CHAPTER 1

Electric Charges and Fields

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Electric Charges		2Q (1 mark)			
Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges	1Q (1 mark)				
Superposition principle and continuous charge distribution					
Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.			1Q (3 marks)		
Electric flux, statement of Gauss's theorem	1Q (1 mark)				2Q (5 marks)
Gauss's theorems applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside)	1Q (3 marks)	1Q (1 mark), 1Q (3 marks)	1Q (1 mark)	2Q (5 marks)	2Q (3 marks)

On the basis of above analysis, it can be said that from exam point of view Electric Flux, Gauss theorem and its applications , electric force on a charge and Electric Field due Electric Dipole are the most important topics of the chapter.

[Topic 1] Coulomb's law, electrostatic field and electric dipole

Summary

Electric Charge

- Electrostatic charge is a fundamental property of matter due to which it produces and experiences electrical and magnetic effects.
- Properties of atoms, molecules and bulk matter are determined by electric and magnetic forces.
- It can be inferred from simple experiments based on frictional electricity that there are two type of charges in nature: negative and positive; and like charges repel and unlike charges attract.
- By convention, the charge on electron is considered as negative and the charge on proton is considered as positive and the charge present is equal. The S.I. unit of electric charge is coulomb. Its C.G.S unit is stat coulomb.
- The nature and amount of electric charge present in a charged body is detected by Gold-leaf electroscope.
- Total charge on a body is expressed as $q = \pm ne$.

Conductors and Insulators

- Objects that allow charges to flow through them are called Conductors (metals) and objects that do not allow charges to flow through are called Insulators (rubber, wood, and plastic).
- Objects that behave as an intermediate between conductors and insulators are called semi-conductors, for example- silicon.
- The process of sharing charges with the earth, when we bring a charged body in contact with the earth is called grounding or earthing.

Charging by Induction

- Charging by induction means charging without contact.
- If a plastic comb is rubbed with wool, it becomes negatively charged.

Three basic properties of electric charge

- **Quantization:** When the total charge of a body is an integral multiple of a basic quantum of charge, this is known as quantization of electric charge. i.e., $q = ne$ where

  $n = \pm 1, \pm 2, \pm 3,$ ....
- **Additivity:** It means that the total charge of a system is the algebraic sum (adding taking into account negative and positive signs both) of all the charges in the system.
- **Conservation of charge:** Conservation of electric charges means that there will be no change in the total charge of the isolated system with time. There is transfer of the electric charge from one body to another, but no charge will be created or destroyed.

Coulomb's law

The force between two point charges q_1 and q_2 is directly proportional to the product of the two charges(q_1q_2) and inversely proportional to the square of the distance between them(r^2)and it acts along the straight line joining the two charges.

$$F_{12} = \text{force on } q_2 \text{ due to } q_1 = \frac{k\left(q_1q_2\right)}{r^2_{21}}\hat{r}_{21}$$

where $k = \dfrac{1}{4\pi\varepsilon_0}$

The experimental value of the constant ε_0 is

$8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

Therefore, the approximate value of k is

$9 \times 10^9 Nm^2C^{-2}$

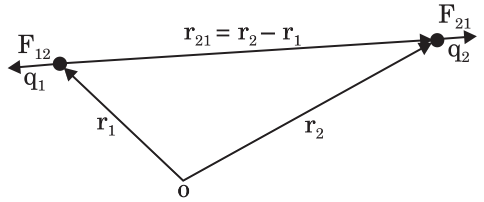


Fig. Depiction of Coulomb's law

Facts about Coulomb's law:

- Coulomb's law is not valid for charges in motion; it should only be used for point charges in vacuum at rest.
- The electrostatic force obeys Newton's third law of motion and acts along the line joining the two charges.
- Presence of other charges in the neighborhood does not affect Coulomb's force.
- The ratio of electric force and gravitational force between a proton and an electron is represented by $\frac{ke^2}{Gm_em_p} \cong 2.4 \times 10^{39}$

Superposition Principle

The presence of an (or more) additional charge does not affect the forces with which two charges attract or repel each other. Superposition principle states that the net force on any charge due to n number of charges at rest is the vector sum of all the forces on that charges, taken one at a time.

i.e. $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ..\vec{F}_{0n}$

- The force on a small positive test charge q placed at the point divided by the magnitude of the charge is the electric field E at a point due to charge configuration.

Electric Field

- The space around a charge up to which its force can be experienced is called electric field.
- Electric field due to a point charge q has a magnitude $E(r) = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$
  - ➢ It is radially outwards if q is positive.
  - ➢ It is radially inwards if q is negative.
- Electric field satisfies the superposition principle.
  - ➢ The unit of electric field is N/C.
  - ➢ Electric field inside the cavity of a charged conductor is zero.

Electric Field lines

- The tangent at each point on the curve of electric field line, gives the direction of electric field at that point.
- The relative strength of electric field at different points is indicated by the relative closeness of field lines.
  - ➢ In regions of strong electric field, they crowd near each other.
  - ➢ In regions of weak electric field, they are far apart.
  - ➢ In regions of constant electric field, the field lines formed are uniformly spaced parallel straight lines.
- Field lines are continuous curves. There will be no breaks.

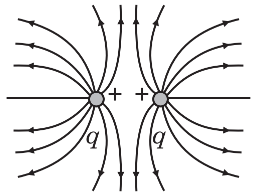


Fig. Electric field lines

- Field lines are not intersecting. They cannot cross each other.
- Electrostatic field lines begin at positive charges and terminate at negative charges.
- No closed loop can be formed by them.

Electric Dipole

- A pair of equal and opposite charges q and –q separated by small distance 2a is known as electric dipole. The magnitude of its dipole moment vector is 2qa and is in the direction of the dipole axis from –q to q.

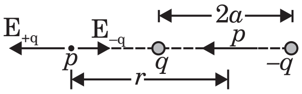


Fig. Electric dipole

- Field of an electric dipole in its equatorial plane at a distance r from the center:

$$E = \frac{-p}{4\pi\varepsilon_o}\frac{1}{\left(a^2 + r^2\right)^{3/2}}$$

$$\cong \frac{-p}{4\pi\varepsilon_o r^3} \quad \text{for } r >> a$$

- Dipole electric field on the axis at a distance r from the center:

$$E = \frac{2pr}{4\pi\varepsilon_o\left(r^2 - a^2\right)^2}$$

$$\cong \frac{2p}{4\pi\varepsilon_o r^3} \quad \text{for } r >> a$$

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $1/r^2$ dependence of electric field due to a point charges.

- In a uniform electric field E, a dipole experiences a torque τ given by

$\tau = p \times E$

But no net force will be experienced by it.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

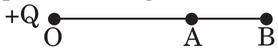
1. What is the geometrical shape of equipotential surface due to a single isolated charge?

[DELHI 2014]

2. Why do the electric field lines never cross each other?

[ALL INDIA 2014]

3. A point charge +Q is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero?



[DELHI 2016]

4. In which orientation, a dipole placed in a uniform electric field is in (i) stable, (ii) unstable equilibrium?

[DELHI 2018]

5. Draw a graph to show the variation of E with perpendicular distance r from the line of charge.

[DELHI 2018]

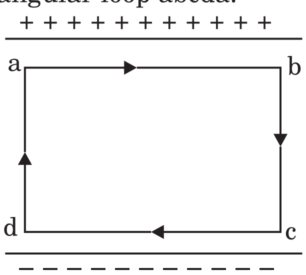
2 Marks Question

6. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has charge ± 8 nC. **[DELHI 2014]**

3 Marks Questions

7. (a) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(b) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop abcda.



OR

(a) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d.

(b) Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting wire acquire charge q_1 and q_2 respectively. Find the ratio of their surface charge densities in terms of their radii.

[DELHI 2014]

8. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.

[DELHI 2016]

5 Marks Question

9. (a) Derive an expression for the electric field E due to a dipole of length "2a' at a point distant r from the centre of the dipole on the axial line.

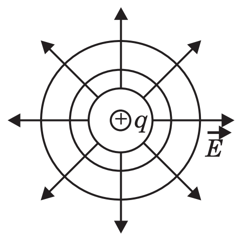
(b) Draw a graph of E versus r for r >> a.

(c) If this dipole were kept in a uniform external electric field E_0,diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

[ALL INDIA 2017]

Solutions

1. The equipotential surfaces of an isolated charge are concentric spherical shells(co-centric shells) and potential will be inversely proportional to distance. **[½]**

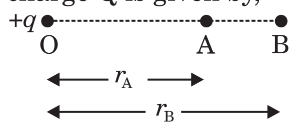


[½]

Fig. Equipotential surfaces of an isolated charge

2. If two electric fields cross each other then there would be two different values of electric field with individual directions at that location which is impossible, hence electric field lines never cross each other. **[1]**

3. Potential at a distance r from a given point charge Q is given by,



[½]

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$

$$V_A = \frac{Q}{4\pi\varepsilon_o r_A}$$

$$V_B = \frac{Q}{4\pi\varepsilon_o r_B}$$

Since, $r_A < r_B \Rightarrow V_A > V_B$

Hence, $V_A - V_B$ is positive. **[½]**

4. A dipole placed in a uniform electric field is in:

(i) Stable Equilibrium: When the electric field is directed along the direction of the dipole i.e. when $\vec{E}$ is parallel to $\vec{p}$. **[½]**

(ii) Unstable Equilibrium: When the electric field is directed at an angle of $180°$ with the direction of the dipole i.e. when $\vec{E}$ is anti-parallel to $\vec{p}$. **[½]**

5.

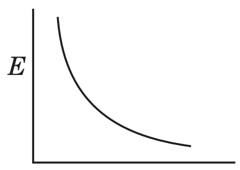


Fig: graph to show the variation of E with perpendicular distance r from the line of charge. **[1]**

6. As $\tau = pE \sin\theta$

$\therefore 4\sqrt{3} = pE\sin\theta$ **[1]**

$\Rightarrow pE \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$

$\Rightarrow$ pE = 8

Potential energy of dipole,

$U = -pE \cos\theta$ **[1]**

$U = -pE \cos 60°$

$U = -4$ J

7. (*a*) Let us consider a parallel-plate capacitor of plate area A. If separation between plates is d metre (meter), capacitance C in given by

$C = \frac{\varepsilon_o A}{d} F$ **[1]**

We know that the magnitude of the electric field between the charged plates of the capacitor in

$E = \frac{\sigma}{\varepsilon_o}$

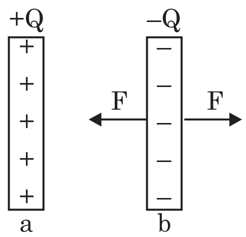
Where, σ is the surface density of either plate. Therefore, the plate charge in is $Q = \sigma A = \varepsilon_0 EA$ Now, the energy stored in the capacitor in

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(\varepsilon_o EA)^2}{\varepsilon_o A/d}$$

$U = \frac{1}{2}\varepsilon_o E^2 (Ad) J$ **[½]**

The volume between the plates is Ad metre^3. Therefore, the energy per unit volume is given by,

$$\boldsymbol{U = \frac{U}{Ad} = \frac{1}{2}\varepsilon_o E^2 J / m^3}$$



[½]

Fig.: Parallel plate capacitor

(*b*) Work done, $\boldsymbol{W = F.d}$

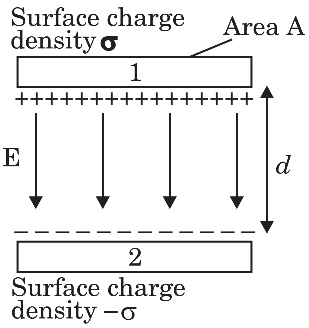
Here, F is the exerted on the charge (q) due to electric field (E) and is given by, $F = qE$

Net displacement, d = 0 **[1]**

Hence, W = 0

OR

(*a*) Derivation for the capacitance of parallel plate capacitor:



[½]

Fig. Capacitance of a parallel plate capacitor

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance d. The two plates have charges q and -1 and distance between them is d.

Plate 1 has charge density $\sigma = \frac{q}{A}$

Plate 2 has charge density $\sigma = -\frac{q}{A}$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up

$$E = \frac{q}{2\varepsilon_o} + \frac{q}{2\varepsilon_o} = \frac{q}{\varepsilon_o} = \frac{q}{A\varepsilon_o}$$ **[½]**

For this electric field, potential difference between the plates in given by,

$$V = Ed = \frac{1}{\varepsilon_o}\frac{qd}{A}$$

The capacitance C of the parallel plate capacitor is then, $C = \frac{Q}{V} = \frac{\varepsilon_o A}{d}$ **[½]**

(*b*) The surface charge density for a spherical conductor of radius R_1 is given by:

$$\sigma = \frac{q_1}{4\pi R_1^2}$$

Similarly, for spherical conductor R_2, the surface charge density is given by:

$$\sigma_2 = \frac{q_2}{4\pi R_2^2}$$ **[½]**

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1 R_2^2}{q_2 R_1^2} \quad (1)$$

As the spheres are connected so the charges will flow between the spherical conductors till their potential become equal.

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2}$$ **[½]**

$$\frac{q_1}{R_1} = \frac{q_2}{R_2} \quad (2)$$

Using (2) in (1)

We have,

$$\frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2}\cdot\frac{R_2^2}{R_1^2} \Rightarrow \frac{R_2}{R_1}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$ **[½]**

8. Suppose we have a ring of radius a that carries a uniformly distributed positive charge q.

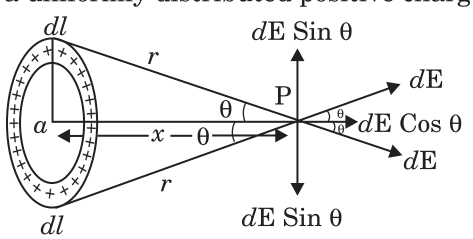


Fig. Uniform distribution of a charge over a ring **[½]**

As the total charge q is uniformly distributed, the charge dq on the element dl is

$$dq = \frac{q}{2\pi a}\cdot dl$$

∴ The magnitude of the electric field produced by the element dl at the axial point P is

$$dE = k\cdot\frac{dq}{r^2} = \frac{kq}{2\pi a}\cdot\frac{dl}{r^2}$$

(*i*) The axial components dEcosθ and

(*ii*) The perpendicular component dEsinθ. **[½]**

Since the perpendicular component of any two diametrically opposite elements are equal and opposite, they cancel out in pairs. Only the axial components will add up to produce the resultant field.

E at point P is given by,

$$E = \int_0^{2\pi a} dE\cos\theta$$ **[½]**

($\because$ Only the axial components contribute towards E)

$$E = \int_0^{2\pi a} \frac{kq}{2\pi a}\cdot\frac{dl}{r^2}\cdot\frac{x}{r}$$

$$E = \frac{kqx}{2\pi a}\cdot\frac{1}{r^3}\int_0^{2\pi a} dl \qquad \left(\because \cos\theta = \frac{x}{r}\right)$$

$$E = \frac{kqx}{2\pi a}\cdot\frac{1}{r^3}(l)_o^{2\pi a}$$

$$E = \frac{kqx}{2\pi a}\cdot\frac{1}{\left(x^2+a^2\right)^{3/2}}\cdot 2\pi a$$ **[½]**

$$\because r^2 = x^2 + a^2$$

$$E = \frac{kqx}{\left(x^2+a^2\right)^{3/2}}$$ **[½]**

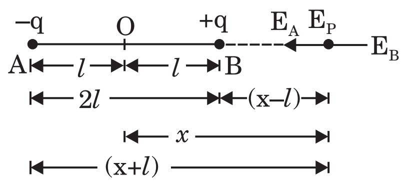
If $x >> a$, then $x^2 + a^2 \approx x^2$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{qx}{\left(x^2\right)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{x^2}$$ **[½]**

This expression is similar to electric field due to point charge.

9.



[½]

Electric field intensity due to as electric dipole

(*a*) Dipole at a point on the axial wire: we have to a calculate the field intensity (E) at a point P on the axial line of the dipole and dt a distance OP = π from the centre O of the dipole. Resultant electric field intensity at the point P, $E_P = E_A + E_B$ The vectors E_A and E_B are collinear at opposite. $\therefore E_P = E_A - E_B$

Here,

$$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(x-l)^2} \text{ and } E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(x+l)^2}$$

[½]

Thus,

$$E_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(x-l)^2} - \frac{q}{(x+l)^2}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4qlx}{\left(x^2-l^2\right)^2}$$

Hence, $E_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4px}{\left(x^2-l^2\right)^2}[\therefore p = q\times 2l]$ **[½]**

In vector form, $E_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4px}{\left(x^2-l^2\right)^2}$

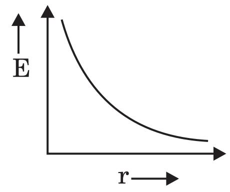
If the dipole is short,i.e. , 2l << x, then **[½]**

$$E_p = \frac{2}{4\pi\varepsilon_0}\cdot\frac{|P|}{x^3} \qquad \text{...... (i)}$$

The direction of E_P is long BP produced clearly,

$E_P \propto \frac{1}{x^3}$ **[½]**

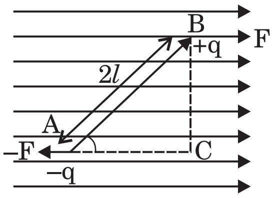
(*b*) Graph of E versus r for r >> a



[1]

Fig.: E versus r

(*c*) Torque on an electric dipole in uniform electric field :-



[½]

Consider an electric dipole considering of two changes –q and +q placed is a uniform external electric field of intensity E. The dipole moment P makes an angle θ with the direction of the electric field. The net force is zero. Since, the two forces are equal in magnitude and opposite in direction and act at different points therefore they constitute a couple. A net torque τ acts on the dipole about an axis passing through the mid-point of the couple. Now τ = force × perpendicular distance BC between the parallel force qE (2l sin θ)

$\tau = (\theta \times 2l)\, E \sin\theta$ or $\tau = pE\sin\theta$ **[½]**

In vector notation,

$\tau = p \times E$

SI unit of torque is newton-meter (N-m) and its dimensional formula is $[ML^2T^2]$

Case-I: If θ = 0° then τ = 0,

The dipole is in stable equilibrium
Case-II: If θ = 90°, then τ = PE (maximum value)
The torque acting on dipole will be maximum.

Case-III: If θ = 180° then τ = 0 **[½]**

The dipole is in unstable equilibrium

Topic 2: Electric Flux

Summary

- Electric flux is proportional to number of lines leaving a surface, outgoing lines with positive sign, incoming lines with negative sign.

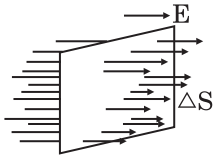


Fig. Electric flux

- Through a small area element ΔS, the flux $\Delta\phi$ of electric field E is given by

  $\Delta\phi = E.\Delta S$

  And the vector area element ΔS is

  $\Delta S = \Delta S\hat{n}$

Where ΔS is the magnetic of the area element and $\hat{n}$ is normal to the area element, which can be considered planar for the sufficiently small ΔS.

Gauss's Law and its application

- The flux of electric field through any closed surface S is $1/\varepsilon_0$ times the total charge enclosed by S.

$$\phi = E\int dA = \frac{q_{enclosed}}{\varepsilon_0}$$

- The law is mainly useful in determining electric field E, when the source distribution has simple symmetry:
  - ➢ Thin infinitely long straight wire of uniform linear charge density λ

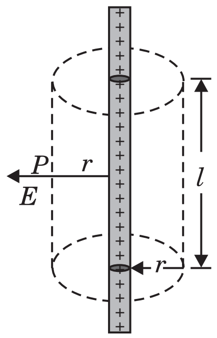


Fig. Thin infinitely long Straight wire

$$E = \frac{\lambda}{2\pi\varepsilon_o r}\hat{n}$$

Where, r is the radial (perpendicular) distance of the point from the wire and $\hat{n}$ is the radial unit vector in the plane normal to the wire passing through the point.

- Infinite plane sheet (thin) of uniform surface charge density σ

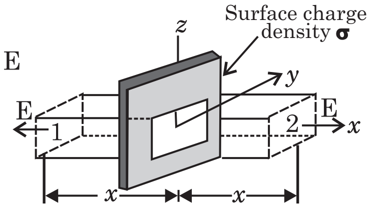


Fig. Infinite plane sheet (thin)

$$E = \frac{\sigma}{2\varepsilon_o}\hat{n}$$

Where $\hat{n}$ is a unit vector normal to the plane and going away from it.

- Thin spherical shell of uniform surface charge density σ

$$E = \frac{q}{4\pi\varepsilon_o r^2}\hat{r} \qquad (r \geq R)$$

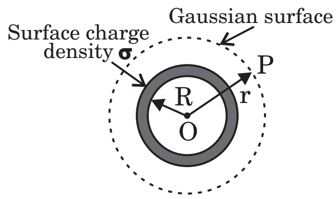


Fig.: Thin uniformly surface charged spherical shell ($r > R$)

(For $r > R$)

$E = 0$ ($r < R$)

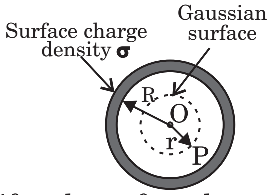


Fig.: Thin uniformly surface charged spherical shell ($r < R$)

(For $r < R$)

Where r is the distance of the point from the center of the shell whose radius is R with the total charge q. The electric field outside the shell is the same as the total charge is concentrated at the center. A solid sphere of uniform volume charge density shows the same result. Inside the shell at all the points, the field is zero.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

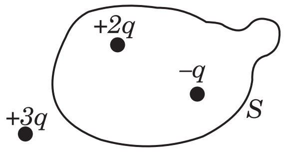
1 Mark Questions

1. What is the electric flux through a cube of side 1 cm which encloses an electric dipole?

[DELHI 2015]

2. Figure shows three point charges +2q, -q and +3q. Two charges +2q and –q are enclosed within a surface 'S'. What is the electric flux due to this configuration through the surface 'S'?

[DELHI 2015]



3. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased?

[DELHI 2016]

2 Marks Questions

4. Given a uniform electric field $\vec{E} = 5\times10^3\hat{i}N/C$, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the y-z plane. What would be the flux through the same square if the plane makes a 30° angle with the x-axis?

[DELHI 2014]

5. Given a uniform electric field $\vec{E} = 2\times10^3\hat{i}\,N/C$. Find the flux of this field through a square of side 20 cm, whose plane is parallel to the y-z plane. What would be the flux through the same square, if the plane makes an angle of 30° with the x-axis?

[DELHI 2014]

6. Given a uniform electric field $\vec{E} = 4\times10^3\hat{i}\,N/C$, find the flux of this field through a square of 5 cm on a side whose plane is parallel to the y-z plane. What would be the flux through the same square, if the plane makes an angle of 30°with the x-axis?

[DELHI 2014]

3 Marks Question

7. Using Gauss's law to obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for r > R and r < R.

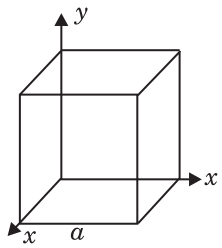
[ALL INDIA 2011]

5 Marks Questions

8. (a) An electric dipole of dipole moment $\vec{p}$ consists of point charges +q and –q separated by a distance 2a apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on ts axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit

(b) Given the electric field in the region $\vec{E} = 2\hat{x}l$,

find the net electric flux though the cube and the charge enclosed by it.



OR

(a) Explain, using suitable diagrams, the difference in the behavior of a (i) conductor and (ii) a dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.

(b) A thin metallic spherical shell of radius a carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and another charge +2Q is placed outside the shell at a distance x from the centre as shown in the figure. Find (i) the force on the charge at the centre of shell and at the point A, (ii) the electric flux through the shell.

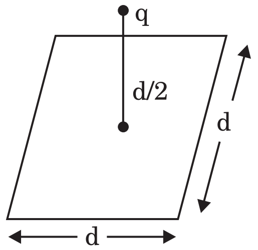
[DELHI 2015]

9. (a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet.

(b) An infinitely large thin plane sheet has a uniform surface charge density +σ. Find the amount of work done in bringing a point charge q from infinity to a point, distance r, in front of the charged plane sheet

[ALL INDIA 2017]

10. (a) Define electric flux. Is it a scalar or a vector quantity? A point charge q is at a distance of $\frac{d}{2}$ directly above the centre of a square of side 'd', as shown in the figure. Use Gauss's theorem to obtain the expression for the electric flux through the square.



(b) If the point charge is now moved to a distance 'd' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

OR

Use Gauss' law to derive the expression for the electric field $\left(\vec{E}\right)$ due to a straight uniformly charged infinite line of charge density λC/m.

[ALL INDIA 2018]

Solutions

1. From Gauss law the net flux passing through a surface is proportional to the charge enclosed within the surface. Since , net charge enclosed by electric dipole is zero hence flux will be zero. **[1]**

2. From gauss law net flux is ratio of total charge enclosed divided by $(S) = \frac{q}{\varepsilon_o}$ from the figure total charge enclosed is +2q – q = q. Hence

$(S) = \frac{q}{\varepsilon_o}$ **[1]**

3. According to Gauss's law,

$\phi = \int \varepsilon \cdot ds = \frac{q_{en}}{\varepsilon_o}$ **[½]**

Where q_{en} is the total charge enclosed by the surface. From above formula it is clear that electric flux does not depend on radius, hence it remains constant.

Flux depends only on the charge enclosed.

Hence, the electric flux remains constant.

4. When the plane is parallel to the y-z plane:

Electric flux, ϕ = EA

Here, $\vec{E} = 5 \times 10^3 \, jN/C$

$A = 10cm^2, \hat{i} = 10^{-2}\hat{i}m^2 = 10^{-2}im^2$ **[1]**

$\phi = 5 \times 10^3 \hat{i} 10^{-2} \hat{i} \Rightarrow \phi = 50$ Weber or Nm^2C^{-1}

When the plane makes a 30° angle with the x-axis, the area vector makes 60° with the x-axis.

$\phi = \vec{E}.\vec{A} \Rightarrow \phi = EA\cos\theta$

$\phi = 5 \times 103.10^{-2} \cos 60°$

$\phi = \frac{50}{2}$

$\Rightarrow \phi = 25$ Weber or $Nm^2 C^{-1}$ **[1]**

5. When the plane is parallel to the y-z plane:

$\phi = \vec{E}.\vec{A}$

$\vec{E} = 2 \times 10^3 \hat{i}$

$A = (20cm)^2 \hat{i} = 0.04M^2\hat{i}$

$A = (20 \text{ cm})^2 \text{ i} = (20 \times 10^{-2})^2 = 0.04m^2i$

$\therefore \phi = \left(2 \times 10^3 \hat{i}\right) \cdot \left(0.04\hat{i}\right)$

$\Rightarrow \phi = 82$

Weber or 80 Nm^2C^{-1} **[1]**

When the plane makes an 30°angle with the x-axis, the area vector makes an 60° angle with the x-axis.

$\phi = \vec{E} \cdot \vec{A} \Rightarrow \phi = EA\cos\theta$

$\phi = 2 \times 10^3 \times 0.04 \cos 60^o$

$\phi = 2 \times 10^3 \times 0.04 \cos 30°$

$\phi = 2 \times 10^3 \times 0.04 \times \frac{1}{2}$

$\Rightarrow \phi = 40$ Weber or 40 Nm^2C^{-1} **[1]**

6. When the plane is parallel to the y-z plane:

Electric flux, $\phi = \vec{E} \cdot \vec{A}$

Here, $\vec{E} = 4 \times 10^3 \hat{i} \, N/C$

$\vec{A} = (5\,cm)^2 \hat{i} = 0.25 \times 10^{-2} \hat{i}m^2$

$\phi = \left(4 \times 10^3 \hat{i}\right) \cdot \left(25 \times 10^{-4} \hat{i}\right)$

$\Rightarrow \phi = 10$ Weber or Nm^2C^{-1} **[1]**

When the plane makes an angle of 30° with the x-axis, the area vector makes an angle of 60° with the x-axis.

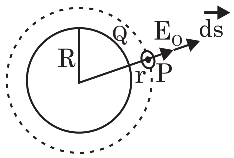
$\phi = \vec{E} \cdot \vec{A} \Rightarrow \phi = EA\cos\theta$

$\Rightarrow \phi = (4 \times 10^3)\,(25 \times 10^{-4}) \cos 60°$

$\Rightarrow \phi = \dfrac{10}{2}$

$\Rightarrow \phi = 5$ Weber or Nm^2C^{-1} **[1]**

7.



