be the outer and inner radii of a ring.

Then, area of the ring = $\pi(R^2 - r^2)$.

Rotating wheels

(i) Distance moved by a wheel in 1 rotation = circumference of the wheel

(ii) Number of rotations in 1 minute

$$= \frac{\text{distance moved in 1 minute}}{\text{circumference}}$$

Rotation of the hands of a clock

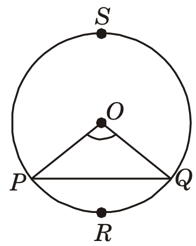
(i) Angle described by the minute hand of a clock in 60 minutes = 360°.

(ii) Angle described by the hour hand of a clock in 12hours = 360°.

Area of Sector and Segment of a Circle

Length of arc, area of sector and segment

Let an arc *AB* make an angle $\theta° < 180°$ at the centre of a circle of radius *r*. Then, we have



(i) Length of the arc $\widehat{PQ} = \frac{2\pi r\theta}{360} = \ell$

(ii) (a) Area of the sector *OPRQO*

$$= \frac{\pi r^2\theta}{360}$$

$$= \left(\frac{1}{2} \times \frac{2\pi r\theta}{360} \times r\right) = \left(\frac{1}{2} \times l \times r\right)$$

(b) Perimeter of the sector *OPRQO*

$$= OP + OQ + \text{length of arc } \widehat{PRQ} = \left(2r + \frac{2\pi r\theta}{360}\right)$$

(iii)(a) Area of the minor segment *PRQP*

= (area of the sector *OPRQO*) – (area of ΔOPQ)

$$= \left(\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2\sin\theta\right)$$

(b) Area of the major segment *QSPQ*

= (area of the circle)

– (area of the minor segment *PRQP*)

PREVIOUS YEARS'

EXAMINATION QUESTIONS

1 Mark Questions

1. The circumference of a circle is 22 cm. The area of its quadrant (in cm^2) is

(a) $\frac{77}{2}$ (b) $\frac{77}{4}$

(c) $\frac{77}{8}$ (d) $\frac{77}{16}$

[TERM 2, 2012]

2. If the difference between the circumference and the radius of a circle is 37cm, then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:

(a) 144 (b) 44

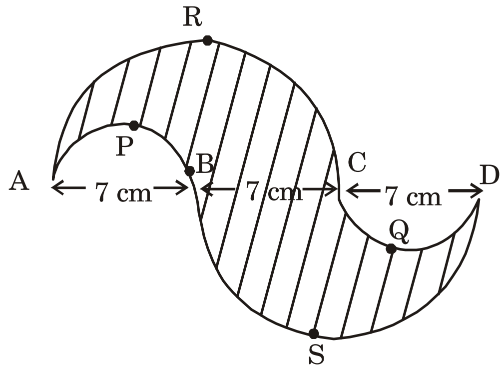
(c) 14 (d) 7

[TERM 2, 2013]

2 Marks Questions

3. In the given figure, APB and CQD are semi circles of diameter 7 cm each, while ARC and BSD are semi circles of diameter 14 cm each. Find the perimeter of the shaded region.

[Use $\pi = \frac{22}{7}$]

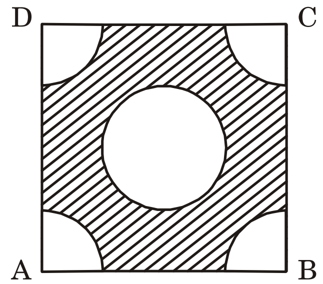


[TERM 2, 2011]

4. Find the area of a quadrant of a circle, where the circumference of circle is 44 cm. [Use $\pi = \frac{22}{7}$]

[TERM 2, 2011]

5. In the given figure, ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region. (Use $\pi = 3.14$)

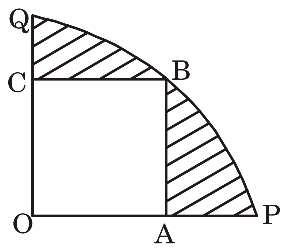


[TERM 2, 2012]

6. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions $14cm \times 7cm$. Find the area of the remaining card board. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[TERM 2, 2013]

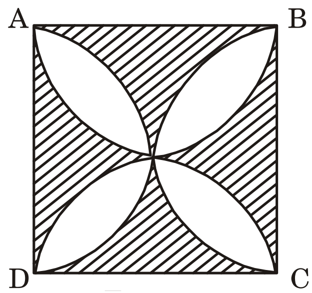
7. In the given figure, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20cm, find the area of the shaded region. (Use $\pi = 3.14$)



[TERM 2, 2014]

8. In the given figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.

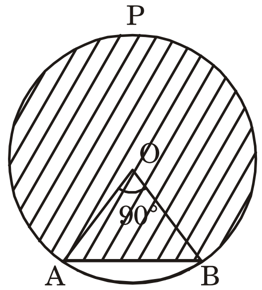
$\left(\text{Use } \pi = \frac{22}{7}\right)$



[TERM 2, 2016]

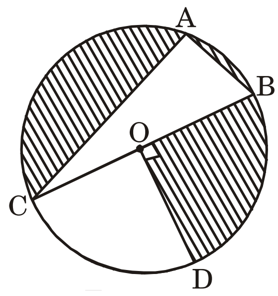
3 Marks Questions

9. Find the area of the major segment APB, in the given figure, of a circle of radius 35 cm and $\angle AOB = 90°$. [Use $\pi = \frac{22}{7}$]



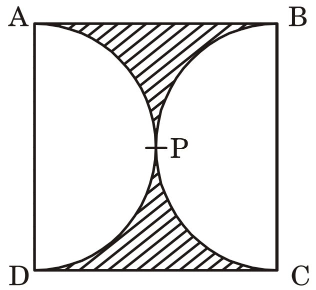
[TERM 2, 2011]

10. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$cm and $\angle BOD = 90°$. Find the area of the shaded region.



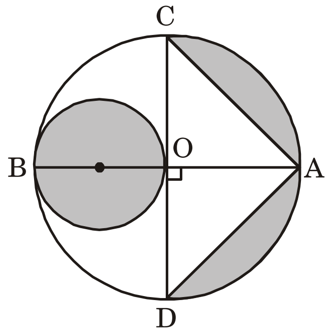
[TERM 2, 2012]

11. In the given figure, find the area of the shaded region, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



[TERM 2, 2012]

12. In the given figure, AB and CD are two diameters of a circle with centre O, which are perpendicular to each other. OB is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

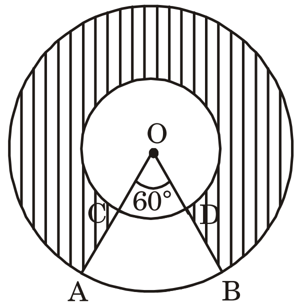


[TERM 2, 2013]

13. In a circle of radius21 cm, an arc subtends an angle of $60°$ at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[TERM 2, 2013]

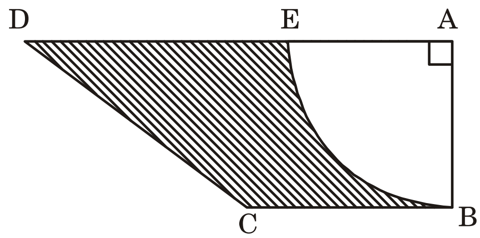
14. In the given figure, two concentric circles with centre O, have radii 21 cm and42 cm. If $\angle AOB = 60°$, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



[TERM 2, 2014]

15. In the given figure, ABCD is a trapezium of area 24.5 sq. cm In it, $AD \parallel BC$

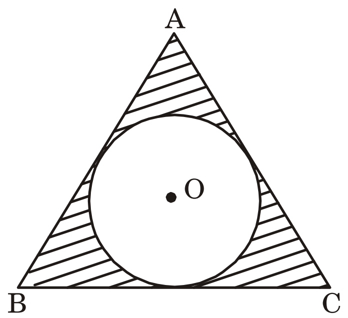
$\angle DAB = 90°$, $AD = 10$ cm and $BC = 4$ cm .If ABE is a quadrant of a circle, find the area of the shaded region. [Take $\pi = \frac{22}{7}$]



[TERM 2, 2014]

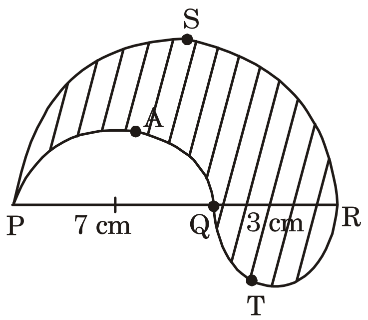
16. In the given figure, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region.

Use $\pi = 3.14$ and $\sqrt{3} = 1.73$



[TERM 2, 2014]

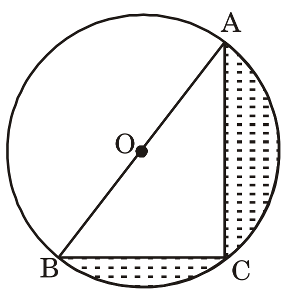
17. In the given figure, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region. Use $\pi = 3.14$



[TERM 2, 2014]

18. In the given figure, O is the centre of a circle such that diameter $AB = 13\text{cm}$ and $AC = 12\text{cm}$. BC is joined. Find the area of the shaded region

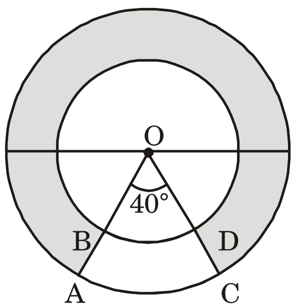
(Take $\pi = 3.14$)



[TERM 2, 2011]

19. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where

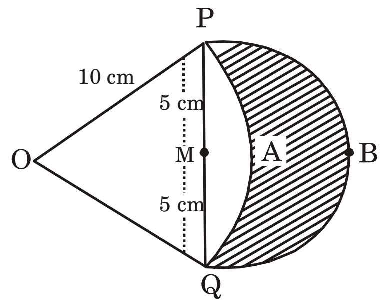
$\angle AOC = 40^o$. $\left(\text{Use } \pi = \frac{22}{7}\right)$



[TERM 2, 2016]

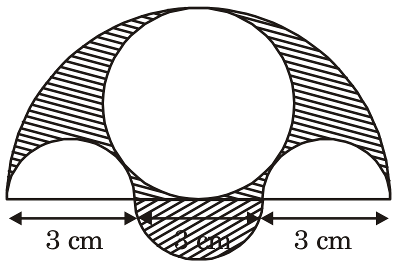
20. In the given figure, are shown two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If OP = PQ = 10 cm show that area of shaded region is

$25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm^2



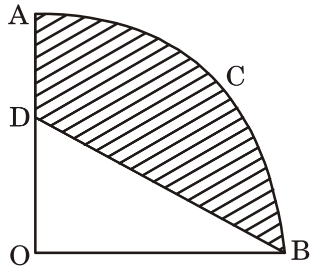
[TERM 2, 2016]

21. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



[TERM 2, 2017]

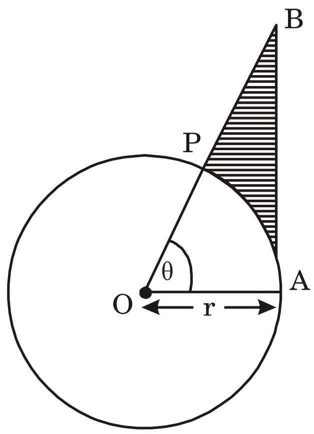
22. In the given figure, OACB is a quadrant of a circle with centre O and radius3.5cm. If OD=2cm, find the area of the shaded region.



[TERM 2, 2017]

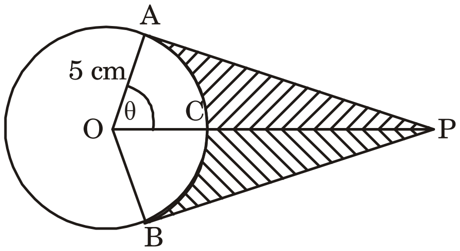
4 Marks Questions

23. In the given figure, is shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta+\sec\theta+\frac{\pi\theta}{180}-1\right]$



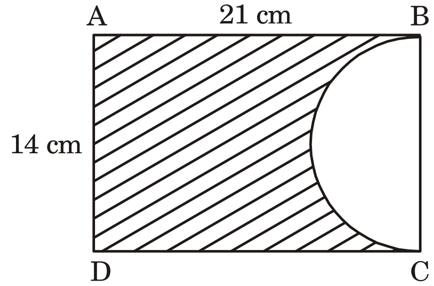
[TERM 2, 2016]

24. An elastic belt is placed around the rim of a pulley of radius 5 cm. (given figure). From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P,10cmfrom the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. $\left(Use\,\pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$



[TERM 2, 2016]

25. In the given figure, ABCD is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.

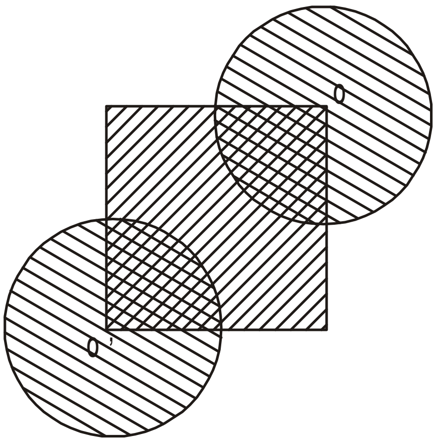


[TERM 2, 2017]

26. A chord PQ of a circle of radius 10 cm subtends an angle of $60°$ at the centre of circle. Find the area of major and minor segments of the circle.

[TERM 2, 2017]

27. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where O and O′ are centres of the circles. Find the area of shaded region.



[TERM 2, 2017]

Solutions

1. Circumference of a circle $= 2\pi r$

$\Rightarrow 22 = 2\pi r$

$\Rightarrow 11 = \pi r$

$\Rightarrow \frac{22}{7} \times r = 11$

$\Rightarrow r = \frac{7}{2}$ cm [½]

Now, area of a quadrant of a circle $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{8} \text{cm}^2$ [½]

2. Let the radius of the circle be 'r'.

Then the circumference of the circle $= 2\pi r$

According to the question,

$2\pi r - r = 37$

$\Rightarrow r(2\pi - 1) = 37$

$\Rightarrow r\left(2 \times \frac{22}{7} - 1\right) = 37$

$\Rightarrow r\left(\frac{37}{7}\right) = 37$

$\Rightarrow r = 7$

$\Rightarrow$ Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ [1]

3. Let r and R be the radii of the semicircle APB and ARC respectively.

$r = \frac{7}{2} cm$ and $R = \frac{14}{2} = 7\, cm$

The given shape is symmetric.

Required perimeter of the shaded region $= 2(\pi R - \pi r)$

(Hint: Perimeter of the semicircle $= \pi r$**)**

$\Rightarrow 2\pi(R - r)$

$\Rightarrow 2 \times \pi\left[7 - \left(\frac{7}{2}\right)\right]$ [1]

$\Rightarrow 2 \times \pi\left(\frac{7}{2}\right)$

$\Rightarrow 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)$

$\Rightarrow 22\, cm$

Therefore, the perimeter of the shaded region is **22 cm** [1]

4. Let r be the radius of the circle.

Given that circumference of circle is 44 cm.

$\Rightarrow 2\pi r = 44$

$\Rightarrow r = \frac{44}{2\pi}$

$\Rightarrow r = \frac{44 \times 7}{2 \times 22}$

$\Rightarrow r = 7$ cm [1]

Now, the area of the quadrant of the circle

$= \frac{\pi r^2}{4}$

$\Rightarrow \frac{22 \times (7)^2}{7 \times 4} = \frac{77}{2} = 38.5$

Therefore, the area of the quadrant of the circle is $38.5\ cm^2$ [1]

5. Let A be the area of each quadrant of the circle of radius 1 cm.

$\therefore$ Area of a quadrant $= \frac{1}{4}\pi r^2$

$\Rightarrow A = \frac{1}{4} \times 3.14 \times 1 \times 1$

$\Rightarrow A = 0.785 \text{ cm}^2$ [1]

Therefore, area of the 4 quadrants

$= 4A = 4 \times 0.785 = 3.14 \text{cm}^2$

Area of the circle inside the square $= \pi r^2 = 3.14 \times 1 \times 1 = 3.14 \text{ cm}^2$

Now, Area of the square $= (\text{side})^2 = (4)^2 = 16 \text{ cm}^2$

So, Area of the shaded part of the square

$= \text{Area of the square} - \left(\begin{array}{l}\text{Area of 4 quadrants} + \\ \text{Area of the circle}\end{array}\right)$

$= 16 - (3.14 + 3.14)$

$= 16 - 6.28$

$= 9.72 \text{ cm}^2$

the area of the shaded region is $9.72 cm^2$ [1]

6. The area of the rectangular sheet

$(A_1) = 14 \times 7 = 98 \text{cm}^2$

According to the question,

Two circular pieces of equal radii which are cut out from rectangular sheet have the maximum area.

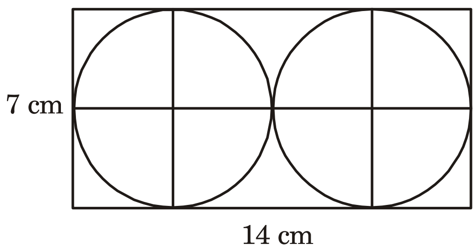
Hence,

The diameter of each circle is, $(d = \frac{14}{2} = 7$ cm

Therefore, radius of each circular sheet,

Thus the correct answer is (C). $r = \frac{d}{2} = \frac{7}{2}$ cm

[1]



Area of a circular piece

$= \pi \times \left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2} \text{cm}^2$

$\Rightarrow$ Area of both the circular pieces

$(A_2) = 2 \times \frac{77}{2} = 77 \text{cm}^2$

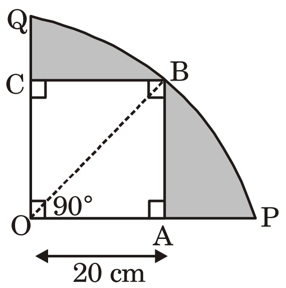
Now,

Area of the remaining card board $= A_1 - A_2$

$= 98 - 77 = 21 \text{cm}^2$

Thus the area of the remaining card board is 21 cm^2 [1]

7. In Δ OAB



$OB^2 = OA^2 + AB^2 = (20)^2 + (20)^2 = 2 \times (20)^2$

$OB = 20\sqrt{2}$

Radius of the circle,

$r = 20\sqrt{2}$

Area of quadrant OPBQ

$= \frac{\theta}{360^o} \times \pi r^2$ [1]

$= \frac{90^o}{360^o} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$= 0.25 \times 3.14 \times 800 \text{cm}^2$

$= 628 \, cm^2$

Area of square

OABC $= (\text{Side})^2 = (20)^2 \text{ cm}^2 = 400 \text{ cm}^2$

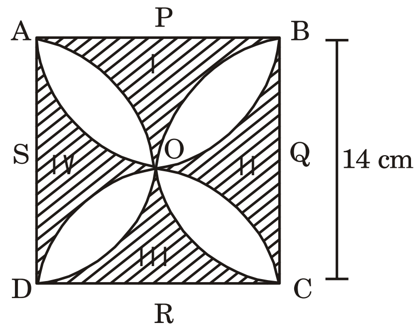
Area of the shaded region = Area of quadrant OPBQ – Area of square OABC

$= 628 - 400$

$= 228 \, cm^2$ [1]

Hence, the area of shaded area is $228 \, cm^2$

8.



We have marked the four shaded region as I, II, III and IV and the centres of the semicircles as P, Q, R and S in the figure above.It is given that the side of the square is 14cm.

Now,

Area of the region I + Area of the region III = Area of the square – Area of the semicircles with centres S and Q and radius $= \mathbf{7cm}$

$= 14 \times 14 - 2 \times \frac{1}{2} \times \pi \times 7^2$

$= 196 - 49 \times \frac{22}{7}$

$= 196 - 154$

$= 42 \text{cm}^2$ [1]

Similarly, Area of the region II + Area of the region IV = Area of the square – Area of the semicircles with centres P and R and radius = 7cm

$= 14 \times 14 - 2 \times \frac{1}{2} \times \pi \times 7^2$

$= 196 - 49 \times \frac{22}{7}$

$= 196 - 154$

$= 42\text{cm}^2$

So, the area of the shaded region = Area of region I + Area of region III + Area of region II + Area of region IV

$= 42\text{cm}^2 + 42\text{cm}^2$

$= 84\text{cm}^2$

Hence, the area of the shaded region is 84cm^2

[1]

9. Area of the given major sector PAOB with

$\angle 270° = \frac{270°}{360°} \times \pi r^2 = \frac{3}{4} \times \frac{22 \times 35 \times 35}{7}$

$= \frac{3}{2} \times 11 \times 5 \times 35$

$= \frac{5775}{2}$

$= 2887.5\ cm^2$ [1]

Area of right triangle

$\text{AOB} = \frac{1}{2}bh = \frac{1}{2} \times 35 \times 35 = 612.5\text{cm}^2$

Now the area of the complete major segment APB = area of major sector PAOB + area of right triangle AOB

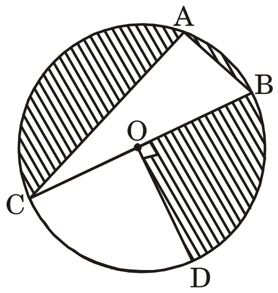
$= 2887.5 + 612.5$ [1]

$= 3500\text{cm}^2$

Hence the area of the major segment $\text{APB} = 3500\text{cm}^2$ [1]

10. Since O is the centre of the circle, therefore, BC will be the diameter of the circle.We know that, an angle in a semi-circle is always a right angle.

$\therefore \angle BAC = 90°$



Thus, ΔABC is a right-angled triangle, right-angled at A.

$\therefore$ By Pythagoras theorem,

$AB^2 + AC^2 = BC^2$

$\Rightarrow 7^2 + 24^2 = BC^2$

$\Rightarrow BC^2 = 625$

$\Rightarrow BC = 25$ cm [1]

Now, Area of the $\Delta ABC = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 24 \times 7$

$= 12 \times 7$

$= 84\ \text{cm}^2$

Again, since O is the centre of the circle

and $\angle BOD = 90°$

$\Rightarrow \angle COD = 90°$

BC is the diameter. Therefore,

$\text{OC} = \frac{1}{2}\text{BC} = \frac{25}{2}$ cm is the radius of the circle.