[½]

Fig.: Spherical Gaussian surface

Consider a spherical Gaussian surface of radius r (›R), concentric with given shell. If E is electric field outside the shell, then by symmetry, electric field strength has same magnitude E_o on the Gaussian surface and is directed radially outward. Also the direction of normal at each point is radially outward, so angle between E_o and ds is zero at each point. Hence, electric flux through Gaussian surface **[½]**

$= \phi_s E_o\, ds$

$= \phi_s E_o\, ds \cos 0° = E_o\, 4\,\pi r^2 ds$

Now, Gaussian surface is outside the given charged shell, so charge enclosed by the Gaussian surface is Q. Hence, by Gauss's theorem

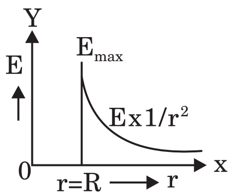
$\phi_s E_o \cdot ds = \dfrac{1}{\varepsilon_o} \times \text{charge} - \text{enclosed}$

Add sign of integration in this formula

$\Rightarrow E_o \cdot 4\pi r^2 = \dfrac{1}{\varepsilon_o} \times Q$

$\Rightarrow E_o = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$ **[1]**

Thus, electric field outside a charged thin spherical shell is same as if the whole charge Q is concentrated at the centre. Graphically,



[½]

E is proportional to $1/r^2$ not multiple as shown in the figure.

For ***r<R***, there is no strength of electric field inside a charged spherical shell.

For ***r > R***, electric field outside a charged thin spherical shell is same as if the whole charge Q is concentrated at the centre.

8. (*a*) Electric field at a point on the axial line

$\left|\overline{E_{+q}}\right| = \dfrac{kq}{(x-a)^2} \left|\overline{E_{-q}}\right| = \dfrac{kq}{(x+a)^2}$ **[1]**

$\left[\because k = \dfrac{1}{4\pi\varepsilon_o}\right]$

$\left[\because \vec{P} = 2aq\right]$

If x>>>a,

In vector form, $\vec{E} = \dfrac{2p}{4\pi\varepsilon_o x^3}$ **[1]**

(*b*) Since, the electric field is parallel to the faces parallel to xy and xz planes, the electric flux through them is zero.

Electric flux through the left face, **[1]**

$\phi_L = (E_L)\,(a^2) \cos 180°$

$\phi_L = (0\,)(a^2) \cos 180° = 0$

Electric flux through the right face,

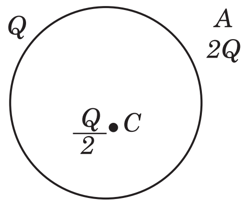
$\phi_R = (E_R)\,(a^2) \cos 0°$

$\phi_R = (2a)\,(a^2) \times 2a^3$

Total flux $(\phi) = 2a^3 = \dfrac{q_{enclosed}}{\varepsilon_o}$ **[1]**

$\therefore q_{enclosed} = 2a^3\varepsilon_o$

OR



[1]

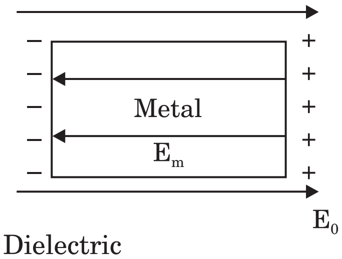
(*a*) (*i*) Conductor $E_o \rightarrow$ External field

$E_m \rightarrow$ Internal field created by the redistribution of electrons inside the metal

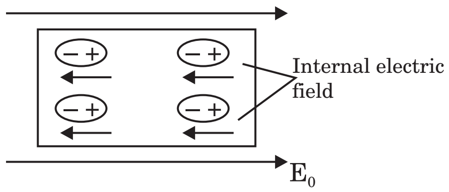
When a conductor like a metal is subjected to external electric field, the electrons experience a force in the opposite direction collecting on the left side. **[1]**

A positive charge is therefore induced on the right hand side. This creates an opposite electric field (E_m) that balances out (E_o).

Hence, the net electric field inside the conductor becomes zero. **[1]**



(*ii*) Dielectric



When external electric field is applied, dipoles are created (in case of non-polar dielectrics). The placement of dipoles is as shown in the given figure. An internal electric field is created which reduces the external electric field.

Polarization of dielectric (P) is defined as the dipole moment per unit volume of the polarized dielectric.

$P = \chi_e \varepsilon_0 E$

Where, χ_e susceptibility **[1]**

$E \rightarrow$ Electric field

(*b*) Net force on the charge $\dfrac{Q}{2}$, placed at the centre of the shell, is zero.

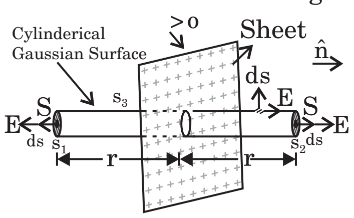
Force on charge 2Q kept at a point A **[1]**

$$F = E \times 2Q = \frac{I\left(\frac{3Q}{2}\right)2Q}{4\pi\varepsilon_o r^2}$$

$$F = \frac{k3Q^2}{r^2}$$ **[1]**

Electric flux through the shell, $\phi = \dfrac{Q}{\varepsilon_o}$

9. (*a*) Gaussian surface for a thin infinite plane sheet of uniform charge density



[1]

Let σ be the surface charge density of the sheet. From symmetry, E on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet. We take a cylindrical cross-sectional area A and length 2r as the Gaussian surface. On the curved surface of the cylinder E and $\hat{n}$ are perpendicular to each other. Therefore flux through curved surface = 0. Flux through the flux surface = EA + EA = 2EA

∴Total electric flux over the centre surface of cylinder $\phi = 2EA$ **[1]**

Total charge enclosed by the cylinder, $q = \sigma A$

acc. to Gauss' law, $\phi_E = \dfrac{q}{\varepsilon_0}$

$\therefore 2EA = \dfrac{\sigma A}{\varepsilon_0}$ or $E = \dfrac{\sigma}{2\varepsilon_0}$ **[1]**

(*b*) Let V_0 be the potential on the surface at sheet that at a distance r from it

$dV = \vec{E} \cdot \overrightarrow{dr}$

$$V_0 - V = \frac{\sigma}{2\varepsilon_0} r$$

$$V = V_0 - \frac{\sigma}{2\varepsilon_0} r$$

10. (*a*) Electric flux is defined as, $\phi_\varepsilon = E.\, ds$

It is scalar quantity. Electric flux through square is $\phi_\varepsilon = \dfrac{q}{\varepsilon_o 6}$ **[3]**

(*b*) Flux will not change, i.e. $\phi_\varepsilon = \dfrac{q}{\varepsilon_o 6}$ **[2]**

OR

To calculate the field, imagine a cylindrical Gaussian surface,as shown in the figure. Since the field is every where radial,flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, E is normal to the surface at every point,and its magnitude is constant, since it depends on r. The surface area of the curve if 2πrl, where l is the length of the cylinder. **[1]**

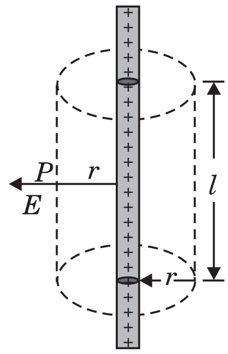
Flux through the Gaussian surface

= flux through the curved cylindrical part of the surface = E ×2πrl

The surface includes charge equal to λl. Gauss's law then gives $E \times 2\pi rl = \dfrac{\lambda l}{\phi_o}$ **[1]**

i.e., $E = \dfrac{\lambda}{2\pi r\phi_o}$ **[1]**

Vectorially, E at any point is given by $E = \dfrac{\lambda}{2\pi r\phi_o}\hat{n}$



[2]

Smart Notes ....

Smart Notes ....

CHAPTER 2

Electrostatic Potential and Capacitance

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Electric potential, potential difference					
electric potential due to a point charge, a dipole and system of charges					
Equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field					
Conductors and insulators, free charges and bound charges inside a conductor					
Dielectrics and electric polarization					
capacitors and capacitance, combination of capacitors in series and in parallel	1Q (3 marks)		1Q (2 marks)		
capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor		1Q (3 marks), 1Q (5 marks)	1Q (3 marks)	1Q (3 marks)	

On the basis of above analysis, it can be said that from exam point of view, Potential Difference, Dielectric and Capacitor, Electric Dipole and Parallel Plate Capacitor are the most important concepts of the chapter.

Topic 1: Electrostatic Potential and Electrostatic Potential Energy

Summary

Electrostatic potential:

- The amount of work done by an external force in moving a unit positive charge from one point to another in electrostatic field is called electrical potential.
- Such that $V = \frac{1}{4\pi\varepsilon}\frac{q}{r}$
- Where, q = charge causing the field, ε = permittivity, r = separation between centre of charge point.
- Electrostatic force is a conservative force.
- Work done by an external force (equal and opposite to the electrostatic force) in bringing a charge q from a point R to a point P is $V_P - V_R$, which is the difference in potential energy of charge q between the final and initial points.

Potential difference:

When the work is done upon a charge to change its potential energy then the difference between the final and the initial location is called electric potential difference.

Electric Potential due to a dipole:

- The electrostatic potential at a point with distance r due dipole at a point making an angle θ with dipole moment p placed at the origin is given by $V(r) = \frac{1}{4\pi\varepsilon_o}\cdot\frac{p\cdot\hat{r}}{r^2}$.

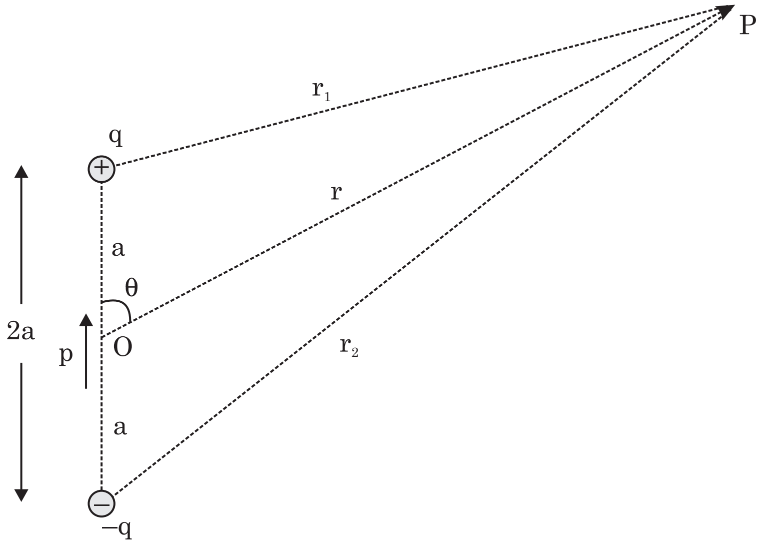


Fig. Electrical potential due to dipole

- It is a scalar quantity.
- Let A and B be the initial and final location for a single charge q then the potential difference between A and B is given by:

$$\Delta V = V_B - V_A = -\int_A^B E\times ds = -\int_A^B E ds\cos\theta = -\int_A^B E\times ds$$

Where, E is the field due to a point charge, ds = dr, so that

$$V_B - V_A = \int_{r_A}^{r_B}\frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B}-\frac{1}{r_A}\right]$$

- The result is true also for a dipole (with charges –q and q separated by 2a for r >> a.

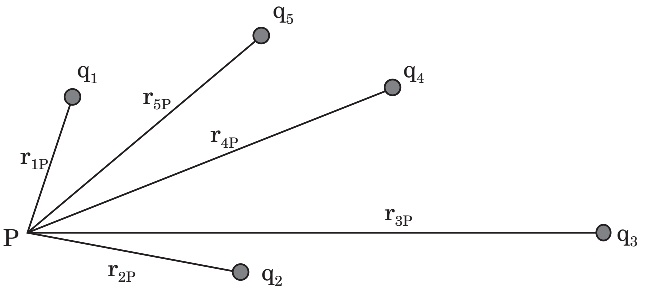
Dipole and System of charges

- For a charge configuration q_1, q_2, ..., q_n with position vectors r_1, r_2, r_3, ..., r_n, then the potential V_1 at point P due to charge q_1 will be,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

And the sum of potentials due to individual charges is given by the superposition principle,

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_{1P}}+\frac{q_2}{r_{2P}}+....+\frac{q_n}{r_{nP}}\right)$$



- In a uniformly charged spherical shell, the electric field outside the shell with outside potential is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Equipotential surfaces

- A surface over which potential has a constant value is known as an equipotential surface.
- The amount of work done in moving a charge over an equipotential surface is zero.
- Concentric spheres centered at a location of the charge act as equipotential surfaces for a point charge.
- The electric field E, at a point and equipotential surface are mutually perpendicular to each other through the point. The direction of the steepest decrease of potential is in E.
- Regions of strong and weak fields are located because of the spacing among equipotential surfaces.

Potential Energy of a System of Charges:

Potential energy stored in a system of charges is the work done by an external agency in assembling the charges at their locations. Total work done in assembling the charges is given by $U = \frac{1}{4\pi\varepsilon_o}\cdot\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$ where r_{12} is distance between q_1 and q_2, r_{13} is distance between q_1 & q_3 and r_{23} is distance between q_2 & relabel q_3.

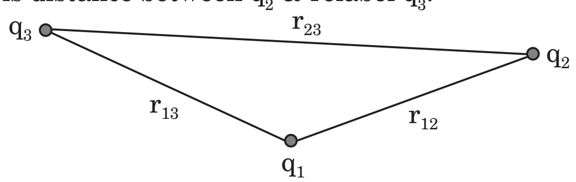


Fig. Potential energy due to System of charges

Electric potential energy of system of two point charges

- Here the work done doesn't depend on path.
- In this system the two charges q_1 and q_2 when separated by distance r, will either repel or attract each other.
- Electrical potential of charges q_1 and q_2 is given by:

$$U = \frac{1}{2}\sum_{i=1}^{2} q_i V_i$$

Potential Energy in an External Field:

- The potential energy of a charge q in an external potential V(r) is qV(r). The potential energy of a dipole moment p in a uniform electric field E is –p.E.
- Electric dipole in an electrostatic field: Electric potential due to a dipole at a point at distance r and making an angle θ with the dipole moment p is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Electrostatics of conductors:

- Electrostatic field is zero inside a conductor.
- Electrostatic field at the surface of a charged conductor must be normal to the surface at every point.
- In the static situation, there cannot be any excess charge in the interior of a conductor.
- Throughout the volume of the conductor, the electrostatics potential is constant and has same value on its surface.
- Electrostatics field E is zero in the interior of a conductor; just outside the surface of a charged conductor, E is normal to the surface given by $E = \frac{\sigma}{\varepsilon_o}\hat{n}$ where $\hat{n}$ is the unit vector along the outward normal to the surface and σ is the surface charge density.
- **Electrostatic shielding:** A field which is inside the cavity of a conductor is always zero and it remains shielded from the electric field, which is known as electrostatic shielding.

Dielectrics and Polarization:

- **Dielectrics:** A non-conducting substance which has a negligible number of charge carriers unlike conductors is called dielectrics.
- **Electric polarization:** The difference between induced electric field and imposed electric field in dielectric due to bound and free charges is known as electric polarization. It is written as:

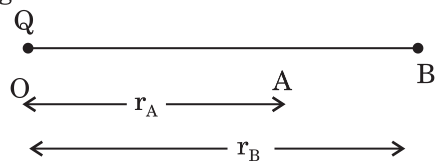
$$P = \frac{D - E}{4\pi}$$

Note: Polarisation can also be written as polarization (with 'z' in place of 's')

PREVIOUS YEARS' EXAMINATION QUESTIONS
TOPIC 1

1 Mark Questions

1. A point charge Q is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero, if Q is (i) positive (ii) negative?



[ALL INDIA 2011]

2. For any charge configuration, equipotential surface through a point is normal to the electric field. Justify.

[DELHI 2014]

3. What is the amount of work done in moving a point charge Q around a circular arc of radius 'r' at the centre of which another point charge 'q' is located ?

[ALL INDIA 2016]

2 Mark Questions

4. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the X-Z plane at a distance 'd' apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge '$-q$' remains stationary between the plates, what is the magnitude and direction of the field?

Or

Two small identical electrical diploes AB and CD, each of dipole moment 'p' are kept an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field E directed along $+X$ direction, what will be the magnitude and direction of the torque acting on this?

[ALL INDIA 2011]

5. An electric dipole of length 2 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $6\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has a charge of $\pm 2\,nC$.

[DELHI 2014]

6. An electric dipole of length 2 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $8\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has a charge of $\pm 4\,nC$.

[DELHI 2014]

7. (i) Can two equipotential surfaces intersect each other? Give reasons.

(ii) Two charges $-q$ and $+q$ are located at points A (0, 0, –a) and B (0, 0, + a) respectively. How much work is done in moving a test charge from point P (97, 0, 0) to Q(–3, 0, 0)?

[DELHI 2017]

3 Mark Questions

8. (a) Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

(b) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

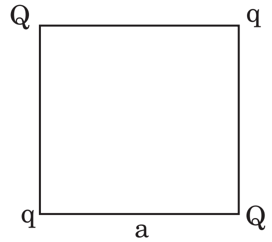
[ALL INDIA 2017]

9. (i) Draw equipotential surfaces for a system of two identical positive point charges placed a distance 'd' apart.

(ii) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r_1}$ and $\vec{r_2}$ respectively in the presence of external electric field $\vec{E}$.

[DELHI 2017]

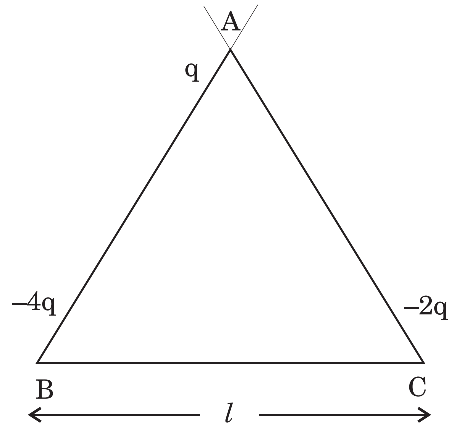
10. Four point charges Q, q, Q and q are place d at the corners of a square of side 'a' as shown in the figure.



Find the (a) resultant electric force on a charge Q, and (b) Potential energy of the system

OR

(a) Three point charge sq, $-4\,q$ and 2q are place d at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.



(b) Find out the amount of the work done to separate the charges a t infinite distance.

[ALL INDIA 2018]

5 Mark Questions

11. (*a*) State Gauss's law in electrostatics. Show, with the help of a suitable example along with the figure, that the outward flux due to a point charge '*q*'. in vacuum within a closed surface, is independent of its size or shape and is given by $\frac{q}{\varepsilon_0}$

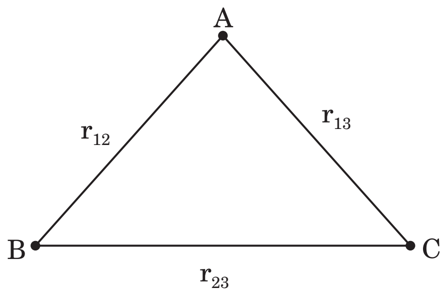
(*b*) Two parallel uniformly charged infinite plane sheets, '1' and '2', have charge densities + σ and –2 σ respectively. Give the magnitude and direction of the net electric field at a point.

(*i*) in between the two sheets and

(*ii*) outside near the sheet '1'.

Or

(*a*) Define electrostatic potential at a point. Write its S.I. unit. Three point charges q_1, q_2 and q_3 are kept respectively at points A, B and C as shown in the figure, Derive the expression for the electrostatic potential energy of the system.

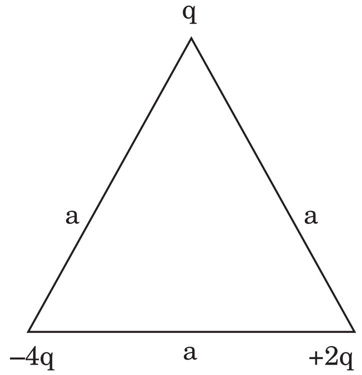


(*b*) Depict the equipotential surface due to (i) an electric dipole, (ii) two identical positive charges separated by a distance.

[DELHI 2015]

12. (*a*) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point. Draw a sketch of equipotential surfaces due to a single charge (–*q*), depicting the electric field lines due to the charge

(*b*) Obtained an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.

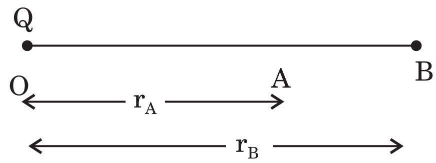


[ALL INDIA 2016]

Solutions

1. Potential at a point: $V = \frac{kq}{r}$,Given $q = Q$

$$V_A - V_B = kQ\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$



[½]

Where, $r_A < r_B$

So, $\frac{1}{r_A} > \frac{1}{r_B}$

$$\therefore \left[\frac{1}{r_A} - \frac{1}{r_B}\right] > 0$$

If Q at O is positive, $V_A - V_B$ will be positive

[½]

If Q at O is negative, $V_A - V_B$ will be negative.

2. If the electric field were not normal to equipotential surface, it would have non-zero component along the surface. To move a charge against this component, work would have to be done. But no work is needed to move a test charge on an equipotential surface. Hence electric field must be normal to the equipotential surface at every point. **[1]**

3. Given charge 'q' is located at centre and charge 'Q' on surface. Then, work done

$$W = \frac{1}{4\pi\varepsilon_0}\frac{q\cdot Q}{r^2}$$ **[½]**

Work done to move the charge over the circular arc is zero, because it is moving over an equipotential surface. **[½]**

4.

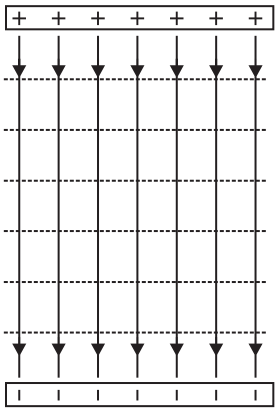


Fig. Two uniformly charged plates **[½]**

Here the dotted lines represent the parallel equipotential surfaces along X-Z plane.

If a charge q has to be held stationary between the two plates, it will have to be balanced by two forces.

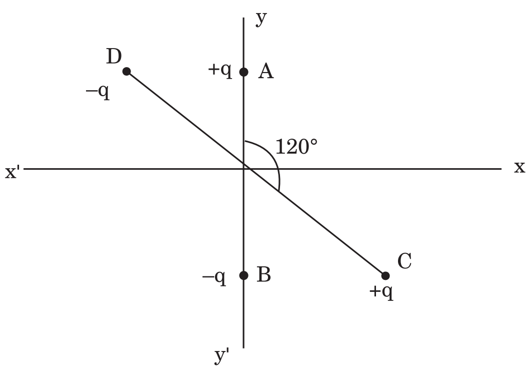
Gravitational force: mg, downwards

Electrostatic force = $2qE$, acting upwards.

This implies, that in X-Z plane, the upper plate is + charged plate & lower plate is –charged plate.

So, E field lines have to be directed along –y axis. **[½]**

Or



[½]

Fig. An Electric dipole

Resultant dipole moment,

$$\vec{P}_{res} = \vec{p}_1 + \vec{p}_2$$

$$p_{res} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 120^\circ}$$

$$P_{res} = p$$

Direction of resultant dipole moment:

$$\tan\theta = \frac{p\sin 120^\circ}{p + p\cos 120^\circ}$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

That is, 30 degrees with +x axis.

Given applied E is along +x axis,

So torque on resultant dipole will be

$$\tau = pE\sin 30^\circ = \frac{pE}{2}$$ **[½]**

Direction will be along –Z-axis.

5. As $\tau = PE\sin\theta$

$$6\sqrt{3} = pE\sin 60^o$$

$$= pE \times \frac{\sqrt{3}}{2} \text{ or } PE = 12$$

$U = -PE\cos\theta$ **[1]**

Potential energy of a dipole,

$= -12\cos 60^\circ$

$$= -12 \times \frac{1}{2}$$

$U = -6J$ **[1]**

6. As $\tau = PE\sin\theta$

$$8\sqrt{3} = pE\sin\theta$$

$$8\sqrt{3} = pE\sin 60^o$$

$$8\sqrt{3} = \frac{pE\sqrt{3}}{2}$$

$\Rightarrow pE = 16$ **[1]**

Also Potential energy of the dipole,

$\Rightarrow U = -PE\cos\theta$

$U = -PE\cos 60^\circ$

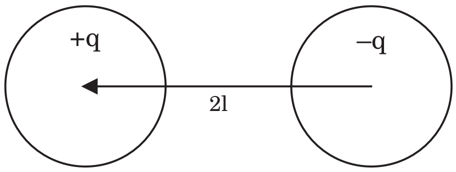
$$U = \frac{-16.1}{2} = -8J$$

$U = -8J$ **[1]**

7. (*i*) Two equipotential surfaces cannot intersect each other because when they will intersect, the electric field will have two directions, which is impossible. **[1]**

(*ii*) Charge P moves on the perpendicular bisector of the line joining $+q$ and $-q$. Hence, this perpendicular bisector is equidistant from both the charges. Thus, the potential will be same everywhere on this line. Therefore, work done will be zero. **[1]**

8. (*a*) Electric dipole moment of an electric dipole is defined as the product of the magnitude of either charge of the electric dipole and the dipole length.



[½]

Fig. An Electric Dipole

i.e., $\vec{p} = q(2\vec{l})$

The magnitude of dipole moment is

$p = q \times 2l$

Dipole moment is a vector quantity. SI Unit of dipole moment $(\vec{p})$ is coulomb metre (Cm)

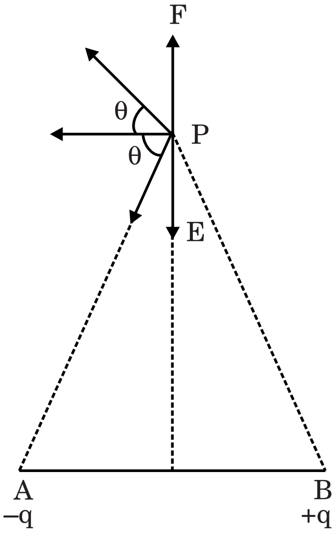


Fig: Electrical field of a dipole at a point on equatorial plane of the dipole

Electric field intensity due to +q charge is given by

$$E_+ = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2} \quad \text{..........(i)}$$ **[½]**

Electric field intensity due to –q charge is given by

$$E_- = \frac{1}{4\pi\varepsilon_o}\frac{q}{AP^2}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q}{\left(r^2 + l^2\right)} \quad \text{............(ii)}$$

From (i) and (ii),

$$E_+ = E_- = \frac{1}{4\pi\varepsilon_o}\frac{q}{\left(r^2 + l^2\right)} \quad \text{............(iii)}$$ **[½]**

The net electric field intensity due to the electric dipole.

$$\therefore E = \sqrt{E_+^2 + E_-^2 + 2E_+E_-\cos 2\theta}$$

$$= \sqrt{E_+^2 + E_+^2 + 2E_+^2\cos 2\theta}\ \left(E_+ = E_-\right)$$

$$= \sqrt{2E_+^2 + 2E_+^2\cos 2\theta}$$

$$= \sqrt{2E_+^2\left(1 + \cos 2\theta\right)}$$

$$= \sqrt{2E_+^2 \times 2\cos^2\theta}\left(1 + \cos 2\theta = 2\cos^2\theta\right)$$

$$\therefore E = 2E_+\cos 2\theta = 2 \times \frac{1}{4\pi\varepsilon_o}\frac{q}{\left(r^2 + l^2\right)}\cos\theta$$ **[½]**

[Using equation (iii)]

$$\cos\theta = \frac{l}{\sqrt{\left(r^2 + l^2\right)}}$$

$$E = \frac{q}{4\pi\varepsilon_o\left(r^2 + l^2\right)^{\left(\frac{3}{2}\right)}}$$

If l << r, $E = \dfrac{q}{4\pi\varepsilon_o r^3}$

(*b*) **[1]**

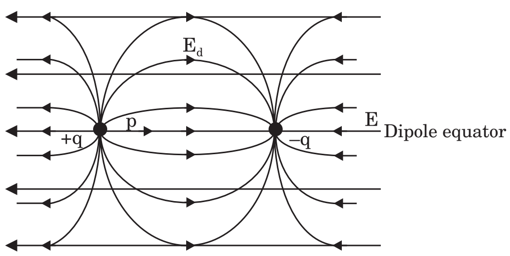


Fig. Equipotential Surface due to dipole

Electrical potential is zero at all points in the plane passing through the dipole equator.

9. (*i*) **[1]**

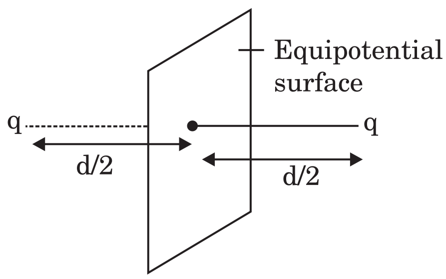


Fig. Equipotential Surface for System of Charges (*ii*) The work done in bringing charge q1 from infinity to $\vec{r_1}$ is $q_1V(\vec{r_1})$.

Work done on q2 against external field

$= q_2V(\vec{r_2})$

Work done on q2 against the field due to

$q_1 = \dfrac{q_1q_2}{4\pi\varepsilon_o r_{12}}$ **[1]**

Where, r_{12} is the distance between q_1 and q_2. By the superposition principle for fields,

Work done in bringing q_2 to

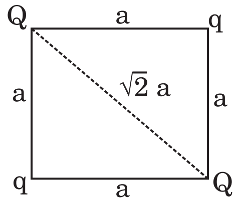
$\vec{r_2}$ is $\left(q_2V(\vec{r_2}) + \dfrac{q_1q_2}{4\pi\varepsilon_o r_{12}}\right)$

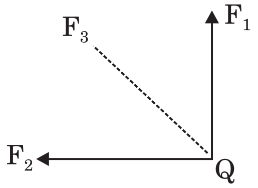
Thus,

Potential energy of system = The total work done in assembling the configuration

$= q_1V(\vec{r_1}) + q_2V(\vec{r_2}) + \dfrac{q_1q_2}{4\pi\varepsilon_o r_{12}}$ **[1]**

10. (a) There will be three forces on charge Q





$F_1 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{qQ}{a^2}$

$F_2 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{qQ}{a^2}$

$F_3 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{QQ}{\left(\sqrt{2}a\right)^2} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q^2}{2a^2}$

F_1 and F_2 are perpendicular to each other so their resultant will be

$F' = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90°}$

$F_1 = F_2$

$F' = \sqrt{F_1^2 + F_1^2 + 2F_1F_1 \times 0}$

$F' = \sqrt{2F_1^2} = \sqrt{2}F_1$

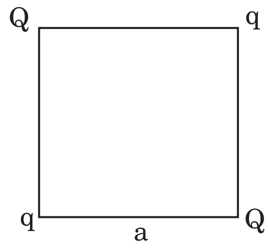
F_3 and resultant of F_1 and F_2 will be in same direction

Net force $F = F' + F_3$

$F = \sqrt{2}\dfrac{1}{4\pi\varepsilon_o}\dfrac{qQ}{a^2} + \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q^2}{2a^2}$

$F = \dfrac{1}{4\pi\varepsilon_o}\dfrac{qQ}{a^2}\left(q\sqrt{2} + \dfrac{Q}{2}\right)$ **[1]**

(*b*)

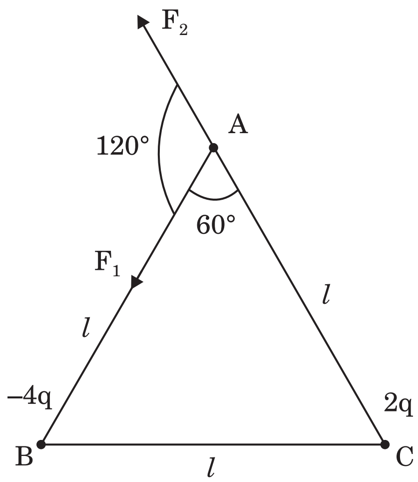


$w = \dfrac{4KQq}{a} + \dfrac{Kq^2}{\sqrt{2}a} + \dfrac{KQ^2}{\sqrt{2}a}$

$K = \dfrac{1}{4\pi\varepsilon_o}$

[½]

Or



(*a*)

$\left|\vec{F_1}\right| = \dfrac{1}{4\pi\varepsilon_o}\dfrac{(4q)(q)}{l^2}$

$F_1 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{4q^2}{l^2}$

$F_1 = \dfrac{1}{\pi\varepsilon_o}\dfrac{q^2}{l^2}$

$\left|\vec{F_2}\right| = \dfrac{1}{4\pi\varepsilon_o}\dfrac{(q)(2q)}{l^2}$

$$F_2 = \frac{1}{2\pi\varepsilon_o}\frac{q^2}{l^2}$$

Angle between F_1 and F_{12} is 120°

$$\overline{F} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

$$F_1 = 2\,F_2$$

$$F = \sqrt{(2F_2)^2 + F_2^2 + 4F_2^2\cos 120°}$$

$$F = \sqrt{4F_2^2 + F_2^2 - 2F_2^2}$$

$$F = \sqrt{3F_2^2}$$

$$F = \sqrt{3}F_2$$

$$F = \frac{\sqrt{3}}{2\pi\varepsilon_o}\frac{q^2}{l^2}$$

[1]

(*b*) The amount of work done to separate the charges at infinity will be equal to potential energy.

$$U = \frac{1}{4\pi\varepsilon_o l}\left[q\times(-4q)+(q\times 2q)+(-4q\times 2q)\right]$$

$$U = \frac{1}{4\pi\varepsilon_o l}\left[-4q^2+2q^2-8q^2\right]$$

$$U = \frac{1}{4\pi\varepsilon_o l}\left[-10q^2\right]$$

$$U = -\frac{1}{4\pi\varepsilon_o l}\left[10q^2\right]\ unit$$

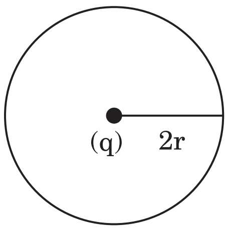
[½]

11. (*a*) Statement: The electric flux linked with a closed surface is equal to ε times the net charge enclosed by a closed surface.

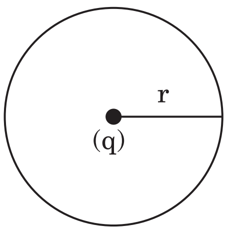
Mathematical expression:

$$\phi_E = \oint E\cdot ds = \frac{1}{\varepsilon_o}(q_{net})$$ **[½]**

Consider two spherical surfaces of radius r and $2r$ respectively and a charge 1 is enclosed in it. According to gauss theorem the total electric flux linked with a closed surface depends on the charge enclosed in it so For fig (a) **[½]**



(a)



(b)

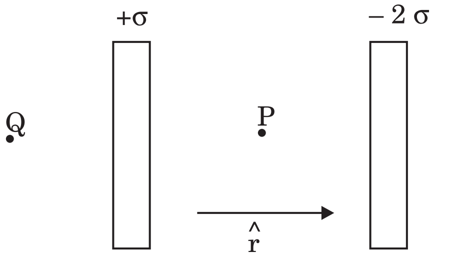
[½]

$$\phi_E = \frac{q}{\varepsilon_o}$$

And for fig (b)

$$\phi_E = \frac{q}{\varepsilon_o}$$ which is same in both the cases so it is independent of size and shape of closed surface.

[½]



(*b*) Plate 1 Plate 2 **[½]**

Let $\hat{r}$ be the unit vector directed from left to right

Let P and Q are two points in the inner and outer region of two plates respectively charge densities on plates are +σ and –2σ

[½]

(*i*) Electric field at point P in the inner region of the plates

$$E = E_1\,E_2$$

$$E = \left(\frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{\varepsilon_o}\right)r$$

$$E = \frac{3\sigma}{2\varepsilon_o}r$$

[½]

(*ii*) Electric field at point Q in the outer region of plate 1

$$E_1 = \frac{\sigma}{2\varepsilon_o}(-r) \text{ and } E_2 = \frac{2\sigma}{2\varepsilon_o}r$$

[½]

∴ Net electric field in the outer region of the plate 1(i.e, at Q) is

$$E = E_1 + E_2 = \left(\frac{\sigma}{\varepsilon_o} - \frac{\sigma}{2\varepsilon_o}\right)r$$

[½]

$$E = \frac{\sigma}{2\varepsilon_o}r$$

[½]

Or

(a) The electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. Its S.I. unit is Volt. The potential energy of a system of three charges q_1, q_2 and q_3 located at r_1, r_2 and r_3 respectively. To bring q_1 first from infinity to r_1 no work is required. Next we bring q_2 from infinity to r_2. Work done is **[1]**

$$q_2V_1(r_2) = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r_{12}} \quad \ldots (1)$$

The charge q_1 and q_2 produce at potential, which at any point P is given by

$$V_{1\cdot2} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}}\right) \quad \ldots. (2)$$

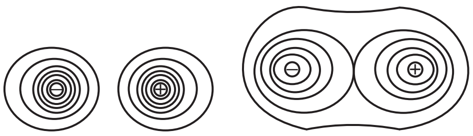
Work done next in bringing q_3 from infinity to the point r_3 is q_3 times V_{12} at

$$q_3V_{1,2}(r_3) = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}}\right) \ldots. (3) \quad \textbf{[1]}$$

The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps [Eq. (1) and Eq. (3)] and gets stored in the form of potential energy.

$$U = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

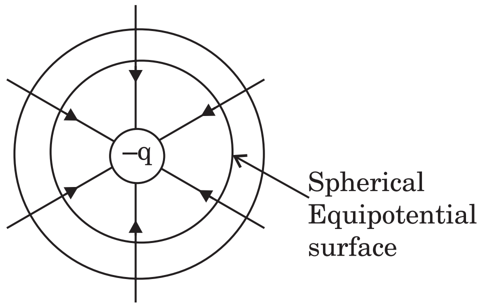
(b) Equipotential surfaces for (a) a dipole and (b) two identical positive charges are shown in Figure.



(a) (b)

Some equipotential surfaces for (a) a dipole, (b) two identical positive charges. **[1]**

12. (a) If the field were not normal to the equipotential surface, it would have non-zero component along the surface. To move a unit test charge against the direction of the component of the field, work would have to be done. But this is in contradiction to the definition of an equipotential surface: there is no potential difference between any two points on the surface and no work is required to move a test charge on the surface. **[2]**



[1]

(b) Total electrostatic potential energy of system

$U = U_{12} + U_{23} + U_{31}$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{q(-4q)}{a} + \frac{(-4q)(2q)}{a} + \frac{q(2q)}{a}\right]$$

$$= -\frac{1}{4\pi\varepsilon_o}\left[\frac{10q^2}{a}\right] \quad \textbf{[1]}$$

$\therefore$ Work done to dissociate the system

$W = -U$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{10q^2}{a}\right] \quad \textbf{[1]}$$

Topic 2: Capacitance

Summary

Capacitor and Capacitance

- **Capacitor:** The system of two conductors separated by an insulator is called capacitor.

  The device which is used to store charge is known as capacitor. The applied voltage and size of capacitor decides the amount of charge that can be stored i.e., $Q = CV$

  Two similar connecting plates are placed in capacitor in the front of each other where one plate is connected to the positive terminal and other plate is connected to the negative terminal.
- **Capacitance:** The ratio of magnitude of charge stored on the plate to potential difference between the plates is called capacitance. It is written as:

  $$C = \frac{Q}{\Delta V}$$

  Size, shape, medium and other conductors in surrounding influence the capacitance of a conductor.

  Its S.I. unit is farad.

  $1F = 1CV^{-1}$ For a parallel plate capacitor (with vacuum between the plates), $C = \varepsilon_o \frac{A}{d}$ where A is the area of each plate and d in the separation between the parallel plates.

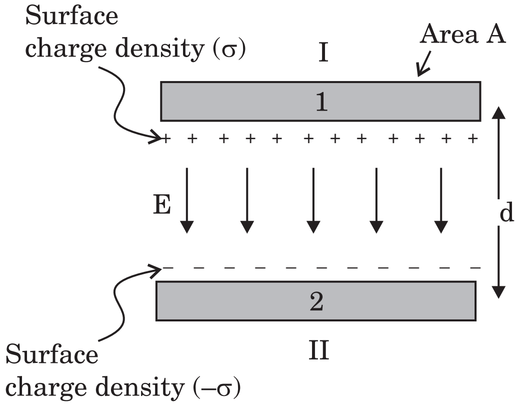


Fig. Capacitor

Effect of Dielectric on Capacitance:

- If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect, called ***polarization***, gives rise to a field in the opposite direction.
- The dielectric is polarised by the field and also the effect is equivalent to two charged sheets with surface charge densities σ_p and $-\sigma_p$.
- The net electric field inside the dielectric and hence the potential difference between the plates is thus reduced. Consequently, the capacitance C increases from its value C_o when there is no medium (vacuum),

  $C = KC_o$ where $K = \frac{\varepsilon}{\varepsilon_0}$ is the ***dielectric constant*** of the insulating substance.

Types of capacitor:

- Parallel plate capacitor: $C = K\varepsilon_0 \frac{A}{d}$
- Cylindrical capacitor: $C = 2\pi K\varepsilon_0 \frac{1}{\ln(b/a)}$
- Spherical capacitor: $C = 4\pi K\varepsilon_0 \left(\frac{ab}{b-a}\right)$

Combination of Capacitors

- For capacitors in the series combination, the total capacitance C is given by

  $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \frac{1}{C_n}$$
- In the parallel combination, the total capacitance C is $C = C_1 + C_2 + C_3 \dots\dots C_n$, where $C_1, C_2, C_3 \dots\dots$ are individual capacitances.
- Capacitors connected in series have the same charges and when connected in parallel have the same voltage.
- Potential across capacitor remains same if the battery is connected but if it is disconnected then charge remains the same which is stored in capacitor.

Electrical Energy Stored in a Capacitor:

- The energy U stored in a capacitor of capacitance C, with charge Q and voltage V is $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$.
- The electric energy density (energy per unit volume) in a region with electric field is $\frac{1}{2}\varepsilon_o E^2$.
- Electric density is alternatively known as ***electrostatic pressure***.

Van De Graaff Generator:

- A Van de Graaff generator consists of a large spherical conducting shell (a few meters in diameter).
- There are two pulleys, one at ground level and one at the center of the shell. Both of them are wounded around by a long and narrow endless belt of insulating material.
- The motor drives the lower pulley which keeps moving this belt continuously.
- At ground level to the top, it continuously carries the positive charge and sprayed on to it by a brush. Then the positive charge is transferred by it to another conducting brush connected to the large shell.
- After the transferring of the positive charge is done, it spreads out uniformly on the outer surface. It can build the voltage difference of as much as 6 to 8 million volts.

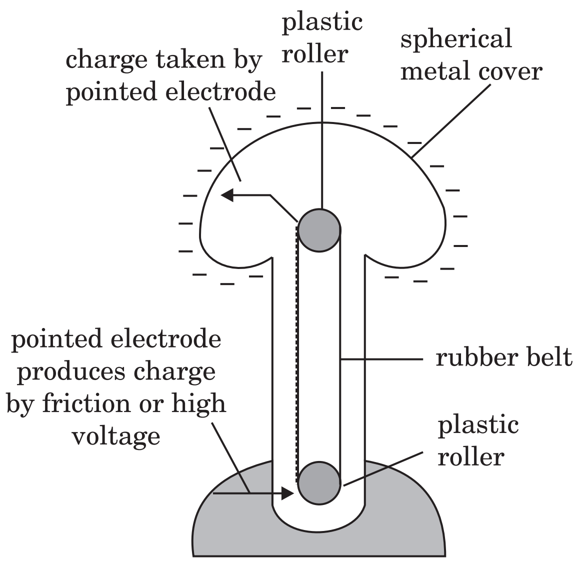


Fig. Yande Graff Generator

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

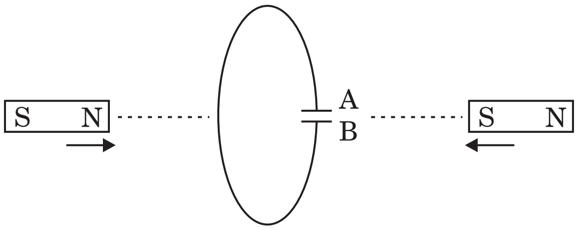
1 Mark Questions

1. Why should electrostatic field be zero inside a conductor?
[All INDIA 2012]

2. A capacitor has been charged by a dc source. What are the magnitudes of conduction and displacement current, when it is fully charged?
[All INDIA 2013]

3. Define dielectric constant of a medium. What is its S.I. unit?
[DELHI 2015]

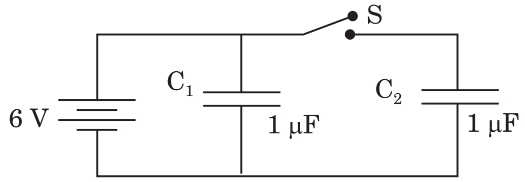
4. Predict the polarity of the capacitor in the situation described below:



[All INDIA 2017]

2 Mark Questions

5. Figure shows two identical capacitors, C_1 and C_2 each of $1\mu F$ capacitance connected to a battery of 6V. Initially switch 'S' is closed. After sometime 'S' is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?



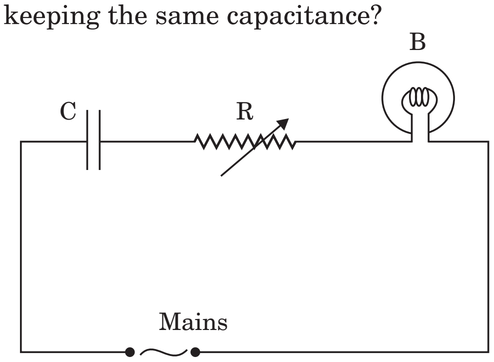
[DELHI 2011]

6. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $2\frac{d}{3}$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.
[DELHI 2011]

7. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC.
Calculate:
(i) The potential V and the unknown capacitance C.
(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V?
[DELHI 2011]

8. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor
[All INDIA 2014]

9. A capacitor '*C*' a variable resistor '*R*' and a bulb '*B*' are connected in series to the ac mains in circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be the same; (ii) the resistance R is increased keeping the same capacitance?



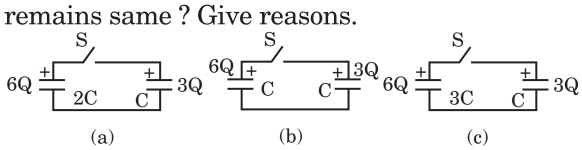
[DELHI 2014]

10. Two capacitors of unknown capacitance $C1$and $C2$ are connected first in series and then in parallel across a battery of 100 *V*. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of $C1$ and $C2$. Also calculate the charge on each capacitor in parallel combination.

[DELHI 2015]

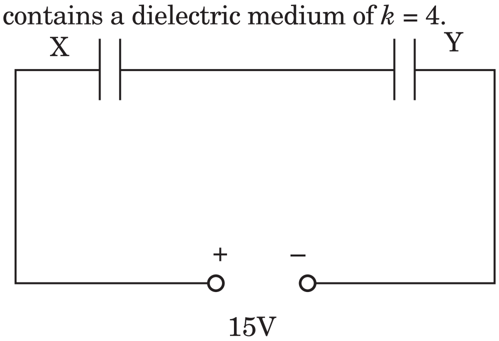
3 Mark Questions

11. Three circuits, each consisting of a switch '*S*' and two capacitors, are initially charged, as shown in the figure. After the switch has been closed, in which circuit will the charge on the left-hand capacitor (i) increase, (ii) decrease and (iii) remains same ? Give reasons.



[All India 2015]

12. Two parallel plate capacitors *X* and *Y* have the same area of plates and same separation between them. *X* has air between the plates while *Y* contains a dielectric medium of $k = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μ*F*.

(ii) Calculate the potential difference between the plates of *X* and *Y*.

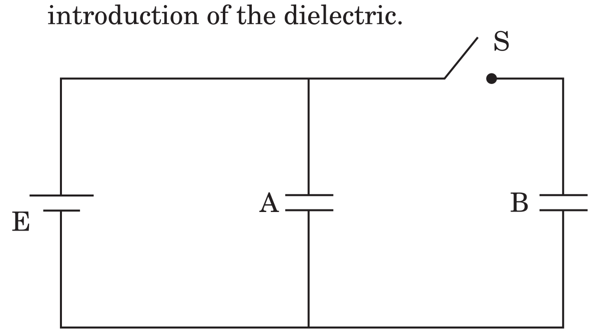
(iii) Estimate the ratio of electrostatic energy stored in *X* and *Y*.

[DELHI 2015]

13. The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. Bywhat factor does the applied potential difference change?

[All INDIA 2017]

14. Two identical parallel plate capacitors A and B are connected to a battery of *V* volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant *K*. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.

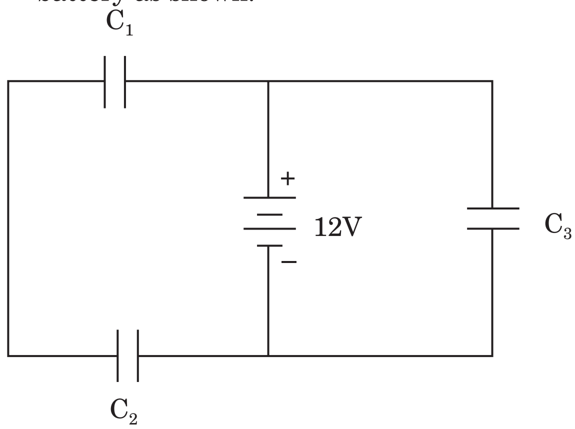


[All INDIA 2017]

15. A thin conducting spherical shell of radius *R* has charge *Q* spread uniformly over its surface. Using Gauss's law; derive an expression for an electric field at a point outside the shell. Draw a graph of electric field E(r) with distance r from the centre of the shell for $0 \le r \le \infty$

[DELHI 2017]

16. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 p*F* each are connected to a 12 *V* battery as shown.



Find (i) charge on each capacitor (ii) equivalent capacitance of the network (iii) energy stored in the network of capacitors

[DELHI 2017]

5 Mark Questions

17. Draw a labeled diagram of Van de Graff generator. State its working principle to show how by introducing a small charged sphere into a larger sphere, a large amount of charge can be transferred to the outer sphere. State the use of this machine and also point out its limitations.

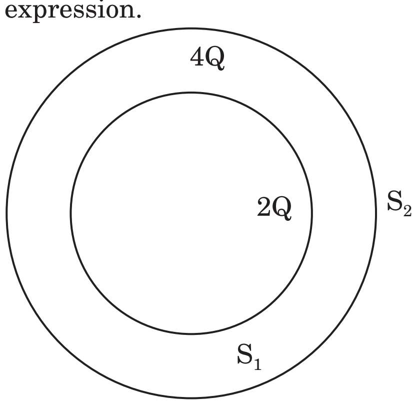
Or

(a) Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$

(b) Consider two hollow concentric spheres, S_1 and S_2, enclosing charges $2Q$ and $4Q$ respectively as shown in the figure

(i) Find out the ratio of the electric flux through them.

(ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



[DELHI 2014]

18. (a) Distinguish, with the help of a suitable diagram, the difference in the behavior of a conductor and a dielectric placed in an external electric field. How does polarized dielectric modify the original external field?

(b) A capacitor of capacitance C is charged fully by connecting it to a battery of emf E. It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change?

(i) Charge stored by the capacitor.

(ii) Field strength between the plates.

(iii) Energy stored by the capacitor.

Justify your answer in each case.

[All INDIA 2016]

19. A parallel-plate capacitor is charged to a potential difference V by a dc source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, a state with reason how the following change:

(i) The electric field between the plates,

(ii) Capacitance and

(iii) Energy stored in the capacitor

[DELHI 2017]

20. Explain the principle of a device that can build up high voltage of the order of a few million volts.

Draw a schematic diagram and explain the working of this device.

Is there any restriction on the upper limit of the high voltage set up in this machine? Explain.

[DELHI 2012]

Solutions

1. In a conductor charges reside on its surface, there are no charges present inside a conductor. Hence electric field inside is zero. **[1]**

2. When capacitor is fully charged it maintains a constant voltage and charge will also be constant. Since current is defined as rate of change of charge it will be zero.

Conduction current, $I_C = C\frac{dV}{dt}$

Since, V is constant, $\frac{dV}{dt} = 0$ **[½]**

$\Rightarrow I_C = C\cdot\frac{dV}{dt} = 0$

Displacement current, $I_D = \varepsilon_o \frac{d\left(\frac{q}{\varepsilon_o}\right)}{dt}$

Since q is constant, $d\left(\frac{q}{\varepsilon_o}\right) = 0$

$\Rightarrow I_D = \varepsilon_o \frac{d\left(\frac{q}{\varepsilon_o}\right)}{dt} = 0$ **[½]**

3. Dielectric constant (or relative permittivity) of a dielectric is the ratio of the absolute permittivity (ε) of a medium to the absolute permittivity of free space(ε_o) $K = \frac{\varepsilon}{\varepsilon_o}$, It is unit less quantity. **[1]**

4. According to Lenz law the polarity of induced emf is such that it opposes the cause of its production so the polarity of the capacitor is as shown

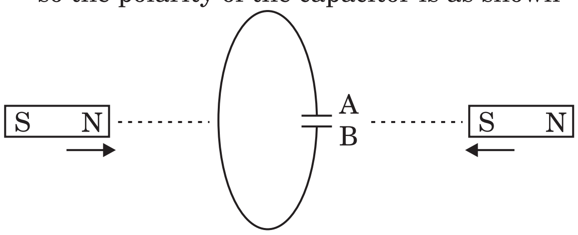


Fig.: Lenz Law

[½]

5. In C_2 :

Charge $Q_o = C_D V_D$

Where, $C_D = KC$ = increases K times

$V_D = \frac{V}{K}$ = decreases K times **[1]**

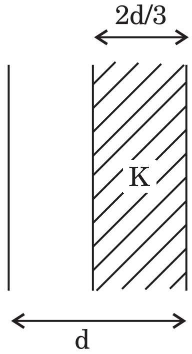
In C_1:

Charge $Q_o = C_D V$

Potential V remains the same as 6V

Charge $Q_D = KCV = KQ$, increases K times **[1]**

6. **[1]**



$$C = \frac{\varepsilon_o A}{\frac{2d/3}{K} + \frac{d/3}{1}} = \frac{3\varepsilon_o AK}{d(2+K)}$$ **[1]**

7. (i) Initial voltage, $V_1 = V$ volts and charge stored, $Q_1 = 360\ \mu C$

$Q_1 = CV_1$ (1)

Charged potential, $V_2 = V - 120$

$Q_2 = 120\ \mu C$

$Q_2 = CV_2$ (2)

By dividing (2) from (1), we get

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2}$$

$$\frac{360}{120} = \frac{V}{V-120}$$

$V = 180$ Volts

$$C = \frac{q_1}{V_1} = \frac{360 \times 10^{-6}}{180}$$

$= 2 \times 10^{-6} F = 2\mu F$ **[1]**

(ii) If the voltage applied had increased by 120 V, then

$Va = 180 + 120 = 300\ V$

Hence, charge stored in the capacitor,

$Q_a = CV_a = 2 \times 10^{-6} \times 300 = 600\ \mu\ C$ **[1]**

8. $U_{initial} = \frac{1}{2}CV^2 + 0 = \frac{1}{2}CV^2$

After connecting common potential

$$V_{common} = \frac{(C_1V_1 + C_2V_2)}{C_1C_2} = \frac{V}{2}$$

$$U_{Final} = \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2$$ **[1]**

$$\therefore \frac{U_{Initial}}{U_{Final}} = \frac{\left(\frac{1}{4}CV^2\right)}{\left(\frac{1}{2}CV^2\right)}$$

$$\frac{U_{Initial}}{U_{Final}} = 1:2$$ **[1]**

9. (i) As the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease ($V= Q/C$). As a result, the potential drop across the bulb will increase (since both are connected in series'). So, its brightness will increase.