Therefore, area of the quadrant $\text{COD} = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

$= \frac{13750}{112}$

$= 122.76\ \text{cm}^2$ [1]

Lastly, area of the circle $= \pi r^2$

$= \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

$= \frac{13750}{28}$

$= 491.07\ \text{cm}^2$

Therefore, Area of the shaded region of the circ

$= \text{Area of circle} - \begin{pmatrix} \text{Area of Quadrant} + \\ \text{Area of } \Delta ABC \end{pmatrix}$

$= 491.07 - (122.76 + 84)$

$= 491.07 - 206.76$

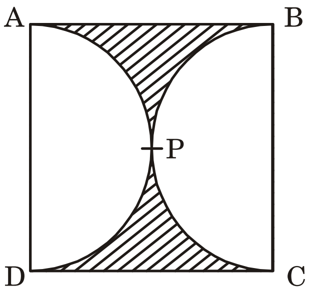
$= 284.31 \text{ cm}^2$

The area of the shaded region is 284.31 cm² [1]

11. Given that, side of the square is **14 cm**

$\Rightarrow$ Diameter of the semi-circles APD and BPC $= 14$ cm

$\Rightarrow$ Radius of the semi-circle APD and BPC $= 7$ cm



[1]

Area of the square $= (side)^2 = (14)^2 = 196 \text{ cm}^2$

Area of the semi-circle APD will be equal to the area of the semi-circle BPC,

$= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77 \text{ cm}^2$ [1]

Therefore, area of the shaded region is

$\text{Area of square} - (\text{Area of two semicircles})$

$= 196 - (77 + 77)$

$= 196 - 154$

$= 42 \ cm^2$

The area of shaded region is $42 \, cm^2$ [1]

12. We know that AB and CD are the diameters of a circle.

Hence,

OA = OB = OC = OD = 7cm (Radius)

Area of the shaded region = Area of the circle with diameter OB + (Area of the semi-circle ACDA " Area of Δ ACD)

$= \pi r^2 + \left(\frac{1}{2}\pi r^2 - \frac{1}{2} \times CD \times OA\right)$ [1]

Substituting the values,

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 + \left(\frac{1}{2} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 14 \times 7\right)$

$= \frac{77}{2} + \left(\frac{22 \times 7}{2} - \frac{14 \times 7}{2}\right)$ [1]

$= 38.5 + 77 - 49 = 66.5 \ cm^2$

Hence the area of the shaded part is 66.5 cm² [1]

13. We know that,

Radius of the circle $= 21 \ cm$

Angle subtended by the arc is $60°$

(i) Length of the arc

$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$ [1]

$= \frac{1}{6} \times 2 \times 22 \times 3 = 22 \ cm$

Hence the length of the arc is $22 \ cm$

(ii) Area of sector $= \frac{\theta}{360°} \times \pi r^2$

Substituting the values again,

$= \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21$ [1]

$= \frac{1}{6} \times 22 \times 3 \times 21 = 231 \ cm^2$

Hence the area of the sector is 231 cm² [1]

14. It is given that radius of inner circle $= 21$cm and radius of outer circle $= 42$cm

Area between the two circles $= \pi\left(R^2 - r^2\right)$ [1]

Area between the two circles

$= \frac{22}{7}\left(42^2 - 21^2\right)$

Area between the two circles

$= \frac{22}{7}(1764 - 441)$

Area between the two circles

$= \frac{22}{7}(1323) = 4158 \text{ cm}^2$

Area covered by the sector of $60°$ in the outer circle is $\frac{60°}{360°} \times \pi R^2$

Area covered by the sector of $60°$ in the outer circle $= \frac{60°}{360°} \times \left(\frac{22}{7}\right) \times 42^2$

Area covered by the sector of $60°$ in the outer circle $= 22 \times 42 = 924 \text{ cm}^2$

Area covered by the sector of $60°$ in the inner circle is $\frac{60°}{360°} \times \pi r^2$ [1]

Area covered by the sector of $60°$ in the inner circle $= \frac{60°}{360°} \times \left(\frac{22}{7}\right) \times 21^2$

Area covered by the sector of $60°$ in the inner circle $= 21 \times 11 = 231 \text{ cm}^2$

Area covered by the sector in between the circles

$924 - 231 = 693 \text{ cm}^2$

Area of shaded portion

$= \begin{pmatrix}\text{Area between} \\ \text{the two circles}\end{pmatrix} - \begin{pmatrix}\text{Area covered by the sector} \\ \text{in between the circles}\end{pmatrix}$

Area of shaded portion $= 4158 - 693 = 3465 \text{ cm}^2$

Thus, the area of shaded region is 3465 cm^2 [1]

15. It is given that area of trapezium is 24.5 cm^2

Area of trapezium

$= \frac{1}{2}(\text{Sum of parallel sides}) \times \text{Height}$

Area of trapezium $= \frac{1}{2}(AD + BC) \times AB$ [1]

$24.5 = \frac{1}{2}(10 + 4) \times AB$

$AB = \frac{49}{14} = 3.5 \text{ cm}$

Radius of the quadrant $= 3.5$ cm

Area of the quadrant $= \frac{\pi r^2}{4}$ [1]

Area of the quadrant $= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$

Area of the quadrant $= 9.625 \text{ cm}^2$

Area of shaded region

$=$ Area of trapezium $-$ Area of quadrant

Area of shaded region

$= 24.5 - 9.625 = 14.875 \text{ cm}^2$

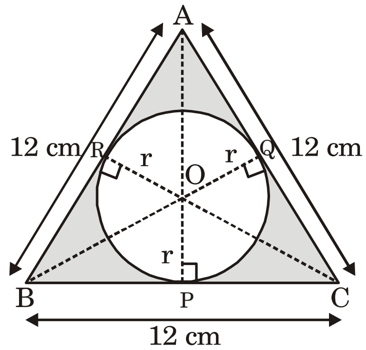
The area of shaded region is 14.875 cm^2 [1]

16. It is given that ABC is an equilateral triangle of side 12 cm Construction: Join OA, OB and OC Draw

$OP \perp BC$

$OQ \perp AC$

$OR \perp AB$



[1]

Let the radius of the circle be r cm.

Area of $\triangle$ AOB + Area of $\triangle$ BOC + Area of $\triangle$ AOC = Area of $\triangle$ ABC

$\frac{1}{2} \times AB \times OR + \frac{1}{2} \times BC \times OP$

$+ \frac{1}{2} \times AC \times OQ = \frac{\sqrt{3}}{4} \times (Side)^2$

$$\frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r$$

$$+\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times(12)^2$$

$$3\times\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times 12\times 12$$ [1]

$$r=2\sqrt{3}=2\times 1.73=3.46$$

Therefore the radius of the inscribed circle is $3.46\ cm$ Now,

area of the shaded region = Area of Δ ABC – Area of the inscribed circle

$$=\left[\frac{\sqrt{3}}{4}\times 12^2-\pi\left(2\sqrt{3}\right)^2\right]cm^2$$

$$=\left[36\sqrt{3}-12\pi\right]cm^2$$

$$=\left[36\times 1.73-12\times 3.14\right]cm^2$$

$$=24.6\,cm^2$$

Therefore, the area of the shaded region is $24.6\,cm^2$ [1]

17. Radius of Semicircle PSR

$$=\frac{1}{2}\times 10\ cm=5\ cm$$ [½]

Radius of Semicircle RTQ

$$=\frac{1}{2}\times 3=1.5\ cm$$

Radius of semicircle PAQ

$$=\frac{1}{2}\times 7\ cm=3.5\ cm$$ [½]

Perimeter of the shaded region = Circumference of semicircle PSR + Circumference of semicircle RTQ + Circumference of semicircle PAQ

$$=\left[\frac{1}{2}\times 2\pi(5)+\frac{1}{2}\times 2\pi(1.5)+\frac{1}{2}\times 2\pi(3.5)\right]cm$$ [1]

$$=\pi(5+1.5+3.5)\,cm$$

$$=3.14\times 10$$

$$=31.4\,cm$$ [1]

18. According to the given figure,

$\angle ACB=90^o$ (Angle in a semi-circle)

Now, in ΔACB, using Pythagoras theorem, we have

$$AB^2=AC^2+BC^2$$

$$(13)^2=(12)^2+(BC)^2$$

$$(BC)^2=169-144=25$$

$$BC=\sqrt{25}=5\,\text{cm}$$ [1]

Now, area of the shaded region= Area of the circle – area of the triangle

Area of the shaded region [1]

$$=\frac{1}{2}\pi r^2-\frac{1}{2}\times BC\times AC$$

$$=\frac{1}{2}\times(3.14)\times(6.5)^2-\frac{1}{2}\times 5\times 12$$

$$=66.33-30$$

$$=36.33\ \text{cm}^2$$

Thus, area of the shaded region is $36.33\ \text{cm}^2$ [1]

19. According to the figure,

Area of the region ABDC = Area of the sector AOC – Area of the sector BO

$$=\left(\frac{40^o}{360^o}\times\frac{22}{7}\times 14\times 14\right)-\left(\frac{40^o}{360^o}\times\frac{22}{7}\times 7\times 7\right)$$

$$=\left(\frac{1}{9}\times 22\times 14\times 2\right)-\left(\frac{1}{9}\times 22\times 7\right)$$

$$=\frac{1}{9}\times 22\times(28-7)$$

$$=51.33\,\text{cm}^2$$ [1]

Now, area of the circular ring =

$$\frac{22}{7}\times 14\times 14-\frac{22}{7}\times 7\times 7$$

$$=22\times 14\times 2-22\times 7$$

$$=22\times(28-7)$$

$$=462\,\text{cm}^2$$ [1]

Therefore, required shaded area = Area of the circular ring – Area of the region ABDC

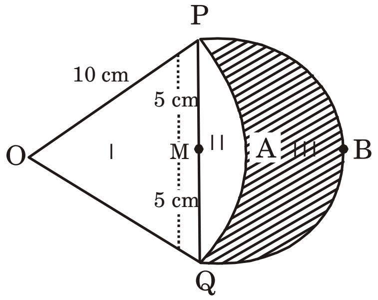
$= 462 - 51.33$

$= 410.67\,\text{cm}^2$

Thus, the area of the shaded region is $410.67\,\text{cm}^2$

[1]

20.



It is given that OP=OQ=10 cm .And we know the fact that tangents drawn from an external point to a circle are equal in length such that OP=OQ=10 cm

This makes ΔPOQ an equilateral triangle.And

$\angle POQ = \angle PQO = \angle QPO = 60^\circ$ [1]

Now, Area of part II = Area of the sector – Area of the equilateral triangle POQ

$$= \frac{\angle POQ}{360^\circ} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (10)^2$$

$$= \frac{60^\circ}{360^\circ} \times \pi (10)^2 - \frac{\sqrt{3}}{4} \times (10)^2$$

$$= 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{ sq units}$$

Area of the semicircle on diameter PQ = Area of part II + Area of part III

$$= \frac{1}{2} \times \pi (5)^2 = \frac{25}{2}\pi \text{ sq units}$$ [1]

Area of the shaded region i.e. part III = Area of the semicircle on diameter PQ – Area of part II

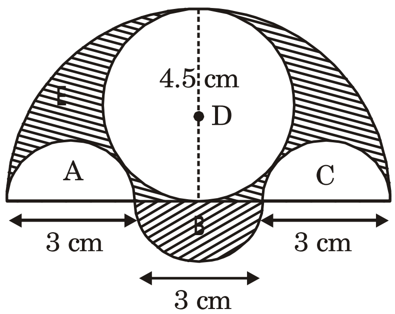
$$= \frac{25}{2}\pi - 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{25}{2}\pi - \frac{100}{6}\pi + 25\sqrt{3}$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ sq units}$$

Hence, we have shown that area of the shaded region is $= 25\left(\sqrt{3} - \frac{\pi}{6}\right)$ sq units [1]

21. From the figure,



Area of the semi-circle.

Diameter = 9 cm

Radius = 4.5 cm

$$\text{Area} = \frac{\pi r^2}{2}$$

$$= \frac{\pi(4.5)^2}{2} = \frac{\pi(4.5)^2}{2}\, cm^2$$ [1]

Area of circle D with Radius, $\frac{(4.5)}{2}$ cm

$$\text{Area} = \pi r^2$$

$$= \pi\left(\frac{4.5}{2}\right)^2$$

$$= \pi\left(\frac{4.5}{2}\right)^2 cm^2$$

Area of region $(A + C)$

$$r = \left(\frac{3}{2}\right)$$

$$= \pi\left(\frac{3}{2}\right)^2 cm^2$$

$\text{Area} = 2.25\pi\ cm^2$ [1]

Area of region B

$$r = \left(\frac{3}{2}\right)$$

$$Area = \frac{\pi}{2}\left(\frac{3}{2}\right)^2$$

$$= \frac{\pi}{2}(2.25)\ cm^2$$

Area of shaded region = area of semi-circle – area of circle D – area of region (A+C) +area of region B

$$= \frac{\pi(4.5)^2}{2} - \pi\left(\frac{4.5}{2}\right)^2 - 2.25\pi + \frac{\pi}{2}(2.25)\text{cm}^2$$

$$= 10.125\pi - 5.0625\pi - 2.25\pi + 1.125\pi$$

$$= 3.9375\pi$$

$$= 12.375\text{cm}^2$$ [1]

22. We know that the area of a circle is given by the formula

$$A = \pi r^2$$

The given quadrant is quarter of a circle. So, the area of the given quadrant will be

$$= \frac{1}{4}\pi r^2$$ [1]

Putting the values, we'll get

$$\Rightarrow \text{Area} = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$$

$$\Rightarrow \text{Area} = \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$$

$$\Rightarrow \text{Area} = 9.625\ \text{cm}^2 \text{.....................(i)}$$

Now, the area of the triangle ODB will be given by

$$= \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= 3.5\ \text{cm}^2 \text{.......................(ii)}$$ [1]

Subtracting equation (ii) from (i) to find the area of the shaded region, we'll get

$$= 9.625 - 3.5$$

$$= 6.125\ \text{cm}^2$$

Hence, the area of the shaded region is $6.125\ \text{cm}^2$ [1]

23. From the figure,

Perimeter of shaded region

$$= AB + PB + AP \quad \text{... }(i)$$

$$\angle OAB = 90^\circ$$

$$\begin{bmatrix}\text{Tangent is perpendicular to} \\ \text{radius through point of contact}\end{bmatrix}$$

In ΔOAB,

$$\tan\theta = \frac{AB}{OA}$$ [1]

$$AB = OA \times \tan\theta$$

$$AB = r \times \tan\theta \quad \text{......}(ii)$$

In ΔOAB,

$$\sec\theta = \frac{OB}{OA}$$

$$OB = OA \times \sec\theta$$

$$OB = r \times \sec\theta$$

$$\therefore\ PB = OB - OP$$ [1]

$$= r\sec\theta - r \quad \text{....}(iii)$$

Therefore, the perimeter is

$$= AB + PB + AP$$

$$\Rightarrow r \times \tan\theta + r \times \sec\theta - r + \frac{\theta}{180^\circ} \times \pi r$$ [1]

On solving it further,

$$\Rightarrow r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$$

Hence Proved. [1]

24. We are given that OA=5cm and OP=10cm

Now we know that the tangent at any point of a circle from an external point, P is perpendicular to the radius through the point of contact.

So, ΔOAP is a right-angled triangle,

$\Rightarrow \angle OAP = 90^\circ$

Now, using the Pythagoras theorem,

$OP^2 = OA^2 + AP^2$

$\Rightarrow 10^2 = 5^2 + AP^2$

$\Rightarrow AP^2 = 75$

$\Rightarrow AP = 5\sqrt{3}cm$ [1]

Now, $\cos\theta = \frac{OA}{OP} = \frac{5}{10}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

So, $\angle AOP = \angle BOP = 60^\circ$
$(\text{as } \Delta OAP \cong \Delta OBP)$

$\Rightarrow \angle AOB = 120^\circ$

Length of the belt still in contact with the pulley = Circumference of the circle – Length of the arc

$AC = 2\times 3.14\times 5 - \frac{120^\circ}{360^\circ}\times 2\times 3.14\times 5$

$= 2\times 3.14\times 5\times\left(1-\frac{1}{3}\right)$

$= 2\times 3.14\times 5\times\frac{2}{3}$

$= 20.93cm$ [1]

Now, Area of $\Delta OAP = \frac{1}{2}\times AP\times OA =$

$\frac{1}{2}\times 5\sqrt{3}\times 5 = \frac{25\sqrt{3}}{2}cm^2$

Area of $\Delta OBP = \frac{25\sqrt{3}}{2}cm^2$

So, Area of ΔOAP + Area of ΔOBP

$= 25\sqrt{3}cm^2 = 25\times 1.73 = 43.25cm^2$

Area of sector OACB

$= \frac{120^\circ}{360^\circ}\times 3.14\times(5)^2$

$= \frac{1}{3}\times 3.14\times 25$

$= 26.17cm^2$ [1]

Therefore, Area of the shaded region = (Area of ΔOAP + Area of ΔOBP) – Area of the sector OACB

$= 43.25cm^2 - 26.17cm^2$

$= 17.08cm^2$

Hence, the length of the belt is 20.93 cm and the area of the shaded region is 17.08 cm^2 [1]

25. Area of shaded region = Area of rectangle – Area of semicircle [1]

$= 21\times 14 - \left\{\frac{1}{2}\times\pi\times\left(\frac{14}{2}\right)^2\right\}$

$= 294 - \left\{\frac{1}{2}\times\frac{22}{7}\times 7\times 7\right\}$

$= 294 - 77$

$= 217\,cm^2$ [1]

So, the area of shaded region is $217\,cm^2$.

Perimeter of shaded region

$= AB + AD + DC + \overline{BC}$

$= AB + AD + DC + \frac{1}{2}\times 2\pi\left(\frac{14}{2}\right)$ [1]

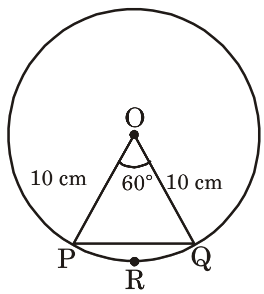
$= 21 + 14 + 21 + \frac{22}{7}\times 7$

$= 56 + 22$

$= 78cm$

So, the perimeter of shaded region is **78 cm**. [1]

26. To find the area of minor arc we need to calculate the area of the sector OPRQ



[1]

ΔOPQ is a isosceles triangle as $OP = OQ$ We know that in an isosceles triangle base angles are equal.

$\therefore \angle OPQ = \angle OQP$.........(i)

Now, $\angle POQ + \angle OPQ + \angle OQP = 180°$

Using equation (i)

$60° + \angle OPQ + \angle OPQ = 180°$

$2\angle OPQ = 120°$

$\angle OPQ = 60°$ [1]

$\therefore$ All angles are of $60°$ which is a property of equilateral triangleSo, ΔOPQ is an equilateral triangle.Area of an equilateral is given by the formula

$$A = \frac{\sqrt{3}}{4} \times (\text{Length of the side})^2$$

$$A = \frac{\sqrt{3}}{4} \times (10)^2$$

$$A = \frac{50\sqrt{3}}{2} = 43.30\,cm^2 ....(a)$$

Also, we know that the area of sector is given by the formula,

$$A = \frac{\theta}{360°} \times \pi r^2$$

So, Area of sector OPRQ

$$= \frac{60°}{360°} \times \pi(10)^2$$

$$= 52.33\,cm^2 ....(b)$$ [1]

Now, to find the area of minor segment, we need to subtract the area of ΔOPQ from the area of the sector $OPRQ$.

so, area of minor segment is
Subtracting equation (a) from (b)

$= 52.33 - 43.30$

$= 9.03\,cm^2$

Area of the circle is given by the formula

$A = \pi r^2 = 3.14 \times (10)^2$

$= 3.14 \times 10 \times 10$

$= 314\,cm^2$

Area of the major segment = Area of circle – Area of minor segment

$A = 314 - 9.03$

$A = 304.97\,cm^2$

Hence, the area of minor segment is 9.03 cm^2 and the area of major segment is 304.97 cm^2 [1]

27. Area of the square is given by the formula

$A = (side)^2$

$= (28)^2$

$= 784\,cm^2$

Area of the circle is given by the formula

$A = \pi r^2 = \pi(14)^2$ [1]

We can see that 2 quadrants are overlapping with the area of the square.

$\therefore$ Area of the shaded region = Area of the square + 2 × Area of circle – Area of two quadrants [1]

$$= 784 + \frac{22}{7} \times 14 \times 14\left(2 - \frac{1}{2}\right)$$

$$= 784 + 22 \times 2 \times 14 \times \left(\frac{3}{2}\right)$$ [1]

$= 784 + 22 \times 14 \times 3$

$= 784 + 924$

$= 1708\,cm^2$

Hence, the area of the shaded region is 1708 cm^2 [1]

Smart Notes ....

CHAPTER 13

Surface Areas and Volumes

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Problem related to Area	3,4 marks	3,4 marks		3 marks	2,3 marks	3 marks
Problem related to Volume			1,3,4,4 marks	4 marks	3,3 marks	3,3,4 marks
Frustum of Cone				3 marks	4 marks	
Converting one type of metallic solid into another				3 marks		

[TOPIC 1] Surface Area & Volume of a Solid

Summary

Surface Area and Volume of Solids

CUBOIDS AND CUBES

Cuboid : A cuboid is a solid figure, held by six rectangular plane regions

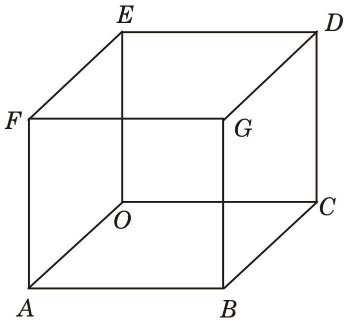
Here, In cuboid we have six faces namely

$AFGB, BGDC, GFED, OCDE, OEFA, OABC.$

We also have 12 edges, where two sides meet namely

$OA, AB, BC, OC, FG, EF, ED, OG, AF, OE, BG, CD.$

Cube: A cuboid in which all length, breadth, height are of equal lengths, is called a cube.



It also has six faces and twelve edges.

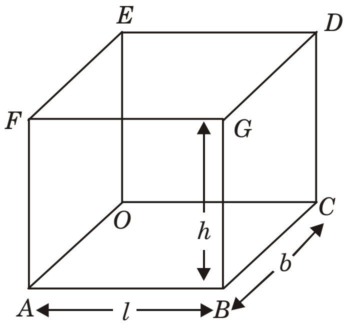
SURFACE AREA OF A CUBOID AND A CUBE

Total surface area of cuboid

$$= Ar(ABCO) + Ar(EFGD) + Ar(AOEF) + Ar(BCDG) + Ar(ABGF) + Ar(OCDE)$$

$$= \ell b + \ell b + bh + bh + \ell h + \ell h$$

$$= 2(\ell b + bh + \ell h)$$



Lateral surface area of cuboid

$$= 2(bh + \ell h)$$

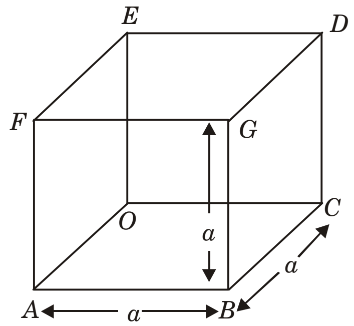
Where ℓ = length of cuboid

b = breadth of cuboid

h = height of cuboid

Total surface area of cube

Since cube is a cuboid in which length (ℓ) = breadth (b) = height (h) side of cube (a) i.e. $\ell = b = h = a$



$\Rightarrow$ Total surface area of cube

$$= 2(a \times a + a \times a + a \times a)$$

$$= 2(a^2 + a^2 + a^2)$$

$$\text{Area} = 2(3a^2) = 6a^2$$

Lateral surface area of cube

$$\text{Area} = 2(a \times a + a \times a)$$

$$= 2(a^2 + a^2) = 2 \times 2a^2$$

$$= 4a^2$$

So, lateral surface area of cube = $4a^2$

Where a = length of a side.

Length of diagonal of a cuboid

Length of diagonal $= OG = AD = BE = CF$

$$= \sqrt{\ell^2 + b^2 + h^2}$$

ℓ = length; b = breadth; h = height

Length of diagonal of a cube

Length of diagonal $= OG = AD = BE = CF$

$$= \sqrt{a^2 + a^2 + a^2}$$

$$= \sqrt{3a^2} = \sqrt{3}a \text{ unit}$$

Where a = length of a side.