[1]

(ii) As the resistance (R) is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease (since both are connected in series). So, its brightness will decrease. **[1]**

10. When the capacitors are connected in parallel,

Equivalent capacitance, $C_p = C_1 + C_2$

The energy stored in the combination of the capacitors, $E_p = \frac{1}{2}C_pV^2$

$$E_p = \frac{1}{2}(C_1 + C_2)(100)^2 = 0.25J$$ **[½]**

$$\Rightarrow (C_1 + C_2) = 5 \times 10^{-5}$$ (1)

When the capacitors are connected in series.

Equivalent Capacitance, $C_s = \frac{C_1C_2}{C_1 + C_2}$

The energy stored in the combination of the capacitors, $E_s = \frac{1}{2}C_sV^2$

$$\Rightarrow E_s = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(100)^2 = 0.045J$$

$$\Rightarrow \frac{1}{2}\frac{C_1C_2}{5\times10^{-5}}(100)^2 = 0.045J$$ **[½]**

$$\Rightarrow C_1C_2 = 0.045\times10^{-4}\times5\times10^{-5}\times2$$

$= 4.5 \times 10^{-10}$

$$\Rightarrow (C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2$$

$$\Rightarrow (C_1 - C_2) = \sqrt{7\times10^{-10}} = 2.64\times10^{-5}$$

$C_1 = C_2 = 2.64 \times 10^{-5}$ (2) **[½]**

Solving (1) and (2), we get

$C_1 = 38.2\ \mu F$ And $C_2 = 0.12\ \mu F$

When the capacitors are connected in parallel, the charge on each of them can be obtained as follows:

$Q_1 = C_1 V = 382.2 \times 10^{-6} \times 100 = 38.2\ 10^{-4}\ C$

$Q_2 = C_2 V = 0.12 \times 10^{-6} \times 100 = 0.12\ 10^{-4}\ C$ **[½]**

11. When charged capacitors are connected to each other then the charge will flow from the capacitor with higher potential towards the capacitor with lower potential untill a common potential is reached.

(*a*) In fig. (a) the potential of both the capacitor is same so the charge on left hand capacitor remains the same **[1]**

(*b*) In fig. (b) the potential of left hand capacitor is high so charge from 6*Q* to 3*Q*. Therefore charge on left hand capacitor will decrease. **[1]**

(*c*) In fig. (c) the potential of left hand capacitor is low so charge will flow from 3*Q* to 6*Q*. Therefore charge on left hand capacitor will increase. **[1]**

12. (*i*) Let the capacitance of *X* be C_1 and capacitance of *Y* be C_2

$$C_1 = \frac{\varepsilon_o A}{d}$$

$$C_2 = \frac{\varepsilon_r \varepsilon_o A}{d}$$

$$\frac{C_1}{C_2} = \frac{1}{\varepsilon_r}$$

$$\Rightarrow C_2 = \varepsilon_r C_1$$

$C_1 = C$

$C_2 = 4\ C$ $(\because \varepsilon_r = 4)$ **[½]**

Since two capacitance are connected in series so, equivalent capacitance will be

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$4\propto F = \frac{C\times4C}{C+4C}$$

$$\Rightarrow C = 5\propto F$$

So, $C_1 = 5\mu F$ and $C_2 = 20\mu F$ **[½]**

(*ii*) $C_{eq}\ V_{net} = Q_{Total}$

$Q_{Total} = 60\ \mu C$

Since in series configuration charge on each capacitor is equal.

Hence, $Q_1 = Q_2 = Q_{Total} = 60C$ **[½]**

Using $Q = CV$

$$V_1 = \frac{Q_1}{C_1} = \frac{60\propto C}{5\propto F} = 12V$$

$$V_2 = \frac{Q_2}{C_2} = \frac{60\propto C}{20\propto F} = 3V$$ **[½]**

(*iii*) $$U_1 = \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{1}{2}\frac{(60\propto C)^2}{5\propto F} = 360\propto J$$

$$U_2 = \frac{1}{2}\frac{Q_2^2}{C_2} = \frac{1}{2}\frac{(60\propto C)^2}{20\propto F} = 900\propto J$$ **[½]**

$$\Rightarrow \frac{U_1}{U_2} = \frac{4}{1}$$

$V_1 : V_2 :: 4 : 1$ **[½]**

13. Let the heat dissipated per unit time.

$$H = \frac{(V)^2}{R}$$ **[1]**

$$H = \frac{(12)^2}{6} = 24J/sec$$ **[1]**

The new heat dissipated per unit time ($H = H X$ 9 = 216J/Sec

Let the new voltage be V'

$$\frac{(V)^2}{R} = 216$$ **[1]**

$(V)^2 = 216 \times 6$

$V = 36$ volt

14. Net capacitance before filling the gap with dielectric slab is given by **[½]**

$C_{Initial} = A + B$ ... (i)

Net capacitance at here filling the gap with dielectric slab of dielectric constant

$C_{final} = K(A + B)$ ... (ii) **[½]**

Energy stored by capacitor is given by $U = \frac{Q^2}{2C}$

[½]

So energy stored in capacitor Combination before introduction of dielectric slab

$$U_{initial} = \frac{Q^2}{(A+B)}$$ **[½]**

Energy stored in combination after introduction of dielectric slab

$$U_{final} = \frac{Q^2}{K(A+B)}$$

Ratio of energy stored $\frac{U_{initial}}{U_{final}} = \frac{K}{1}$ **[½]**

15. According to Gauss law:

$$\varepsilon_o E \oint dA = q$$

Where, q is the point charge E is electric field due to the point charge dA is a small area on the Gaussian surface at any distance and ε_o is the proportionality constant.

For a spherical shell at distance r from the point charge, the integral $\oint dA$ is merely the sum of all differential of dA on the sphere. **[1]**

Therefore, $\oint dA = 4\pi r^2$

$$\varepsilon_o E\left(4\pi r^2\right) = q$$

or,

$$E = \frac{q}{\varepsilon_o\left(4\pi r^2\right)}$$ **[1]**

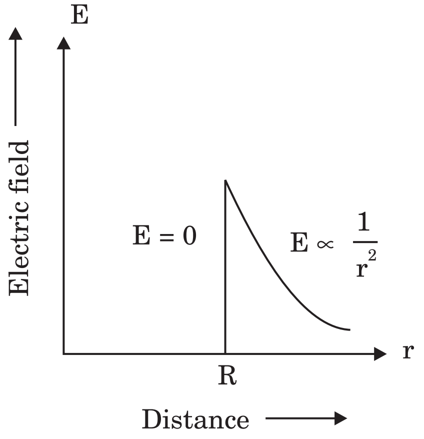
Therefore, for a thin conducting spherical shell of radius R and charge Q. spread uniformly over its surface, the electric field at any point outside the shell is

$$E = \frac{Q}{\varepsilon_o\left(4\pi r^2\right)}$$

Where r is the distance of the point from the centre of the shell

$$E = \frac{q}{\varepsilon_o\left(4\pi r^2\right)}$$

The graph of electric field $E(r)$ with distance r from the centre of the shell for $0 \le r \le \infty$ **[½]**



[½]

Fig. Valuation of Electric field with respect to distance

16. The 12 V battery is in parallel with C_1, C_2 and C_3. C_1, C_2 are in series with each other while C_3 is in parallel with the combination formed by C_1 and C_2. Total voltage drop across

$C_3 = 12\ V$

$q_3 = CV$

Where, q = Charge on the capacitor C_1, C_2, C_3 = 6 μF (Given in the question)

$q_3 = 6 \times 12 = 72\ \mu\ C$ **[1]**

Voltage drop across C_1 and C_2 combined will be 12 V.

Let the voltage drop at $C_1 = V_1$

Let the voltage drop at $C_2 = V_2$

Then, $V = V_1 + V_2$

$$V_2 = \frac{q_2}{C}$$

$$V_1 = \frac{q_1}{C}$$

$$\frac{q_1}{6} + \frac{q_2}{6} = 12$$ **[1]**

As both the capacitors are in series.

$q_1 = q_2 = q$

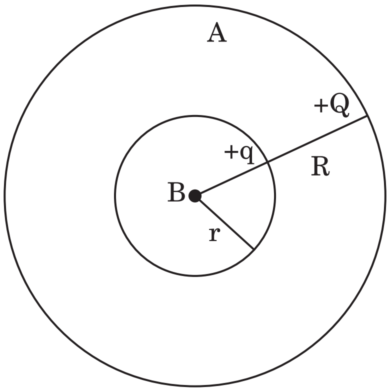
Then,

$$q\left\{\frac{1}{6} + \frac{1}{6}\right\} = 12$$

$q \times \frac{1}{3} = 12$

q = 36 micro coulombs Thus. charge on each of is 36 coulombs. **[1]**

17. Theory. Consider a large spherical conducting shell A having radius R and charge $+Q$. Potential inside the shell, $V_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R}$, Assume that a small conducting sphere B of radius r and carrying charge $+q$ is placed at the centre of shell A, **[1 + 1]**



Potential due to A at the surface of B

$$V_2 = V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{Q}\ V_2 - V_1$$

Potential due to B at the surface of B,

$$V = V_B - V_A = \frac{1}{4\pi\varepsilon_o r} = \frac{q}{r}$$

Total potential at the surface of shell A,

$$V_A = V_1 + V_2 = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{R} + \frac{q}{r}\right]$$

Total potential at the surface of B,

$$V_B = V_1 + V_3 = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{R} + \frac{q}{R}\right]$$

Potential difference,

$$V = V_B - V_A = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{R} + \frac{q}{r} - \frac{Q}{R} - \frac{q}{R}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r} - \frac{1}{R}\right].$$ **[1]**

The potential difference is independent of charge Q on the shell. If the sphere is connected to the shell by a wire, the charge will flow to the shell because the shell is at a lower potential. **[½]**

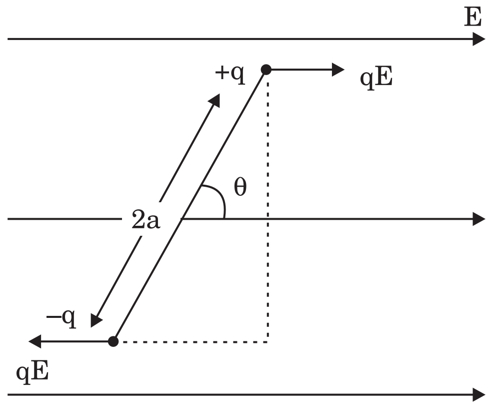
Application.

It can be used to accelerate particles like protons, deuterons, α- particles and other ions. These accelerated particles are called ‘’projectiles”. These particles are used in nuclear physics for collision experiments. **[½]**

Limitations of Van de Graff generator: Due to very high electric field at sphere, sparking & leakage of charge takes place so high pressure gasses are used around sphere. But still **leakage takes place** at higher electric field so **highest potential is limited.** **[1]**

Or

(*a*) Consider an electric dipole placed in uniform electric field then



[1]

Fig. Electric Dipole in Uniform electric field

τ = Force × Perpendicular distance

= $(qE)\ (2a \sin \theta)$

τ = $PE \sin \theta$

τ = PE **[1]**

(*b*) (i) According to Gauss theorem

$$\varphi_{net} = \frac{\sum q}{\varepsilon_o \varepsilon_r} \propto \sum q$$

$$\frac{\varphi S_1}{\varphi S_2} = \frac{2Q}{2Q + 4Q} = \frac{1}{3}$$ **[2]**

(ii) If medium is filled in S_1 then

$$\varphi_{s_1} = \frac{\sum q}{\varepsilon_o \varepsilon_r} = \frac{2Q}{\varepsilon_o \varepsilon_r}$$ **[1]**

18. (*a*) When conductor is placed in an external electric field then induction phenomena occur, due to which induced charge get develop on the conductor surface.

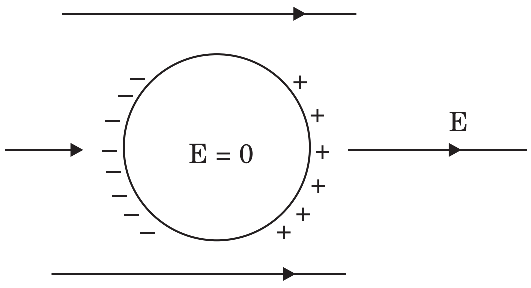
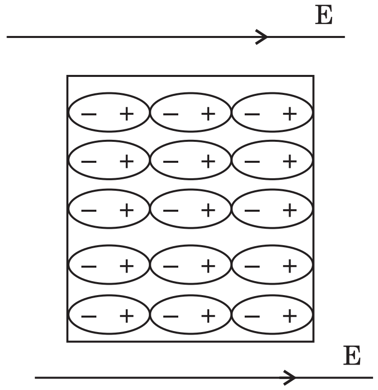


Fig. Conductor in external field

When dielectric is placed in an external electric field then polarization phenomena will occur. **[1]**



[1]

Fig. Polarization

(*b*) After disconnection from battery and doubling the separation between two plates

(*i*) charge on capacitor remains same.

i.e., $CV = C'V'$

$\Rightarrow CV = \left(\frac{C}{2}\right)V'$

$\Rightarrow V' = 2V$ **[1]**

(*ii*) Q Electric field between the plates

$$E' = \frac{\nu'}{d'} = \frac{2\nu}{2d}$$

$$E' = \frac{\nu}{d} = E$$

$\Rightarrow$ Electric field between the two plates remains same. **[1]**

(*iii*) Energy stored in capacitor when connected from battery

$$U_1 = \frac{q^2}{2\times\frac{c}{2}} = \frac{q^2}{c}$$

$$U_2 = 2\left(\frac{q^2}{2c}\right) = 2U_1$$

$U_2 = 2U_1$ **[1]**

Energy stored in capacitor gets doubled to its initial value.

19. $Q = CV$

$$Q = \left(\frac{\varepsilon_o A}{d}\right)(Ed)$$

$Q = \varepsilon_o AE$

$$\therefore E = \frac{Q}{\varepsilon_o A}$$

[1]

Therefore, the electric field between the parallel plates depends only on the charge and the plate area. It does not depend on the distance between the plates.

Since, the charge as well as the area of the plates does not change, the electric field between the plates also does not change. **[1]**

Let the initial capacitance be 'C' and the final capacitance be 'C''. Accordingly,

$$C = \frac{\varepsilon_o A}{d}$$

And $C' = \frac{\varepsilon_o A}{d}$

$\frac{C}{C'} = 2$ or $C' = \frac{C}{2}$ **[1]**

Hence, the capacitance of the capacitor gets halved when the distance between plates is doubled.

(*iii*) Energy of a capacitor, $U = \frac{1}{2}\frac{Q^2}{C}$

Since, Q remains the same but the capacitance decreases,

$$U' = \frac{1}{2}\frac{Q^2}{\left(\frac{C}{2}\right)}$$

[1]

$$\frac{U}{U'} = \frac{1}{2}$$

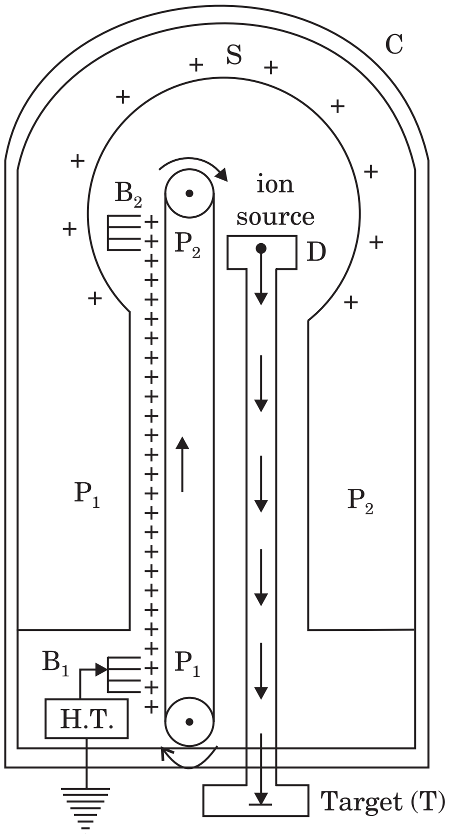
$U' = 2U$

The energy stored is in the capacitor gets doubled when the distance between the plates is doubled.

[1]

20. Van de Graff generator is the device used for building up high voltages of the order of a few million volts. Such high voltages are used to accelerate charged particles such as electrons, protons, ions, etc. It is based on the principle that charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it. **[1]**

Construction:



[1]

Fig.: Van de Graff Generator

It consists of large spherical conducting shell (S) supported over the insulating pillars. A long narrow belt of insulating material is wound around two pulleys P_1 and P_2, B_1 and B_2 are two sharply pointed metal combs. B_1 is called the spray comb and B_2 is called the collecting comb.

Working: The spray comb is given a positive potential by high tension source. The positive charge gets sprayed on the belt. As the belt moves and reaches the sphere, a negative charge is induced on the sharp ends of collecting comb B2 and an equal positive charge is induced on the farther end of B_2 . This positive charge shifts immediately to the outer surface of S. Due to discharging action of sharp points of B_2, the positive charge on the belt is neutralized. The uncharged belt returns down and collects the positive charge from B_1, which in turn is collected by B_2. This is repeated. Thus, the positive charge of S goes on accumulating. In this way, potential differences of as much as 6 or 8 million volts (with respect to the ground) can be built up. The main limiting factor on the value of high potential is the radii. If the electric field just outside the sphere is sufficient for dielectric breakdown, of air, no more charge can be transferred to it. **[1 + 1]**

For a conducting sphere, electric field just outside sphere

$$E = \frac{Q}{4\pi\varepsilon R^2}$$

And electric potential

$$V = \frac{Q}{4\pi\varepsilon_o R}$$

Thus, $E = \dfrac{V}{R}$

Now, for $E = 3 \times 10^6 V/m$ (dielectric breakdown)

Radius of sphere should be 1 m.

Thus, the maximum potential of a sphere of radius 1 m would be $3 \times 10^6 V$. **[1]**

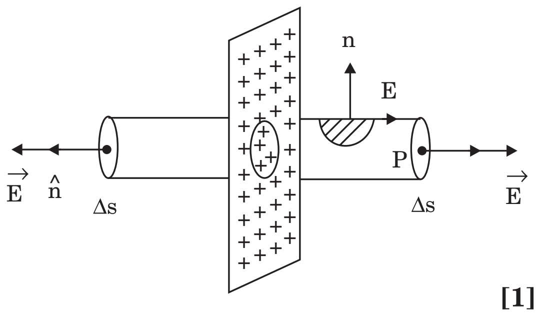
OR

(*a*) Electric Flux: It is the number of electric field lines passing through a surface normally.

$$\phi = \vec{E} \cdot \vec{A}$$

Where E electric field, A = Area

S.I. unit of flux is Nm^2C^{-1} **[1]**



[1]

(*b*) Consider a uniformly infinite plane sheet of charge density σ .

We have to find electric field E at point P as shown in figure in the form of cylinder.

Applying Gauss's law,

$$\phi = \oint \vec{E} \cdot d\vec{s} = \frac{\sigma \Delta s}{\varepsilon_o}$$

$$\Rightarrow E\Delta s + E\Delta s + 0 = \frac{\sigma \Delta s}{\varepsilon_o}$$

[1]

$$= 2E\Delta s = \frac{\sigma \Delta s}{\varepsilon_o}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_o}$$

[1]

It shows that electric field is uniform due to charged infinite plane sheet. Also, we can say that E is independent of distance from the sheet.

(*c*) $E = \frac{\sigma}{2\varepsilon_o}$ **[½]**

(*i*) Direction of field will be away from the sheet if sheet is positively charged.

(*ii*) $E = \frac{-\sigma}{2\varepsilon_o}$

Direction of field will be towards the sheet if sheet is negatively charged. **[½]**

Smart Notes ....

CHAPTER 3

Current Electricity

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Electric current; flow of electric charges in a metallic conductor; drift velocity, mobility and their relation with electric current	1Q (5 marks)	1Q (2 marks)	1Q (5 marks)		
Ohm's law, electrical resistance, V-I characteristics (linear and non-linear)				1Q (1 mark)	1Q (2 marks)
electrical energy and power, electrical resistivity and conductivity				1Q (3 marks)	1Q (3 marks), 1Q (4 marks)
Carbon resistors, colour code for carbon resistors; series and parallel combinations of resistors; temperature dependence of resistance					
Internal resistance of a cell,potential difference and emf of a cell, combination of cells in series and in parallel, measurement of internal resistance of a cell	1Q (2 marks)	1Q (3 marks)			1Q (2 marks)
Kirchhoff's laws and simple applications; Wheatstone bridge, metre bridge			1Q (5 marks)	1Q (3 marks)	
Potentiometer - principle and its applications to measure potential difference and for comparing emf of two cells	1Q (5 marks)		1Q (3 marks)		

On the basis of above analysis, it can be said that from exam point of view Kirchhoff's Rule, Drift Velocity, Relaxation Time Meter Bridge, Potentiometer, Resistance in series and parallel, emf of a cell and Power loss are most important concepts of the chapter. This is one of the important chapters in Class XII Physics from exam point of view.

Topic 1: Electricity conduction, Ohm's law and resistance

Summary

Electric Current: Net charge flowing across a given area of conductor per unit time is defined as electric current.

$I = \frac{q}{t}$, S.I. unit of current is Ampere (A).

A steady current is generated in a closed circuit where electric charge moves from lower to higher potential. ***Electromotive force*** or ***emf*** is the work done by the source in taking the charge from higher to lower potential energy.

Drift velocity: The free electrons drift with some velocity towards the positive terminal when a potential difference is applied across the ends. The average velocity with which the electrons move is termed as drift velocity.

Drift velocity, $v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$

Where e = charge on electron

E = Electric field intensity

V = Potential difference across the ends of the conductor

τ = Relaxation time

m = Mass of electron

Relation between current and drift velocity: Current is directly proportional to the drift velocity.

$I \propto v_d$

When the number of electrons are less, current is less so the drift velocity is small.

When the number of electrons are large, high current flows so the drift velocity is large.

Ohm's law: The voltage across the ends of the conductor is directly proportional to the electric current flowing through the conductor.

$V \propto I$

Or V = IR, where R is the electrical resistance of the conductor

Resistance: The property that resists the flow of current through any conductor is called the resistance of the conductor.

$R = \frac{V}{I}$

It varies directly with the length of the conductor while depends inversely on the area of cross section of the conductor.

$R = \frac{\rho l}{A}$, ρ being the resistivity of the material of the conductor.

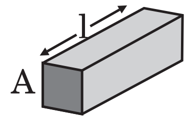


Fig.: Resistance in a conductor

Resistivity: It depends on the nature of the material and temperature. It is also termed as specific resistance.

$\rho = \frac{m}{ne^2\tau}$ gives the relation between resistivity and relaxation time.

There is an increasing order of resistivity as we go from metal to insulator.

$\rho_{metals} < \rho_{semiconductors} < \rho_{insulators}$

Conductivity and conductance: The reciprocal of resistivity is conductivity (σ).

$\sigma = \frac{1}{\rho}$ and its S.I. unit is $\Omega^{-1}m^{-1}$.

The reciprocal of resistance is the conductance of the conductor. Its S.I. unit is mho.

Current Density: The amount of charge flowing per unit area per second is called the current density.

$J = mqv_d$, where v_d is the drift velocity of the charge carriers, n is the number of charge carriers and q is the charge.

The relation between current density and conductivity is

$J = \sigma E$

Mobility: Mobility is the ratio of drift velocity to the applied electric field. Mobility is symbolized by μ .

$\mu = \frac{v_d}{E} = \frac{q\tau}{m}$

Its S.I. unit is $m^2s^{-1}V^{-1}$.

Resistors: The objects which resist the flow of charge are called resistors which can be of two types, i.e. wire bound resistors and carbon resistors.

Resistors can combine in two different ways; either in series or in parallel.

- Consider n number of resistors connected in series, then the combined resistance will be as follows:

  $R_{eqv} = R_1 + R_2 + R_3 + ... + R_n$

Same amount of current will flow through each resistor connected in series while the potential difference would be different for every resistor.

- Consider n number of resistors connected in parallel, then the combined resistance will be as follows:

$$\frac{1}{R_{eqv}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

The current flowing through each resistor would be different in this case while the potential difference would be same for all the resistors.

Internal resistance: It is the resistance on the current offered by the electrolyte and the electrodes. It is symbolize by r.

Let us assume a cell with 2 electrodes connected by an external resistance R. Then current is, $I = \frac{\varepsilon}{R + r}$

where ε = emf, r = Internal resistance

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

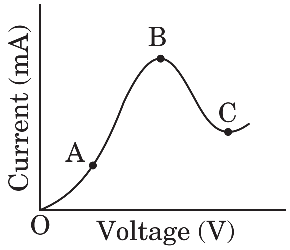
1. When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction?

[ALL INDIA 2012]

2. Show on a graph, the variation of resistivity with temperature for a typical semiconductor.

[ALL INDIA 2012]

3. The graph shown in the figure represents a plot of current versus voltage for a given semiconductor. Identify the region, if any, over which the semiconductor has a negative resistance.



[ALL INDIA 2013]

4. Define the term 'Mobility' of charge carriers in a conductor. Write its S.I. unit.

[DELHI 2014]

5. Plot a graph showing variation of current versus voltage for the material Ge.

[DELHI 2014]

6. Define the term 'drift velocity' of charge carriers in a conductor and write its relationship with the current flowing through it.

[DELHI 2014]

7. Define the term 'electrical conductivity' of a metallic wire. Write its S.I. unit.

[DELHI 2014]

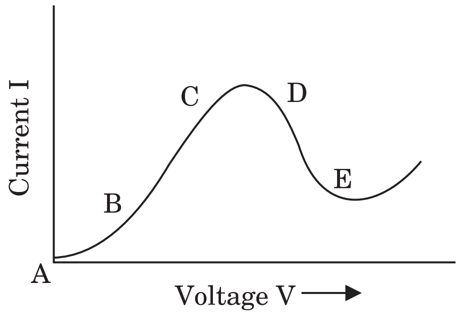
8. Show variation of resistivity of copper as a function of temperature in a graph.

[DELHI 2014]

9. Graph showing the variation of current versus voltage for a material GaAs is shown in the figure, Identify the region of:

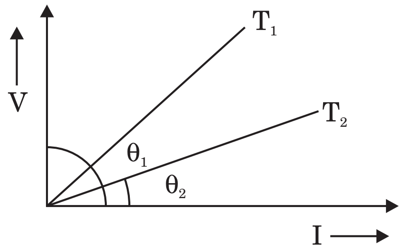
(i) Negative resistance

(ii) Where Ohm's law is obeyed



[DELHI 2015]

10. *V-I* graph for a metallic wire at two different temperature T_1 and T_2 is as shown in the figure. Which of the two temperatures is higher and why ?



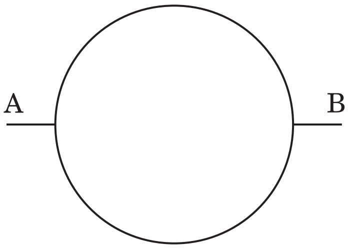
[ALL INDIA 2015]

11. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer.

[ALL INDIA 2017]

2 Marks Questions

12. A wire of resistance $8R$ is bent in the form of a circle. What is the effective resistance between the ends of diameter AB?



[DELHI 2018]

13. Explain the term 'drift velocity' of electrons in a conductor. Hence obtain the expression for the current through a conductor in terms of 'drift velocity'. **[ALL INDIA 2013]**

14. Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 2.5×10^{-7} m^2 carrying a current of 2.7 A. Assume the density of conduction electrons to be 9×10^{28} m^{-3}.

[ALL INDIA 2014]

15. When $5V$ potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m/s. If the electron density in the wire is 8×10^{28} m^{-3}, calculate the resistivity of the material of wire.

[ALL INDIA 2018]

3 Marks Questions

16. Define the terms

(i) drift velocity, (ii) relaxation time.

A conductor of length L is connected to a source of emf e. If this conductor is replaced by another conductor of same material and same area of cross-section but of length $3L$, how will the drift velocity change?

[ALL INDIA 2011]

17. A potential difference V is applied across a conductor of length L and diameter D. How is the drift velocity, v_1 of charge carriers in the conductor affected when (i) V is halved, (ii) L is doubled and (iii) D is halved? Justify your answer in each case.

[ALL INDIA 2015]

18. Define the terms (i) drift velocity, (ii) relaxation time. A conductor of length L is connected to a *dc* source of emf 8. If this conductor is replaced by another conductor of same material and same area of cross-section but of length $3L$, how will the drift velocity change?

[ALL INDIA 2011]

19. Derive an expression for drift velocity of free electrons in a conductor in terms of relaxation time.

[DELHI 2017]

20. (a) Define the term 'conductivity' of a metallic wire. Write its SI unit.

(b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence obtain the relation between current density and the applied electric field E.

[ALL INDIA 20168]

5 Marks Questions

21. (i) Define the term drift velocity.

(ii) On the basis of electron drift, derive an expression for resistivity of a conductor in terms of number density of free electrons and relaxation time. On what factors does resistivity of a conductor depend?

(iii) Why alloys like constantan and manganin are used for making standard resistors?

[DELHI 2016]

Solutions

1. No, when electric field is applied the electrons will have net drift from lower to higher field but locally electrons may collide with ions and may change its direction of motion. **[1]**

2. The following curve shows the variation of resistivity with temperature for a typical semiconductor.

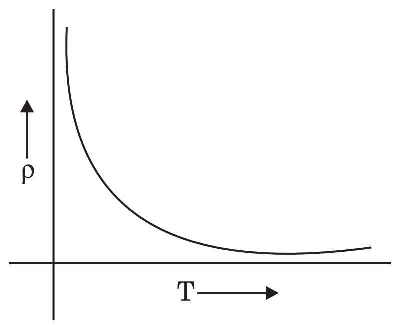


Fig.: Variation of resistivity with respect to temperature. **[½]**

This is because for a typical semiconductor, resistivity decreases rapidly with increasing temperature. **[½]**

3. From ohm's law:

In the region BC, ΔV is positive and ΔI is negative, hence resistance is negative. **[1]**

4. Mobility of charge carriers in a conductor is defined as the magnitude of their drift velocity per unit applied electric field. **[½]**

Mobility, $\mu \rightarrow$ Drift of electric field

$$\mu = \frac{V_d}{E}$$ **[½]**

S.I. unit of mobility is $m^2V^{-1}s^{-1}$ or $ms^{-1}N^{-1}C$.

5. Current—Voltage characteristics graph for :

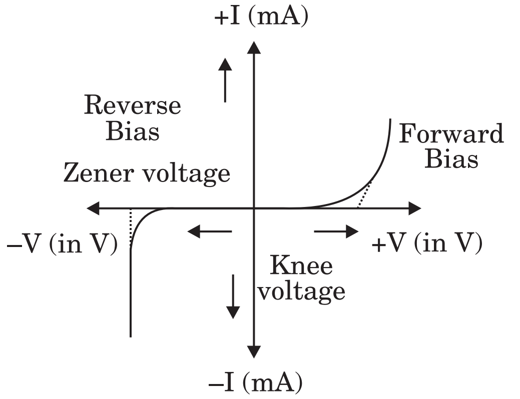


Fig.: V-I Characteristics of material Ge **[1]**

6. The net speed achieved by an electron due to a current carrying conductor is called as drift velocity. **[1]**

i = neAV

n is number of electron per unit volume

e is charge per unit electron

A be cross sectional area

V be drift velocity

7. The electric conductivity of a metallic wire is defined as the ratio of the current density to the electric field it creates electrical conductivity.

$\sigma = \frac{J}{E}$, S.I. Unit = ohm m^{-1}. **[½]**

Or, the reciprocal of resistivity of a material is called its electrical conductivity.

$$\text{Conductivity} = \frac{1}{\text{resistivity}} \text{ or } \sigma = \frac{1}{\rho}$$ **[½]**

8. The variation of resistivity of copper with temperature is parabolic in nature. This is shown in the following graph:

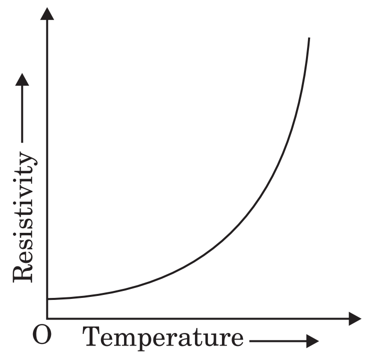


Fig.: Variation of resistivity of Cu with Temperature **[1]**

9. (*i*) DE is the region of negative resistance because the slope of curve in this part is negative. **[½]**

(*ii*) BC is the region where Ohms law is obeyed because in this part, the current varies linearly with the voltage. **[½]**

10. The slope of V-I graph gives the resistance of the metallic wire and the slope is higher at temperature T_1 and we know that on increasing the temperature of metallic wire resistance of the wire increases, so T_1 temperature is higher. **[½]**

11. Heat dissipated in a wire is

$H = I^2RT$

$$H = I^2 \frac{\rho l}{A} \times t \qquad \left(\therefore R = \frac{\rho l}{A}\right)$$ **[½]**

For same current "I" length (l) and Area (A), H depends on ρ and t.

$\rho_{nichrome} > \rho_{copper}$.

So, $H_{Nichrome} > H_{Copper}$. **[½]**

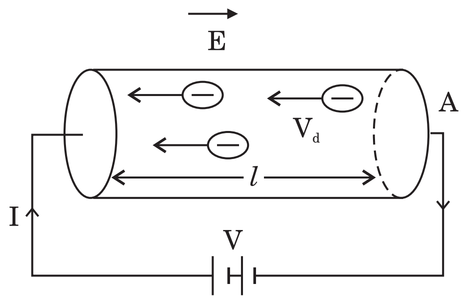
12. When it is bent to form circle the ends of diameter separates the circle into semicircles in such a way that the two semicircles make a parallel connection in the circuit. Also both the semicircles have equal circumference therefore they have equal resistance. We get $4R$ as the resistances of the semi circles $\{8R = 4R + 4R\}$

[1]

Therefore 1/effective resistance $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$,

Effective resistance **[1]**

13. Drift velocity: Drift velocity is defined as the average velocity with which free electrons in a conductor get drifted in a direction opposite to the direction of the applied electric field. Let n be the number of free electrons per unit volume of the conductor. Then, total number of free electrons in the conductor = n × volume of the conductor = $n \times Al$ **[1]**



If e is the magnitude of charge on each electron, then the total charge in the conductor,

$Q = (nAl)e$ ------- (1)

The time taken by the charge to cross the conductor length is given by

$$t = \frac{1}{v_d}$$

Where v_d is the drift velocity of the electrons.

According to the definition of electric current,

$$I = \frac{Q}{t} = \frac{nAle}{l/v_d} = neAv_d$$

14. $I = ne\,Av_d$ **[1]**

$$\therefore V_d = \frac{1}{neA} = \frac{2.7}{9\times10^{28}\times1.6\times10^{-19}\times2.5\times10^{-7}}$$

$= 7.5 \times 10^{-4}$ m/s **[1]**

15. Given $p.dV = 5V$

$l = 0.1\ m$

$v_d = 0.1\ m$

$n = 8 \times 10^{28}$

$E = \rho J$

$$\frac{V}{l} = \rho nev_d$$ **[1]**

$$\frac{V}{nelv_d} = \rho$$

$$= \frac{5}{0.1\times8\times10^{28}\times1.6\times10^{-19}\times2.5\times10^{-4}}$$ **[1]**

$= 1.56 \times 10^{-5}\ \Omega m$

16. The average velocity of electrons, independent of time, although accelerated through a conductor is called drift velocity. **[1]**

The average time interval between successive collisions is called relaxation time. **[1]**

Drift velocity

$$V_d = \frac{e\tau E}{m} = \frac{e\tau V}{mL}$$ **[1]**

Where V is the potential difference applied across the length of the conductor.

Keeping V constant, if Length L of the conductor is made 3 times, the drift velocity will become 1/3rd.

17. We know drift velocity is given by

$$|v_d| = \frac{e\tau}{m}E$$ **[1]**

Also, $E = \frac{V}{L}$

So, $|v_d| = \frac{e\tau}{m}\left(\frac{V}{L}\right)$ **[1]**

(*i*) When V is halved drift velocity (v_d) gets halved

(*ii*) When L is doubled drift velocity (v_d) gets halved

(*iii*) When D is halved drift velocity (v_d) remains same. **[1]**

18. (*i*) Drift velocity: The average velocity with which the free electrons drift towards positive terminal under the influence of an external field is called drift velocity. **[1]**

(*ii*) Relaxation time: Average time interval between two successive collisions of an electron with the ions / atoms of the conductor. **[1]**

The drift velocity will be inversely proportional to L, i.e. $\nu_d \propto \frac{1}{L}$ hence it will become one third of its initial value.

$$\nu_d = \frac{\nu_d}{3}$$

19. If there are N electrons and the velocity of the electron at a given time is v,

where, $i = (1, 2, 3, \ldots N)$, then $\frac{1}{N}\sum_{i=1}^{N}\nu_i = 0$

(If there is no external field)

When an external electric field is present, the electrons will be accelerated due to this field by

$$\vec{a} = \frac{-e\vec{E}}{m}$$

Where, $-e$ = Negative charge of the electron

E = External field **[1]**

m = Mass of an electron Let v_i be the velocity immediately after the last collision after which external field was experienced by the electron. If v_i is the velocity at any time t then from the equation $V = u +$ at, we obtain

$$\vec{V_i} = \vec{\nu_i} - \frac{-e\vec{E}}{m}t \quad \ldots\ldots\ldots\ldots\ldots\text{(i)}$$ **[1]**

For all the electrons in the conductor, average value of v_i is zero. The average of v_i is v_d or drift velocity.

This is the average velocity experienced by an electron in an external electric field. There is no fixed time after which each collision occurs. Therefore, we take the average time after which one collision takes place by an electron.

Let this time, also known as relaxation time τ be.

Substituting this in equation (i)

$$\vec{\nu_i} = 0$$

$$t = \tau$$

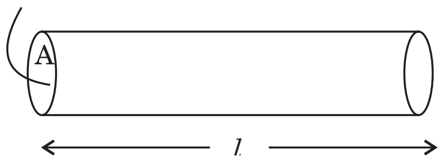
$$\vec{\nu_i} = \vec{\nu_d}$$

Then,

$$\vec{\nu_d} = \frac{-e\vec{E}}{m}\tau$$ **[1]**

Negative sign shows that electrons drift opposite to the applied field.

20. (*a*) Conductivity



$$R = \rho\frac{l}{A}$$

$$\rho = \frac{RA}{l}$$

$$\frac{1}{\rho} = \frac{l}{R \cdot A}$$ **[1]**

Conductivity of a wire is defined as reciprocal of resistivity. S.I. unit mho / m

(*b*) As we know that

$$R = \frac{\rho l}{A}$$

$$V = IR$$

$$V = \frac{I\rho l}{A}$$

$$\frac{I}{A} = j = \text{current density}$$

$$V = j\,\rho l$$

$$V = E\,l$$

$$E, l = j\,\rho l$$

$$E = j\,\rho$$

$$j = \frac{E}{\rho}$$ **[1]**

$$V = u + at$$

$$a = \frac{F}{m}$$

$$a = \frac{-eE}{m}$$

$$I \cdot \Delta t = neA\left(V_d\right)\Delta t$$

$$j = \frac{i}{A}\Delta t$$

$$j = neA\frac{eE}{m}$$

$$j = \frac{ne^2E}{m}$$ **[1]**

21. (*i*) Drift velocity is defined as the average velocity which the electrons are drifted towards the positive terminal under the effect of applied electric field. Thermal velocities are randomly distributed and average thermal velocity is zero.

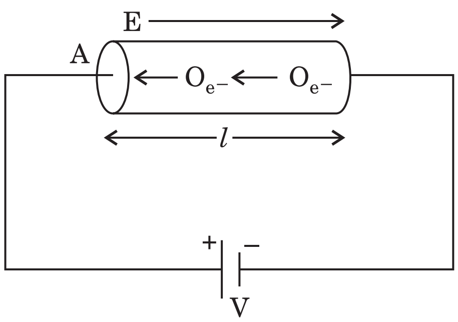
$$\frac{\overrightarrow{u_1} + \overrightarrow{u_2} + \dots\dots + \overrightarrow{u_N}}{N} = 0$$ **[1]**

$$v_d = -\frac{eE\tau}{m}$$

(*ii*) We know that the current flowing through the conductor is:

$$\mathrm{I} = Aev_d$$

$$I = neA\left(-\frac{eE\tau}{m}\right)$$ **[1]**



Using, $E = -\frac{V}{I}$

$$I = neA\left(\frac{eV}{ml}\right)\tau$$

$$= \left(\frac{ne^2A\tau}{ml}\right)V = \frac{1}{R}V$$ **[1]**

$I \propto V \rightarrow$ Ohm's Law

Where, $R = \frac{ml}{nAe^2\tau}$ is a constant for a particular conductor at a particular temperature and is called the resistance of the conductor.

$$R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A} = \frac{\rho l}{A}$$ **[1]**

$$\rho = \left(\frac{m}{ne^2\tau}\right)$$

Where P is the specific resistance or resistivity of the material of the wire. It depends on number of free electron per unit volume and temperature.

(*iii*) They are used to make standard resistors because: **[1]**

(a) They have high value of resistivity

(b) Temperature coefficient of resistance is less.

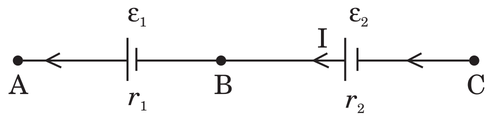
(c) They are least affected by temperature.

Topic 2: Kirchhoff's Laws, cells and their combinations

Summary

Cells in series and in parallel

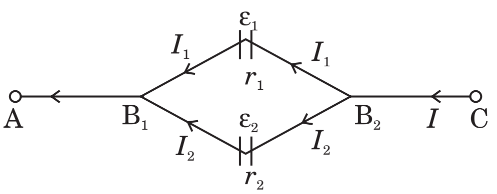
- The equivalent emf of a series combination of n cells is just the sum of their individual emfs
- The equivalent internal resistance of a series combination of n cells is the sum of their internal resistances.



$\varepsilon = \varepsilon_1 + \varepsilon_2$

- In a parallel connection,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \ldots\ldots + \frac{1}{r_n} \text{ and } \frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \ldots\ldots + \frac{\varepsilon_n}{r_n}$$



Kirchhoff's law:

- **Junction Rule:** The sum of currents entering a junction would be equal to the sum of currents leaving the junction.
- **Loop Rule:** The sum of changes in potential around any loop that is closed should be zero.

  Wheatstone bridge: It is an arrangement of four resistors in a way so that a galvanometer is placed between the two opposite arms.

  There is a null-point condition in the wheat stone bridge where current is zero which can be represented as follows:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

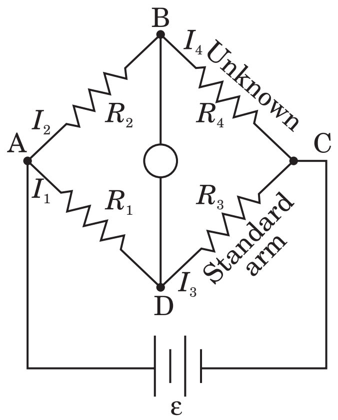


Fig.: Wheastone bridge

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. A cell of emf E and internal distance r draws a current 'I'. Write the relation between terminal voltage 'V' in terms of E, I, r.

   [DELHI 2013]

2. A heating element is marked 210 V, 630 W. What is the value of current drawn by the element when connected to a 210 V, dc source?

   [DELHI 2013]

3. Two identical cells, each of emf E, having negligible internal resistance, are connected in parallel with each other across an external resistance R. What is the current through this resistance?

   [ALL INDIA 2013]

2 Marks Questions

4. A cell of emf E and internal resistance r is connected to two external resistances R_1 and and R_2 a perfect ammeter. The current in the circuit is measured in four different situations:

   (i) without any external resistance in the circuit

   (ii) with resistance R_1 only

   (iii) with R_1 and R_2 in series combination

   (iv) with R_1 and R_2 in parallel combination

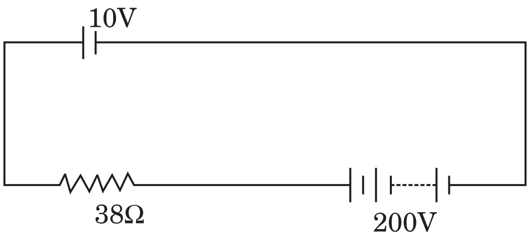
The currents measured in the four cases are 0.42 A, 1.05 A, 1.4 A and 4.2 A, but not necessarily in that order. Identify the currents corresponding to the four cases mentioned above.

[ALL INDIA 2012]

5. An ammeter of resistance 1 Ω can measure current up to 1.0 A. (i) What must be the value of the shunt resistance to enable the ammeter to measure up to 5.0 A ? (ii) What is the combined resistance of the ammeter and the shunt?

[DELHI 2013]

6. A 10 V battery of negligible internal resistance is connected across a 200 V battery and a resistance of 38Ω as shown in the figure. Find the value of the current in circuit.



[DELHI 2013]

7. The emf of a cell is always greater than its terminal voltage. Why?

[DELHI 2013]

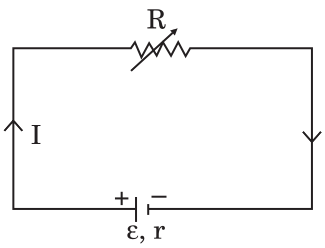
8. A cell of emf 'E' and internal resistance is connected across a variable resistor 'R'. Plot a graph showing the variation of terminal potential 'V' with resistance 'R'. Predict from the graph the condition under which 'V' becomes equal to 'E'.

[DELHI 2017]

9. A variable resistor R is connected across a cell of emf ε and internal resistance r as shown in the figure. Draw a plot showing the variation of:

(i) Terminal voltage V

(ii) the current I, as a function of R.



[ALL INDIA 2015]

10. State Kirchhoff's rules. Explain briefly how these rules are justified.

[DELHI 2014]

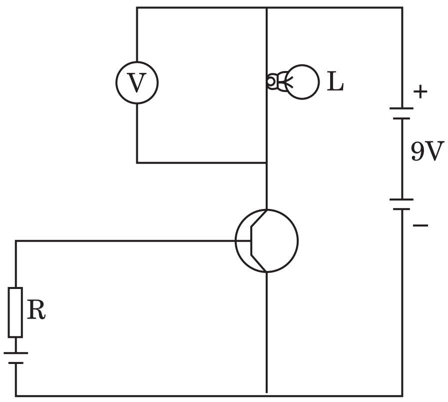
11. An ammeter of resistance 0.80 Ω can measure current up to 1.0 A.

(i) What must be the value of shunt resistance to enable the ammeter to measure current up to 5.0 A?

(ii) What is the combined resistance of the ammeter and the shunt?

[DELHI 2013]

12. In the given circuit diagram a voltmeter 'V' is connected across a lamp 'L'. How would (i) the brightness of the lamp and (ii) voltmeter reading 'V' be affected, if the value of resistance 'R' is decreased? Justify your answer.



[DELHI 2013]

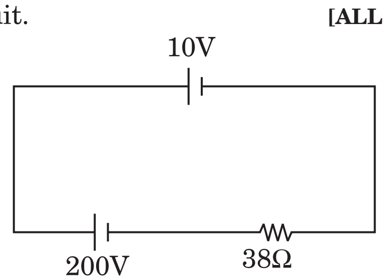
13. A cell of emf 'E' and internal resistance 'r' is connected across a variable resistor 'R'. Plot a graph showing variation of terminal voltage 'V' of the cell versus the current 'I'. Using the plot, show how the emf of the cell and its internal resistance can be determined.

[ALL INDIA 2014]

14. Two cell of emfs 1.5 V and 2.0 V having internal resistances 0.2 Ω and 0.3 Ω respectively are connected in parallel. Calculate the emf and internal resistance of the equivalent cell.

[DELHI 2016]

15. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200V and internal resistance 38Ω as shown in the figure. Find the value of current in the circuit. **[ALL INDIA 2018]**

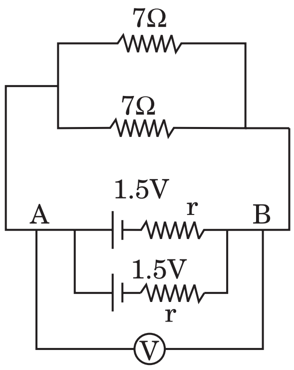


3 Marks Questions

16. A cell of emf 'E' and internal resistance 'r' is connected across a variable load resistor R. Draw the plots of the terminal voltage V versus (i) R and (ii) the current i. It is found that when $R = 4\Omega$, the current is $1A$, when R is increased to 9Ω, the current reduces to $0.5\,A$. Find the values of the emf E and internal resistance r.

[DELHI 2016]

17. Two identical cells of emf 1.5 V each joined in parallel supply energy to an external circuit consisting of two resistances of 7Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell.



[ALL INDIA 2016]

18. A cell of emf E and internal resistance r is connected to two external resistances R_1 and R_2 and a perfect ammeter. The current in the circuit is measured in four different situations:

(i) without any external resistance in the circuit

(ii) with resistance R_1 only

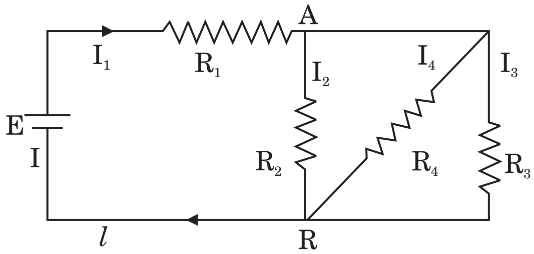
(iii) with R_1 and R_2 in series combination

(iv) with R_1 and R_2 in a parallel combination

The currents measured in the four cases are 0.42 A, 1.05 A, 1.4 A and 4.2 A, but not necessarily in the order. Identify the currents corresponding to the four cases mentioned above.

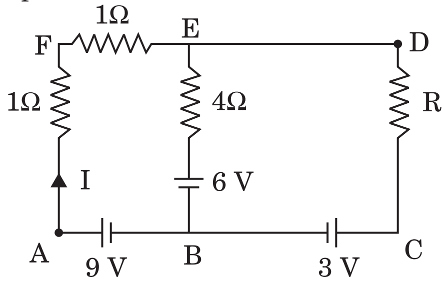
[DELHI 2014]

19. In the circuit shown, $R_1 = 4\,R_2 = R_3 = 15, R_4 = 30$ and $E = 10$V. Calculate the equivalent resistance of the circuit and the current in each resistor.



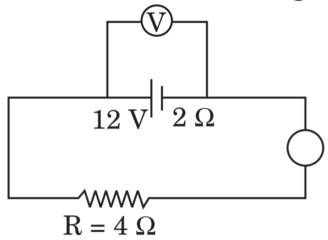
[ALL INDIA 2011]

20. Using Kirchhoff's rule, determine the value of unknown resistance R in the circuit so that no current flows through 4Ω resistance. Also, find the potential difference between A and D.



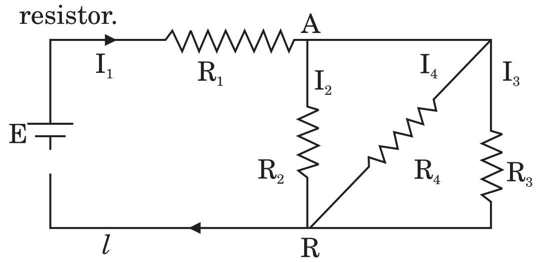
[ALL INDIA 2012]

21. In the figure shown, an ammeter 'A' and a resistor of 4 Ω are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of 2 Ω. Calculate the voltmeter and ammeter reading.



[ALL INDIA 2017]

22. In the circuit shown, $R_1 = 4\ \Omega$, $R_2 = R_3 = 15\ \Omega$, $R_4 = 30\Omega$ and $E = 10$ V. Calculate the equivalent resistance of the circuit and the current in each resistor.



[ALL INDIA 2011]

5 Marks Questions

23. (a) State Kirchhoff's rules for an electric network. Using Kirchhoff's rules, obtain the balance condition in terms of the resistance of four arms of Wheatstone bridge.

[DELHI 2013]

Solutions

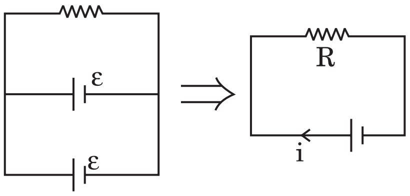
1. When the current I draws from a cell of emf E and internal resistance r, then the terminal voltage is $V = E - IR$ **[1]**

2. In dc source, $P = VI$

   Given that, $P = 630$ Wand $V = 210\ V$

So, $I = \frac{P}{V} = \frac{630}{210} = 3A$ **[1]**

3. **[1]**



$i = \frac{\varepsilon}{R}$

4. The current relating to corresponding situations is as follows:

   (i) Without any external resistance in the circuit:

   $$I_1 = \frac{E}{r}$$

   The current in this case would be maximum.
   So $I_1 = 4.2A$ **[½]**

   (ii) With resistance R_1 only:

   $$I_2 = \frac{E}{r + R_1}$$

   The current in this case will be second smallest value, so $I_2 = 1.05A$ **[½]**

   (iii) With R_1 and R_2 in series combination

   $$I_3 = \frac{E}{r + (R_1 + R_2)}$$

   The current in this case will be minimum as the resistance will be maximum, so $I_3 = 0.42A$ **[½]**

   (iv) With R_1 and R_2 in parallel combination

   $$I_4 = \frac{E}{r + \left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

   The current in this case would be the second largest value so $I_3 = 0.42A$ **[½]**

5. We have,
Resistance of ammeter, $R_A = 1\ \Omega$
Maximum current across ammeter, $I_A = 1.0A$
So, voltage across ammeter,
$V = IR = 1.0 \times 1.0 = 1V$

   (i) Resistance of ammeter with shunt,

   $$R = \frac{R_A x}{R_A + x} = \frac{x}{1 + x}$$

   Current through ammeter, $I = 5A$

   Now, $\left(\frac{x}{1+x}\right) \times 5 = 1.0$ **[1]**

   $x \times 5 = 1 + x$

   $4x = 1$

   $x = 0.25$

   Thus the shunt resistance is 0.25 Ω

   (ii) Combined resistance of the ammeter and the shunt,

   $$R = \frac{x}{1 + x}$$

   $$R = \frac{0.25}{1 + 0.25}$$

   $$R = \frac{0.25}{1.25}$$

   $= 0.2\ \Omega$ **[1]**

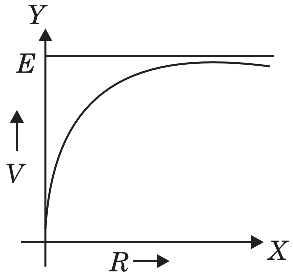
6. Since, the positive terminal of the batteries are connected together, so the equivalent emf of the batteries is given by
   $E = 200–10 = 190\ V$ **[1]**

   $$I = \frac{E}{R} = \frac{190}{38}$$ **[1]**

   Hence, the current in the circuit is given by $I = 5A$

7. The emf of a cell is greater than its terminal voltage because there is some potential drop across the cell due to its small internal resistance. **[2]**

8. V becomes equal to E when no current flows through the circuit. **[1]**



   The condition under which V will be equal to E is $E = \infty$ **[1]**

9. (i) Terminal voltage across a cell as a function of R **[1]**

   As resistance R increases current (I) in the circuit decreases and terminal voltage (V) increases. We know, $V = \varepsilon – Ir$, Where ε is emf of the cell.

   (ii) Current I as a function of R.

   The current across a cell is given by

   $$I = \frac{\varepsilon}{R + r}$$ **[1]**

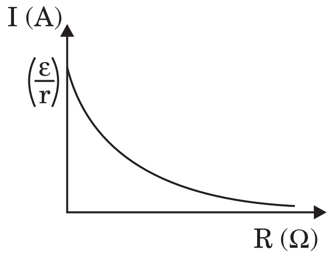
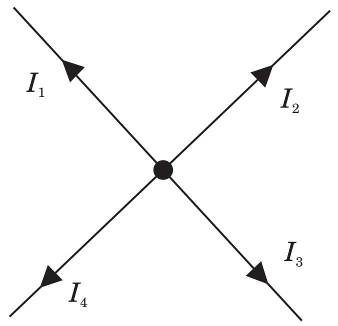


Fig.: I versus R

10. Kirchhoff s first Law—Junction Rule: In an electrical circuit, the algebraic sum of the currents meeting at a junction is always zero.