Surface Area of a Right Circular Cylinder

CURVED SURFACE AREA OF A CYLINDER

$= 2\pi rh$

Where r = radius of base

h = height of cylinder.

TOTAL SURFACE AREA OF A CYLINDER

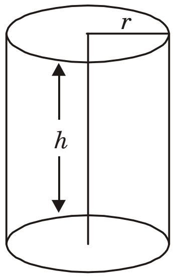
$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

r = radius of base

h = height of cylinder

$\pi = \frac{22}{7}$ or 3.14 approx.



VOLUME OF CYLINDER

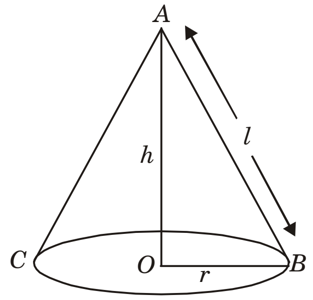
$V = \pi r^2 h$

Where r = radius of base

h = height of cylinder

Surface Area of a Right Circular Cone

CURVED SURFACE AREA OF A CONE



$C = \pi r\ell$

C = curved surface area

r = radius of base of cone

ℓ = slant height

$\ell = \sqrt{h^2 + r^2}$

TOTAL SURFACE AREA OF A CONE

$T = \pi r\ell + \pi r^2 = \pi r(r + \ell)$

Here, T = total surface area

r = radius of base of cone

ℓ = slant height of cone

VOLUME OF RIGHT CIRCULAR CONE

$V = \frac{1}{3}\pi r^2 h$

Where V = volume of cone

r = radius of base of cone

h = height of cone

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by

(*a*) 3

(*b*) 4

(*c*) 5

(*d*) 6

[TERM 2, 2011]

2. Volume and surface area of a solid hemisphere are numerically equal.

What is the diameter of hemisphere?

[TERM 2, 2017]

2 Marks Question

3. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. [Take $\pi = \frac{22}{7}$]

[TERM 2, 2014]

3 Marks Questions

4. A hemispherical bowl of internal radius 9 cm is full of water. Its contents are emptied in a cylindrical vessel of internal radius 6 cm. Find the height of water in the cylindrical vessel.

[TERM 2, 2012]

5. A vessel is in the form of hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm Find the total surface area of the vessel. $\left[\pi = \frac{22}{7}\right]$

[TERM 2, 2013]

6. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm find the volume of wood in the toy. $\left[\pi = \frac{22}{7}\right]$

[TERM 2, 2013]

7. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. [Use $\pi = \frac{22}{7}$]

[TERM 2, 2014]

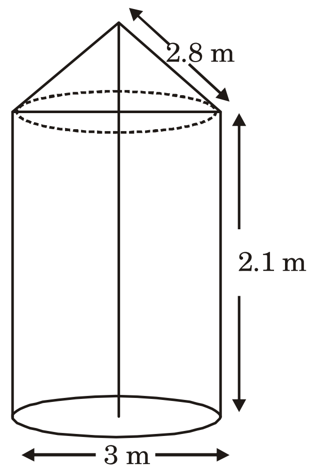
8. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}\text{cm}^3$. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of Rs 10 percm^2. $\left[Use \pi = \frac{22}{7}\right]$

[TERM 2, 2015]

9. A cubical block of side10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs 5 per 100 sq. cm. [Use $\pi = 3.14$]

[TERM 2, 2015]

10. In the given figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs. 500/sq. metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$



[TERM 2, 2016]

11. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel.

[TERM 2, 2016]

12. A sphere of diameter 12 cm is dropped in a right circular cylindrical vessel partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

[TERM 2, 2016]

13. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left(Use \pi = \frac{22}{7}\right)$

[TERM 2, 2016]

14. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

[TERM 2, 2017]

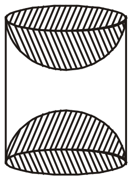
15. The $\frac{3}{4}$ th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

[TERM 2, 2017]

16. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

[DELHI, 2018]

17. A wooden article was made by scooping out a hemisphere from each of the solid cylinder, as shown in fig. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.



[DELHI, 2018]

4 Mark Questions

18. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take π = 3.14]

[TERM 2, 2011]

19. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?

[TERM 2, 2012]

20. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

[TERM 2, 2013]

21. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

[TERM 2, 2014]

22. From a solid cylinder of height 2.8cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

$\left[\text{Take } \pi = \frac{22}{7}\right]$

[TERM 2, 2014]

Solutions

1. Let r and R denote the radii of sphere and cylinder respectively.

Let H denote the rise in the water level.

$\therefore\ r = \frac{d}{2} = \frac{18}{2} = 9\text{cm} \text{ and } R = \frac{D}{2} = \frac{36}{2} = 18\text{cm}$

According to the question,

Volume of sphere = volume of water rising up

$\frac{4}{3}\pi r^3 = \pi R^2 H$ [½]

Substituting the given values in the above equation.

$\Rightarrow \frac{4}{3}\pi(9)^3 = \pi(18)^2 H$

$\Rightarrow H = \frac{4\pi(9)^3}{3\pi(18)^2}$

$\Rightarrow H = 3$

Hence, the correct option is (a). [½]

2. We know that volume of a solid hemisphere is given by

$V = \frac{2}{3}\pi r^3$

Also, surface area of a solid hemisphere is given by

$S = 3\pi r^2$ [½]

Where, r is the radius of the solid hemisphere

According to the question,

Volume and surface area of a solid hemisphere are numerically equal

$\frac{2}{3}\pi r^3 = 3\pi r^2$

$\Rightarrow 2\ r = 9$

We know that $2\ r$ = diameter

⇒ diameter = 9 units

Hence, the diameter of the solid hemisphere is 9 units. [½]

3. Total surface area of a solid hemisphere = $3\ \pi r^2$

$\Rightarrow 3\pi r^2 = 462$

$\Rightarrow 3\left(\frac{22}{7}\right) r^2 = 462$

$\Rightarrow r^2 = \frac{462 \times 7}{3 \times 22}$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7\text{cm}$ [1]

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{2156}{3} = 718.67\,\text{cm}^3$

So the volume of hemisphere is 718.67 cm^3. [1]

4. Let R and r be the radii of hemispherical bowl and cylindrical vessel respectively and h be the height of water present in the cylindrical vessel.

Volume of water in the hemispherical bowl

$= \frac{2}{3}\pi R^3 = \frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9$

$= \frac{10692}{7}\ \text{cm}^3$ [1]

This whole volume of water is emptied in a cylindrical vessel of internal radius 6 cm.

Therefore, Volume of water in the cylindrical vessel $= \frac{10692}{7}\ \text{cm}^3$

$\Rightarrow \pi r^2 h = \frac{10692}{7}$ [1]

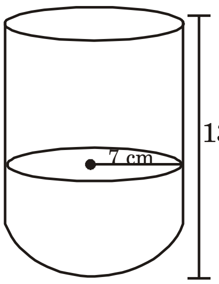
$\Rightarrow \frac{22}{7} \times 6 \times 6 \times h = \frac{10692}{7}$

$\Rightarrow h = \frac{10692}{792}$

$\Rightarrow h = 13.5$ cm

Therefore, the height of water in the cylindrical vessel is 13.5 cm. [1]

5.



Let us assume the radius of the cylinder be r and the height be h.

The radius of hemispherical bowl $= \frac{14}{2} = 7\text{cm}$

Vessel Height = 13 cm [1]

Height of the cylinder = Total height of the vessel- Radius of the hemispherical bowl So,

$\Rightarrow 13 - 7 = 6$ cm

Total surface area of the vessel = Curved surface area of the cylinder + Surface area of the hemisphere = $2\pi rh + 2\pi r^2$

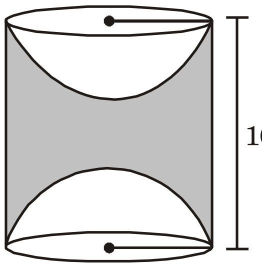
$= 2\pi r(h + r)$ [1]

Substituting the values,

$= 2 \times \frac{22}{7} \times 7(6 + 7) = 572\ \text{cm}^2$

Hence the surface area of the vessel is 572 cm^2. [1]

6.



We know that,

Height of the cylinder (h) = 10 cm

Radius of hemisphere (r) = 3.5 cm

Hence, Radius of cylinder (r) = 3.5 cm

Volume of wood in the toy = Volume of the cylinder – 2 × Volume of each hemisphere. [1]

$= (\pi r^2 h) - \left(2 \times \frac{2}{3} \times \pi r^3\right)$ [1]

Substituting the values,

$= \left(\frac{22}{7} \times (3.5)^2 \times 10\right) - \left(\frac{4}{3} \times \frac{22}{7} \times (3.5)^3\right)$

$= 385 - 179.66 = 205.33\ \text{cm}^3$

Hence the volume of the wood in the toy is 205.33 cm^2. [1]

7. Side of cube = 7 cm

Volume of cube = s^3

$= 7 \times 7 \times 7 = 343\ \text{cm}^3$ [1]

Radius of the sphere carved out $= \frac{7}{2} = 3.5\text{cm}$

Volume of the sphere of radius 3.5 cm $= \frac{4}{3}\pi r^3$ [1]

Volume of the sphere

$= \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.7\ \text{cm}^3$

Volume of wood left = Volume of cube – Volume of sphere carved out

Volume of wood left = $343 - 179.7 = 163.3\ \text{cm}^3$

Thus, 163.3 cm^3 of wood was left. [1]

8. Let h represent the height of the cone and r represent the radius of the base of the cone.

It is given that the radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}cm^3$

Therefore the total volume of the toy is $166\frac{5}{6}cm^3$. [1]

Volume of the toy = Volume of the hemisphere + Volume of the cone.

$\Rightarrow$ Volume of the toy $=\frac{2}{3}\pi r^3+\frac{1}{3}\pi r^2 h$

$$=\frac{1}{3}\pi r^2(2r+h)$$

$$\Rightarrow 166\frac{5}{6}=\frac{1}{3}\times\frac{22}{7}\times(3.5)^2(2\times 3.5+h)$$

$$\Rightarrow \frac{1001}{6}=\frac{269.5}{21}(7+h)$$

$$\Rightarrow \frac{1001\times 21}{6\times 269.5}=7+h$$

$$\Rightarrow \frac{21021}{1617}=7+h$$

$\Rightarrow 13=7+h$

$\Rightarrow h=13-7=6$ [1]

Therefore the height of the toy = height of cone + radius of hemisphere

= 6 + 3.5 = 9.5 cm

To paint the hemispherical part we need to find out the curved surface area of the hemisphere.

Curved surface area of the hemisphere = $2\pi r^2$

$$=2\times\frac{22}{7}\times(3.5)^2$$

$$=\frac{44}{7}\times 12.25$$

= 77 cm^2

Now, cost to paint 1 cm^2 area the cost is Rs 10

$\Rightarrow$ Cost to paint 77 cm^2 the cost = 77 × 10 = 770

Therefore, the cost of painting the hemispherical part is Rs 770. [1]

9. Let r be the radius of the hemisphere and a be the length of the sides of the cubical block.

The largest diameter the hemisphere can have is 10 cm.

Total surface area of the solid = Total Surface area of the cube + Curved surface area of the hemisphere – area of the base of the hemisphere

$=6a^2+2\pi r^2-\pi r^2$ [1]

$$=6\times 10^2+2\times\frac{22}{7}\times 5^2-\frac{22}{7}\times 5^2$$

$$=6\times 10\times 10+\frac{22}{7}\times 25$$

= 600 + 78.57

= 678.57 cm^2 [1]

The cost of painting the total surface area of the solid so formed, at the rate of Rs 5 per 100 sq. cm

$$=\frac{678.57}{100}\times 5=33.93$$

Hence, the amount is Rs. 33.93. [1]

10. Given,

Height of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 3 m

Slant height (l) of the conical part = 2.8 m

According to the figure, [1]

Total canvas used = CSA of the cylindrical part + CSA of the conical part

$= 2\pi rh+\pi rl$

$$=\frac{22}{7}\times\frac{3}{2}((2\times 2.1)+2.8)$$

$$=\frac{22}{7}\times\frac{3}{2}(4.2+2.8)$$

$$=\frac{22}{7}\times\frac{3}{2}\times 7$$

= 33 m^2 [1]

Cost of 1m^2 canvas = Rs 500

Cost of 33m^2 canvas = 33 × 500 = 16,500

Therefore, the cost of the canvas needed to make the tent is Rs. 16,500. [1]

11. Radius of the conical vessel = r_1 = 5 cm

Height of the conical vessel = h_1 = 24

Radius of the cylindrical vessel = r_2 = 10 cm

Let the water rise upto the height of h_2 cm in the cylindrical vessel.

Now, volume of water in conical vessel = Volume of water in cylindrical vessel [1]

Hence, $\frac{1}{3}\pi r_1^2 h_1=\pi r_2^2 h_2$

$r_1^2 h_1 = 3 r_2^2 h_2$

$5^2 \times 24 = 3 \times 10^2 \times h_2$ [1]

$h_2 = \frac{5 \times 5 \times 24}{3 \times 10 \times 10} = 2\,\text{cm}$

Thus, the water will rise up to the height of 2 cm in the cylindrical vessel. [1]

12. Diameter of sphere is 12 cm,

Therefore, radius of sphere is 6 cm.

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi\,\text{cm}^3$

Let R be the radius of cylindrical vessel. [1]

Rise in the water level of the cylinder $= h = 3\frac{5}{9}$ cm

$= \frac{32}{9}$ cm

Rise in the volume of water in the cylindrical vessel $= \pi R^2 h = \pi R^2 \frac{32}{9} = \frac{32}{9}\pi R^2$ [1]

Now, volume of water displaced by the sphere is equal to volume of the sphere.

Therefore, $\frac{32}{9}\pi R^2 = 288\pi$

$R^2 = \frac{288 \times 9}{32} = 81$

R = 9 cm

Hence, diameter of the cylindrical vessel = $2R$ = $2 \times 9 = 18$ cm. [1]

13. Let suppose the base and height of the solid right circular cylinder be r cm and h cm, respectively.

According to the question,

$r + h = 37$ ....(1)

Now the Total surface area = 1628 sq. cm

We know that Total Surface area of the cylinder is $2\pi r(r + h)$.

So, $2\pi r(r + h) = 1628$ ....(2) [1]

Form (1) and (2),

$2\pi r(37) = 1628$

$\Rightarrow 2\pi r = 44$

$\Rightarrow 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow r = \frac{44 \times 7}{2 \times 22}$

$\Rightarrow r = 7$ cm [1]

Substituting the value of r in (1), we get,

$7 + h = 37$

$\Rightarrow h = 30$ cm

Now, we know the formula for the Volume of the Cylinder as $\pi r^3 h$

So, Volume of the Cylinder

$= \frac{22}{7} \times 7 \times 7 \times 30 = 4{,}620\,cm^3$

Hence, the volume of the cylinder is 4,620 cm^3. [1]

14. Given, Perimeter of lower end, $c = 6$ cm

Perimeter of upper end, $C = 18$ cm

Slant height, $l = 4$ cm

Let radius of upper end be R and radius of lower end be r

As, We know

$C = 2\pi R$

$\Rightarrow 2\pi R = 18$ [1]

$\Rightarrow R = \frac{18}{2\pi}$

$\Rightarrow R = \frac{9}{\pi}\,cm$

As, $c = 6$ cm

$\Rightarrow 2\pi R = 6$

$\Rightarrow r = \frac{6}{2\pi}$

$\Rightarrow r = \frac{3}{\pi}\,cm$ [1]

Curved surface area of frustum = $\pi(R + r)\,l$

$\Rightarrow \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$

$\Rightarrow \pi \times \left(\frac{12}{\pi}\right) \times 4$

$\Rightarrow 48\,\text{cm}^2$

Therefore, the curved surface area of frustum is 48 cm^2. [1]

15. Given: Height (h) of conical vessel = 24 cm

Radius (r) of the conical vessel = 5 cm

Radius (R) of the cylindrical vessel = 10 cm

Volume of the conical vessel is given by

$V = \frac{1}{3}\pi r^2 h$

According to the question the height of water is $\frac{3}{4}$ th of the height of the conical vessel. [1]

Therefore, volume of water $= \frac{3}{4}$ of volume of the conical vessel

$= \frac{3}{4} \times \frac{1}{3}\pi r^2 h$

Putting the values

$= \frac{3}{4} \times \frac{1}{3}\pi (5)^2 \times 24$

$= \pi \times 25 \times 6$

$= 150\pi$ ....(i) [1]

Now, it is given that the water is emptied into a cylindrical vessel with internal radius 10 cm.

Volume of a cylinder is given by the formula

$V = \pi R^2 h$

Putting the values in the above equation

$V = \pi \times (10)^2 \times h$

$V = 100\pi h$ ....(ii)

Equating (i) and (ii), we'll get

$100\pi h = 150\pi$

$\Rightarrow h = \frac{150}{100}$

$h = 1.5$ cm

Hence, height of water in cylindrical vessel is 1.5 cm. [1]

16. Radius of the heap $(r) = \frac{d}{2} = \frac{24}{2} = 12\text{m}$

Height if the heap $(h) = 3.5$ m

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ Volume of the heap of rice $= \frac{1}{3}\pi (12)^2 \times 3.5$ [1]

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 144 \times 3.5$

$\Rightarrow 528\ \text{m}^3$

The amount canvas required = Surface area of the cone made by the rice heap.

Surface area of a cone $= \pi rl$

where $l = \sqrt{r^2 + h^2}$

Surface area of the heap $= \pi r\sqrt{r^2 + h^2}$ [1]

$\Rightarrow \frac{22}{7} \times 12\sqrt{12^2 + 3.5^2}$

$\Rightarrow \frac{22}{7} \times 12 \times 12.5$

$\Rightarrow 471.42\ \text{m}^2$

Thus 471.42 m^2 area of canvas required to cover the heap. [1]

17. Let r be the radius of the hemisphere and the cylinder and h be the height of the cylinder.

For the hemisphere

Radius (r) = 3.5 cm

Surface area $= 2\pi r^2$

$\Rightarrow 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$\Rightarrow 77\ \text{cm}^2$ [1]

For the cylinder.

Radius $(r) = 3.5$cm

Height $(h) = 10$cm

Curved surface area of cylinder $= 2\pi rh$ [1]

$\Rightarrow 2 \times \frac{22}{7} \times 3.5 \times 10$

$\Rightarrow 220\ \text{cm}^2$

Total surface area of the article = 2 × surface area of the hemisphere + curved surface area of the cylinder.

Total surface area of the article $= (2 \times 77 + 220)\text{cm}^2$

$= 374\ \text{cm}^2$ [1]

18. Total surface area of remaining portion will be = Curved surface area of cylinder + Curved surface area of cone + Area of the base of the cylinder

$= 2\pi rh + \pi rl + \pi r^2$ [1]

Here, $h = 15$ cm,

Slant height of cone, $l = \sqrt{h^2 + r^2}$

$l = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289}$

$l = 17$ cm [1]

Now total surface area of remaining portion is,

TSA $= \pi rl + \pi r^2 + 2\pi rh$

$\Rightarrow \pi r(l + r + 2h)$

$\Rightarrow 3.14 \times 8\ (17 + 8 + 2 \times 15)$ [1]

$\Rightarrow 3.14 \times 8\ (55)$

$\Rightarrow 3.14 \times 440$

$\Rightarrow 1381.6$

Hence the required total surface area is 1381.6.[1]

19. Diameter of the base of the hemispherical tank = 3 m

$\therefore$ The radius of the hemispherical tank $= \frac{3}{2}$ m

volume of the hemispherical tank $= \frac{2}{3}\pi r^2$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$

$= \frac{99}{14}$ m^3 [1]

$\therefore$ The amount of water in the hemispherical tank $= \frac{99}{14} \times 1000 \left[1\ m^3 = 1000\ L\right]$

$= \frac{99000}{14}$ litres [1]

Volume of water to be emptied $= \frac{1}{2} \times$ Volume of the tank

$= \frac{1}{2} \times \frac{99000}{14}$ litres

$= \frac{99000}{28}$ litres [1]

Now, it is given that tank is emptied at the rate of $\frac{25}{7}$ litres per second.

$\therefore$ Time taken to empty $\frac{99000}{28}$ litres $= \frac{\frac{99000}{28}}{\frac{25}{7}}$

$= \frac{7}{25} \times \frac{99000}{28}$ seconds

$= \frac{693000}{700}$ seconds

= 990 seconds

$= \frac{990}{60}$ minutes

= 16.5 minutes

Therefore, it will take 16.5 minutes to empty half the tank. [1]

20. We know,

Internal diameter of circular end of pipe = 2cm

$\therefore$ Radius (r_1) of circular end of pipe

$= 1\,cm = \frac{1}{100}\text{m} = 0.01\ m$ [1]

Speed of water = 0.4 m/s = 0.4×60 = 24 metre/min

Length of water column in one minute = 24 m

Volume of water that flows through the pipe in 1 minute $= \pi r_1^2 h$ [1]

$= \pi(0.01)^2 \times 24 = 0.0024\pi\, m^3$

Volume of water that flows through the pipe in 30 minutes = 30 × 0.0024 $\pi\, m^3$ = 0.072$\pi\, m^3$

$\therefore$ Radius (r_3) of base of cylindrical tank = 40 cm = 0.4 m

Let the rise in level of the water in the cylindrical tank filled in 30 minutes be h m.

Now, Volume of water filled in tank in 30 minutes = Volume of water flowed in 30 minutes from the pipe. [1]

$\therefore \pi \times (r_2)^2 \times h = 0.072\pi$

$\Rightarrow (0.4)^2 \times h = 0.072$

$\Rightarrow 0.16 \times h = 0.072$

$\Rightarrow h = \frac{0.072}{0.16}$

$\Rightarrow h = 0.45\ m = 45\ cm$

Hence the rise in the level in half an hour is 45 cm. [1]

21. Let h be the rise in the level of water in the vessel.

Diameter of spherical marble = 1.4 cm

Radius of spherical marble $= \frac{1.4}{2} = 0.7$ cm

Volume of spherical marble $= \frac{4}{3}\pi r^3$

Volume of spherical marble $= \frac{4}{3} \times \frac{22}{7}(0.7)^3$

Volume of spherical marble

$= \frac{30.184}{21} = 1.437$ cm^3 [1]

Volume occupied by 150 spherical marbles

= 150 × 1.437 = 215.6 cm^3

Volume of water increased

$= \pi r^2 h = \frac{22}{7} \times (3.5)^2 \times h$ [1]

Volume of water increased = Volume of 150 spherical marbles.

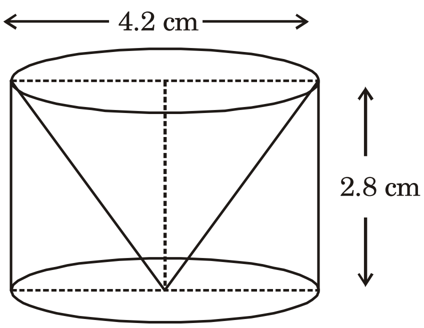
$\frac{22}{7} \times (3.5)^2 \times h = 215.6$

$h = \frac{215.6 \times 7}{22 \times (3.5)^2}$ [1]

$h = \frac{1509.2}{269.5} = 5.6$ cm

The level of water increased by 5.6 cm. [1]

22. [1]



Height of the cone = Height of the cylinder= 2.8cm

Diameter of the cylinder = Diameter of the cone = 4.2 cm

Hence, Radius of the cylinder = Radius of the cone = 2.1cm

Slant height (l) of the conical part = $\sqrt{r^2 + h^2}$

$= \sqrt{(2.1)^2 + (2.8)^2}$ cm

$= \sqrt{4.41 + 7.84}$ cm

$= \sqrt{12.25}$ cm

= 3.5cm [1]

Now, Total Surface Area of the remaining solid = Curved Surface Area of the Cylindrical section + Curved Surface Area of the Conical section + Area of the base of the cylinder

$= 2\pi rh + \pi rl + \pi r^2$ [1]

$= \left(2 \times \frac{22}{7} \times 2.1 \times 2.8 + \frac{22}{7} \times 2.1 \times 3.5 + \frac{22}{7} \times 2.1 \times 2.1 \right) cm^2$

= (36.96 + 23.1 + 13.86) cm^2

= 73.92 cm^2

Hence, the total surface area of the remaining solid = 73.92 cm^2. [1]

[TOPIC 2] Conversion of Solid

Summary

Conversion of Solid from One Shape to Another

For commercial works and for industrial development work, we need to convert a solid into another solid of different shape or more than one solid of similar shape but with reduced size

A cylinder, a cone and a hemisphere are of equal base and have the same height. The ratio of their volume is 3:1:2

PREVIOUS YEARS'

EXAMINATION QUESTION

TOPIC 2

1 Mark Questions

1. The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4cm, is:

   (*a*) 3 (*b*) 5

   (*c*) 4 (d) 6

   [TERM 2, 2014]

2 Marks Questions

2. Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.

   [TERM 2, 2011]

3. A solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. find the number of cones so formed. (Use $\pi = \frac{22}{7}$)

   [TERM 2, 2012]

3 Marks Questions

4. Water in a canal, 6 m wide and 1.5 m deep, is flowing at a speed of 4 km/h. How much area will it irrigate in 10 minutes, if 8 cm of standing water is needed for irrigation?

   [TERM 2, 2014]

5. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely?

   [TERM 2, 2014]

6. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

   [TERM 2, 2015]

7. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. [Use $= \frac{22}{7}$]

   [TERM 2, 2015]

8. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width to form an embankment. Find the height of the embankment.

   [TERM 2, 2016]

9. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

   [TERM 2, 2016]

10. The dimensions of a solid iron cuboid are 4.4 m × 2. 6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

    [TERM 2, 2017]

4 Marks Questions

11. Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\frac{2}{5}$ th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?

[TERM 2, 2014]

12. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

[TERM 2, 2015]

13. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, if the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

[TERM 2, 2015]

14. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 10 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire.

[Use $=\frac{22}{7}$]

[TERM 2, 2015]

Solutions

1. Let r and h be the radius and the height of the cylinder, respectively.

Given: Diameter of the cylinder = 4cm

Radius of the cylinder, (r) = 2 cm

Height of the cylinder (h) = 45 cm

Volume of the solid cylinder

$= \pi r^2 h$

$= \pi \times 2 \times 2 \times 45 \text{ cm}^2$

$= 180\ \pi \text{ cm}^3$ [½]

Suppose the radius of each sphere be R cm.

Diameter of the sphere = 6 cm

Radius of the sphere, R = 3 cm

Let n be the number of solid spheres formed by melting the solid metallic cylinder.

$n \times$ Volume of the solid spheres = Volume of the solid cylinder

$$n \times \frac{4}{3}\pi R^3 = 180\pi$$

$$n \times \frac{4}{3}\pi \times 3^3 = 180\pi$$

$$n = \frac{180 \times 3}{4 \times 27} = 5$$

Thus, the number of solid spheres that can be formed is 5.

Hence, the correct option is (b). [½]

2. Given that, sides of each cube = 4 cm

Now, when the cubes are joined end to end, then

The length of the resulting cuboid = 4 + 4 = 8 cm

Width of the resulting cuboid = 4 cm

Height of the resulting cuboid = 4 cm

We know that, the surface area of the cuboid

= 2 (Ib + bh + hl) [1]

= 2 (8 × 4 + 4 × 4 + 4 × 8)

= 2 (32 + 16 + 32)

= 2 (80)

= 160 cm^2

Hence, the surface area of the resulting cuboid is 160 cm^2. [1]

3. Since the solid sphere has been melted and recast into smaller solid cones, the total volume of all the solid cones will be equal to the volume of the solid sphere. Let n solid cones are formed by melting a solid sphere.

Let R and r be the radii of sphere and cone respectively and h be the height of the cone.

$\therefore$ R = 10.5 cm, r = 3.5 cm and h = 3 cm

Volume of the solid sphere

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$$

$= 4851 \text{ cm}^3$ [1]

Now, volume of one smaller solid cone

$$= \frac{1}{3}\pi r^2 h$$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3$

$= 38.5\ cm^3$

Clearly, volume of solid sphere = n × volume of one solid cone.

$4851 = n \times 38.5$

$\Rightarrow n = \frac{4851}{38.5}$

$\Rightarrow n = 126$

Therefore, 126 cones are formed by melting a solid sphere. [1]

4. It is given that width of canal is 6 m and depth is 1.5 m.

Speed of water in canal is 4 km/h.

Distance covered by water in 1 hour or 60 minutes = 4 km

Distance covered in 1 minute $= \frac{4}{60} = \frac{1}{15}$km

Distance covered in 10 minutes

$\frac{1}{15} \times 10 = \frac{2}{3}\text{km} = \frac{2000}{3}\text{m}$ [1]

Volume of water flowing through canal in 10 minutes = Volume of area irrigated

Volume of water in canal = Area irrigated × Height

Volume of water in canal =

Area irrigated $\times \frac{8}{100}$m [1]

Area irrigated = Volume of water in canal $\times \frac{100}{8}$

Area irrigated $= \frac{2000}{3} \times 6 \times 1.5 \times \frac{100}{8}$

Area irrigated = 75,000 m^2

Therefore, 75,000 m^2 area will be irrigated in 10 minutes. [1]

5. For the given tank.

Diameter = 10 m

Radius, $R = 5$ m

Depth, $H = 2$ m

Internal radius of the pipe, $r = \frac{20}{2} = 10\text{cm}$

Rate of flow of water, $h = 4$ km / $h = 4000\ m/h$ [1]

Let t be the time taken to fill the tank. So, the water flows through the pipe in t hours will equal to the volume of the tank

$\therefore \pi r^2 \times h \times t = \pi R^2 H$ [1]

$\pi \times \left(\frac{1}{10}\right)^2 \times 4000 \times t = \pi \times 5^2 \times 2$

$t = \frac{25 \times 2 \times 100}{4000} = \frac{5}{4}$

Hence, the time taken is $1\frac{1}{4}$ Hours or 1 hour and 15 minutes. [1]

6. A hemispherical bowl of internal diameter 36 cm (Given)

Radius of the hemispherical bowl is 18 cm.

Radius of cylindrical bottles = 3 cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 18 \times 18 \times 18$

$= 12219.43\ cm^3$ [1]

10% liquid is wasted in transfer from hemispherical bowl to cylindrical bottles.

= 10% of volume of the hemisphere

$= \frac{1}{10} \times 12219.43$

$= 1221.943\ cm^3$ is wasted.

The volume of the remaining liquid = 12219.430 – 1221.943 = 10997.487

Let, h represent the height of the cylinder

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 3 \times 3 \times h$ [1]

Volume of 72 cylindrical bottles $= \frac{22}{7} \times 9 \times h \times 72$

Volume of the remaining liquid after wastage = Volume of 72 cylindrical bottles

$\Rightarrow 10997.487 = \frac{22}{7} \times 9 \times h \times 72$

$h = 5.4$ cm

Therefore, the height of each bottle is 5.4 cm. [1]

7. Let r and h be the radius and the diameter of the cone respectively and R be the radius of the sphere.

Diameter and height of each cone 3.5 cm and 3 cm respectively.

Radius of the cone = $\frac{3.5}{2}cm$

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$

$= \frac{77}{8}$ [1]

Volume of 504 cones = $\frac{77}{8} \times 504 = 4851\,cm^3$

They are melted and made into sphere.

$\therefore$ volume of 504 cones = volume of the sphere

$4851 = \frac{4}{3} \times \pi \times r^3$ [1]

$= r^3 = \frac{4851 \times 3 \times 7}{4 \times 22} = 1157.625\,cm$

= r = 10.5 cm

$\therefore$ Diameter = 2 × 10.5 = 21 cm

Total surface area of sphere = 4 πr^2

$= 4 \times \frac{22}{7} \times 10.5^2 = 1386\,cm^2$

Total surface area of sphere = 1386 cm^2 [1]

8. Let suppose r and h be the radius and depth of the well respectively.

$\therefore r = \frac{4}{2} = 2m$ and $h = 21m$ [1]

Let suppose R and H be the outer radius and height of the embankment respectively.

$\therefore$ $R = r + 3 = 2 + 3 = 5$ m

Now, Volume of the earth used to form the embankment = Volume of the earth dug out of the well.

$= \pi\left(R^2 - r^2\right)H = \pi r^2 h$ [1]

$\Rightarrow H = \frac{r^2 h}{R^2 - r^2}$

$\Rightarrow H = \frac{2^2 \times 21}{5^2 - 2^2} = 4m$ [1]

Hence, the height of the embankment is 4 m.

9. Given

Depth of canal = 1.8 m

Width of canal = 5.4 m

Height of standing water = 10 cm =0.1 m

Speed of flowing water = 25 km/h

$= \frac{25000}{60}$

$= \frac{1250}{3} m/\text{min}$ [1]

Volume of water flowing out of canal in 1 min $\Rightarrow$ width × depth × water flowing in 1 minute

$\Rightarrow 5.4 \times 1.8 \times \frac{1250}{3}$

$\Rightarrow 4050\ m^3$ [1]

Volume of water flowing out of canal in 40 min $\Rightarrow$ 40 × 4050 m^3 = 162000 m^3

Area of irrigation × Height of standing water = Volume of water flowing out in 40 minutes.

Area of irrigation

$\Rightarrow \frac{\text{Volume of water out in 40min}}{\text{Height of standing water}}$

$\Rightarrow \frac{162000}{0.1}$

$\Rightarrow 1620000\ m^2$

$\Rightarrow$ 162 hectare ($\because$ 1 hectare = 10000 m^2)

It can irrigate 162 hectare in 40 min. [1]

10. Given,

Volume of solid iron cuboid = 4.4 m × 2.6 m × 1.0 m = 440 cm × 260 cm × 100 cm

Internal radius of pipe, r = 30 cm

External radius of pipe, R = 30 + 5 = 35 cm [1]

Let length of pipe be h cm

Volume of iron in the pipe

$= \pi R^2 h - \pi r^2 h = \pi h\left(R^2 - r^2\right)$

$= \pi h (35^2 - 30^2)$

$= \pi h (35 - 30) (35 + 30)$

$= \pi h (5 - 65)$ [1]

Volume of iron in pipe = Volume of solid iron cuboid

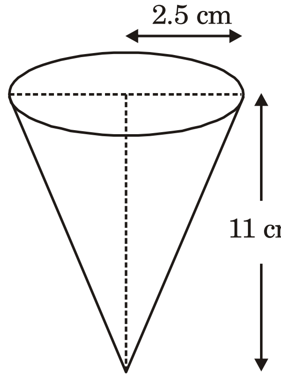
$\Rightarrow \pi h (5 \times 65) = 440 \times 260 \times 100$

$\Rightarrow h = \dfrac{440 \times 260 \times 100 \times 7}{5 \times 65 \times 22}$

$\Rightarrow h = 11,200$ cm = 112 m

Therefore, length of pipe is 112 m. [1]

11. [1]



Given,

The height, h of the conical vessel = 11cm

Radius, r_1 of the conical vessel = 2.5cm

Radius, r_2 of the metallic spherical balls

$= \dfrac{0.5}{2} = 0.25\,cm$ [1]

Let the number of spherical balls dropped in the vessel be 'n'.

Volume of the water spilled = Volume of the total spherical balls dropped.

$\Rightarrow \dfrac{2}{5} \times$ Volume of cone = n × Volume of one spherical ball [1]

$\Rightarrow \dfrac{2}{5} \times \dfrac{1}{3}\pi r_1^2 h = n \times \dfrac{4}{3}\pi r_2^3$

$\Rightarrow r_1^2 h = n \times 10 r_2^3$ [1]

$\Rightarrow (2.5)^2 \times 11 = n \times 10 \times (0.25)^3$

$\Rightarrow 68.75 = 0.15625n$

$\Rightarrow n = 440$

Thus, the number of spherical balls dropped in the vessel is 440.

Sushant has shown a responsible attitude by using the water sensibly by making the arrangement so that water that flows out, irrigates the flower beds. [1]

12. Let h be the depth of the well, $h = 14$ m and let r be the radius of the well,

$r = d / 2 = 4 / 2 = 2$ m [1]

Now let R be the outer radius of the embankment, and h be the height of the embankment i.e. $h' = 40$ cm = 0.4 m,

Now as we know volume of the embankment and volume of the well will be same,

Therefore,

Volume of well = Volume of the embankment

$\Rightarrow \pi r^2 h = \pi R^2 h' - \pi r^2 h'$

$\Rightarrow \pi r^2 h = \pi h'\left(R^2 - r^2\right)$ [1]

Taking π common from both sides,

$\Rightarrow r^2 h = h'\left(R^2 - r^2\right)$

Now substituting values of r, h, h' and solving for R,

$\Rightarrow 2^2 \times 14 = 0.4\left(R^2 - 2^2\right)$

$\Rightarrow \dfrac{4 \times 14}{0.4} = R^2 - 4$

$\Rightarrow 10 \times 14 = R^2 - 4$

$\Rightarrow 140 + 4 = R^2$

$\Rightarrow R^2 = 144$ [1]

$\therefore R = 12$

Now since $R = 12$ m and $r = 2$ m,

Therefore Width of the embankment will be difference of the outer radius(R) and inner radius(r),

Width = R – r = 12 – 2 = 10 m

Hence, width of the embankment is 10 m. [1]

13. Let us suppose the internal radius of the pipe is r m and the distance covered by the water in half an hour will be the length of the cylindrical pipe i.e.

$$H = 2.52 \times \frac{1}{2} = 1.26\,\text{km} = 1.26 \times 1000\text{m} = 1260\text{ m},$$

[1]

Now radius of base of cylindrical tank is 40 cm i.e. 0.4 m and height is 3.15 m,

Hence volume of water filled in tank in half an hour will be,

Volume [1]

$$= \pi r^2 h = \frac{22}{7} \times 0.4 \times 0.4 \times 3.15 = 22 \times 0.16 \times 0.45$$

$$= 22 \times 0.072$$

$\Rightarrow 1.584\text{ m}^3$

Now this volume is same as volume of water released by the pipe,

Volume of water released by pipe = 1.584 m^3

$\Rightarrow \pi r^2 H = 1.584$ [1]

$\Rightarrow \pi r^2 \times 1260 = 1.584$

$$\Rightarrow r^2 = \frac{1.584}{\pi \times 1260} = \frac{1.584 \times 7}{22 \times 1260} = \frac{0.072}{180} = \frac{0.004}{10}$$

$$= 0.0004$$

$\Rightarrow r^2 = 0.0004$

$\Rightarrow r = 0.02$

Hence the internal radius of the pipe is, r = 0.02m

Therefore diameter is 0.04 m or 4 cm. [1]

14. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. (Given)

Let r be the radius of the base of the cylinder and the hemisphere and h be the height of the cylinder.

Let R and H be the radius and the height of the cylindrical wire respectively.

Volume of the cylinder = $\pi r^2 h$ [1]

$$= \frac{22}{7} \times 4.2^2 \times 10$$

$= 554.4\text{ cm}^3$

Volume of the hemispherical part scooped out = 2 × volume of the hemisphere

$$= 2 \times \frac{2}{3} \times \pi \times r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.2^3$$

$= 310.464\text{ cm}^3$ [1]

Remaining volume of the cylinder after scooping out two hemispheres = $554.4\text{ cm}^3 - 310.464\text{ cm}^3$

$= 243.936\text{ cm}^3$ [1]

Diameter of the cylindrical wire is given 1.4 cm

So, the radius of the cylindrical wire is 0.7 cm

The volume of the wire = Remaining volume of the cylinder after scooping out two hemispheres

$$\Rightarrow \pi \times 0.7^2 \times h = 243.936$$

Where, h is the length of the wire.

$$\Rightarrow 243.936 = 1.54 \times h$$

$$\Rightarrow h = \frac{243.936}{1.54}$$

$h = 158.4$ cm [1]

[TOPIC 3] Frustum of a Right Circular Cone

Summary

Frustum of a Right Cicular Cone

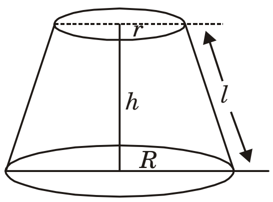
When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone.

Let R and r be the radii of the base and the top of the frustum of a cone.

Let h be its height and ℓ be its slant height.

Then,

VOLUME OF THE FRUSTUM OF THE CONE



$$= \frac{\pi h}{3}[R^2 + r^2 + Rr] \text{ cubic units.}$$

LATERAL SURFACE AREA OF THE FRUSTUM OF THE CONE

$$= \pi\ell(R + r), \text{ where } \ell^2 = h^2 + (R - r)^2 \text{ sq units.}$$

TOTAL SURFACE AREA OF THE FRUSTUM OF THE CONE

= (area of the base) + (area of the top) + lateral surface area)

$$= [\pi R^2 + \pi r^2 + \pi\ell(R + r)]$$

$$= \pi[R^2 + r^2 + \ell(R + r)] \text{ sq units.}$$

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 3

1 Mark Question

1. A solid right circular cone is cut into two parts at the middle of its height by a plane parallel to its base. The ratio of the volume of the smaller cone to the whole cone is

   (*a*) 1 : 2

   (*b*) 1 : 4

   (c) 1 : 6

   (*d*) 1 : 8

   [TERM 2, 2012]

3 Marks Questions

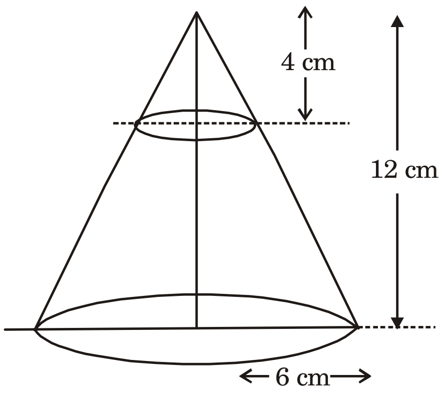
2. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r<14 cm). If the volume of bucket is 5390 cm^3, then find the value of r. [Use $\pi = \frac{22}{7}$]

   [TERM 2, 2011]

3. A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{12}$ cm, find the length of the wire.

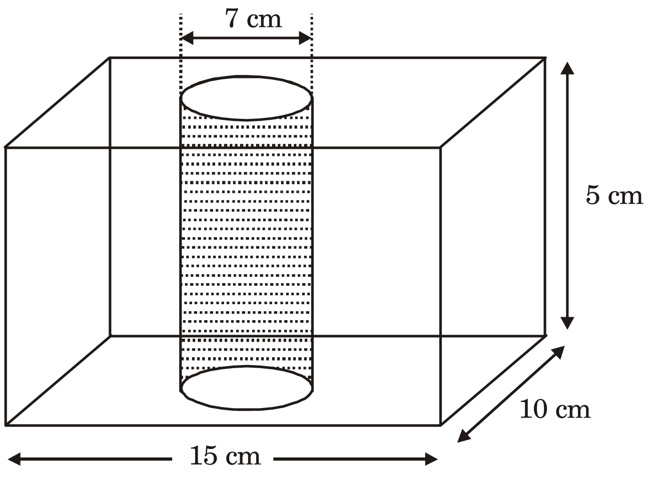
   [TERM 2, 2014]

4. In the given figure, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. Use $\left(\pi = \frac{22}{7} and \sqrt{5} = 2.236\right)$.



[TERM 2, 2015]

5. In the figure below, from a cuboidal solid metallic block, of dimensions 15cm × 10cm × 5cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[Use\ \pi = \frac{22}{7}\right]$

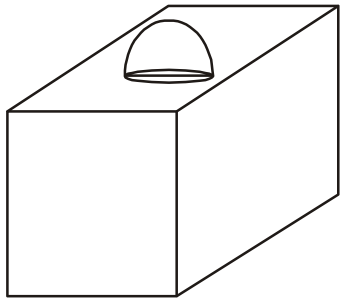


[TERM 2, 2015]

6. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs Rs. 100 per sq. m, find the amount, the associations will have to pay. What values are shown by these associations? [Use $\pi = \frac{22}{7}$]

[TERM 2, 2015]

7. In the figure below, a decorative block, made up of two solids - a cube and a hemi-sphere? The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{use } \pi = \frac{22}{7}\right)$



[TERM 2, 2016]

4 Marks Questions

8. A drinking glass is in the shape of the frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass. [Use $\pi = \frac{22}{7}$]

[TERM 2, 2012]

9. A bucket open at the top and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of Rs. 10 per 100 cm^2. [Use $\pi = 3.14$]

[TERM 2, 2013]

10. A container open at the top, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of 21 per litre. [Use $\pi = \frac{22}{7}$]

[TERM 2, 2016]

11. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use $\pi = 3.14$)

[TERM 2, 2016]

12. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

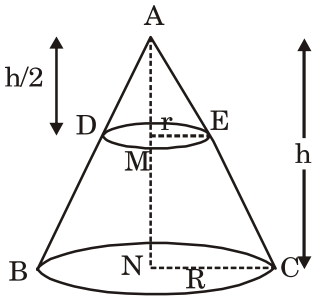
[TERM 2, 2017]

Solutions

1. Let the height of the whole cone be h and its radius be R.

Since, the cone has been cut into two parts at the middle of its height by a plane parallel to its base.

Therefore, height of the smaller cone will be $\frac{h}{2}$.



Let the radius of the smaller cone be r.

Now, we can see that ΔAME and ΔANC are similar triangles.

$$\therefore \frac{AM}{AN} = \frac{ME}{NC}$$

$$\Rightarrow \frac{h/2}{h} = \frac{r}{R}$$

$$\Rightarrow r = \frac{R}{2}$$

Now, Volume of the whole cone = $\frac{1}{3}\pi R^2 h$

$= \frac{\pi R^2 h}{3}$, and [½]

Volume of the smaller cone $= \frac{1}{3}\pi r^2 \frac{h}{2}$

$$= \frac{1}{3}\pi\left(\frac{R}{2}\right)^2 \frac{h}{2}$$

$$= \frac{1}{3} \times \pi \times \frac{R^2}{4} \times \frac{h}{2}$$

$$= \frac{\pi R^2 h}{24}$$

$$\frac{\text{Volume of the smaller cone}}{\text{Volume of the whole cone}} = \frac{\frac{\pi R^2 h}{24}}{\frac{\pi R^2 h}{3}} = \frac{3}{24} = \frac{1}{8}$$

Therefore, the ratio of the volume of the smaller cone to the whole cone is 1 : 8. [½]

Option (d) is correct.