[1]

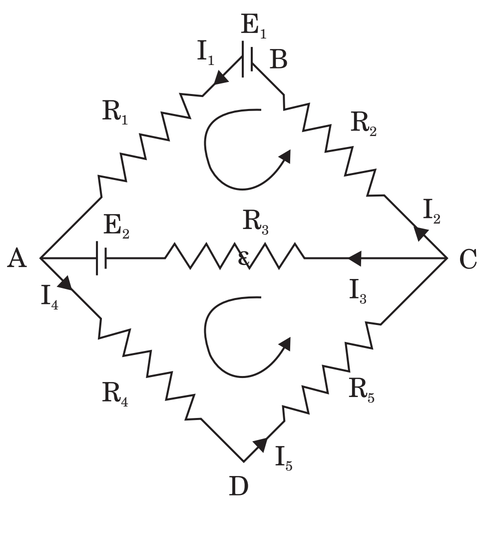
I_1, I_2, I_3 and I_4 are the currents flowing through the respective wires.

Convention: The current flowing towards the junction is taken as positive. The current flowing away from the junction is taken as negative.

$I_3 + (-I_1) + (-I_2) + (-I_4) = 0$

This law is based on the law of conservation of charge.

Kirchhoff's Second Law — Loop rule: In a closed loop, the algebraic sum of the emf 's is equal to the algebraic sum of the products of the resistances and the currents flowing through them.



[1]

For the closed loop *BACB*:

$$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$$

For the closed loop *CADC*:

$$E_2 = I_3R_3 + I_4R_4 + I_5R_5$$

This law is based on the law of conservation of energy.

11. We have, resistance of ammeter, $R_A = 0.80$ ohm

Maximum current across ammeter, $IA = 1.0\ A$.

So, voltage across ammeter, $V = IR = 1 \times 0.80 = 0.8\ V$

Let the value of shunt be x.

(i) Resistance of ammeter with shunt,

$$R = \frac{R_A x}{R_A + x} = \frac{0.8x}{0.8 + x}$$

Current through ammeter, $I = 5A$

$$\therefore \left(\frac{0.8x}{0.8 + x}\right) \times 5 = 0.8$$ **[1]**

$$\Rightarrow 0.8x \times 5 = 0.8\ (0.8 + x)$$

$$4x = 0.64 + 0.8x$$

$$x = \frac{0.64}{3.2}$$

$$x = 0.2$$

Thus, the shunt resistance is 0.2 Ω

(ii) Combined resistance of the ammeter and the shunt,

$$R = \frac{0.8}{0.8 + x} = \frac{0.8 \times 0.2}{0.8 + 0.2} = \frac{0.16}{1}$$ **[1]**

$$R = 0.16\ \Omega$$

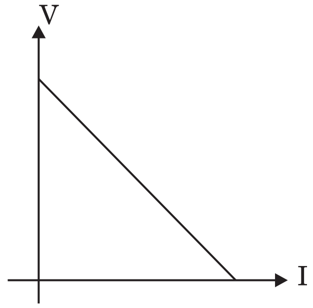
12. The given figure is Common Emitter (*CE*) configuration of an *n*-*p*-*n* transistor is shown in the figure. The input circuit is forward biased and collector circuit is reverse biased. **[½]**

If resistance *R* decreases, forward biased in the input circuit will increase, thus the base current (*IB*) will decrease and the emitter current (IE) will increase. This will increase the collector current (*IC*) as *IE* = *IB* + *IC*. **[1]**

When an *IC* increase which flows through the lamp, the voltage across the bulb will also increase making the lamp brighter and the voltmeter is-connected in parallel with the lamp, the reading in the voltmeter will also increase.

[½]

13. Plot between V and I is straight line of positive intercept and negative slope **[½]**



[½]

(i) value of potential difference corresponding to zero current gives emf of cell **[½]**

(ii) Maximum current is drawn when terminal voltage is zero so

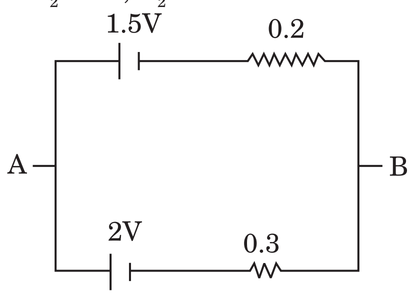
$V = E - Ir$

$0 = E - I_{max.} R$

$\Rightarrow r = \dfrac{E}{I_{\max}}$ **[½]**

14. Given, $E_1 = 1.5V, R_1 = 0.2\ \Omega$

$= E_2 = 2V, R_2 = 0.3\ \Omega$ **[½]**



[½]

Equivalent emf $= \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}$

$= \dfrac{(1.5 \times 0.3) + (2 \times 0.2)}{0.2 + 0.3}$ **[½]**

$= \dfrac{0.45 + 0.4}{0.5}$

$= \dfrac{0.85}{0.5} = 1.7\text{volt}$

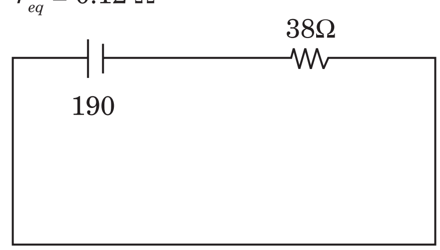
Equivalent internal resistance $= \dfrac{r_1 r_2}{r_1 + r_2}$

$= \dfrac{0.2 \times 0.3}{0.2 + 0.3}$ **[½]**

$= \dfrac{0.06}{0.5}$

$r_{eq} = 0.12\ \Omega$

15.



[1]

$E = 200 - 10 = 190\ V$

$I = \dfrac{V}{P} = \dfrac{190}{30} = 20A$ **[1]**

OR

Answer: Potentiometer at open circuit, $l_1 = 350$

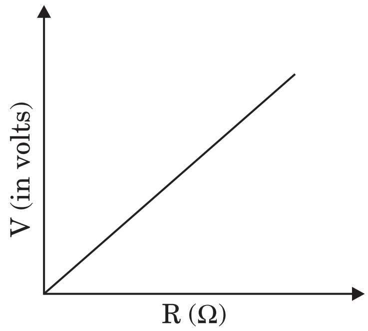
$R = 9$ **[1]**

$l_2 = 300$

$r = 9\left(\dfrac{350}{300} - 1\right)$ **[1]**

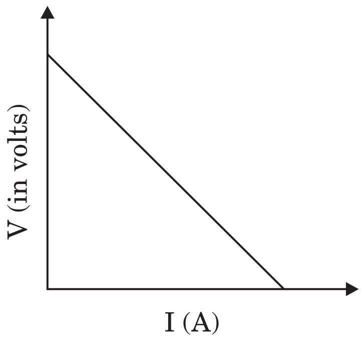
r = 1.5 Ω

16. (i) Graph between terminal voltage (V) and resistance (R)



[1]

(ii) Graph between terminal voltage (V) and current (i)



[1]

(iii) When $R = 4\ \Omega$ and $I = 1A$

Terminal voltage, $V = E - Ir$

So, $V = IR = r = E - Ir$

$E - r = 4 \quad (1)$

When $R = 4\ \Omega$ and $I = 0.5A$

$V = IR = 0.5 \times 9 = E - 0.5r$

$E - 0.5r = 4.5$

Subtracting (1) from (2) we get,

$E - 0.5r = -E + r = 4.5 - 4$

$0.5r = 0.5$

$r = 1\ \Omega$

Substituting value of r in (1)

$E - 1 = 4$

$E = 5V$

Thus

$r = 1\ \Omega$ and $E = 5V$ **[1]**

17. E_{eq} of cells across AB, $E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

$E_1 + E_2 = 1.5V$

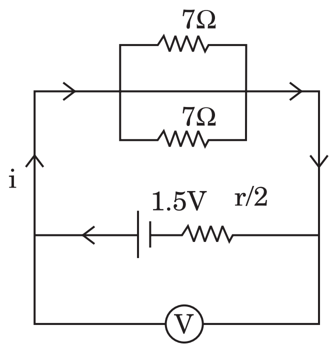
$$E_{eq} = \frac{1.5r + 1.5r}{2r}$$

$E_{eq} = 1.5V$ **[1]**

Equivalent resistance

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \Rightarrow r_{eq} = \frac{r^2}{2r} \Rightarrow r_{eq} = \frac{r}{2}$$

Equivalent circuit



$$i = \frac{1.5}{\frac{7}{2} + \frac{r}{2}} \Rightarrow i = \frac{3}{7 + r}$$ **[1]**

Reading of voltmeter $V = iR$

$$1.4 = i\left(\frac{7}{2}\right)$$

$$\left(\frac{3}{7 + r}\right)\frac{7}{2} = 1.4 \Rightarrow 7 + r = 7.5$$

$r = 0.5\ \Omega$ **[1]**

18. The current relating to corresponding situation are as follows:

(i) without any external resistance in the circuit:

$$I_1 = \frac{E}{r}$$ **[½]**

The current in this case would be maximum. So,

$I_1 = 4.2\ A$

(ii) with resistance R_1 only:

$$I_2 = \frac{E}{r + R_1}$$ **[½]**

The current in this case will be the second smallest value. So, $I_2 = 1.05\ A$

(iii) with R_1 and R_2 in series combination:

$$I_3 = \frac{E}{r + (R_1 + R_2)}$$ **[1]**

The current in this case will be minimum as the resistance will be maximum. So, $I_3 = 0.42\ A$

(iv) with R_1 and R_2 in a parallel combination:

$$I_4 = \frac{E}{r + \left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$ **[1]**

The current in this case would be the second largest value. So, $I_4 = 1.4\ A$

19. R_3, R_4 and R_2:Parallel

Upon calculation comes out to be **[1]**

$R_p = 6\ \Omega$

R_1 and R_p are in series.

$R_{eq} = 6 + 4 = 10\ \Omega$

Using Kirchoff's rules we get the following equations,

$E - I_1 R_1 - I_2 R_2 = 0$

$- I_4 R_4 + I_2 R_2 = 0$

$E - I_1 R_1 - I_3 R_3 = 0$

$- I_3 R_3 + I_2 R_2 = 0$

$I_1 = I_2 + I_3 + I_4$ **[1]**

Solving the above equation we get

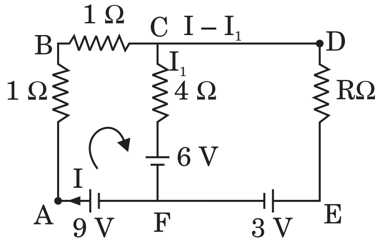
$I_1 = 1A$

$$I_2 = \frac{10}{25}A$$

$$I_3 = \frac{10}{25}A$$

$$I_4 = \frac{5}{25}A$$ **[1]**

20.



Apply Kirchhoff's law in loop ABCFA:

$I + I + 4I_1 = 9 - 6$

$2I + 4I = 3$ ----- (1)

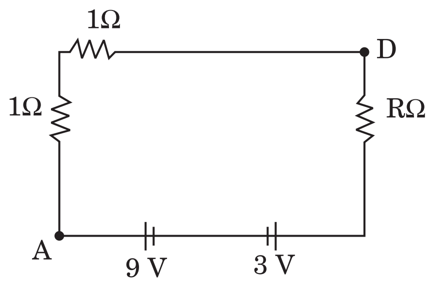
As there is no current flowing through the 4Ω resistance,

$I_1 = 0$

Or $2I = 3$

$I = 1.5A$

Thus current through resistance R is $1.5A$. As there is no current through branch CF, so equivalent circuit will be,

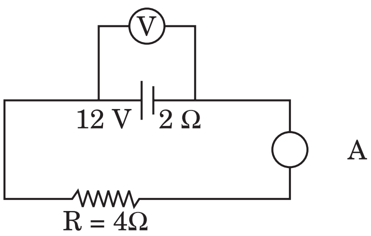


By applying Kirchhof's loop law we get,

$1.5 + 1.5 + R\,(1.5) = -3$ **[1]**

$R = 2\Omega$ Potential Difference between A and D

21.



[1]

Emf $(E) = 12V$

Internal resistance $(r) = 2\Omega$

Exterrnal resistance $(R) = 4\Omega$

Then current (I) $\dfrac{F}{R+r} = \dfrac{12}{4+2} = \dfrac{12}{6} = 2A$ **[1]**

So reading of ammeter will be 2A

We know $V = E - Ir$

$\Rightarrow V = E - Ir$

$\Rightarrow V = 12 - 2 \times 2$

$\Rightarrow V = 12 - 4 = 8V$ **[1]**

So the reading in voltmeter will be $8V$.

22. R_2, R_3, R_4 are in parallel combination

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{15} + \frac{1}{15} + \frac{1}{30}$$

$$= \frac{2+2+1}{30}$$

$$= \frac{5}{30}$$

R_{234} 6Ω **[1]**

Now R_{234} is in series with R_1, so,

$R_{eq} = 4\Omega + 6\,\Omega = 10\,\Omega$

$$\therefore I = \frac{E}{R_{eq}}$$

$$= \frac{10}{10}A = 1A$$

$\therefore I_i = 1A$ **[1]**

Current through $R_1 = 1A$

P.D. across R_1, $V = IR_1 = 1 \times 4 = 4V$

So, P.D. across $R_{234} = 6V$

$\therefore I_2 R_2 = I_3 R_3 = I_4 R_4 = 6V$

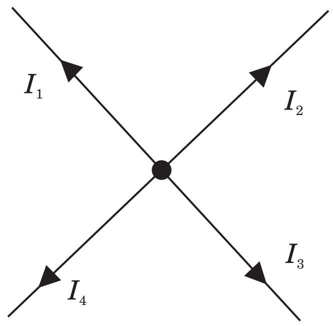
$$I_2 = \frac{6}{15}A = 0.4A$$

$$I_3 = \frac{6}{15}A = 0.4A$$

$$I_4 = \frac{6}{30}A = 0.2A$$

[1]

23. (*a*) Kirchhoff's First Law – Junction Rule: The algebraic sum of the currents meeting at a point in an electrical circuit is always zero.



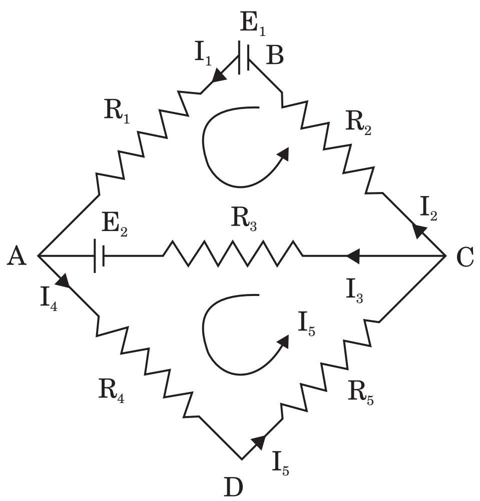
Let the currents be $I_1, I_2, I_3,$ and I_4

Convention: Current towards the junction - positive **[1]**

Current away from the junction – negative

$I_3 + (-I_1) + (-I_2) + (-I_4) = 0$

Kirchhoff's Second Law – Loop Rule: In a closed loop, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistance and current flowing through them.



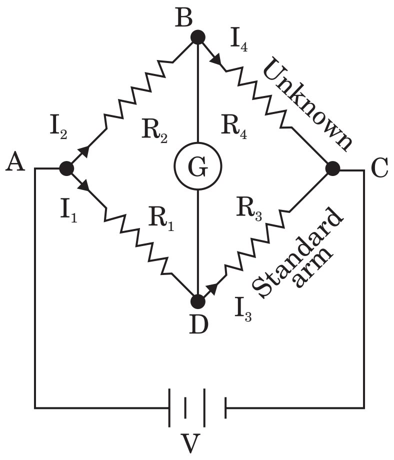
For closed part *BACB*,

$E_1 - E_2 = I_1R_2 - I_3R_3$

For closed part *CADC*,

$E_2 = I_3R_3 + I_4R_4 + I_5R_5$ **[1]**

Wheatstone Bridge is an arrangement of four resistances as shown in the following figure.



[1]

R_1, R_2, R_3 and R_4 are the four resistances.

Galvanometer (G) has a current I_g flowing through it at balanced condition, $I_g = 0$

Applying junction rule at B,

$\therefore I_2 = I_4$

Applying loop rule to closed loop **ADBA**,

$-I_1R_1 + 0 + I_2R_2 = 0$

$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1}$ (1)

Applying loop rule to closed loop *CBDC*,

$I_2R_4 + 0 - I_1R_3 = 0$

$\because I_3 = I_1, I_4 = I_2$ **[1]**

$\therefore \frac{I_1}{I_2} = \frac{R_4}{R_3}$ (2)

From equations (1) and (2),

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

This is the required balanced condition of Wheatstone Bridge.

Topic 3: Electrical devices

Summary

Meter Bridge: Meter Bridge is the simplest form of the Wheatstone bridge which is used for accurate comparison of resistances.

In order to find out an unknown resistance R with the help of a standard known resistance S:

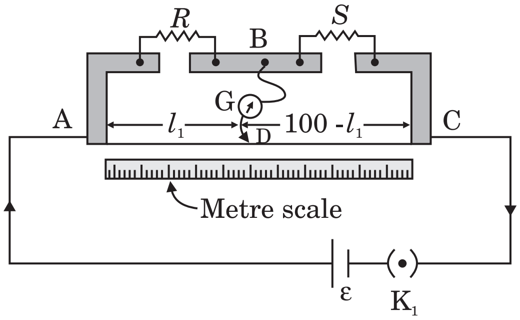


Fig.: Meter bridge

$R = S\frac{l_1}{100 - l_1}$, l_1 being the distance of the jockey from end A at the balance point.

Potentiometer: It is a device which is used to compare potential differences and emf'"s. It also measures the internal resistance of a cell.

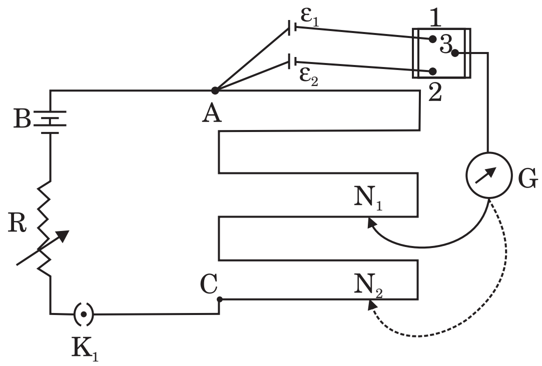


Fig.: Potentiometer

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

Potentiometer does not draw any current from the voltage source being measured. The internal resistance of a given cell can be measured by:

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

PREVIOUS YEARS'
EXAMINATION QUESTIONS
TOPIC 3

1 Mark Questions

1. A resistance R is connected across a cell of emf ε and internal resistance r. *A* potentiometer now measures the potential difference between the terminals of the cell as *V*. Write the expression for '*r*' in terms of ε, *V* and *R*.

[ALL INDIA 2011]

2 Marks Questions

2. Use Kirchhoff's rules to obtain conditions for the balance condition in a Wheatstone bridge.

[DELHI 2015]

3. Describe briefly, with the help of a circuit diagram, how a potentiometer is used to determine the internal resistance of a cell.

[ALL INDIA 2013]

4. Two electric bulbs P and Q have their resistances in the ratio of 1:2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs.

[DELHI 2018]

5. In a potentiometer arrangement for determining the emf of a cell, the balance point of the cell in open circuit is 350cm.When a resistance of 9Ω is used in the external circuit of the cell, the balance point shifts to 300 cm. Determine the internal resistance of the cell.

[ALL INDIA 2018]

3 Marks Questions

6. A potentiometer wire of length 1.0 m has a resistance of 15 Ω. It is connected. o a 5V battery in series with a resistance of 5 Ω. Determine the emf of the primary cell which gives a balance point at 60 cm.

[DELHI 2016]

7. Answer the following:

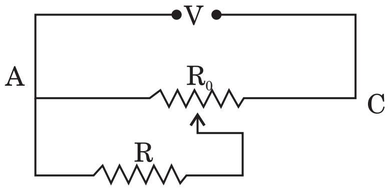
(a) Why are the connections between the resistors in a meter bridge made of thick copper strips ?

(b) Why is it generally preferred to obtain the balance point in the middle of the meter bridge wire?

(c) Which material is used for the meter bridge wire and why?

Or

A resistance of R Ω draws current from a potentiometer as shown in the figure. The potentiometer has a total resistance R_oΩ. A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of the potentiometer.



[ALL INDIA 2014]

8. A potentiometer wire of length 1 m has a resistance of 10Ω. It is connected to a 6 V battery in series with a resistance of 5Ω. Determine the emf of the primary cell which gives a balance point at 40 cm

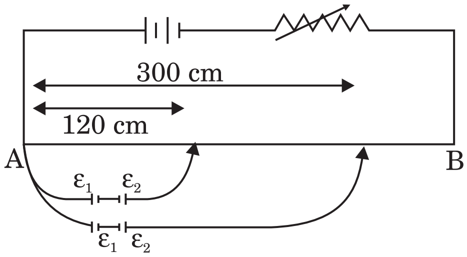
[DELHI 2014]

9. In the figure a long uniform potentiometer wire AB is having a constant potential gradient along its length. The null points for the two primary cells of emf's ε_1 and ε_2 connected in the manner shown are obtained at a distance of 120 cm and 300 cm from the end A. Find

(i) $\frac{\varepsilon_1}{\varepsilon_2}$ and

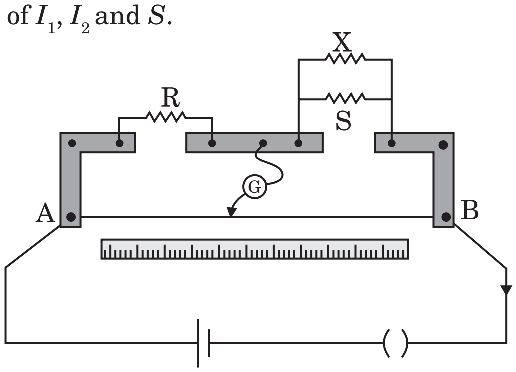
(ii) position of null point for the cell ε_1.

How is the sensitivity of a potentiometer increased?



[All India 2012]

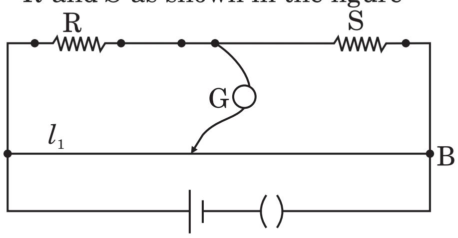
10. In a meter bridge, the null point is found at a distance of I_1 cm from A. If now a resistance of X is connected in parallel with S, the null point occurs at I_2 cm. Obtain a formula for X in terms of I_1, I_2 and S.



[DELHI 2018]

11. (a) Write the principle of working of a metre bridge.

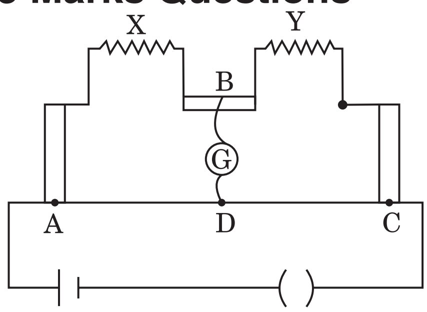
(b) In a metre bridge, the balance point is formed at a distance I_1 unit with resistances R and S as shown in the figure



An unknown resistance X is now connected in parallel to the resistance S and the balance point is found at a distance I_2. Obtain a formula for X in terms of I_1, I_2 and S. **[ALL INDIA 2017]**

5 Marks Questions

12.



The figure shows experimental set up of a meter bridge. When the two unknown resistances X and Y are inserted, the null point D is obtained 40 cm from the end A. When a resistance of 10 0 is connected in series with X. the null point shifts by 10 cm. Find the position of the null point when the 10 0 resistance is instead connected in series with resistance 'Y'. Determine the values of the resistances X and Y.

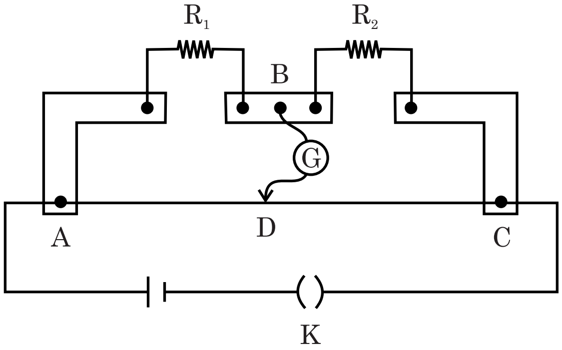
[DELHI 2017]

13. (a) State the working principle of a potentiometer. With the help of the circuit diagram, explain how a potentiometer is used to compare the emfs of two primary cells. Obtain the required expression used for comparing the emfs.

(b) Write two possible causes for one sided deflection in a potentiometer experiment.

OR

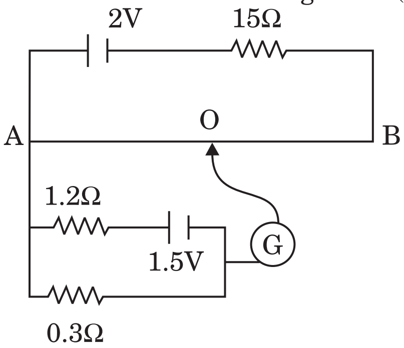
In the meter bridge experimental set up, shown in the figure, the null point 'D' is obtained at a distance of 40 cm from end A of the meter bridge wire. If a resistance of 10 Ω is connected in series with R_1, null point is obtained at AD = 60 cm. Calculate the values of R_1 and R_2.



[DELHI 2013]

14. (i) State the principle of working of a potentiometer.

(ii) In the following potentiometer circuit AB is a uniform wire of length 1 m and resistance 10 Ω. Calculate the potential gradient along the wire and balance length AO (=l).



[DELHI 2016]

15. Write the principle of working of a potentiometer. Describe briefly, with the help of a circuit diagram, how a potentiometer is used to determine the internal resistance of a given cell.

[DELHI 2018]

Solutions

1. $\varepsilon = I(R + r)$ ---------- (1)

$V - IR$ ----------------- (2) **[½]**

From equations (1) and (2), we have

$$\frac{\varepsilon}{V} = \frac{R+r}{R}$$

$$\frac{\varepsilon}{V} = 1 + \frac{r}{R}$$

$$\Rightarrow \frac{r}{R} = \frac{\varepsilon}{V} - 1$$

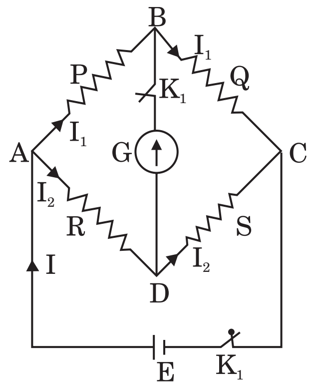
$$\Rightarrow r = \left(\frac{\varepsilon}{V} - 1\right)$$

[½]

2. Let us consider a Wheatstone bridge arrangement as shown below:

Wheat stone bridge is a special bridge type circuit which consists of four resistances, a galvanometer and a battery. It is used to determine unknown resistance. **[1]**

In figure four resistance P, Q, R and S are connected in the form of four arms of a quadrilateral. Let the current given by battery in the balanced position be I. This current on reaching point A is divided into two parts I_1 and I_2. As there is no current in galvanometer in balanced state, therefore, current in resistances P and Q is I_i and in resistances R and S it is I_2.