2. Since the bucket will be in a shape of frustum,

And volume of a frustum is,

$$V = \frac{1}{3}\pi h\left(R^2 + r^2 + R \cdot r\right)$$

Here h is the height of the bucket i.e. h = 15 cm

R is the radius of the larger circular end, R= 14cm,

Volume is given i.e. V = 5390 cm^3

$$V = \frac{1}{3}\pi h\left(R^2 + r^2 + R \cdot r\right)$$

$$\Rightarrow 5390 = \frac{1}{3}\pi \times 15\left(14^2 + r^2 + 14 \cdot r\right)$$ [1]

$$\Rightarrow \frac{5390 \times 3}{\pi \times 15} = \left(14^2 + r^2 + 14r\right)$$

$$\Rightarrow \frac{5390 \times 7}{22 \times 5} = \left(196 + r^2 + 14r\right)$$

$$\Rightarrow 49 \times 7 = \left(r^2 + 196 + 14r\right)$$

$$\Rightarrow 343 = \left(r^2 + 196 + 14r\right)$$

$$\Rightarrow r^2 + 196 + 14r - 343 = 0$$ [1]

$$\Rightarrow r^2 + 14r - 147 = 0$$

$$\Rightarrow r^2 + 21r - 7r - 147 = 0$$

$$\Rightarrow r(r+21) - 7(r+21) = 0$$

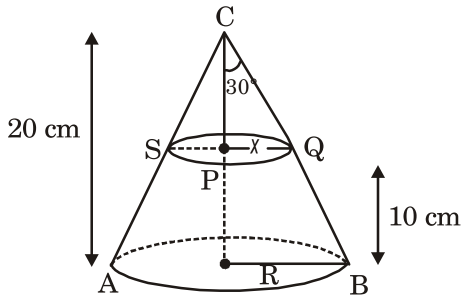
$$\Rightarrow (r+21)(r-7) = 0$$

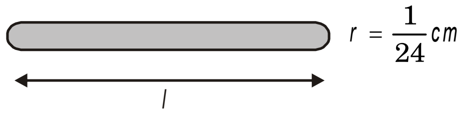
Now r = 7 or r = –21

Since r cannot be negative,

Therefore r = 7 cm. [1]

3.





Let ACB be the cone whose vertical angle $\angle ACB = 60°$.

Let R and x be the radii of the lower and upper end of the frustum.

Here, height of the cone,

OC = 20 cm = H and $CP = h$ = 10 cm

Let us consider P as the mid-Point of OC. After cutting the cone into two parts through P.

$$OP = \frac{20}{2} = 10\,cm$$ [1]

Also, $\angle ACO \text{ and } \angle OCB = \frac{1}{2} \times 60^o = 30^o$

After cutting cone CQS from cone CBA, the remaining solid obtained is a frustum.

Now, in triangle CPQ:

$$\tan 30^o = \frac{x}{10}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$x = \frac{10}{\sqrt{3}}\,cm$$

In triangle COB:

$$\tan 30^o = \frac{R}{CO}$$

$$\frac{1}{\sqrt{3}} = \frac{R}{20}$$

$$R = \frac{20}{\sqrt{3}}\,cm$$ [1]

Volume of the frustum

$$V = \frac{1}{3}\pi\left(R^2 H - x^2 h\right)$$

$$V = \frac{1}{3}\pi\left[\left(\frac{20}{\sqrt{3}}\right)^2 .20 - \left(\frac{10}{\sqrt{3}}\right)^2 .10\right]$$

$$V = \frac{1}{3}\pi\left(\frac{8000}{3} - \frac{1000}{3}\right)$$

$$V = \frac{1}{3}\pi\left(\frac{7000}{3}\right)$$

$$V = \frac{1}{9}\pi \times 7000$$

$$V = \frac{7000}{9}\pi$$

The volumes of the frustum and the wire formed are equal

$$\pi \times \left(\frac{1}{24}\right)^2 \times l = \frac{7000}{9}\pi \left(Volume\, of\, wire = \pi r^2 h\right)$$

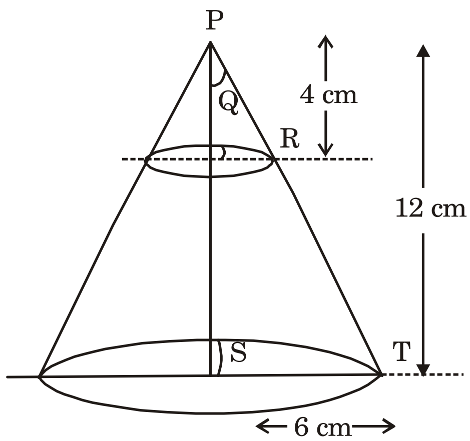
$$l = \frac{7000}{9} \times 24 \times 24$$

l = 448000 cm

l = 4480 m

Hence the length of the wire is 4480 m. [1]

4. When from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base the remaining solid will be a frustum.



The total surface area of the frustum

$= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Where

r_1 is the smaller radius of the frustum.

r_2 is the larger radius of the frustum = 6 cm.

l is the slant height of the frustum.

$\Delta PQR \sim \Delta PST$ by AA similarity criterion.

$$\therefore \frac{QR}{ST} = \frac{PQ}{PS}$$

$$\Rightarrow \frac{r_1}{6} = \frac{4}{12}$$

$$\Rightarrow r_1 = \frac{4}{12} \times 6 = 2 \text{ cm}$$ [1]

Height of the frustum is 12 cm – 4 cm = 8 cm

We know that slant height is given by the formula,

$$l = \sqrt{h^2 + (r_2 - r_1)^2}$$

$$l = \sqrt{8^2 + (6-2)^2}$$

$$\Rightarrow l = \sqrt{64 + 16}$$

$$\Rightarrow l = \sqrt{80} = 4\sqrt{5} \text{ cm}$$

Therefore the total surface area of the frustum

$= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

$$= \frac{22}{7} \times 4\sqrt{5}(2+6) + \frac{22}{7}(2)^2 + \frac{22}{7}(6)^2$$ [1]

$$= \frac{22}{7}\left[32\sqrt{5} + 4 + 36\right]$$

$$= \frac{22}{7}\left[32\sqrt{5} + 40\right]$$

$$= \frac{22}{7}[32 \times 2.236 + 40]$$

$$= \frac{22}{7}[111.552]$$

$$= \frac{2454.144}{7}$$

$= 350.592 \text{ cm}^2$

Hence the surface area of the frustum is 350.592 cm^2. [1]

5. Given the dimensions of the cuboidal box as 15 cm × 10 cm × 5 cm.

Length 'l' of the cuboidal block = 15 cm

Breadth 'b' of the cuboidal block = 10cm

Height 'h' of the cuboidal block = 5cm

Let d and r represent the diameter and radius of the cylindrical hole respectively. [1]

The diameter of the cylindrical hole = 7cm

The radius of the cylindrical hole

$$= \frac{d}{2} = \frac{7}{2} = 3.5 \ cm$$

It is given that from a cuboidal solid metallic block, a cylindrical hole of diameter 7 cm is drilled out.

Therefore the surface area of the remaining block = Surface area of the cuboid + Curved surface area of the cylinder – 2 (Area of the base of cylinder)

$= \text{Remaining area} = 2(lb + bh + lh) + 2\pi rh - 2(\pi r^2)$ [1]

$$2(15 \times 10 + 10 \times 5 + 5 \times 15) + 2 \times \frac{22}{7} \times 3.5$$
$$\times 5 - 2 \times \frac{22}{7}(3.5)^2$$

= 2 (150 + 50 + 75) + 110 – 77

= 550 + 110 – 77

$= 583 \text{ cm}^2$

Hence, the area of the remaining block is 583 cm^2. [1]

6. The height and diameter of the cylinder are 4m and 4.2 m respectively. (Given)

The height and diameter of the cone are 2.8 m and 4.2 m respectively. (Given)

So, the radius of cylindrical and conical part is 2.1 m.

Curved surface area of cylindrical part = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 4 = 52.80 \text{ m}^2$$ [1]

Curved surface area of conical part = πrl

Where, l = Slant height

$$l = \sqrt{h^2 + r^2}$$

Curved surface area of conical part

$= \frac{22}{7} \times 2.1 \times \sqrt{2.8^2 + 2.1^2}$

$= 6.6 \times \sqrt{12.25} = 23.10\ m^2$

Surface area of tent = Curved surface area of cylindrical part + Curved surface area of conical part

Surface area of tent = 52.80 m^2 + 23.10 m^2 = 75.90 m^2 [1]

Surface area of 100 tents = 100 × 75.90 = 7590 m^2

Cost of canvas used is Rs 100 per sq. m

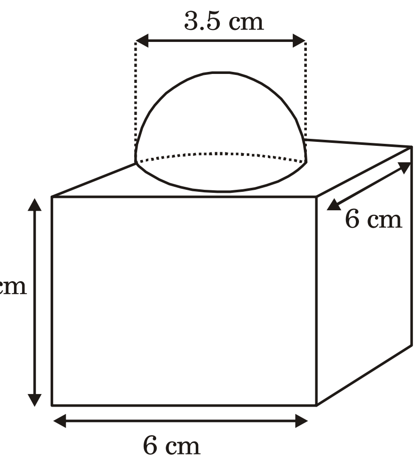
∴ Cost of 100 tents = 100 × 7590 = Rs. 759000

Welfare association is paying of the total amount = 50% of 759000 = Rs. 379500

The welfare associations will pay Rs 3,79,500

The welfare association has shown a sense of responsibility towards the society. [1]

7. [1]



Surface area of the block = Total surface area of the cube – Base area of the hemisphere + Curved surface area of the hemisphere [1]

$= 6 \times (Edge)^2 - \pi r^2 + 2\pi r^2$

$= \left(6 \times (6)^2 + \pi r^2\right)$

$= \left(216 + \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)$

= (216 + 9.625)

= 225.625 cm^2

Hence, the total surface area of the block is 225.625 cm^2. [1]

8. Given that, the height of the frustum of a cone = 14 cm

The diameters of its two circular ends are 4 cm and 2 cm.

∴ The radius of its two circular ends will be 2 cm and 1 cm respectively. [1]

Let us suppose, radius of one end, r_1 = 2 cm, and

Radius of another end, r_2 = 1 cm

Volume of the frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$ [1]

$= \frac{1}{3} \times \frac{22}{7} \times 14\left(2^2 + 1^2 + 2 \times 1\right)$

$= \frac{1}{3} \times 22 \times 2(4 + 1 + 2)$ [1]

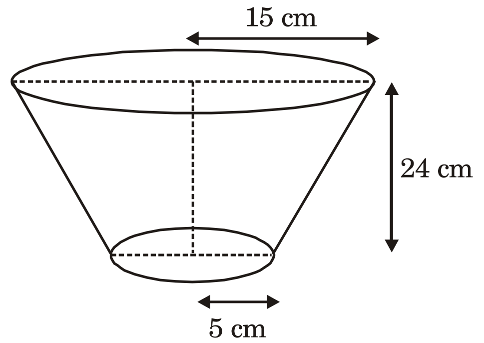
$= \frac{1}{3} \times 22 \times 14$

$= \frac{308}{3}$

= 102.6 cm^3

Therefore, capacity of the glass in the shape of a frustum is 102.6 cm^3. [1]

9. [1]



We know that,

Diameter of upper end of bucket = 30cm

Hence, Radius (r_1) of upper end of bucket = 15 cm

Diameter of lower end of bucket = 10 cm [1]

Hence, Radius (r_2) of lower end of bucket =5 cm

Height (h) of bucket = 24 cm

Slant height (l) of frustum

$= \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(15 - 5)^2 + 24^2}$

$= \sqrt{100 + 576} = \sqrt{676} = 26$ cm

Area of metal sheet required to make the bucket = CSA of frustum + area of the base of bucket

$= \pi(r_1 + r_2)l + \pi r^2$ [1]

$= \pi\ (15\ +\ 5)\ 26\ +\ \pi(5)^2$

$= 520\pi + 25\ \pi = 545\pi\ cm^2$

Cost of 100 cm^2 metal sheet = Rs 10

Cost of 545 π cm^2 metal sheet

$Rs.545 \times 3.14 \times \frac{10}{100} = Rs.171.13$

Hence the total cost is Rs. 171.13. [1]

10. Consider a frustum of cone of height (*h*) 24 cm, radius of lower end (*r*) 8 cm and radius of upper end (*R*) 20 cm.

Volume of frustum of cone $= \frac{\pi}{3} h\left(R^2 + Rr + r^2\right)$

Substitute the values of *R*, *r* and h in the above equation

Volume of frustum of cone

$= \frac{\pi}{3}(24)\left(20^2 + 20 \times 8 + 8^2\right)$ [1]

Volume of frustum of cone

$= \frac{\pi}{3}(24)(400 + 160 + 64)$ [1]

Volume of frustum of cone $= \frac{\pi}{3}(24)(624)$

Volume of frustum of cone = 15689.14 cm^3

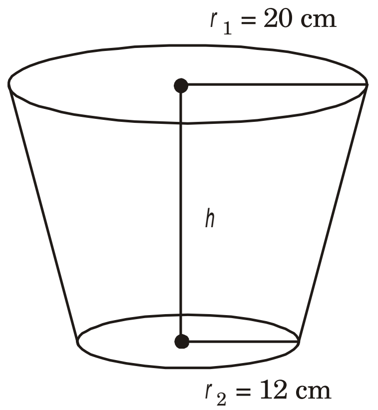
The cost of milk for 1 litre is Rupees 21. [1]

Since,

$1L = 1{,}000\ cm^3, 15689.14\ cm^3 = \frac{15689.14}{1{,}000} = 15.69L$

Cost of 15.68 L of milk is $15.69 \times 21 =$ ₹329.49 [1]

11. Consider the figure.



Volume of the frustum = 12308.8 cm^3

Radii, $r_1 = 20\,cm$ and $r_2 = 12\,cm$

Now we know the formula for the volume of the frustum,

$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$ [1]

$\Rightarrow 12308.8 \times 3 = \pi h\left(20^2 + 12^2 + 20 \times 12\right)$

$\Rightarrow 12308.8 \times 3 = \pi h(400 + 144 + 240)$

$\Rightarrow 12308.8 \times 3 = \pi h(784)$

$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784}$

h = 15 cm [1]

Thus, the height of the frustum is 15cm.

Now,

Slant height of the frustum of cone is given by;

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

So, $l = \sqrt{15^2 + (20 - 12)^2}$

$\Rightarrow l = \sqrt{225 + 64}$

$\Rightarrow l = \sqrt{289} = 17\,cm$

Now, Curved Surface area of the frustum

$= \pi(r_1 + r_2)l$

$= \pi(20 + 12)17$

$= 544 \times 3.14 = 1708.16\ cm^2$ [1]

Area of the base $= \pi r^2 = \pi 12^2 = 452.16\,cm^2$

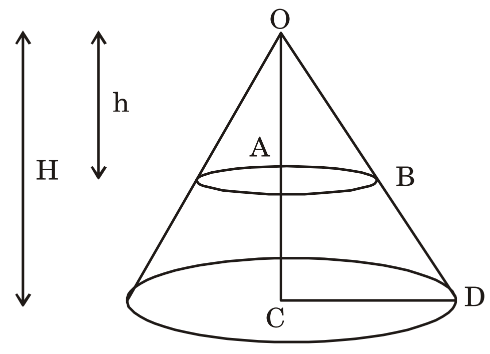
Metal Sheet required to make the frustum = Curved Surface area + Area of the base of the frustum

So, the metal sheet required to make the frustum = 1708.16 + 452.16 = 2160.32 cm^2

Hence, the height of the bucket is 15 cm and the area of the metal sheet used in making the bucket is 2160.32 cm^2. [1]

12. Let *OA* and *AB* be *h* and *r* respectively.

Also, *OC* and *CD* be *H* and *R* respectively.



According to the question $2h = H$

In ΔOAB and ΔOCD

$\angle OAB = \angle OCD = 90°$

$\angle AOB = \angle COD$ [$\because$ common]

$\therefore$ $\Delta OAB \sim \Delta OCD$ [$\because$ AA similarity] [1]

Now, the ratio of the sides must be equal.

$$\frac{OC}{OA} = \frac{CD}{AB}$$

$$\Rightarrow \frac{H}{h} = \frac{R}{r}$$

$$\Rightarrow \frac{2h}{h} = \frac{R}{r}$$

$\Rightarrow 2r = R$ [1]

We know that the volume of a cone is given by the formula

$$V = \frac{1}{3}\pi r^2 h$$

$\therefore$ Volume of the smaller cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ Volume of the bigger cone

$= \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi(2r)^2(2h)$ [1]

Volume of the remaining portion = volume of bigger cone - volume of smaller cone.

$$\Rightarrow 8 \times \frac{1}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 7 \times \frac{1}{3}\pi r^2 h$$

Now, ratio of the volumes of the two parts will be,

$$= \frac{\frac{1}{3}\pi r^2 h}{7 \times \frac{1}{3}\pi r^2 h} = \frac{1}{7}$$

Hence, the ratio of the volumes of the two parts is 1 : 7. [1]

Value Based

PREVIOUS YEARS'

EXAMINATION QUESTIONS

4 Marks Questions

1. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decide to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs. 120/sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem?

$\left(\text{use } \pi = \dfrac{22}{7}\right)$

[TERM 2, 2016]

2. In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3.5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation.

[TERM 2, 2016]

3. In a hospital, used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park. Write your views on recycling of water.

[TERM 2, 2017]

4. The diameter of lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic? [Use $\pi = 3.14$]

[DELHI, 2018]

Solutions

1. Let the slant height, radius and height of the cone be l, r and h respectively.

Let H be the height of the cylindrical base of the tent.

Slant height is given by $l^2 = h^2 + r^2$

$l^2 = (2.1)^2 + (2.8)^2$

$l^2 = 4.41 + 7.84$

$l^2 = 12.25$

$l = \sqrt{12.25} = 3.5\,\text{m}$ [1]

The canvas used for each tent = CSA of cylindrical base + CSA of conical upper part

The canvas used for each tent = $2\pi rH + \pi rl$

$= \pi r(2H + l)$

$= \pi \times 2.8 \times (2(3.5) + 3.5)$

$= \dfrac{22}{7} \times 2.8 \times 10.5$

$= 92.4\ \text{m}^2$ [1]

So, canvas used for 1 tent is 92.4 m^2

Thus, canvas used for 1500 tents

$= (92.4 \times 1500)\,\text{m}^2$

Cost of canvas is Rs 120/sq. m

So, cost of canvas for 1500 tents

$= \text{Rs}(92.4 \times 1500 \times 120)$ [1]

As 50 schools participated to provide the tents. Therefore, the amount shared by each school to set up the tents

$= \dfrac{92.4 \times 1500 \times 120}{50} = \text{Rs}\,3,32,640$

Thus, the amount shared by each school to set up the tents is Rs. 3, 32, 640. [1]

2. Given:

Width of the roof $b = 20\ m$

Length of the roof $l = 22$ m

Height of cylindrical vessel $H = 3.5\ m$

Base radius of cylindrical vessel $R = \frac{2}{2} = 1m$

Let h cm of rainfall has taken place. [1]

Now,

Total amount of rainfall = Volume of rain water collected in cylindrical vessel

$\Rightarrow lbh = \pi R^2 H$ [1]

$\Rightarrow 22 \times 20 \times h = \frac{22}{7} \times (1)^2 \times 3.5$

$\Rightarrow 440h = \frac{22}{7} \times 3.5$

$\Rightarrow h = \frac{22}{7} \times \frac{3.5}{440}$

$\Rightarrow h = 0.025m$

$\therefore h = 2.5cm$ [1]

It is very important to conserve water for sustainable development. For conserving water different methods can be put in use. Rain water harvesting is one of them it not only avoids wastage of water but also helps in fulfilling all demands of water in summers. [1]

3. We know that the volume of a cylinder is given by the formula,

$V = \pi r^2 H$,

Where r is the radius of the base of the cylinder and H is its height.

Here, $r = \frac{d}{2} = \frac{2}{2} = 1m$ and $h = 5m$

$\therefore V = (3.14)(1)^2(5) = 15.7m^3 \ldots\ldots(i)$ [1]

Also, Volume of a rectangular field is given by the formula,

$V = lbh$,

Where h is the height of the standing water.

Here, $l = 25m$, $b = 20m$

$V = 25 \times 20 \times h = 500\ (h)\ m^3 \quad \ldots.(ii)$ [1]

The height of standing water used for irrigating the park can be found by equating the volume of cylindrical tank and the volume of water used for irrigating the park.

Equating equation (i) and (ii), we'll get

$\Rightarrow 15.7 = 500(h)$

$\Rightarrow \frac{15.7}{500} = h$

$\Rightarrow h = 0.0314m$

Hence, the height of the standing water is $0.0314m$ [1]

Recycling of water is one of the best methods for sustainable development as it reduces wastage and help in reuse of water. It reduces water pollution and also helps in conservation of water. [1]

4. (i) Here, $h = 24$ cm, $r_1 = 30$ cm and $r_2 = 5$ cm

$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{24^2 + (15-5)^2}$

$= \sqrt{676} = 26\,cm$ [1]

Total surface area = Curved surface Area of frustum + Area of base

$= \pi\ (r_1 + r_2)\ L + \pi r_2^2$

$= \pi(15 + 5) \times 26 + \pi\ (5)^2$ [1]

$= 3.14 \times 20 \times 26 + 25 \times 3.14$

$= 3.14\,(520 + 25)$

$= 545 \times 3.14$

$= 1711.3\ cm^2$

Hence, the Area of metal sheet used to make the bucket is $1711.3\ cm^2$ [1]

(ii) Plastics are non biodegradable. That is, plastic material mostly end as harmful waste that pollutes the environment and causes health problems, we should avoid using plastic. [1]

Smart Notes ....

CHAPTER 14

Statistics

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Mean						
Median	3 marks	3 marks				
Mode						
Cumulative Frequency Graph	4 marks	4 marks				

[TOPIC 1] Mean, Median and Mode

Summary

It is generally observed that observations or data on a variable tend to gather around some central value. This gathering of data towards a central value is called **central tendency** or the middle value of the distribution, also known as middle of the data set.

A certain value representative of the whole data and signifying its characteristics is called an **average** of the data.

Three types of averages are useful for analyzing data.

They are : (i) Mean, (ii) Median, (iii) Mode.

Mean for a Grouped Frequency Dis Tribution

DIRECT METHOD

Step 1: For each class, find the class mark x_i, as

$$x_i = \frac{1}{2} \text{ (lower limit + upper limit)}$$

Step 2: Calculate $f_i x_i$ for each i.

Step 3: Use the formula : $= \frac{\sum(f_i x_i)}{\sum f_i}$.

Assumed-Mean Method

Following steps are taken to solve cases by assumed-mean method.

Step 1: For each class interval, calculate the class mark x_i by using the

formula: $x_i = \frac{1}{2}$(lower limit + upper limit).

Step 2: Choose a value of x_i in the middle as the assumed mean and denote it by A.

Step 3: Calculate the deviations $d_i = (X_i - A)$ for each i.

Step 4: Calculate the $(f_i d_i)$ for each i.

Step 5: Find $n = \sum f_i$.

Step 6: Calculate the mean, $\bar{x}$, by using the formula:

$$\bar{x} = A + \frac{\sum f_i d_i}{n}.$$

Step-Deviation Method

Following steps are taken to solve cases by step-deviation method.

Step 1: For each class interval, calculate the class mark x_i by using the

formula: $x_i = \frac{1}{2}$(lower limit + upper limit).

Step 2: Choose a value of x_i in the middle of the x_i column as the assumed mean and denote it by A.

Step 3: Calculate h = [(upper limit) – (lower limit)].

Step 4: Calculate $u_i = \frac{(x_i - A)}{h}$ for each class.

Step 5: Calculate $f_i u_i$ for each class and find $\sum(f_i u_i)$.