Applying Kirchhoff's first law at point A

$I - I_1 - I_2 = 0$ or $I = I_1 + I_2$ (1)

Applying Kirchhoff's second law to closed mesh $ABDA$

$-I\,P + I_2 R = 0$ or $I_1 P = I_2 R$ (2)

Applying Kirchhoff's second law to mesh $BCDB$

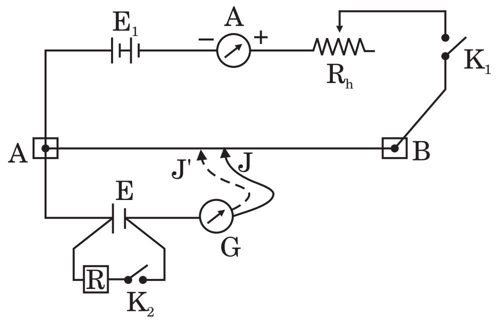
$-I\,Q + I_2 S = 0$ or $I_1 Q = I_2 S$ (3)

Dividing equation (2) by (3), we get

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S} \quad or \quad \frac{P}{Q} = \frac{R}{S}$$ **[1]**

This is condition of balance Wheatstone's bridge.

3.



Close key K_1 keeping key K_2 open. Find the point (say J) on the wire AB such that on pressing the jockey on the wire at J, the galvanometer gives no deflection. At this stage, the potential difference across A and J is equal to the e.m.f. (E) of the cell. If $AJ = l_1$ then, **[1]**

$E = V_{AJ} = \propto l_1$ …….. (i)

$E = Kl_1$

Now close key K_2 so that a known resistance (R) is connected across the cell. Find the point (J', say) such that on pressing the jockey at J', the galvanometer gives no deflection. The terminal potential difference (V) of cell is equal to the potential difference across A and J′.

If AJ' = then $V = V_{AJ} = \propto l_2$ or $V = Kl_2$ … (ii)

Dividing (i) by (ii), we get,

$$\frac{E}{V} = \frac{l_1}{l_2} \quad ………..(iii)$$

We know, internal resistance of a cell is given by,

$$r = \left(\frac{E}{V} - 1\right)R \quad ………..(iv)$$

4. $\frac{R_P}{R_o} = \frac{1}{2}$ **[1]**

$$\frac{P_P}{P_Q} = \frac{I_P^2 R_P}{I_Q^2 R_Q}$$ **[1]**

$$\frac{P_P}{P_Q} = \frac{1}{2}$$

5. Potentiometer at open circuit, $l_1 = 350$

$R = 9$ **[1]**

$l_2 = 300$

$$r = 9\left(\frac{350}{300} - 1\right)$$ **[1]**

r = 1.5 Ω

6. Current through potentiometer wire,

$$I = \frac{V}{R_{AB} + R}$$

$$= \frac{5}{15+5} = 0.25A$$ **[½]**

$P.D.$ across the potentiometer wire,

$V = IR_{AB}$

$= 0.25 \times 5 = 1.25V$ **[½]**

Potential gradient, $K = \frac{V}{I}$ **[½]**

$$K = \frac{V}{I}$$

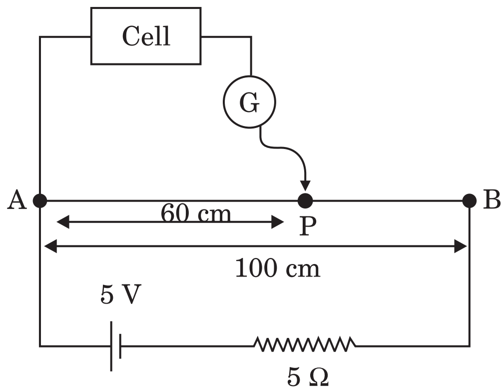
$$= \frac{1.25V}{1m}$$ **[½]**

$0.0125V\ cm^{-1}$

So, unknown e.m.f. balanced against 60 cm of the wire.

$$\xi = kl = 0.0125\,Vcm^{-1} \times 60cm$$

$= 0.75V$ **[½ + ½]**



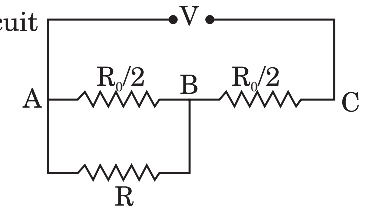
7. (*a*) For making gaps thick copper strips are used due to their negligible resistance. **[1]**

(*b*) Meter bridge is most sensitive when all four resistances are of same order. **[1]**

(*c*) Alloy Magnanin or Constantun are used for making meter bridge wire due to low temperature Coefficient of resistance & high resistivity. **[1]**

Or

Equivalent circuit is as shown



So equivalent resistance of circuit

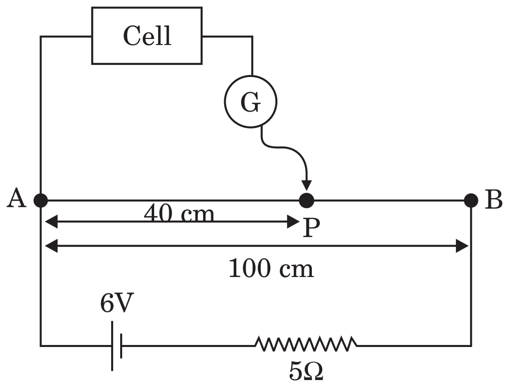
$$\frac{R_o}{2}+\frac{\frac{RR_o}{2}}{R+\frac{R_o}{2}}$$ **[1]**

$$\therefore I_{circuit}=\frac{V}{R_{eq}}$$ **[1]**

$\therefore$ *PD* across *R*

$$I\left(\frac{\frac{RR_o}{2}}{R+\frac{R_o}{2}}\right)=\frac{VR}{2\left(R+\frac{R_o}{4}\right)}$$ **[1]**

8. From the figure below: **[½]**



Total resistance of the circuit,

$R = (R_{AB} + 5) = 15\ \Omega$

[½]

Current in the circuit,

$$i=\frac{V}{R}=\frac{6}{15}A$$

$$V_{AB}=6-\frac{6}{15}\times 5$$

$= 6 - 2 = 4V$ **[½]**

Hence, voltage across *AB*,

Emf of the cell, $e=\left(\frac{l}{L}\right)V_{AB}$ **[½]**

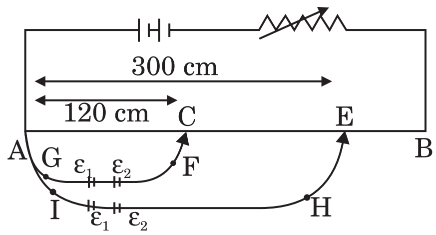
Here, $l = 40$m (balance point) **[½]**

$AB = L = 1m = 100$cm (total length of the wire)

$$\therefore e=\left(\frac{40}{100}\right)4$$ **[½]**

$e = 1.6V$

9.



Apply Kirchhoff's law in loop ACFGA:

$\phi(120) = \varepsilon_1 - \varepsilon_2$ **[1]**

ϕ = potential drop per unit length

Or, $\varepsilon_1 = \varepsilon_2 + \phi(120)$ -------- (1)

Loop AEHIA:

$\phi(300)\ \varepsilon_1 + \varepsilon_2$

Or, $\varepsilon_2+(\varepsilon_2+\phi(120))=\phi(300)$

[by substituting value of from equation (1)]

$2\varepsilon_2 = (300 - 120)\phi$

$\varepsilon_2 = 90\phi$ -------- (2) **[1]**

From equation (1) and (2),

$\varepsilon_1 = 210\phi$------- (3)

Hence, $\frac{\varepsilon_1}{\varepsilon_2}=\frac{210}{90}=\frac{7}{3}$

As we know, $\varepsilon = \phi I$

Thus, from equation (2) and (3)

Null point for cell ε_2 is 90 cm and for cell ε_1 it is 210 cm. Senstivity of potentiometer can be increased by:

Increasing the length of potentiometer wire,

Decreasing the resistance in the primary circuit.

10. Initially when X is not connected

$$\frac{R}{S}=\frac{I_1}{100-I_1}$$ [Condition for balance] ---------- (1)

The equivalent resistance ($R_{eq.}$) of the combination of *X* and *S* is:

$$\frac{1}{R_{eq.}}=\frac{1}{X}+\frac{1}{S}$$

$$R_{eq.}=\frac{SX}{X+S}$$ **[1]**

$$\frac{R}{R_{eq.}}=\frac{I_2}{100-I_2}$$

$$\frac{R(X+S)}{SX}=\frac{I_2}{100-I_2}$$ --------- (2)

On dividing (1) by (2), we get

$$\frac{R}{S}\times\frac{SX}{R(X+S)}=\frac{I_1(100-I_2)}{I_2(100-I_1)}$$ **[1]**

$$\frac{X}{(X+S)}=\frac{I_1(100-I_2)}{I_2(100-I_1)}$$

$$XI_2(100-I_1)=(X+S)I_1(100-I_2)$$

$$XI_2(100-I_1)=XI_1(100-I_2)+SI_1(100-I_2)$$

$$XI_2(100-I_1)-XI_1(100-I_2)=SI_1(100-I_2)$$

$$X\left[I_2(100-I_1)-I_1(100-I_2)\right]=SI_1(100-I_2)$$

$$X=\frac{SI_1(100-I_2)}{\left[I_2(100-I_1)-I_1(100-I_2)\right]}$$

This is the expression for X in terms of l1, l2 and S. **[1]**

11.

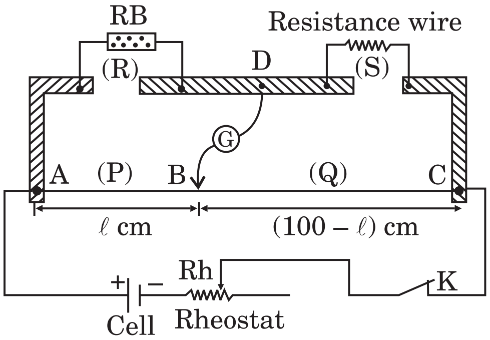
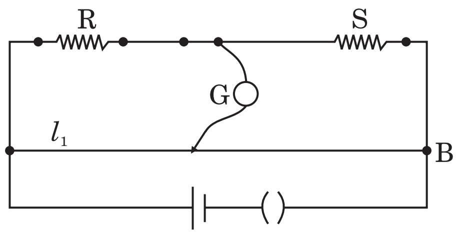


Fig.: Meter Bridge

Principle: It is constructed on the principle of balanced wheatstone bridge. i.e., when a wheatstone bridge is balanced $\frac{P}{Q}=\frac{R}{S}$ where the initial value has usual meaning = at balancing situation of bridge **[1]**

$$\frac{P}{Q}=\frac{R}{S}\Rightarrow\frac{1}{100-1}=\frac{R}{S}\Rightarrow S=\frac{100-1}{1}\times R$$



According to question

For first balanced bridge situation

$$\frac{R}{S}=\frac{I_1}{100-I_1}\text{..................(i)}$$

when the π is connected in parallel with s the equivalent resistance is

$$S_{eq}=\frac{XS}{X+S}$$ **[1]**

for the second balanced bridge

$$\frac{R}{S_{eq}}=\frac{I_1}{100-I_2}$$

$$\frac{R}{\frac{XS}{X+S}}=\frac{I_2}{100-I_2}$$

$$\frac{R(X+S)}{XS}=\frac{I_2}{100-I_2}\text{.......(ii)}$$

$$\therefore\ \frac{R}{S}=\frac{I_2}{100-I_2}$$

$$\frac{(X+S)}{S}\times\frac{I_1}{100-I_1}=\frac{I_2}{100-I_2}$$

$$\frac{X+S}{X}=\frac{I_2(100-I_1)}{(100-I_2)I_1}$$ **[1]**

12. For a meter bridge :

$$\frac{X}{Y}=\frac{l_1}{100-l_1}$$

Where, it is given that l_1 = 40 cm **[1]**

$$\frac{X}{Y}=\frac{40}{100-40}=\frac{2}{3}\text{ --------- (1)}$$

When 10 it resistance is added in series to X, null point shifts by 10 cm.

$$\frac{X+10}{Y}=\frac{40+10}{100-(40+10)}$$

$$X+10=\frac{50}{50}$$ **[1]**

$$\frac{X+10}{Y}=1\text{ or }X+10=Y\text{ ------------ (3)}$$

Substituting the value of X from equation (2), we obtain

$$\frac{2}{3}Y+10=Y$$

$$10=Y-\frac{2}{3}Y$$

Or $\frac{Y}{3} = 10$ **[1]**

$Y = 30\ \Omega$

Substituting the value of Y from equation (3), we obtain

$X + 10\ 30$

$X = 20\ \Omega$

Position of null point when 10 Ω resistance is put in series with Y,

$$\frac{20}{20+10} = \frac{l_i}{100 - l_i}$$

$2000 = 20\ l_1 = 40\ l_1$

$60l_1 = 2000$

$$l_1 = \frac{2000}{60}$$ **[1]**

$l_1 = 33.3$ cm

13. (*a*) Working principle of Potentiometer: When a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portion of the wire is directly proportional to the length of that portion.

Applications of Potentiometer for comparing emfs of two cells: The following figure shows an application of the potentiometer to compare the emf of two cells of emf E_1 and E_2

E_1, E_2 are the emf of the two cells. **[1]**

1, 2, 3 form a two way key.

When 1 and 3 are connected, E_1 is connected to the galvanometer (G).

Jokey is moved to N_1, which is at a distance Li from A, to find the balancing length.

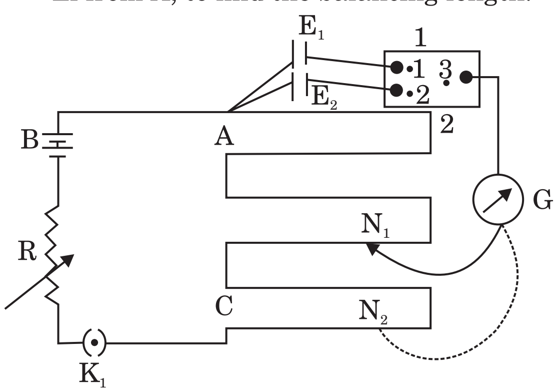


Fig.: Construction of Potentiometer

Applying loop rule to AN_1G31A, **[1]**

$\phi l_1 + 0 - E_1 = 0$ (1)

Where, is the potential drop per unit length

Similarly, for E_2 is balanced against $l_2(AN_2)$,

$\phi l_2 + 0 - E_2 = 0$ (2)

$$\frac{E_1}{E_2} = \frac{I_1}{I_2}$$ **[1]**

Thus, we can compare the emfs of any two sources. Generally, one of the cells is chosen as a standard cell whose emf is known to a high degree of accuracy. The emf of the other cell is then calculated from equation (3).

(*b*) (i) The emf of the cell connected in main circuit may not be more than the emf of the primary cells whose emfs are to be compared. **[1]**

(ii) The positive ends of all cells are not connected to the same end of the wire. **[1]**

OR

Considering both the situations and writing them in the form of equations Let R' be the resistance per unit length of the potential meter wire, **[1]**

$$\frac{R_1}{R_2} = \frac{R' \times 40}{R'(100-40)} = \frac{40}{60} = \frac{2}{3} \quad (1)$$

$$\frac{R_1 + 10}{R_2} = \frac{R' \times 60}{R'(100-60)} = \frac{60}{40} = \frac{3}{2} \quad (2)$$

Putting the value of R_1 from equation (1) and substituting in equation (2)

$$\frac{2}{3} + \frac{10}{R_2} = \frac{3}{2}$$

$R_2 = 12\ \Omega$

Recalling equation (1) again

$$\frac{R_1}{12} = \frac{2}{3}$$

$R_1 = 8\ \Omega$

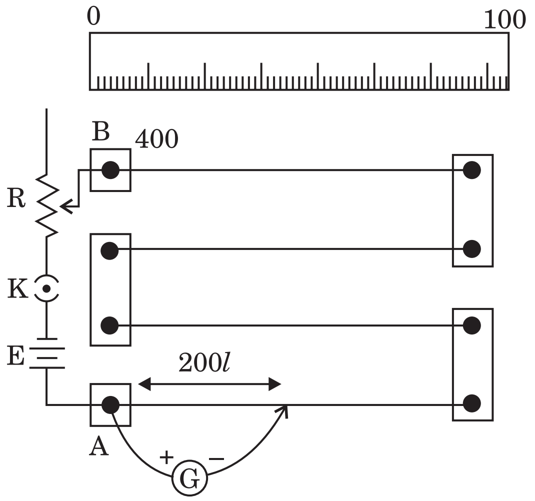
14. (i) Principle of potentiometer: The basic principle of potentiometer is that when a constant current flows through a wire of uniform cross-section area and the composition of the potential drop across any length of the wire is directly proportional to that length. **[1]**

A potentiometer is a device used to measure an unknown emf or potential difference and internal resistance of a cell accurately.

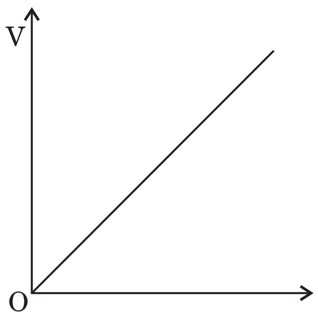
Construction:

1. A potentiometer consists of a long uniform cross-section of wire generally made of manganin or constantan.

2. Usually, 1 m long separate pieces of wire are fixed on a wooden board parallel to each other.
3. The wire are joined in series by thick copper strips.
4. The ends A and B are connected to a battery (called driving all), a plug key and rheostat.
5. A jockey J is provided with the help of which contact can be made at any point on the wire.
6. This circuit sends a constant current I through the wire AB. **[1]**



Construction of Potentiometer



Potential vs. Length of difference

Principle: When a constant current flows through a wire of uniform cross sectional area and composition the potential drop across any length of the wire is directly proportional to that length.

Let V be the potential difference across the portion of the wire of length l whose resistance is R

By Ohm's law, $V = IR = I \cdot \frac{\rho l}{A}$ **[1]**

Where, $\rho \rightarrow$ resistivity of wire:

A = Area of cross-section

$$V = \frac{I\rho l}{A}$$

$V = kl$ (If I ; A and P are constants)

$V \propto l$

Where, $k = \frac{I\rho}{A}$

Here, $\frac{V}{l} = k$ (Potential gradient i.e. potential per unit length of wire)

(ii) Total resistance of the primary circuit

15 v+ 10 = 25 Ω, emf = 2V

∴ Current in the wire AB,

$$I = \frac{2}{25} = 0.08A$$ **[1]**

$P.D.$ across the wire AB = Current × Resistance of wire AB = 0.08 × 10 = 0.8V

Potential gradient

$$= \frac{\text{P.D.}}{\text{Length}} = \frac{0.8}{100} = 0.008 Vcm^{-1}$$

Resistance of secondary circuit

1.2 + 0.3 = 1.5 Ω

e.m.f. =15V

Current in the secondary circuit

$$= \frac{1.5}{1.5} = 1.0A$$ **[1]**

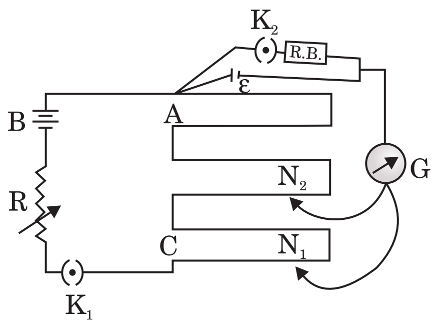
The same is the current in 0.3 Ω resistor

$P.D.$ between points A and O

$P.D.$ across 0.3 Ω resistor in the zero-deflection condition = Current × Resistance = 1.0 × 0.3 = 0.3V

$$\text{Length } AO = \frac{\text{Potential difference}}{\text{Potential gradient}}$$

15. The working principle of a potentiometer is based on Kirchhoff's voltage law. According to this rule, the algebraic sum of changes in voltage around any closed loop involving resistors and cells in the loop is zero. **[1]**



[1]

Fig.: Construction of Potentiometer

Let ϕ be the potential drop per unit length in the potentiometer wire. When only a cell is connected, the balance point is N_1. **[1]**

Applying Kirchhoff's voltage law,

$\varepsilon = \phi l_1$ (l_1 = Length at which the balance point is achieved]

When some current is drawn using the resistance box, the balance point is achieved at N_2.

$V = \phi l_2$ **[1]**

This gives,

$$\frac{\varepsilon}{V} = \frac{l_1}{l_2}$$

$\varepsilon = l\,(r + R)$ [R = Resistance of the resistance box]

$V = IR$, which gives, **[1]**

$$\frac{\varepsilon}{V} = \frac{r + R}{R}$$

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

The internal resistance of the cell can be determined by plugging-in the measured values of l_1 and l_2.

Smart Notes ....

Smart Notes ....

CHAPTER 4

Moving Charges and Magnetism

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Concept of magnetic field, Oersted's experiment. Biot–Savart law and its application to current carrying circular loop		1Q (5 mark)		1Q (3 marks)	1Q (3marks)
Ampere's law & its applications to infinitely long straight wire. Straight and toroidal solenoids	1Q (3 marks)	1Q (3 marks)	1Q (2 marks)		
Force on a moving charge in uniform magnetic and electric fields; Cyclotron		1Q (3 marks)		1Q (2 marks)	
Force on a current-carrying conductor in a uniform magnetic field; force between two parallel current-carrying conductors-definition of ampere	1Q (3 marks)				
Torque experienced by a current loop in uniform magnetic field;	1Q (3 mark)				
Moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter	1Q (1 mark)		1Q (3 marks)		

On the basis of above analysis, it can be said that from exam point of view Moving Coil Galvanometer, Magnetic Field ,Magnetic Lorentz Force and permeability & susceptibility are most important concepts of the chapter.

[Topic 1] Magnetic Field Laws and their Applications

Summary

- The Oersted's law states that an electric current creates a magnetic field.
- The Biot Savart's law states that, the magnitude of magnetic field dB is proportional to the current I, the element length dl and inversely proportional to the square of the distance r. Its direction is perpendicular to the plane containing dl and r. Thus in vector notation, $dB \propto \frac{Idl \times r}{r^3}$, where $\frac{\mu_0}{4\pi}$ is the constant of proportionality and is equal to 10^{-7} Tm/A.

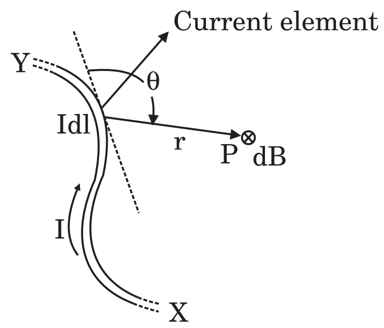


Fig.: Biot Savart's law

Applications of Biot-Savart's Law:

- Magnetic field at a point in circular loop will be $B = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$

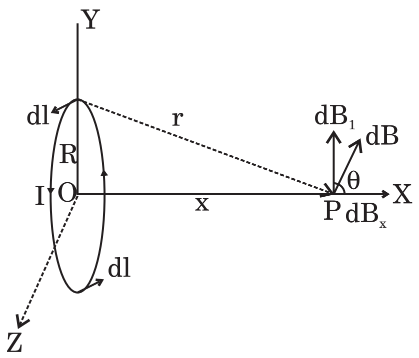


Fig.: Magnetic field at a point in circular loop

- Magnetic field at centre of the coil is $B = \frac{\mu_0 Ni}{2R} \quad (x = 0)$
- Magnetic field due to current carrying circular arc with centre O is $B = \frac{\mu_0 i}{4r}$
- If we curl the palm of our right hand around the circular wire with the fingers pointing in the direction of the current, the right hand thumb rule gives the direction of the magnetic field.
- ***Ampere's circuital law:*** The line integral of the magnetic field around some closed loop is equal to the times the algebraic sum of the currents which pass through the circular loop. For some circuital loop, C, $\oint_C B.dl = \mu_0 I$

Applications of Ampere's Law

Magnetic field due to current carrying solenoid, B = $\mu_0 nI$

At the end of a short solenoid, $B = \frac{\mu_0 nI}{2}$

- The magnetic force produced by a Solenoid as stated by Ampere's law is given as F = $\mu_0 nI$, where n is the number of turns of the wire per unit length, I is the current flowing through the wire and the direction is given using the right hand thumb rule.
- Due to a toroid a magnetic field is given as, $B = \frac{\mu_0 NI}{2\pi r}$ where 'N' is the number of turns of the toroid coil, I is the amount of current flowing and r is the radius of the toroid.
- Antiparallel currents repel and parallel currents attract.
- Magnetic moment on a rectangular current loop in a uniform magnetic field, m = NIA where m is the magnetic moment and N is the number of closely wounded turns and A is the area vector.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

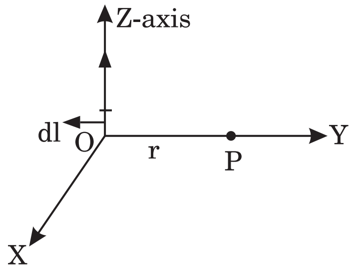
1 Mark Questions

1. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

[DELHI 2017]

2 Mark Questions

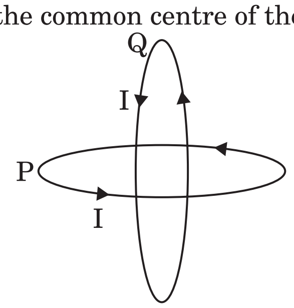
2. State Biot-Savart law. A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element $d\vec{l}$ at point P situated at distance r from the element as shown in the figure.



[DELHI 2017]

3 Mark Questions

3. Two identical circular wires P and Q each of radius R and carrying current 'I' are kept in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils.



[DELHI 2012]

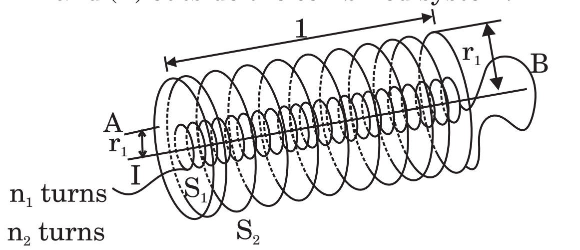
4. **(a)** **Sta**te Biot – Savart law and express this law in the vector form.

(b) Two identical circular coils, P and Q each of radius R, carrying currents 1A and $\sqrt{3}$ A respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils.

[DELHI 2017]

5. **(a)** State Ampere's Circuital law, expressing it in the integral form.

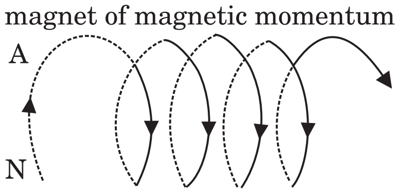
(b) Two long coaxial insulated solenoids, S_1 and S_2 of equal lengths are wound one over the other as shown in the figure. A steady current "I" flows through the inner solenoid S_1 to the other end B, which is connected to the outer solenoid S_2 through which the same current "I" flows in the opposite direction so as to come out at end A. If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point (i) inside on the axis and (ii) outside the combined system.



[DELHI 2014]

6. (a) **Stat**e Ampere's Circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius r, having 'n' turns per unit length and carrying a steady current I.

(b) An observer to the left of a solenoid of N turns each of cross section area A observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic momentum $M = NIA$.



OR

(a) Define mutual inductance and write its S.I. units.

(b) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.

[DELHI 2015]

7. A long straight wire of a circular cross-section of radius 'a' carries a steady current 'I'. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point 'r' in the region for (i) $r < a$ and (ii) $r > a$.

[DELHI 2018]

5 Mark Questions

8. (a) Using Biot-Savart's law, derive the expression for the magnetic field in the vector form at a point on the axis of a circular current loop.

(b) What does a toroid consist of?