Step 6: Calculate the mean, by using the formula:

$$\bar{x} = A + \left[h \frac{\sum(f_i u_i)}{\sum f_i}\right].$$

Mode

It is that value of a variate which occurs most often. More precisely, mode is that value of the variable at which the concentration of the data is maximum.

Modal Class : In a frequency distribution, the class having maximum frequency is called the modal class.

Formula for Calculating Mode:

We have:

Mode, $M_0 = \ell + h.\left[\frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}\right]$, where

ℓ = lower limit of the modal class interval;

f_1 = frequency of the modal class;

f_0 = frequency of the class preceding the modal class;

f_2 = frequency of the class succeeding the modal class;

h = width of the class interval.

Method for Finding the Median for Grouped Data

Median of a distribution is the value of the variable which divides it into two equal parts. It is the value which exceeds and is exceeded by the same number of observation i.e., it is the value such that the number of observation above it is equal to the number of observation below it.

In case of grouped frequency distribution, the class corresponding to the cumulative (c.f) just greater than $\frac{N}{2}$ is called the median class.

Following steps are involved in finding the median of the given frequency distribution.

Step 1: For the given frequency distribution, prepare the cumulative frequency table and obtain $N = \Sigma f_i$.

Step 2: Find $(N/2)$.

Step 3: Find the cumulative frequency just greater than $(N/2)$ and find the corresponding class, known as median class.

Step 4: Use the formula:

Median, $Me = \ell + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f} \right]$, where

ℓ = lower limit of median class,

h = width of median class,

f = frequency of median class,

c = cumulative frequency of the class preceding the median class, $N = \Sigma f_i$.

Relationship Among Mean, Median and Mode

We have, Mode = 3(Median) –2(Mean)

or

Median = Mode $+\frac{2}{3}$ (Mean – Mode)

or

Mean = Mode $+\frac{3}{2}$ (Median – Mode)

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. If the mode of some data is 7 and their mean is also 7, then their median is:

   (*a*) 10 (*b*) 9

   (*c*) 8 (*d*) 7 **[TERM 1, 2011]**

2. Relationship among mean, median and mode is:

   (*a*) 3 Median = Mode + 2 Mean

   (*b*) 3 Mean = Median + 2 Mode

   (*c*) 3 Mode = Mean = 2 Median

   (*d*) Mode = 3 Mean – 2 Median **[TERM 1, 2012]**

3. Monthly pocket money of 50 students of a class are given in the following distribution:

Monthly pocket money (*in* Rs)	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250	250 – 300
Number of students	2	7	8	30	12	1

   Find modal class and also give class rank of the modal class. **[TERM 1, 2014]**

4. Write an empirical relationship between the three measures of central tendency i.e. mean, median and mode.

[TERM 1, 2015]

2 Marks Questions

5. Find the mode of the following distribution of marks obtained by 50 students.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	4	8	10	20	8

[TERM 1, 2011]

6. Find the mode of the following frequency distribution:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	8	9	10	3

[TERM 1, 2012]

7. Data regarding weights of students of class X of a school is given below. Calculate the average (Mean) weight of the students.

Weight (in Kg)	50 – 52	52 – 54	54 – 56	56 – 58	58 – 60	60 – 62	62 – 64
Number of students	18	21	17	28	16	35	15

[TERM 1, 2014]

8. In a class test, 50 students obtained marks are as follows. Find the modal class and the median class

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Numbers	4	6	25	10	5

[TERM 1, 2016]

3 Marks Questions

9. Find the mean of the following frequency distribution using assumed mean method.

Classes	2-8	8-14	14-20	20-26	26-32
Frequency f:	6	3	12	11	8

[TERM 1, 2011]

10. 200 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in English alphabets in the surnames was obtained as follows

No. of letters	1-5	5-10	10-15	15-20	20-25
No. of surnames	20	60	80	32	8

Evaluate the median of it. **[TERM 1, 2011]**

11. The mean of the following frequency distribution is 52. Find the missing frequency.

C.I.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	3	4	f	2	6	13

[TERM 1, 2011]

12. Calculate the median for the following distribution:

Marks Obtained	Number of students
Below 10	6
Below 20	15
Below 30	29
Below 40	41
Below 50	60
Below 60	70

[TERM 1, 2012]

13. Compute the arithmetic mean for the following data:

Marks obtained	Number of students
Less than 10	14
Less than 20	22
Less than 30	37
Less than 40	58
Less than 50	67
Less than 60	75

[TERM 1, 2012]

14. Find the mean of the following data:

Classes	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
Frequency	6	10	16	15	24	8	7

[TERM 1, 2013]

15. Find the median of the following data:

Marks	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Number of Students	5	15	25	20	7	8	10

[TERM 1, 2013]

16. In annual examination, marks (out of 90) obtained by students of class IX in mathematics are given below:

Marks	0-15	15-30	30-45	45-60	60-75	75-90
Number of students	2	4	5	20	9	10

Find the mean marks of the student.

[TERM 1, 2014]

17. In a hospital, age record of diabetic patients was recorded as follows:

Age (in years)	0-15	15-30	30-45	45-60	60-75
Number of patients	5	20	40	50	25

Find the median age.

[TERM 1, 2014]

18. The following table gives the ages of 1000 persons who visited a shopping centre on Sunday:

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons	105	222	220	138	102	113	100

Find the mean number of the people who visited a shopping centre on Sunday

[TERM 1, 2015]

19. A school conducted a test (of 100 marks) in English for students of class X. The marks obtained by students are shown in the following table:

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	1	2	4	15	15	25	15	10	2	1

Evaluate modal marks. **[TERM 1, 2015]**

20. Find the mean of the following distribution:

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	7	5	10	12	2

[TERM 1, 2016]

21. The following table gives the literacy rate of 40 cities:

Literacy Rate (in %)	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Number of odes	6	7	10	6	8	3

Find the modal literacy rate.

[TERM 1, 2016]

22. Find the mean of the data by step deviation method.

C.I	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Frequency	6	11	7	4	4	2	1	10

[TERM 1, 2017]

4 Marks Questions

23. If the median of the following data is 525. Find the values of x and y if the sum of the frequencies is 100.

C.I.	0-100	100-200	200-300	300-400	400-500
Frequency	2	5	x	12	17

C.I.	500-600	600-700	700-800	800-900	900-1000
Frequency	20	y	9	7	4

[TERM 1, 2011]

24. Find the missing frequency f_1 and f_2 in the following distribution table, if $N = 100$ and median is 32.

Class:	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency:	10	f_1	25	30	f_2	10	100

[TERM 1, 2012]

25. Find mode of the following data:

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency	6	8	10	12	6	5	3

[TERM 1, 2013]

26. The mean of the following data is 42. Find the missing frequencies x and y if the total frequency is 100.

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	10	x	13	y	10	14	9

[TERM 1, 2013]

27. Weights of new born babies in a hospital are as follows:

Weight (in kg)	1.3-1.5	1.5-1.7	1.7-1.9	1.9-2.1	2.1-2.3	2.3-2.5	2.5-2.7	2.7-2.9
Number of new born babies	1	4	6	9	10	x	8	3

If the mode of the data is 2.2 kg, find the unknown frequency x.

[TERM 1, 2015]

28. Following distribution gives the marks obtained out of 200 by the students of class IX in their class test.

Marks	0-25	25-50	50-75	75-100	100-125	125-150	150-175	175-200
Number of students	10	15	22	30	28	27	12	6

Find the mean and mode of the data.

[TERM 1, 2015]

29. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of houses preceding the house numbered X is equal to sum of the numbers of houses following X.

[TERM 1, 2016]

Solutions

1. The relation between mean, median and mode is

   Mode = 3 Median – 2 Mean

   Substitute mode = 7 and mean = 7 in above equation,

   $7 = 3 \times \text{Median} - 2 \times 7$

   $\Rightarrow 7 = 3 \times \text{Median} - 14$

   $\Rightarrow 3 \times \text{Median} = 21$

   Divide the above equation by 3,

   $$\text{Median} = \frac{21}{3} = 7$$

   Hence, the correct option is (d). [1]

2. The empirical relationship between the three measures of central tendency is Median = Mode + 2 Mean

   Hence, the correct option is (a). [1]

3. Since we can see from the given grouped frequency table the highest frequency is in the group 150 – 200

   So the modal class is 150 – 200 and its rank is 30 as there are 30 students in the modal class. [1]

4. Empirical relationship between the three measures of central tendency i.e. mean, median and mode is:

   3 Median – 2 Mean = Mode [1]

5. $$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

   The modal class with highest frequency = 30 – 40 where l = Lower limit of modal class = 30

   f_1=Frequency of modal class=20

   f_0=Frequency of class before modal class=10

   f_2=Frequency of class after modal class=8

   h=Class Interval=40-30=10 [1]

   Substituting the above values in formula of mode,

   $$\text{Mode} = 30 + \left(\frac{20-10}{2(20)-10-8}\right) \times 10$$

   $$\text{Mode} = 30 + \left(\frac{10}{22}\right) \times 10 = 30 + 4.545 \approx 34.55$$

   Hence, the mode is 34.55. [1]

6. Here, the maximum frequency is 10 and the corresponding class is 30 – 40. So, 30 – 40 is the modal class such that

   Lower limit of the modal class, l = 30

   Width of the modal class, h = 10

   Frequency of the modal class, f = 10

   Frequency of the class preceeding the modal class, f_1 = 9

   Frequency of the class following the modal class, f_2 = 3

   We know that,

   $$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$ [1]

   Substituting values in the formula, we get,

   $$\text{Mode} = 30 + \frac{10-9}{20-9-3} \times 10$$

   $$\Rightarrow \text{Mode} = 30 + \frac{10}{8}$$

   $$\Rightarrow \text{Mode} = 30 + 1.25 = 31.25$$

   Therefore, the mode of given frequency distribution is 31.25 [1]

7. To find the mean of the given data, we can find the midpoint of the given class.

Weight (in Kg)	Number of students (f)	Mid-point of the class w	Product wf
50 – 52	18	51	918
52 – 54	21	53	1113
54 – 56	17	55	935
56 – 58	28	57	1596
58 – 60	16	59	944
60 – 62	35	61	2135
62 – 64	15	63	945

[1]

   Here total number of students

   = 18 + 21 + 17 + 28 + 16 + 35 + 15

   = 150

   Total weight $= \sum wf = 8586$ kg

   $$\text{Mean} = \frac{\text{Total weight}}{\text{Number of students}} = \frac{8586}{150} = 57.24$$

   Mean weight of the students is 57.24 kg. [1]

8. Modal class is the class with highest frequency.

Here, 25 students got their marks between

Thus,

Modal class = 40 – 60

Class interval	frequency	Cumulative frequency
0 – 20	4	4
20 – 40	6	4 + 6 = 10
40 – 60	25	10 + 25 = 35
60 – 80	10	35 + 10 = 45
80 – 100	5	45 + 5 = 50
	Total (N) = 50	

[1]

We have,

N = 50

$$\frac{N}{2} = \frac{50}{2} = 25$$

The cumulative frequency with just greater 25 and this belongs to the class 40 – 60.

Hence, median class 40 – 60.

Thus both the modal class and the median class are 40 – 60. [1]

9. Construct the following table:

Classes	f_i	x_i	$d_i = x_i - A$	$d_i \times f_i$
2 – 8	6	5	–12	–72
8 – 14	3	11	–6	–18
14 – 20	12	17 = A	0	0
20 – 26	11	23	6	66
26 – 32	8	29	12	96

[2]

Here, A = 17

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i}$$

$$\sum f_i d_i = 72 \text{ and } \sum f = 40$$

$$\text{Mean} = 17 + \frac{72}{40}$$

Mean = 17 + 1.8 = 18.8

Hence, the mean is 18.8. [1]

10. Construct the following table: [1]

Class Interval	Frequency	Cumulative Frequency
1 – 5	20	20
5 – 10	60	80
10 – 15	80	160
15 – 20	32	192
20 – 25	8	200

$$\frac{N}{2} = \frac{200}{2} = 100$$

So, the median class is 10 – 15.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$ [1]

Here cf = cumulative frequency of class preceding median class, f = frequency of median class, l = lower limit of median class and h = size of class.

$l = 10, cf = 80, f = 80$ and $h = 5$

$$\text{Median} = 10 + \frac{100 - 80}{80} \times 5$$

Median = 10 + 1.25 = 11.25

Hence, median is 11.25. [1]

11. Construct the following table:

Class Interval	Frequency (f_i)	x_i	$f_i \times x_i$
10 – 20	5	15	75
20 – 30	3	25	75
30 – 40	4	35	140
40 – 50	f	45	$45f$
50 – 60	2	55	110
60 – 70	6	65	390
70 – 80	13	75	975

[2]

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f}$$

$$52 = \frac{1765 + 45f}{33 + f}$$

$$\Rightarrow 52(33 + f) = 1765 + 45f$$

$$\Rightarrow 1716 + 52f = 1765 + 45f$$

$$\Rightarrow 7f = 49$$

Dividing both sides by 7,

$$f = \frac{49}{7} = 7$$

Hence, the missing frequency is 7. [1]

12. The given frequency table is of less than type represented with upper class limits. Therefore, the class intervals with their respective cumulative frequency can be defined as below:

Marks Obtained	Number of students (f_i)	Cumulative frequency (cf)
0- 10	6	6
10- 20	15 – 6 = 9	15
20- 30	29 – 15 = 14	29
30- 40	41 – 29 = 12	41
40- 50	60 – 41 = 19	60
50- 60	70 – 60 = 10	70
Total(??)	70	

[1]

From the table, it can be observed that $n = 70$

$$\Rightarrow \frac{n}{2} = \frac{70}{2} = 35$$

Also, cumulative frequency(cf) just greater than $\frac{n}{2}$ (i.e. 35) is 41, which belongs to the interval 30 – 40. [1]

Therefore, median class = 30 – 40

Lower limit(l) of median class = 30.

Class size (h) = 10

Frequency (f_t) of median class = 12

Cumulative frequency (cf) of class preceding the median class = 29

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 30 + \left(\frac{35 - 29}{12}\right) \times 10$$

$$= 30 + \left(\frac{6}{12}\right) \times 10$$

$$= 30 + 5$$

$$= 35$$

Hence, the median is 35. [1]

13. The given frequency table is of less than type represented with upper class limits. Therefore, the class intervals with their respective cumulative frequency can be defined as below:

Marks Obtained	Number of students (f_i)	Cumulative frequency (cf)	x_i	$f_i x_i$
0- 10	14	14	5	70
10- 20	22 – 14 = 8	22	15	120
20- 30	37 – 22 = 15	37	25	375
30- 40	58 – 37 = 21	58	35	735
40- 50	67 – 58 = 9	67	45	405
50- 60	75 – 67 = 8	75	55	440
Total(n)	75			$\sum f_i x_i$ =2145

[2]

The mean is given by the formula:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{2145}{75} = 28.6$$

Therefore, mean = 28.6. [1]

14. Using direct method:

Classes	Frequency (f_1)	Class Mark (x_1)	$f_i x_i$
5 – 15	6	10	60
15 – 25	10	20	200
25 – 35	16	30	480
35 – 45	15	40	600
45 – 55	24	50	1200
55 – 65	8	60	480
65 – 75	7	70	490
Total	$\sum f_i = 86$		$\sum f_i x_i = 3510$

[2]

The sum of the values in the last column gives $\sum f_i x_i$. So, the mean is given by

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3510}{86} = 40.81$$

Hence, the mean of data is 40.81. [1]

15. [1]

Marks	Number of students	Cumulative Frequency
20 – 30	5	5
30 – 40	15	$5 + 15 = 20$
40 – 50	25	$25 + 20 = 45$
50 – 60	20	$20 + 45 = 65$
60 – 70	7	$7 + 65 = 72$
70 – 80	8	$8 + 72 = 80$
80 – 90	10	$10 + 80 = 90$

From the table, it can be observed that $n = 90$

$$\Rightarrow \frac{n}{2} = \frac{90}{2} = 45$$

Also, cumulative frequency(cf) just greater than $\frac{n}{2}$ (i.e. 45) is 65, which belongs to the interval 50 – 60

Therefore, median class 50 – 60

Lower limit(l) of median class = 50. [1]

Class size (h) = 10

Frequency (f_i) of median class = 20

Cumulative frequency (cf) of class preceding the median class = 45

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\text{Median} = 50 + \left(\frac{45 - 45}{20}\right) \times 10$$

Median = 50 + 0 = 50

Hence, the median is 50. [1]

16. To find the mean of the given data, we can find the midpoint of the given class.

Marks	Number of students (f_i)	x_i	$f_i x_i$
0-15	2	7.5	15
15-30	4	22.5	90
30-45	5	37.5	187.5
45-60	20	52.5	1050
60-75	9	67.5	607.5
75-90	10	82.5	825

[2]

Here total number of students = 50

Total weight $= \sum f_i x_i = 2775$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2775}{50} = 55.5$$

Mean marks of the students 55.5. [1]

17. [1]

Age (in years)	Number of patients (fi)	Cumulative frequency
0-15	5	5
15-30	20	25
30-45	40	65
45-60	50	115
60-75	25	140

$$\text{Median } m = L + \left(\frac{\frac{N}{2} - F}{f_m}\right) C$$ [1]

Here, L= lower boundary of median class

N = Total frequency

F = Cumulative frequency before median class

C = Class size

fm = frequency of median class,

$\frac{N}{2} = 70$, so median class is 45 – 60.

f_m = 50, C = 1,5, F = 65, L = 45 and N = 140

Now substituting these values,

$$m = 45 + \left(\frac{\frac{140}{2} - 65}{50} \right) 15 = 45 + \left(\frac{70 - 65}{50} \right) 15$$

$$= 45 + \frac{75}{50}$$

= 45 + 1.5

m = 46.5

Hence median age is 46.5 years. [1]

18. [2]

Age (in years)	Class marks (x_i)	Number of persons (f_i)	$f_i x_i$
0-10	5	105	525
10-20	15	222	3330
20-30	25	220	5500
30-40	35	138	4830
40-50	45	102	4590
50-60	55	113	6215
60-70	65	100	6500

It is given that there are 1000 persons.

Therefore $\sum f_i = 1000$

$$\sum f_i x_i = 525 + 3330 + 5500 + 4830 + 4590 + 6215 + 6500 = 31490$$

Therefore mean = $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{31490}{1000} = 31.490$

[1]

Hence, the mean number of the people who visited a shopping centre on Sunday is 31 rounded to nearest whole number.

19. To find the modal marks look for the group that has the highest frequency. This is because the mode is the number that comes up the most times.

From the given table it can be observed that there are 25 students who have obtained the marks in the range 50 – 60.

Therefore the modal class is 50 – 60.

The formula to estimate the Mode is:

$$\text{Mode} = L + \frac{(f_m - f_{m-1})}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$ [2]

Where,

L = the lower boundary of the modal group = 50

f_{m-1} = the frequency of the group before the modal group = 15

f_m = the frequency of the modal group = 25

f_{m+1} = the frequency of the group after the modal group =15

w = the group width =10

$$\text{Mode} = 50 + \frac{(25 - 15)}{(25 - 15) + (25 - 15)} \times 10$$

$$\text{Mode} = 50 + \frac{10}{10 + 10} \times 10$$

⇒ Mode = 50 + 5 = 55

Hence, the modal marks are 55. [1]

20. Mean can be found using the step-deviation method,

Class interval	Mid value (x_i)	$d_i = x_i - 15$	$u_i = \frac{(x_i - 15)}{6}$	Frequency f_i	$f_i u_i$
0 – 6	3	–12	–2	7	–14
6 – 12	9	–6	–1	5	–5
12 – 18	15	0	0	10	0
18 – 24	21	6	1	12	12
24 – 30	27	12	2	2	4
				$\sum f_i = 36$	$\sum f_i u_i = -3$

[2]

Hear, $a = 15$ and the class interval $(h) = 6$

The mean of the data is given by,

$$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 15 + \left(\frac{-3}{36}\right) \times 6$$

$$= 15 - \frac{1}{2}$$

$= 14.5$

Thus the mean of the following distribution is 14.5. [1]

21.

Literacy Rate (in %)	Number of odes (frequency) f_i	Cumulative frequency
30 – 40	6	6
40 – 50	7	$6+7=13$
50 – 60	10	$13+10=23$
60 – 70	6	$23+6=29$
70 – 80	8	$29+8=37$
80 – 90	3	$37+3=40$

[1]

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2 \times f_1 - f_0 - f_2}\right) \times h$$ [1]

Where, l = Lower class limit of the modal class

h = Class size

f_1 = stands for the frequency of the modal class.

f_0 = Frequency of the class preceding the modal class.

f_2 = Frequency of the class succeeding the modal class.

Modal class is the class with highest frequency.

Thus,

Modal class = 50 – 60

$$\text{Mode} = 50 + \left(\frac{10-7}{2\times 10 - 7 - 6}\right) \times 10$$

$$\text{Mode} = 50 + \left(\frac{3}{20-13}\right) \times 10$$

$$\text{Mode} = 50 + \left(\frac{3}{7}\right) \times 10$$

Mode = 54.29

Thus the modal literacy rate is 54.29%. [1]

22. We will first find the mid values and take middle value as assumed mean.

Here, let assumed mean $(a) = 60$ and h = upper limit – lower limit = 10

[2]

$C.I$	$x_i = \frac{u.l + l.l}{2}$	f_i	$d_i = x_i - a$	$u_i = \frac{d_i}{h}$	$f_i u_i$
15 – 25	20	6	–40	–4	–24
25 – 35	30	11	–30	–3	–33
35 – 45	40	7	–20	–2	–14
45 – 55	50	4	–10	–1	–4
55 – 65	60	4	0	0	0
65 – 75	70	2	10	1	2
75 – 85	80	1	20	2	2
85 – 95	90	10	30	3	30
		$\sum f_i = 45$			$\sum f_i u_i = -41$

Mean $\bar{x} = a + h\left[\frac{\sum f_i u_i}{\sum f_1}\right]$

$$\bar{x} = 60 + 10\left[\frac{-41}{45}\right]$$

$$\bar{x} = 60 + 10(-0.91)$$

$$\bar{x} = 60 - 9.1, \quad \bar{x} = 50.9$$

Therefore, the mean of the given data is 50.9. [1]

23. Construct the following table.

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	$2+5=7$
200 – 300	x	$7+x$
300 – 400	12	$12+7+x=19+x$
400 – 500	17	$19+x+17=36+x$
500 – 600	20	$20+36+x=56+x$
600 – 700	y	$56+x+y$
700 – 800	9	$9+56+x+y=65+x+y$
800 – 900	7	$7+65+x+y=72+x+y$
900 – 1000	4	$4+72+x+y=76+x+y$

[2]

As median is 525, the median class is 500 – 600.

l = Lower limit of median class = 500

h = Class Size = 100

n = Sum of frequencies = 100

cf = Cumulative frequency of class before median class = $36 + x$

f = Frequency of median class = 20

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$525 = 500 + \frac{\frac{100}{2} - (36+x)}{20} \times 100$$

$$\Rightarrow 525 - 500 = (50 - (36+x)) \times 5$$

$$\Rightarrow 25 = (14 - x) \times 5$$

$$\Rightarrow 5 = (14 - x)$$

$$\Rightarrow x = 14 - 5 = 9$$ [1]

The sum of frequencies is 100.

$$\Rightarrow 100 = 76 + x + y$$

Substituting $x = 9$, we get

$100 = 76 + 9 + y$

$$\Rightarrow 100 = 85 + y$$

$$\Rightarrow y = 100 - 85 = 15$$

Hence, the value of x is 9 and value of y is 15. [1]

24. We know that the frequency of the class 10 – 20 is f_1 and that of the class 40 – 50 is f_2.

Also, the total frequency is 100.

Therefore, $10 + f_1 + 25 + 30 + f_2 + 10 = 100$

$f_1 + f_2 = 100 - 75$

$f_1 + f_2 = 25$ ....(i) [1]

Class	Frequency	Cumulative Frequency
0 – 10	10	10
10 – 20	f_1	$10+f_1$
20 – 30	25	$25+10+f_1=35+f_1$
30 – 40	30	$30+35+f_1=65+f_1$
40 – 50	f_2	$f_2+65+f_1=65+f_1+f_2$
50 – 60	10	$10+f_2+65+f_1=75+f_1+f_2$

[1]

We know that median is 32, so it lies in the class 30 – 40.

Hence 30 – 40 is the median class.

We have, l = Lower limit of median class = 30, h = 10, f = Frequency of median class = 30, cf = Cumulative frequency of class preceding median class = $35 + f_1$, $N = 100$

We know,

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$ [1]

$$32 = 30 + \frac{\frac{100}{2} - (35+f_1)}{30} \times 10$$

$$\Rightarrow 2 = \frac{50 - (35+f_1)}{3}$$

$$\Rightarrow 6 = 15 - f_1$$

$$\Rightarrow f_1 = 9$$

We know that, $f_1 + f_2 = 25$ (Using (i))

$$\therefore f_2 = 16$$

The missing frequency $f_1 = 9$ and $f_2 = 16$. [1]

25. Find out the modal group (the group with highest frequency), which is 60 – 80.

The formula to estimate the Mode is:

$$\text{Mode} = L + \frac{(f_m - f_{m-1})}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w \qquad [2]$$

Where,

L = the lower boundary of the modal group = 60

f_{m-1} = the frequency of the group before the modal group = 10

f_m = the frequency of the modal group = 12

f_{m+1} = the frequency of the group after the modal group =6

w = the group width =20

$$\text{Mode} = 60 + \frac{(12-10)}{(12-10)+(12-6)} \times 20 \qquad [1]$$

$$\text{Mode} = 60 + \frac{2}{2+6} \times 20$$

$\Rightarrow$ Mode = 60 + 5 = 65

Hence, mode is 65. [1]

26. From question, total frequency is 100.

7 + 10 + x + 13 + y + 10 + 14 + 9 = 100

$\Rightarrow$ $x + y = 37$ ....(i)

Classes	x_i	Frequency (f_i)
0 – 10	5	7
10 – 20	15	10
20 – 30	25	x
30 – 40	35	13
40 – 50	45	y
50 – 60	55	10
60 – 70	65	14
70 – 80	75	9

[1]

The Mean will be given by the following formula:

$$\frac{\left[\begin{array}{l}(\text{Midpoint of Ist interval} \times \text{frequency1}) + \\ \text{.........................} + \left(\begin{array}{l}\text{Midpoint of 10th interval} \\ \times \text{frequency10}\end{array}\right)\end{array}\right]}{(\text{sum of frequencies})}$$

$$= \frac{\begin{array}{c}(5 \times 7) + (15 \times 10) + (25 \times x) + (35 \times 13) \\ + (45 \times y) + (55 \times 10) + (65 \times 14) + (75 \times 9)\end{array}}{(7 + 10 + x + 13 + y + 10 + 14 + 9)}$$

[1]

Simplifying we get:

$$\Rightarrow \frac{25x + 45y + 2775}{x + y + 63} = 42$$

$$\Rightarrow 25x + 45y + 2775 = 42(x + y + 63)$$

$$\Rightarrow 25x + 45y + 2775 = 42x + 42y + 2646$$

$$\Rightarrow 17x - 3y = 129 \quad ----------(ii)$$

Multiply (i) by 3,

$$\Rightarrow 3x + 3y = 111 \quad ----------(iii) \qquad [1]$$

Add (ii) and (iii),

$20x = 240$

Divide the above equation by 20,

$\Rightarrow x = 12$

Putting the above value in $x + y = 37$,

We get $y = 25$

Therefore, $x = 12$ and $y = 25$. [1]

27. Here the unknown frequency is x So, $l = 2$, $h = 0.2$, $f = 10$, $f_1 = 9$, $f_2 = x$

As we know

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h \qquad [2]$$

$$2.2 = 2.1 + \frac{10-9}{20-9-x} \times 0.2$$

$$2.2 - 2.1 = \left(\frac{1}{11-x}\right) \times 0.2 \qquad [1]$$

$$0.1 = \left(\frac{1}{11-x}\right) \times 0.2$$

$$\Rightarrow 11 - x = 2$$

$$\Rightarrow x = 9$$

Hence, the missing frequency is 9. [1]

28. Calculation of Mean

Marks	Frequency	Mid Values x_i	$d_i = x_i - A$ $= x_i - 112.5$	$u_i = \dfrac{x_i - A}{h}$	$f_i u_i$
0 – 25	10	12.5	–100	–4	–40
25 – 50	15	37.5	–75	–3	–45
50 – 75	22	62.5	–50	–2	–44
75 – 100	30	87.5	–25	–1	–30
100 – 125	28	112.5	0	0	0
125 – 150	27	137.5	25	1	27
150 – 175	12	162.5	50	2	24
175 – 200	6	187.5	75	3	18

[1]

$A = 112.5$ is the assumed Mean,

$h = 25$

Total of the frequency $N = 150$

$\sum f_i u_i = -90$

Using the formula

$$\text{Mean} = \overline{X} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$$

$$\overline{X} = 112.5 + 25\left(\frac{1}{150} \times -90\right)$$

$$= 112.5 + 25\left(\frac{-90}{150}\right)$$ [1]

$$= 112.5 + 25\left(\frac{-3}{5}\right)$$

$$= 112.5 - 15$$

Mean = 97.5

Calculation of Mode

Marks	0-25	25-50	50-75	75-100	100-125	125-150	150-175	175-200
Number of students	10	15	22	30	28	27	12	6

Here the maximum frequency is 30 and corresponding class is 75 – 100

$l = 75$ Lower limit of modal class

$f = 30$ Frequency of modal class

$h = 25$ Width of modal class

$f_1 = 22$ Frequency of class preceding the modal class

$f_2 = 28$ Frequency of class following the modal class [1]

Putting value is the formula

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 75 + \frac{30 - 22}{2 \times 30 - 22 - 28} \times 25$$

$$= 75 + \frac{8}{60 - 22 - 28} \times 25$$

$$= 75 + \frac{8}{10} \times 25$$

$$= 75 + 20$$

Mode = 95

Hence, the mean is 97.5 and mode is 95. [1]

29. Assume a value of x such that the sum of houses preceding the house numbered X is equal to sum of the numbers of houses following X.

That is, $1 + 2 + 3 \ldots + (x - 1) = (x + 1) + (x + 2) + \ldots + 49$ [1]

So,

$1 + 2 + 3 + \ldots + (x - 1) = \{1 + 2 + \ldots + x + (x + 1) + \ldots + 49\} - (1 + 2 + 3 + \ldots + x)$

$$\frac{(x-1)}{2}(1 + x - 1) = \frac{49}{2}(1 + 49) - \frac{x}{2}(1 + x) \quad [1]$$

$$x(x-1) = 49 \times 50 - x(1 + x)$$

$$x(x-1) + 1(1 + x) = 49 \times 50$$

$$x^2 - x + x + x^2 = 49 \times 50$$

$x^2 = 49 \times 25$ [1]

Taking square root,

$x = 7 \times 5 = 35$

Since x is not a fraction, the value of x satisfying the given condition exists and has a value of 35. [1]

[TOPIC 2] Cumulative Frequency Distribution

Summary

Graphical Representation of Cumulative Fequency Distribution

Let a grouped frequency distribution be given to us.

FOR A 'LESS THAN' SERIES

On a graph paper, mark the upper class limits along the x-axis and the corresponding cumulative frequencies along the y-axis.

(i) On joining these points successively by line segments, we get a polygon, called cumulative frequency polygon.

(ii) On joining these points successively by smooth curves, we get a curve, known as cumulative frequency curve or an ogive.

- ***Taken a point $A\left(0, \frac{N}{2}\right)$ on the y-axis and draw AP II x-axis, cutting the above curve at a point P.***

  Draw PM ⊥ x-axis, cutting the x-axis at M. Then, median = length of OM.

FOR A 'GREATER THAN' SERIES

On a graph paper, mark the lower class limits along the x-axis and the corresponding cumulative frequencies along the y-axis.

(i) On joining these points successively by line segments, we get a polygon, called cumulative frequency polygon.

(ii) On joining these points successively by smooth curves, we get a curve, known as cumulative frequency curve or an ogive.

- ***Let P be the point of intersection of 'less than' and 'more than' curves. Draw PM ⊥ x-axis, cutting x-axis at M. Then, median = length of OM.***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day.
   Write the above distribution as less than type cumulative frequency distribution.

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Cumulative frequency	60	42	55	70	53	20

[TERM 1, 2011]

2. If the 'less than' type ogive and 'more than' type ogive intersect each other at (20.5, 15.5) then the median of the given data is

(*a*) 36.0 (*b*) 20.5

(*c*) 15.5 (*d*) 5.5

[TERM 1, 2013]

2 Marks Questions

3. Convert the following distribution to a 'more than type' cumulative frequency distribution:

Class	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	4	8	10	12	10

[TERM 1, 2012]

4. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day

Age(in years)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of patients	60	42	55	70	53	20

Form 'less than type' cumulative frequency distribution.

[TERM 1, 2013]

5. Given below is a frequency distribution table showing daily income of 100 workers of a factory:

Daily income of workers (in Rs)	200-300	300-400	400-500	500-600	600-700
Number of workers	12	18	35	20	15

Convert this table to a cumulative frequency distribution table of 'more than type'.

[TERM 1, 2015]

4 Marks Questions

6. For the following frequency distribution, draw a cumulative frequency curve of less than type.

Class:	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
Frequency:	30	15	45	20	25	40	10	15

[TERM 1, 2012]

Solutions

1. The following is less than type cumulative frequency distribution.

Age(in years)	Cumulative frequency
Less than 20	60
Less than 30 Less than 40	$60 + 42 = 102$ $102 + 55 = 157$
Less than 50	$157 + 70 = 227$
Less than 60	$227 + 53 = 280$
Less than 70	$280 + 20 = 300$

[1]

2. The less than ogive curve gives cumulative frequency (probability) for $x \leq a$.

The more than ogive curve gives cumulative frequency (probability) for $x \geq a$.

They intersect exactly at the median because cumulative frequency will be 50% of the total at median. So, median is 20.5

Hence the correct option is (b). [1]

3. The following is more than type cumulative frequency distribution.

More than type	Cumulative Frequency
More than or equal to 10	$4 + 40 = 44$
More than or equal to 20	$32 + 8 = 40$
More than or equal to 30	$22 + 10 = 32$
More than or equal to 40	$10 + 12 = 22$
More than or equal to 50	10

[2]

4. The following is less than type cumulative frequency distribution.

Age (in years)	Cumulative frequency
Less than 20	60
Less than 30	$60 + 42 = 102$
Less than 40	$102 + 55 = 157$
Less than 50	$157 + 70 = 227$
Less than 60	$227 + 53 = 280$
Less than 70	$280 + 20 = 300$

[2]

5. The table can be re-written in 'more than type' as:

Daily income of the workers (In Rs)	Cumulative Frequency (C.F.)
More than 200	12 + (18 + 35 + 20 + 15) = 100
More than 300	18 + (35 + 20 + 15) = 88
More than 400	35 + (20 + 15) = 70
More than 500	20 + 15 = 35
More than 600	15

[2]

6. The less than type cumulative frequency for the given frequency distribution is:

Marks	Frequency	Cumulative Frequency
Less than 200	0	0
Less than 250	30	30
Less than 300	15	$30 + 15 = 45$
Less than 350	45	$45 + 45 = 90$
Less than 400	20	$20 + 90 = 110$
Less than 450	25	$25 + 110 = 135$
Less than 500	40	$40 + 135 = 175$
Less than 550	10	$10 + 175 = 185$
Less than 600	15	$15 + 185 = 200$

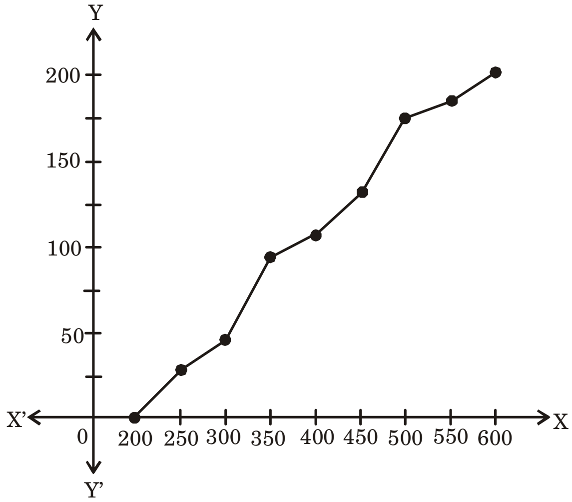
[2]

Mark the upper class limit on the x-axis and cumulative frequency on the y axis.

Then, plot the points (200, 0), (250, 30), (300, 45), (350, 90), (400, 110), (450, 110), (450, 135), (500, 175), (550, 185), (600, 200) and join the points.

Hence, the cumulative frequency curve of less than type is given below [1]

[1]



Smart Notes ....

Smart Notes ....

CHAPTER 15

Probability

Chapter Analysis of Previous Three Years' Board Exams

Number of Questions asked in Exams

	2018		2017		2016	
	Delhi	**All India**	**Delhi**	**All India**	**Delhi**	**All India**
Question based on Dice	2 marks	2 marks	3, 4 marks	4 marks	3 marks	
Question based on Cards					1 mark	1 mark
Question based on Coin						3 marks
Mix question on Probability	2 marks	2 marks	1 mark	1, 3 marks	4 marks	4 marks

Summary

Probability

Probability is a concept which numerically measures the degree of certainty of the occurrence of events.

EXPERIMENT

An operation which can produce some well-defined outcomes is called an experiment.

I. Tossing a coin. When we throw a coin, either a head (H) or a tail (T) appears on the upper face.

II. Throwing a die. A die is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5 and 6, or having 1, 2, 3, 4, 5 and 6 dots.

III. A deck of playing cards has in all 52 cards.

(i) It has 13 cards each of four suits, namely Spades, clubs, hearts and diamonds.

(a) Cards of spades and clubs are black cards.

(b) Cards of hearts and diamonds are red cards.

(ii) Kings, queens and jacks are known as face cards.

EVENT

The collection of all or some of the possible outcomes is called an event.

Examples:

(i) In throwing a coin, H is the event of getting a head.

(ii) Suppose we throw two coins simultaneously and let E be the event of getting at least one head. Then, E contains HT, TH, HH.

EQUALLY LIKELY EVENTS

A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

Probability of Occurrence of an Event

Probability of occurrence of an event E, denoted by $P(E)$ is defined as:

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes}}$$

COMPLEMENTARY EVENT

Let E be an event and (not E) be an event which occurs only when E does not occur.

The event (not E) is called the complementary event of E.

Clearly, $P(E) + P(\text{not } E) = 1$.

$\therefore\ P(E) = 1 - P(\text{not } E)$.

Some Special Sample Spaces

A die is thrown once: $S = \{1, 2, 3, 4, 5, 6\}; n(S) = 6$

A coin is tossed once: $S = \{H, T\};\ n(S) = 2$

A coin is tossed twice or Two coins are tossed simultaneously: $S = \{HH, HT, TH, TT\};\ n(S) = 4 = 2^2$

A coin is tossed three times or Three coins are tossed simultaneously:

$$S = \begin{Bmatrix} HHH, HHT, HTH, THH \\ TTT, TTH, THT, HTT \end{Bmatrix}; n(S) = 8 = 2^3$$

Two dice are thrown together or A die is thrown twice:

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$n(S) = 6^2$

- *1.* $P(E) = \dfrac{\textit{number of favourable cases}}{\textit{total number of cases}}$

  2. $P(E) + P(\text{not } E) = 1$

  3. $0 \le P(E) \le 1$

  *4. **Sum of the probabilities of all the outcomes of random experiment is 1.***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

1 Mark Questions

1. Which of the following cannot be the probability of an event?

(*a*) 1.5 (*b*) $\frac{3}{5}$

(*c*) 25% (*d*) 0.3

[TERM 2, 2011]

2. Cards bearing numbers 2, 3, 4 ... 11 are kept in a bag. A card is drawn at random from the bag. The probability of getting a card with a prime number is

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{5}$

(c) $\frac{3}{10}$ (*d*) $\frac{5}{9}$

[TERM 2, 2012]

3. The probability of getting an even number, when a die is thrown once, is:

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{5}{6}$

[TERM 2, 2013]

4. A box contains discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime-number less than 23, is:

(*a*) $\frac{7}{90}$ (*b*) $\frac{10}{90}$

(*c*) $\frac{4}{45}$ (*d*) $\frac{9}{89}$

[TERM 2, 2013]

5. In a family of 3 children, the probability of having at least one boy is

(*a*) $\frac{7}{8}$ (*b*) $\frac{1}{8}$

(*c*) $\frac{5}{8}$ (*d*) $\frac{3}{4}$

[TERM 2, 2014]

6. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4, is

(*a*) $\frac{4}{15}$ (*b*) $\frac{2}{15}$

(*c*) $\frac{1}{5}$ (*d*) $\frac{1}{3}$

[TERM 2, 2014]

7. A number is selected at random from the numbers 1 to 30. The probability that it is a prime number is:

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{6}$

(*c*) $\frac{1}{3}$ (*d*) $\frac{11}{30}$

[TERM 2, 2014]

8. If two different dice are rolled together, the probability of getting an even number on both dice, is:

(*a*) $\frac{1}{36}$ (*b*) $\frac{1}{2}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{1}{4}$

[TERM 2, 2014]

9. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

[TERM 2, 2016]

10. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.

[TERM 2, 2016]

11. A number is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What will be the probability that square of this number is less than or equal to 1?

[TERM 2, 2017]

2 Marks Questions

12. A coin is tossed two times. Find the probability of getting at least one head.

[TERM 2, 2011]

13. A card is drawn at random from a well-shuffled pack of cards. Find the probability of getting
    (i) A red king
    (ii) A queen or a jack

[TERM 2, 2012]

14. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

[TERM 2, 2013]

15. Rahim, tosses two different coins simultaneously. Find the probability of getting at least one tail.

[TERM 2, 2014]

16. Two different dice are tossed together. Find the probability
    (i) That the number on each die is even.
    (ii) That the sum of numbers appearing on the two dice is 5.

[TERM 2, 2014]

17. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

[TERM 2, 2017]

3 Marks Questions

18. Two dice are rolled once. Find the probability of getting such numbers on two dice, whose product is a perfect square.

[TERM 2, 2011]

19. A game consists of tossing a coin three times and noting its outcome each time. Hanif wins if he gets three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

[TERM 2, 2011]

20. All kings, queens and aces are removed from a pack of 52 cards. The remaining cards are well shuffled and then a card is drawn from it. Find the probability that the drawn card is
    (i) A black face card
    (ii) A red card

[TERM 2, 2012]

21. Three different coins are tossed together. Find the probability of getting
    (i) Exactly two heads
    (ii) At least two heads
    (iii) at least two tails

[TERM 2, 2016]

22. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice? (ii) a total of 9 or 11?

[TERM 2, 2016]

23. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

[TERM 2, 2017]

24. Two different dice are thrown together. Find the probability that the numbers obtained
    (i) have a sum less than 7
    (ii) have a product less than 16
    (iii) is a doublet of odd numbers.

[TERM 2, 2017]

4 Marks Questions

25. A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is
    (i) extremely patient
    (ii) extremely kind or honest.

    Which of the above values you prefer more?

[TERM 2, 2013]

26. Red queens and black jacks are removed from a pack of playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is
    (i) a king (ii) of red colour
    (iii) a face card (iv) a queen

[TERM 2, 2014]

27. A bag contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:
    (i) an odd number
    (ii) a multiple of 5
    (iii) a perfect square
    (iv) an even prime number

[TERM 2, 2014]

28. Two different dice are thrown together. Find the probability that the numbers obtained have
    (i) Even sum, and
    (ii) Even product.

[TERM 2, 2017]

29. Peter throws two different dice together and finds the product of the two numbers as 25. Rina throws a die and squares the number obtained. Who has the better chance to get the number

[TERM 2, 2017]

Solutions

1. Let E be any event and $P(E)$ be the probability of the happening of that event.
   The value of $P(E)$ will lie in the range $0 \le P(E) \le 1$
   Therefore, its value can never be greater than 1.
   Hence, the correct option is (a).
2. Total cards in the bag = 2, 3, 4, ..., 11 = 10 [1]
   The cards bearing prime numbers
   = 2, 3, 5, 7, 11 = 5
   Therefore, the probability of getting a card with a prime number
   $$= \frac{\text{Number of cards bearing prime numbers}}{\text{Total number of cards}}$$
   $$= \frac{5}{10}$$
   $$= \frac{1}{2}$$ [1]
   Hence, the correct option is (a).
3. In an event of throwing a die,
   Total number of possible outcomes = 6
   In a die, even numbers are 2, 4 and 6
   Thus, number of favourable outcomes = 3
   ∴ Probability of getting an even number
   $$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
   $$\Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}$$
   Thus the probability of getting an even number, when a die is thrown once is $\frac{1}{2}$ [1]
   Hence the correct option is (a).
4. There are 90 discs in the box and one disc is drawn at random.
   So, the total number of possible outcomes = 90
   Prime numbers less than 23 are 2, 3, 5, 7, 11, 13, 17 and 19.
   ∴ Number of favourable outcomes = 8
   ∴ Probability of getting a prime numbers less than 23 [½]
   $$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
   $$\Rightarrow P(E) = \frac{8}{90} = \frac{4}{45}$$
   Thus the probability of getting a disc with prime number less than 23 is $\frac{4}{45}$
   Hence the correct answer is (c). [½]
5. Probability of having a boy child is $\frac{1}{2}$
   Probability of having a girl child is $\frac{1}{2}$
   Probability of having no boy in the family
   $$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$ [½]
   Thus, the probability of having at least 1 boy
   $$= 1 - \frac{1}{8}$$
   $$= \frac{8-1}{8} = \frac{7}{8}$$
   The correct answer is (a). [½]
6. It is given that the numbers are from 1 to 15.
   Total possibilities = 15
   Out of the first 15 numbers chosen, multiples of 4 are 4, 8, 12
   Favorable outcomes = 3
   The probability that a number selected at random [½]
   from the numbers 1, 2, 3, ..., 15 is a multiple of 4,
   $$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}.$$
   $$= \frac{3}{15} = \frac{1}{5}$$
   Thus the correct answer is (c). [½]
7. Total number of possible outcomes = 30
   Prime numbers between 1 to 30 are 2, 3, 5, 7, 11, 13, 17,19, 23 and 29.
   Total number of favourable outcomes = 10
   Probability of selecting a prime number from 1 to 30
   $$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}}$$
   $$= \frac{10}{30} = \frac{1}{3}$$
   Hence, the correct option is (c). [1]

8. Possible outcomes on rolling the two dice are given below:

$$\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$$ [½]

Total number of outcomes = 36

Favorable outcomes are given below:

$\{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}$

Total number of favourable outcomes = 9

$\therefore$ Probability of getting an even number on both dice

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}}$$

$$= \frac{9}{36} = \frac{1}{4}$$

Hence, the correct option is (d). [½]

9. There are 26 red cards and 26 black cards in a pack of 52 playing cards.

Total number of outcome = 52

Favorable number of outcome (neither a red card nor a queen) = 52 – (26 + 2) = 24

Probability of getting neither a red card nor a queen

$$= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{24}{52} = \frac{6}{13}$$

Hence, the probability of getting neither a red card nor a queen is $\frac{6}{13}$. [1]

10. It is given that the box contains cards marked with numbers 3, 4, 5, ..., 50

Therefore, the total number of outcomes = 48

The perfect squares between numbers 3 and 50 are 4, 9, 16, 25, 36 and 49.

So, the number of favourable outcomes = 6

Hence, the probability that a card drawn at random is a perfect square

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{48} = \frac{1}{8}$$ [1]

11. Here, the sample space S is

S = {–3, –2, –1, 0, 1, 2, 3}

And, the favourable outcomes E is E = {–1, 0, 1}

$$\therefore P(E) = \frac{\text{Number of elements in } E}{\text{Number of elements in } S}$$

$$\Rightarrow P(E) = \frac{3}{7}$$ [½]

Hence, the probability of getting a number whose square is less than or equal to 1 is $\frac{3}{7}$. [½]

12. When a coin is tossed two times, the elementary outcomes are: $\{HH, HT, TH, TT\}$ [1]

Total outcomes = 4

Favorable outcome for at least one head

= {HT, TH, HH} = 3

Probability of getting at least one head

$$= \frac{\text{Favourable outcome}}{\text{Total outcome}} = \frac{3}{4}$$

Hence, the probability of getting at least one head is $\frac{3}{4}$. [1]

13. (i) In a deck of well-shuffled pack of 52 playing cards, there are 2 red cards with 'king' face cards.

Therefore, the probability of getting a red king

$$= \frac{2}{52} = \frac{1}{26}$$ [1]

(ii) In a deck of well-shuffled pack of 52 playing cards, there are 4 'queen' cards and 4 'jack' cards.

Therefore, the probability of getting a queen or a jack $= \frac{8}{52} = \frac{2}{13}$ [1]

14. Total number of possible outcomes = 52

Number of cards that are king or queen= 4 + 4=8

$\therefore$ Number of other cards = 52 – 8 = 44

Thus the number of cards which are neither a king nor a queen = 44

Total number of favourable out comes = 44

$\therefore$ Probability of getting a card which is neither a king nor a queen [1]

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{44}{52} = \frac{11}{13}$$

Thus the probability of getting a card which is neither a king nor a queen is $\frac{11}{13}$ [1]

15. Rahim tosses two coins simultaneously. The sample space of the experiment is {HH, HT, TH and TT}

Total number of outcomes = 4 [1]

Number of outcomes which are in favour of getting at least one tail on tossing the two coins = {HT, TH, TT}

Number of outcomes in favour of getting at least one tail = 3

Probability of getting at least one tail on tossing the two coins

$$= \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}}$$

$$= \frac{3}{4}$$ [1]

16. Two dice are tossed together, $n(S) = 6^2 = 36$

(i) Let P be the event of getting an even number

$$n(P) = \left\{\begin{matrix}(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),\\(6,2),(6,4),(6,6)\end{matrix}\right\} = 9$$

[1]

Probability that the number on each die is even

$$= \frac{n(P)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) Let Q be the event of getting a sum of 5 on the two dice

$$n(Q) = \{(1,4),(2,3),(3,2),(4,1)\} = 4$$

Probability that the sum of numbers appearing on the two dice is $= \frac{n(Q)}{n(S)} = \frac{4}{36} = \frac{1}{9}$ [1]

17. Let A be the event of selecting rotten apples.

Let n be the number of rotten apples from the heap.

Probability of an event

$$A = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$ [1]

$$P(A) = \frac{n}{900}$$

$$\Rightarrow 0.18 = \frac{n}{900}$$

$$\Rightarrow n = 162$$

Thus, there are 162 rotten apples in the heap. [1]

18. When two dice are rolled, then the possible outcomes are:

$$\begin{pmatrix}(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6)\\(2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6)\\(3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6)\\(4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6)\\(5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6)\\(6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\end{pmatrix} = 36$$ [1]

outcomes

Favorable outcomes for the numbers on two dice, whose product is a perfect square are

$$\{(1,1)\,(1,4)\;(2,2)\;(3,3)\;(4,1)\;(4,4)\;(5,5)\;(6,6)\},$$

i.e. 8 outcomes. [1]

Therefore, probability of getting such numbers on two dice, whose product is a perfect square

$$= \frac{\text{Favourable outcome}}{\text{Total outcome}} = \frac{8}{36} = \frac{2}{9}$$ [1]

19. When a coin is tossed three times, then the possible outcomes are:

{(THH), (TTT), (TTH), (THT), (HHH), (HTT), (HTH), (HHT)} [1]

$\therefore$ Total outcomes = 8

Favorable outcome for three heads or three tails are (HHH), (TTT) i.e. 2 outcomes.

Probability of Hanif winning the game

$$= \frac{\text{Favourable outcome for three heads or three tails}}{\text{Total outcome}}$$ [1]

$$= \frac{2}{8} = \frac{1}{4}$$

Clearly, probability of Hanif losing the game

= 1 – Probability of winning

$$\Rightarrow 1 - \frac{1}{4}$$

$$\Rightarrow \frac{3}{4}$$

Therefore, the probability that Hanif will lose the game is $\frac{3}{4}$. [1]

20. In a pack of 52 playing cards, there are 4 kings (2 red + 2 black), 4 queens (2 red + 2 black) and aces (2 black + 2 red).

So, when all the kings, queens and aces are removed from a pack of 52 cards, the remaining cards are 52 – 12 = 40 [1]

Now, in a standard deck of 52 playing cards, there are four suits: clubs, diamonds, hearts and spades. Each suit has one Jack, Queen, and King as the face cards. Hence there are 12 face cards in a deck of 52 playing cards.

So, after removing all the kings and queens cards from the pack, there will be 4(2 red + 2 black) face cards remaining in the deck of 40 cards.

(i) Probability of getting a black face card is

$= \frac{2}{40} = \frac{1}{20}$. [1]

(ii) In a deck of 52 cards, 26 cards are black cards and 26 cards are red cards. When all the kings, queens and aces are removed, out of the m 2 kings, 2 queens and 2 aces are red cards. So, the total number of red cards remaining in the deck of cards is 26 – 6 = 20.

Probability of getting a red card $= \frac{20}{40} = \frac{1}{2}$ [1]

21. The possible outcomes, when three coins are tossed together are

{HHH, HHT, HTT, HTH, THH, TTH, THT, TTT}

Therefore, total number of possible outcomes= 8 [1]

(i) Favorable outcomes of exactly two heads are HHT, HTH, THH

Therefore, total number of possible outcomes= 3

Probability of getting exactly two head

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{3}{8}$$

[1]

(ii) Favorable outcomes of at least two heads are HHH, HHT, HTH, THH

Therefore, total number of possible outcomes= 4

Probability of getting at least two heads

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{4}{8} = \frac{1}{2}$$

(iii) Favorable outcomes of at least two tails are HHT, TTH, THT, TTT

Therefore, total number of possible outcomes= 4

Probability of getting at least two tails

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{4}{8} = \frac{1}{2}$$

[1]

22. We know that the total number of outcomes on throwing a pair of dice = 6 × 6 = 36

(i) Let A be the event of getting a prime number on each dice.

So, the favourable outcomes

$$= \begin{Bmatrix} (2,\ 2),(2,\ 3),(2,\ 5),(3,\ 2),(3,\ 3),(3,\ 5), \\ (5,\ 2),(5,\ 3),(5,\ 5) \end{Bmatrix}$$ [1]

Number of favourable outcomes = 9

Now,

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{9}{36} = \frac{1}{4}$$

Thus, the probability of getting a prime number on each dice is $\frac{1}{4}$. [1]

(ii) Let B be the event of getting a total of 9 or 11.

So, the favourable outcomes

$$= \{(3,\ 6),(4,\ 5),(5,4),(6,\ 3),(5,\ 6),(6,\ 5)\}$$

Number of favourable outcomes = 9

Now, $P(B) = \frac{6}{36} = \frac{1}{6}$

Thus, the probability of getting a total of 9 or 11 is $\frac{1}{6}$. [1]

23. Let us assume that the number of black ball = x

Number of white balls = 15

$P(\text{Black Ball}) = 3 \times P(\text{White Balls})$ [1]

$\Rightarrow \frac{x}{15+x} = 3 \times \frac{15}{15+x}$ [1]

$\Rightarrow x = 3 \times 15$

$\Rightarrow x = 45$

Therefore, number of black balls are 45. [1]

24. We know that the total number of outcome when two dice are thrown together is 36

(i) Have a sum less than 7

The favourable outcomes are

$$= \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(2,1), \\ (2,2),(2,3),(2,4),(3,1),(3,2),(3,3), \\ (4,1),(4,2),(5,1) \end{Bmatrix}$$

Number of favourable outcomes = 15

So, probability of getting a sum less than 7

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{15}{36}$$

$$= \frac{5}{12}$$ [1]

(ii) Have a product less than 16

The favourable outcomes are

$$= \left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),\\ (2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),\\ (3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(5,1),\\ (5,2),(5,3),(6,1),(6,2)\end{matrix}\right\}$$

Number of favourable outcomes = 25

So, probability will be

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{36}$$ [1]

(iii) A doublet of odd numbers.

The favourable outcomes are

= (1, 1), (3, 3), (5, 5)

Number of favourable outcomes = 3

So, probability will be

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

$$= \frac{1}{12}$$ [1]

25. Number of people in the group = 12

Number of possible outcomes = 12

Let us assume, E_1 as an event for selecting extremely patient people and E_2 be the event for selecting extremely king or honest.

Number of outcomes for E_1 = 3

Number of extremely honest people = 6

Extremely kind people = 12 – (6 + 3) = 3

Number of outcomes for E_2 = 9 [2]

(i) P (Extremely Patient)

$$= P(E_1) = \frac{\text{Outcomes for } E_1}{\text{Total possible outcomes}} = \frac{3}{12} = \frac{1}{4}$$ [1]

(ii) P (Kind and Honest)

$$= P(E_2) = \frac{\text{Outcomes for } E_2}{\text{Total possible outcomes}} = \frac{9}{12} = \frac{3}{4}$$ [1]

26. Total cards in a pack = 52

Number of black jacks = 2

Number of red queens = 2

Number of cards remaining after removing black jacks and red queens = 52 – 2 – 2 = 48

(i) Probability that the card drawn is a king

$$= \frac{\text{Number of kings}}{\text{Total number of cards}}$$

Probability that the card drawn is a king

$$= \frac{4}{48} = \frac{1}{12}$$ [1]

The probability of drawing a king after removing black jacks and red queens is $\frac{1}{12}$

(ii) Probability that the card drawn is of red color

$$= \frac{\text{Number of red cards}}{\text{Total number of cards}}.$$

There are 26 red cards in a pack out of which 2 red cards are removed

Number of red cards remaining = 26 – 2 – 24

Probability that the card drawn is of red color

$$= \frac{24}{48} = \frac{1}{2}$$

The probability of drawing a red card after removing black jacks and red queens is $\frac{1}{2}$ [1]

(iii) Probability that the card drawn is a face card

$$= \frac{\text{Number of face cards}}{\text{Total number of cards}}$$

There are 12 face cards in a pack out of which 4 are removed.

Number of face cards remaining = 12 – 4 = 8

Probability that the card drawn is a face card

$$= \frac{8}{48} = \frac{1}{6}$$

The probability of drawing a face card after removing black jacks and red queens is $\frac{1}{6}$ [1]

(iv) Probability that the card drawn is a queen

$$= \frac{\text{Number of queens}}{\text{Total number of cards}}$$

There are 4 queens in a pack of cards out of which 2 are removed.

Number of queens remaining = 4 – 2 = 2

Probability that the card drawn is a queen

$$= \frac{2}{48} = \frac{1}{24}$$

The probability of drawing a queen after removing black jacks and red queens is $\frac{1}{24}$. [1]

27. (i) Total number of cards = 49

Odd numbers from 1 to 49 are

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

Total number of favourable outcomes = 25

Hence the require probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{25}{49}$$ [1]

(ii) Sample space or total number of outcome=49

Multiples of 5 that can be considered as the favourable number of outcomes are

5, 10, 15, 20, 25, 30, 35, 40, 45

The number of favourable outcomes = 9

Hence the required probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{9}{49}$$ [1]

(iii) Sample space or total number of outcomes=49

The numbers less than or equal to 49 that are perfect squares are

1, 4, 9, 16, 25, 36, 49

Total number of favourable outcomes = 7

Hence the required probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{7}{49}$$ [1]

(iv) Sample space or total number of out20 comes = 49

Only one even prime number exists and that is, 2

Total number of favourable outcomes = 1

Hence the required probability

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total possible outcomes}} = \frac{1}{49}$$ [1]

28. The outcomes when two dices are thrown together

$$\begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$ [2]

There are 36 total outcomes

(i) When sum of numbers is even

Let B be the event of getting even sum.

$$\begin{Bmatrix} (1,1), (1,3), (1,5) \\ (2,2),(2,4),(2,6) \\ (3,1),(3,3),(3,5) \\ (4,2),(4,4),(4,6) \\ (5,1),(5,3),(5,5) \\ (6,2),(6,4),(6,6) \end{Bmatrix}$$ [1]

There are 18 favourable outcomes.

Probability for even sum outcomes

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

(ii) Even product outcome

Let B be the event of getting even product. [1]

29. We know that the total number of outcome when two dice are thrown together is 36

Favorable out comes = (5, 5) [1]

Probability of getting two numbers having product as $25 = \frac{1}{36}$ [1]

Rina throws the die only once. So, the total number of outcomes will be 6 only.

Favorable outcomes = (5) [1]

Probability of getting a number whose square is $25 = \frac{1}{6}$

Clearly, $\frac{1}{6} > \frac{1}{36}$

Hence, Rina has the better chance to get the number 25. [1]

Smart Notes ....

Smart Notes ....

CBSE
Sample Question Paper 1

Mathematics
Class X

Time : 3 hrs **MM : 80**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 30 questions divided into four sections A, B, C and D.

(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

(1 × 6 = 6)

1. 'The product of two consecutive positive integers is divisible by 2'. Is this statement true or false? Give reasons.

2. For all real values of c, the pair of equations

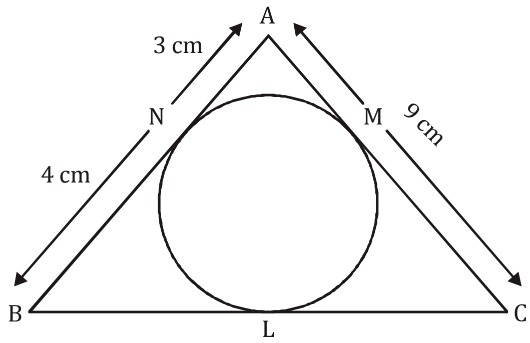
$$x - 2y = 8 \text{ and } 5x - 10y = c$$

has a unique solution. Justify whether it is true or false.

3. Can the quadratic polynomial $x^2 + kx + k$ have equal zeros for odd integer $k > 1$?

4. Find the 7th term from the end of the A.P.: 7, 10, 13, …, 184.

5. In Fig. ΔABC is circumscribing a circle. Find the length of BC.



6. If a dice is rolled, find the probability that number which turnup is even prime.

SECTION B

(2 × 6 = 12)

7. Show that 21^n cannot end with the digits 0, 2, 4, 6 or 8 for any natural number n.

8. If α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

9. Solve the quadratic equation $x^2 - 4x + 1 = 0$ by the method of completing the square.

10. Prove that : $(\text{cosec}\,\theta + \cot\theta)^2 = \dfrac{\sec\theta + 1}{\sec\theta - 1}$.

11. Evaluate : $\frac{4}{3}\cot^2 30° + 3\sin^2 60° - 2\text{cosec}^2 60° - \frac{3}{4}\tan^2 30°$.

12. A chord of a circle of radius 12 cm subtends an angle of 120^0 at the centre. Find the area of the corresponding segment of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

SECTION C

(3 × 10 = 30)

13. Show that $5 - \sqrt{3}$ is irrational.

OR

Prove that $\sqrt{2} + \sqrt{5}$ is irrational.

14. If one zero of the polynomial $3x^2 - 8x - (2k + 1)$ is seven times the other, find both zeros of the polynomial and the value of k.

15. Determine graphically, the vertices of the triangle formed by the lines y = x, 3y = x and x + y = 8.

OR

The area of a rectangle gets reduced by 9 sq. units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, then the area is increased by 67 sq. units. Find the dimensions of the rectangle.

16. Which term of the sequence 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term?

17. If P(9a – 2, – b) divides the line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3: l, find the values of a and b.

18. If the point A (2, – 4) is equidistant from P (3, 8) and Q (– 10, y), find the value of y. Also find distance PQ.

19. Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

OR

D and E are points on the sides AB and AC respectively of ΔABC such that DE is parallel to BC, and AD : DB = 4 : 5. CD and BE intersect each other at F. Find the ratio of the areas of ΔDEF and ΔCBF.

20. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by

$$r = \frac{a + b - c}{2}.$$

OR

T' AT is a tangent drawn to the circle with centre O. If ∠BAC = 50°, find ∠BAT and ∠BCA. Also find ∠DAB + ∠DCB.

21. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be

(i) red ?

(ii) white ?

(iii) not green?

22. The lengths of 40 leaves of a plant are measured correctly to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	118–126	127–135	136–144	145–153	154–162	163–171	172–180
Number of Leaves	3	5	9	12	5	4	2

Find the median length of the leaves.

SECTION D

(4 × 8 = 32)

23. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do it so? If yes, at what distances from the two gates should the pole be erected?

24. Sides AB, AC and median AD of a triangle ABC are respectively proportional to sides PQ, PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

OR

ABC is a triangle in which AB = AC and D is a point on AC such that $BC^2 = AC \times CD$. Prove that BD = BC.

25. Let ABC be a right angled triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90°$. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

26. Prove that:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta.\text{cosec}\,\theta.$$

OR

If $\sin\theta + \sin^2\theta = 1$, find the value of

$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta\cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$

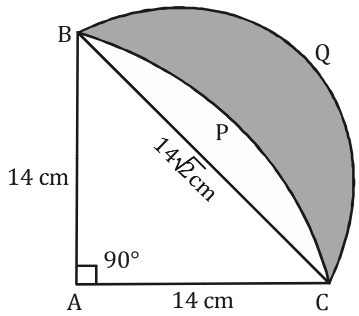
27. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of the two ships as observed from the top of the light house are 60^0 and 45^0. If the height of the light house is 200 m, find the distance between the two ships. [Use $\sqrt{3}$ = 1.73]

28. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

OR

Two types of water tankers are available in a shop at same cost. One is in a cylindrical form of diameter 3.5 m and height 2 m and another is in the form of a sphere of diameter 3 m. Calculate the volume of both tankers. The shopkeeper advises to purchase cylindrical tanker. What value is depicted by the shopkeeper?

29. In Fig. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



30. The median of the following data is 50. Find the values of p and q, if the sum of all the frequencies is 90.

Marks	20–30	30–40	40–50	50–60	60–70	70–80	80–90
Frequency	P	15	25	20	q	8	10

CBSE
Sample Question Paper 2

Mathematics
Class X

Time : 3 hrs **MM : 80**

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 30 questions divided into four sections A, B, C and D.

(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

(1 × 6 = 6)

1. What is L.C.M. of smallest prime number and smallest composite number?

2. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the polynomial?

3. If radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

4. Is 0 a term of the AP 31, 28, 25,...? Justify your answer.

5. For what value of k, will the system of equations $x + 2y = 5$

$3x + ky - 15 = 0$. have no solution?

6. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is a prime number.

SECTION B

(2 × 6 = 12)

7. Find the largest number which divides 70 and 125, leaving remainders 5 and 8 respectively.

8. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then prove that the product of other two zeroes is $b - a + 1$.

9. For what value of K the equation : $x^2 + kx - 1 = 0$ has equal roots?

10. If $x = a \cos \theta$, $y = b \sin \theta$, then find the value of $b^2x^2 + a^2y^2 - a^2b^2$.

11. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

12. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

SECTION C

(3 × 10 = 30)

13. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

14. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

OR

Find the sum of the two middle most terms of the A.P.:

$$\frac{-4}{3}, -1, \frac{-2}{3}, \ldots, 4\frac{1}{3}.$$

15. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 4x + 1$, find a quadratic polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

16. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

17. Find the value of m, if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

OR

The centre of a circle is $(x + 2, x - 1)$. Find x if the circle passes through (2, –2) and (8, –2).

18. In what ratio does the X-axis divide the line segment joining the points (–4, –6) and (–1, 7)? Find the coordinates of the points of division.

19. P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that:

(i) $4\,AQ^2 = 4AC^2 + BC^2$

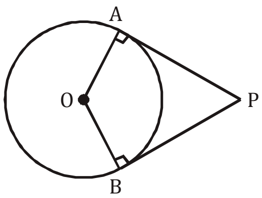
(ii) $4BP^2 = 4BC^2 + AC^2$

(iii) $4(AQ^2 + BP^2) = 5AB^2$

OR

In ΔABC, If $AP \perp BC$ and $AC^2 = BC^2 - AB^2$, then prove that $PA^2 = PB \times CP$.

20. In figure, OP is equal to diameter of the circle. Prove that ABP is a equilateral triangle.



21. A bag contains 35 balls out of which x are blue.

(i) If one ball is drawn at random, what is the probability that it will be a blue ball?

(ii) If 7 more blue balls are put in the bag, the probability of drawing a blue ball will be double than that in (i).

Find x.

OR

A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

22. Find the mean of the following distribution by Assumed Mean Method:

10–20	8
20–30	7
30–40	12
40–50	23
50–60	11
60–70	13
70–80	8
80–90	6
90–100	12

SECTION D

(4 × 8 = 32)

23. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

A shopkeeper buys a number of books for 80. If he had bought 4 more books for the same amount, each book would have cost 1 less. How many books did he buy ?

24. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

25. Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$

OR

If tan A = n tan B and sin A = m sin B, prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$.

26. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

27. Construct two circles of radii 3 cm and 5 cm, such that their centres are 12 cm apart. Join their centres and construct the perpendicular bisector of the line segment thus obtained. Take a point M, 3.5 cm away from the mid-point of the line segment joining the two centres and lying on perpendicular bisector. From M, construct tangents to the bigger circle. Write steps of construction.

28. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem?

$\left[\text{Use } \pi = \frac{22}{7}\right]$.

29. A well of diameter 3 m is dug 14 m deep. The mud taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

OR

Milk in a container, which is in the form of a frustum of a cone of height 30 cm and the radii of whose lower and upper circular ends are 20 cm and 40 cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of 35 per litre and 880 litres of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and what cost will it put on the donor agency for this.

30. 50 students enter a school javelin throw competition. The distance (in metre) thrown are recorded below

Distance (in m)	0-20	20-40	40-60	60-80	80-100
Numbers of students	6	11	17	12	4

(i) Construct a cumulative frequency table.

(ii) Draw a cumulative frequency curve (less than type) and calculate the median distance drawn by using this curve.

(iii) Calculate the median distance by using the formula for median.

(iv) Are the median distance calculated in (ii) and (iii) same?