Find out the expression for the magnetic field inside a toroid for N turns of the coil having the average radius r and carrying a current I. Show that the magnetic field in the open space inside an exterior to the toroid is zero. **[DELHI 2013]**

9. State Biot-Savart law, giving the mathematical expression for it. Use this law to derive the expression for the magnetic field due to a circular coil carrying current at a point along its axis. How does a circular loop carrying current behave as a magnet? **[DELHI 2011]**

Solutions

1. Magnetic field lines form closed loops around a current-carrying wire. The geometry of a straight solenoid is such that magnetic field lines cannot loop around circular wires without spilling over to the outside of the solenoid. The geometry of a toroid is such that magnetic field lines can loop around electric wires without spilling over to the outside. Hence, magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. [1]

2. Biot-Savart's law states that the magnitude of the magnetic field dB is proportional to the current element idl and inversely proportional to the square of the distance. The direction of magnetic field is along the negative X-direction. [1]

3. Magnetic field at the centre of circular loop carrying current 'I' is given by,

$$B = \frac{\mu_o I}{2a}$$

Here, $a = R$ now magnetic field due to loop Q:

$$B_Q = B_x = \frac{\mu_o I}{2R}$$

Magnetic field due to loop P:

$$B_P = B_y = \frac{\mu_o I}{2R}$$ [½]

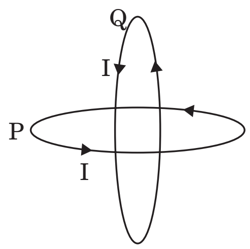


Fig.: Two identical circular wires placed in perpendicular planes. [½]

Net field at centre,

$$B_N = \sqrt{B_P^2 + B_Q^2}$$

$$B_N = \sqrt{\left(\frac{\mu_o I}{2R}\right)^2 + \left(\frac{\mu_o I}{2R}\right)^2}$$

$$B_N = \frac{\mu_o I}{\sqrt{2}R}$$ [½]

Direction of net magnetic field,

$$\tan\theta = \frac{B_P}{B_Q} = 1$$

$$\tan\theta = \tan\frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}$$ [½]

4. (a) Biot-savart's law: This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

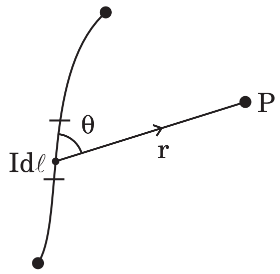


Fig.: Biot-Savart's law [½]

(i) directly proportional to the Idl (current element of the conductor) $dB \propto idl$

(ii) directly proportional to $\sin\theta \propto dB\sin\theta$, θ is the angle b/w dl and r

(iii) inversely proportional to the square of the distance of point p from the current element $dB \propto \frac{1}{r^2}$ [½]

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2} = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$$

where $\frac{\mu_o}{4\pi} = 10^{-7}\,Tm/A$ for free space

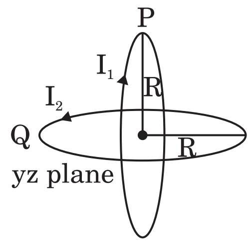
The direction of magnetic field can be obtained using right band thumb rule

$$\mathrm{dB} = \frac{\mu_o}{4\pi}\frac{Idl \times \hat{r}}{|r|^2}$$

In vector form Biot-Savart's law can be written as

$$\mathrm{dB} = \frac{\mu_o}{4\pi}\frac{Idl \times r}{r^3}$$ [½]

(b)



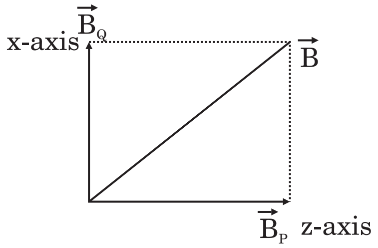
[½]

$I_1 = 1A$

$I_2 = \sqrt{3}A$

Magnetic field due to coil Q at its centre is

$B_q = \frac{\mu_0 I_1}{2R}$ along X-axis



[½]

Resultant magnetic field is

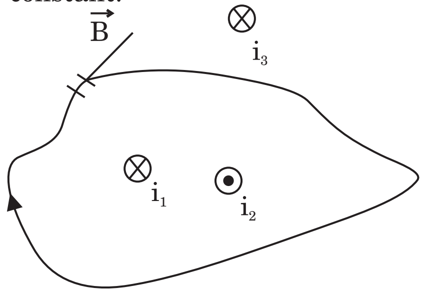
$\sqrt{Bp^2 + Bq^2}$

$\sqrt{\left(\frac{\mu_0 I_1}{2R}\right)^2 + \left(\frac{\mu_0 I_2}{2R}\right)^2} = \sqrt{\left(\frac{\mu_0}{2R}\right)^2 \left(I_1^2 + I_2^2\right)}$

$= \frac{\mu_0}{2R}\sqrt{1^2 + \left(\sqrt{3}\right)^2} = \frac{\mu_0}{2R} \times 2 = \frac{\mu_0}{2R}$

and its direction is in X – Z plane [½]

5. (a) Ampere's Circuital law states that the circulation of the resultant magnetic field along a closed, plane curve is equal to p_o times the total current crossing the area bounded by the closed curve, provided the electric field inside the loop remains constant. [½]



In the above illustration, the Ampere's Circuital Law can be written as follows:

$\oint \vec{B} \cdot \vec{dl} = \mu_o i$

[½]

Where, $i = i_1 - i_2$

(b) (i) The magnetic field due to a current carrying solenoid:

$B = \mu_o N_i$ [½]

Where, N = number of turns per unit length

i = current through the solenoid.

Now, the magnetic field due to solenoid S_1 will be in the upward direction and the magnetic field due to S_2 will be in the downward direction (by right hand screw rule)

$B_{net} = B_{S_1} - B_{S_2}$

$B_{net} = \mu_o n_1 I - \mu_o n_2 I = \mu_o I\left(n_1 - n_2\right)$

[½]

(ii) The magnetic field is zero outside the solenoid. [1]

6. (a) Amperes circuital law in electro magnetism is analogous to Gauss law in electrostatics. This law states that "The line integral of resultant magnetic field along a closed plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus $\oint B.dl = \mu_o I_{enclosed}$, where permeability of free space and $I_{enclosed}$ is the net current enclosed by the loop.

A toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. Consider an air-cored toroid (as shown below) with centre O.

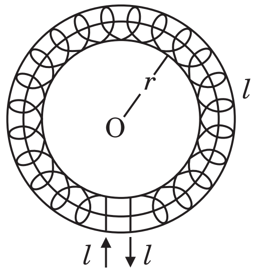


Fig.: Ampere circuital law

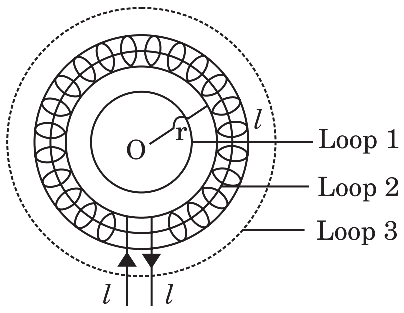
Given:

r = Average radius of the toroid

I = Current through the solenoid.

n = Number of turns per unit length

To determine the magnetic field inside the toroid, we consider three amperian loops (loop 1, loop 2 and loop 3) as show in the figure below.



According to Ampere;s Circuital Law, we have

$\oint B.dl = \mu_o I_{enclosed}$ [½]

Total current for loop 1 is zero because no current is passing through this loop.

So, for loop 1, $\oint \vec{B}.\vec{dl} = 0 = \mu_o$ (Total current)

For loop 3, according to Ampere;s Circuital Law, we have $\oint \vec{B}.\vec{dl} = \mu_o$ (Total current)

Total current for loop 3 is zero because net current coming out of this loop is equal to the net current going inside the loop.

For Loop 2: The total current flowing through the toroid is NI, where N is the total number of turns

$\oint \vec{B}.\vec{dl} = 0 = \mu_o (NI)$ ...(1)

Now, $\vec{B}$ and $\vec{dl}$ are in the same direction

$\oint \vec{B}.\vec{dl} = B\oint dl$

$\Rightarrow \oint \vec{B}.\vec{dl} = B(2\pi r)$ ...(2)

Comparing and we get,

$B(2\pi r) = \mu_o NI$

$\Rightarrow B = \frac{\mu_o NI}{2\pi r}$

Number of turns per unit length is given by

$n = \frac{N}{2\pi r}$

$B = \mu_0 nI$

This is the expression for magnetic field inside air-cored toroid. [1]

(b) Given that the current flows in the clockwise direction for an observer on the left side of the solenoid. This means that left face of the solenoid acts as South Pole and right face acts as North Pole. Inside a bar magnet the magnetic field lines are directed from south to north. Therefore, the magnetic field lines are directed from left to right in the solenoid. [½]

Magnetic moment of single current carrying loop is given by

$m = LA$

Where,

I = Current flowing through the loop A = Area of the loop

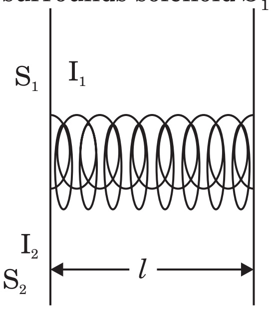
So, Magnetic moment of the whole solenoid is given by

$M = Nw' = N (IA)$ [1]

OR

(a) Mutual inductance is the property of two coils by the virtue of which each opposes any change in the value of current flowing through the other by developing an induced emf. The SI unit of mutual inductance is henry and its symbol is H. [1]

(b) Consider two long solenoids S_1 and S_2 of same length l such that solenoid S_2 surrounds solenoid S_1 completely.



[½]

Let:

$n_1 \rightarrow$ Number of turns per unit length of S_1

$n_2 \rightarrow$ Number of turns per unit length of S_2

$I_1 \rightarrow$ Current passed through solenoid S_1

$\phi_{21} \rightarrow$ Flux linked with S_2 due to current flowing through S1

$\phi_{21} \propto I_1$

$\phi_{21} = M_{21} I_1$

When current is passed through solenoid S_1 an emf is induced in solenoid S_2

Magnetic field produced inside solenoid S_1 on passing current through it is given by $B = \mu_0 nI_1$

Magnetic flux linked with each turn of solenoid S_2 will be equal to B_1 times the area of cross-section of solenoid S_1

Magnetic flux linked with each turn of the solenoid $\phi_{21} = B_1 A$

Therefore, total magnetic flux linked with the solenoid S_2 is given by $\phi_{21} = B_1 A$ ϕ_{21} $A = \mu_o n_1 I_1 A$ [½]

$$M_{21} = \frac{N_2 \phi_{21}}{I_1}$$

$$M_{21} = \frac{N_2 \mu_o n_1 I_1 A}{I_1} = \mu_o n_1 N_2 A$$

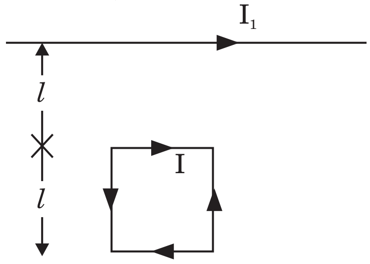
Where N_2 is total number of turns wound over the secondary coil.

$M_{21} = \mu_o n_1 N_2 A$

Similarly the mutual inductance between the two solenoids when current is passed through solenoid S_2 and induced emf is produced in solenoid S_1 is given by

$M_{12} = \mu_o n_2 I_1 A$ [1]

7. (i) For $r < a$,



[½]

$$\oint \vec{B}.\vec{dl} = \mu_o I_{enclosed}$$

$$\frac{I_{enclosed}}{\pi a^2} = \frac{I}{\pi r^2}$$

$$I_{enclosed} = I\frac{r^2}{a^2}$$

$$\vec{B}\cdot\vec{dl} = Bdl\cos\theta$$

$$\vec{B}\cdot\vec{dl} = Bdl \qquad (\because \cos\theta = 1)$$

$$\therefore \oint Bdl = \mu_o I\frac{r^2}{a^2}$$

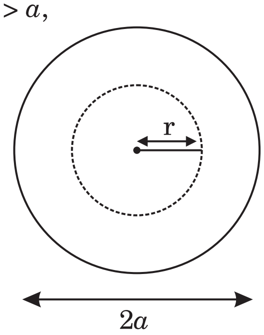
$$B\oint dl = \mu_o I\frac{r^2}{a^2}$$

$$B(2\pi r) = \mu_o I\frac{r^2}{a^2}$$

$$B = \frac{\mu_o}{2\pi}\frac{1}{a^2}r$$

[2]

(ii) For $r > a$,



[½]

From Ampere's Circuital law,

$$\oint \vec{B}.\vec{dl} = \mu_o I_{enclosed}$$

$$\vec{B}.\vec{dl} = Bdl\cos\theta$$

$$\theta = 0°$$

$$\vec{B}.\vec{dl} = Bdl$$

$$I_{enclosed} = I$$

$$\oint Bdl = \mu_o I$$

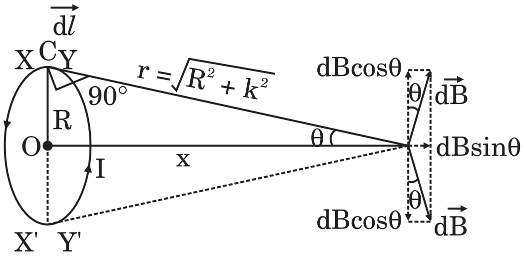
$$B\oint dl = \mu_o I$$

$$B(2\pi r) = \mu_o I$$

$$B = \frac{\mu_o I}{2\pi r}$$

[2]

8.



[1]

(a) According to Biot-Savart's law, magnetic field due to a small element XY at point P is

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\varphi}{r^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin 90°}{r^2} = \frac{Idl}{r^2} = (Q\sin 90° = 1)$$

Resolving dB into two components:

(i) $dB\cos\theta$, which is perpendicular to the axis of the coil

(ii) $dB\sin\theta$ which is along the axis of the coil and away from the centre of the coil.

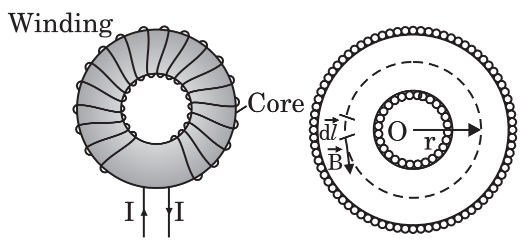
$$B = \int dB\sin\theta \ \ or\ \ B = \frac{\mu_o}{4\pi}\frac{I\sin 90°}{r^2}\int dl$$

$$\therefore B = \frac{\mu_o I\sin\theta \times 2\pi R}{4\pi^2 r^2}$$

$$\therefore B = \frac{\mu_o \times}{4\pi^2}\frac{2\pi IR^2}{4\pi\left(r^2 + x^2\right)^{3/2}}$$

[2]

(b)



$\oint \vec{B} \cdot \vec{dl} = B \times 2\pi r$ ...(i)

According to Ampere's circuital law,

× $\oint \vec{B} \cdot \vec{dl} = \mu_o$ net current enclosed by the circle of radius r

$= \mu_o$ total number of turns × I

$\mu_o (n \times 2\pi r)$ ...(ii)

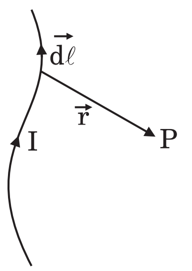
Comparing equation (i) and (ii), we get

$B \times 2\pi r = \mu_o (n \times 2\pi r)I$

$B = \mu_o nI$ [2]

For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field B is zero because the net current enclosed in these spaces is zero. But magnetic field is not exactly zero.

9. For any general shape of the wire, magnetic field due to current carrying wire is given by Biot Savart Law:



$$\vec{B}(P) = \frac{\mu_0 I}{4\pi}\int \frac{\vec{d\ell} \times \vec{r}}{r^3}$$

[1]

Where dl is a vector tangent to the current wire equal to $dy\,j$ for a straight wire along y-axis.

Consider a circular loop of radius r carrying a current I.

Since $dl \perp r$

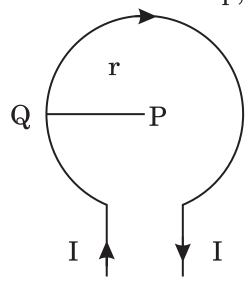
$\rightarrow \theta = 90°$

Applying Biot Savart law:

$$dB = \frac{\mu_o}{4\pi}\frac{Idl \sin 90°}{r^2}$$

[1]

For entire closed circular loop;



$$dB = \int_0^{2\pi r} \frac{\mu_o}{4\pi}\frac{Idl \sin 90°}{r^2}$$

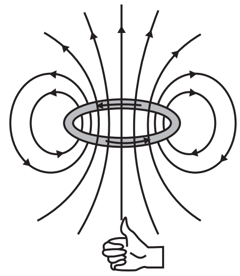
$$B = \frac{\mu_o}{4\pi}\frac{I}{r^2}\int_0^{2\pi r} dl = \frac{\mu_o}{4\pi}\left(\frac{2\pi l}{r}\right)$$

[1]

For n turns of a coil; $B = \frac{\mu_o}{4\pi}\left(\frac{2\pi nl}{r}\right)$

The magnetic field lines due to a circular wire form closed loops.

The direction of the magnetic field is given by right hand thumb rule. [1]



The current carrying loop produces a magnetic field around it, whose magnetic moment is given as $I \times A$ (here, I is the current through the loop and A is the area of cross-section; hence it behaves like a magnet. [1]

Topic 2: Lorentz Force and Cyclotron

Summary

- The electric field, E produced by the source of the field Q, is given as $E = \frac{Q\hat{r}}{(4\pi \epsilon_0) r^2}$, where $\hat{r}$ is the unit vector and the field E is a vector field. A charge 'q' interacts with this field and experiences a force F, expressed as

  $$F = qE = \frac{qQ\hat{r}}{(4\pi \epsilon_0) r^2}$$

- In the presence of both electric field E(r) and magnetic field B(r) there is a point charge q (moving with a velocity v and located at 'r' at a given time t). The force on an electric charge 'q' due to both of them is written as

  $F = q[E(r) + v \times B(r)] = F_{electric} + F_{magnetic}$. This force is called the Lorentz force.

- We can calculate the Lorentz force for a straight rod, if B is the external magnetic field by considering the straight rod as a collection of linear strips dl_j, where l is the length of the rod, j is the current density. Hence, the force can be calculated as $F = \sum_j I dl_j \times B$.

Cyclotron:

- It consists of two D's which are placed in a strong magnetic field. An oscillating electric field is applied from the oscillator which is parallel to the magnetic field.

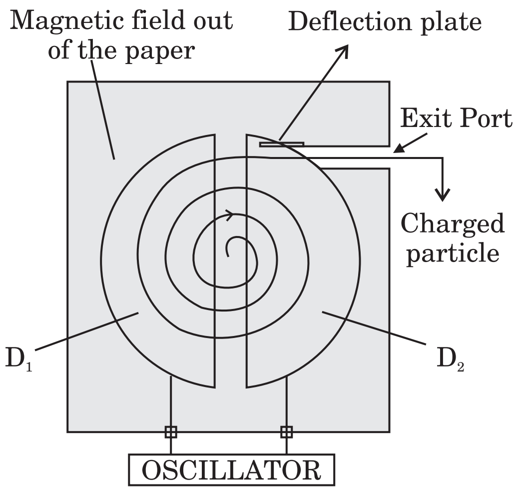


Fig.: Cyclotron

- The charged particle gets accelerated and moves in a circular path whose radius is given by $r = \frac{mv}{q_0 B}$

- The frequency of the cyclotron is given by $v = \frac{1}{T} = \frac{Bq}{2\pi m}$

- A charge of any type in uniform circular motion would have an associated magnetic moment given by $\mu_L = \frac{-e}{2m_e} l$, where l is the magnitude of angular momentum of electron. $\frac{\mu_L}{l} = \frac{e}{2m_e} = 8.8 \times 10^{10} C/kg.$, and this ratio is called Gyro magnetic ratio.

PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 2

1 Mark Questions

1. Write the expression, in a vector form, for the Lorentz magnetic force $\vec{F}$ on a charge –q moving with velocity $\vec{V}$ in a magnetic field $\vec{B}$. What is the direction of the magnetic force?

[DELHI 2014]

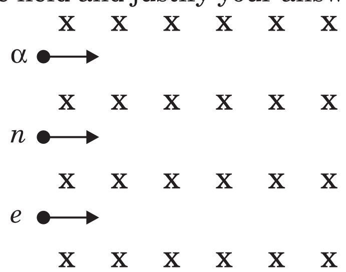
2. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency?

[DELHI 2018]

2 Mark Questions

3. (a) Write the expression for the magnetic force acting on a charged particle moving with velocity $\vec{v}$ in the presence of magnetic field $\vec{B}$.

(b) A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown. Trace their paths in the field and justify your answer.



[DELHI 2016]

4. State the underlying principle of a cyclotron. Write briefly how this machine is used to accelerate charged particles to high energies.

[DELHI 2018]

5. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed.

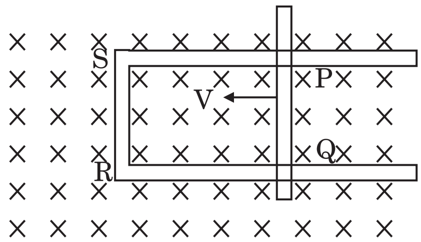
[DELHI 2017]

3 Mark Questions

6. A metallic rod of length L is rotated with a frequency v with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field B parallel to the axis is present everywhere. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtain the expression for it.

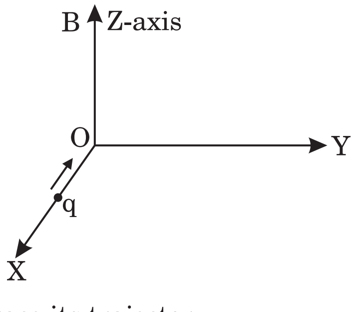
[DELHI 2013]

7. Figure shows a rectangular conducting loop conducting $PQRS$ in which the arm PQ is free to move. A uniform magnetic field acts in the direction perpendicular to the plane of the loop. Arm PQ is moving with a velocity v towards the arm RS Assuming that the arms QR, RS and SP have negligible resistances and the moving arm PQ has the resistance r, obtain the expression for (i) the current in the loop (ii) the force and (iii) the power required to move the arm PQ.



[DELHI 2013]

8. A charge 'q' moving along the X-axis with a velocity c is subjected to a uniform magnetic field B along the Z-axis as it crosses the origin 0.



(i) Trace its trajectory.

(ii) Does the charge gain kinetic energy as it enters the magnetic field? Justify your answer.

5 Mark Questions

9. (a) Draw a schematic sketch of a cyclotron. Explain clearly the role of crossed electric and magnetic field in accelerating the charge. Hence derive the expression for the kinetic energy acquired by the particles.

(b) An α-particle and a proton are released from the centre of the cyclotron and made to accelerate.

(i) Can both be accelerated at the same cyclotron frequency? Give reason to justify your answer.

(ii) When they are accelerated in turn, which of the two will have higher velocity at the exit slit of the dees?

[DELHI 2013]

10. With the help of a labelled diagram, state the underlying principle of a cyclotron.

Explain clearly how it works to accelerate the charged particles.

Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason.

[DELHI 2011]

11. (a) Deduce an expression for the frequency of revolution of a charged particle in a magnetic field and show that it is independent of velocity or energy of the particle.

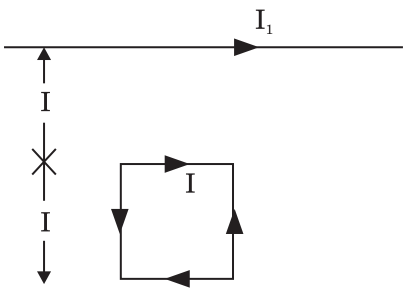
(b) Draw a schematic sketch of a cyclotron. Explain, giving the essential details of its construction, how it is used to accelerate the charged particles.

[DELHI 2014]

12. Explain the principle and working of a cyclotron with the help of a schematic diagram. Write the expression for cyclotron frequency.

[DELHI 2017]

13.



Write the expression for the magnetic moment $(\vec{m})$ due to a planar square loop of side 'l' carrying a steady current 'I' in a vector form. In the given figure this loop is placed in a horizontal plane near a straight long conductor carrying a steady current 'I_1' at a distance of 'l' as shown. Give reason to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop.

[DELHI 2018]

Solutions

1. The Lorentz magnetic force is given by the following relation:

$\vec{F} = q(\vec{V} \times \vec{B})$ [1/2]

Here, q is the magnitude of the moving charge.

The direction of the magnetic force is perpendicular to the plane containing the velocity vector $\vec{V}$ and the magnetic field vector $\vec{B}$. [1/2]

2. As freq. of rev $= \left(\frac{qB}{m}\right)$ here both charged particles will move in circular tracks. As charge on electron and proton is same and both are subjected to same magnetic field so electron will show higher frequency due to less mass. [1]

3. (a) A charge particle having charge q is moving with velocity 'v' in a magnetic field of field strength 'B' then the force acting on it is given by the formula $F = q(\vec{V} \times \vec{B})$ and $F =$ $qvB \sin\theta$ (Where q is the angle between velocity vector of magnetic field).

Direction of force is given by the cross product of velocity and magnetic field. [1]

(b)

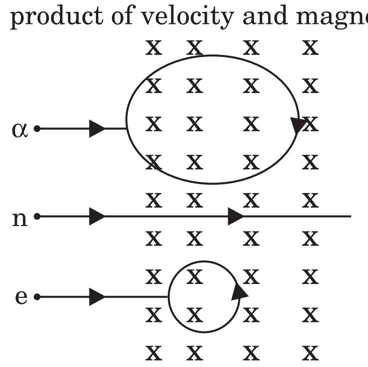


Fig.: Magnetic field

α Particle will trace circular path in clockwise direction as it's deviation will be in the direction of $\left(\vec{v}\times\vec{B}\right)$ i.e. perpendicular to the velocity of particle, neutron will pass without any deviation as magnetic field does not exert force on neutral particle.

Electron will trace circular path in anticlockwise direction as its deviation will be in the direction opposite to $\left(\vec{v}\times\vec{B}\right)$ with a smaller radius due to large charge/mass ratio as $r=\dfrac{mv}{qB}$ [1]

4. The underlying principle of a cyclotron is that an oscillating electric field can be used to accelerate a charge particle to high energy.

A cyclotron involves the use of an electric field to accelerate charge particles across the gap between the two D-shaped magnetic field regions. The magnetic field is perpendicular to the paths of the charged particles that makes them follow in circular paths with in the two *Ds*. An alternating voltage accelerates the charged particles each time they cross the *Ds*. The radius of each particles path increases with its speed. So, the accelerated particles spiral toward the outer wall of the cyclotron. [1/2]

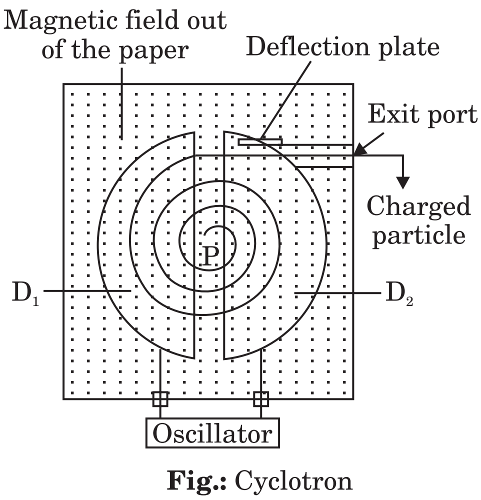


Fig.: Cyclotron

[1/2]

[The *Ds* are the semi-circular structures (D_1 and D_2) between which the charges move. The accelerating voltage is maintained across the opposite halves of the *Ds*.] Square wave electric fields are used to accelerate the charged particles in a cyclotron.

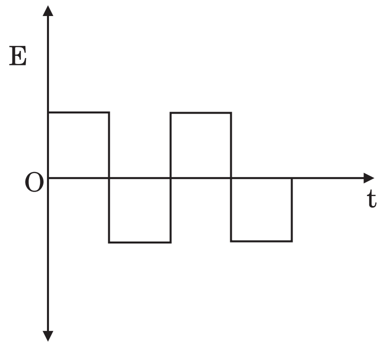


Fig.: Square Wave

[1/2]

The accelerating electric field reverses just at the time the change particle finishes its half circle so that it gets accelerated across the gap between the *Ds*.

The particle gets accelerated again and again, and its velocity increases. Therefore, it attains high kinetic energy.

The positively charged ion adopts a circular path with a constant speed v, under the action of magnetic field B, which is perpendicular to the planes of D's of radius r.

$r=\dfrac{mv}{qB}$ [1/2]

5. Moving uniformly charged particles can be selected under electric and magnetic field vectors form the equation $v=\dfrac{E}{B}$, where E is electric field and B is magnetic field. [2]

6. Suppose the length of the rod is greater than the radius of the circle and rod rotates anticlockwise and suppose the direction of electrons in the rod at any instant be along $+y$ direction.

Suppose the direction of the magnetic field is along $+z$ direction.

Then, using Lorentz law, we get the following:

$$\vec{F}=-e\left(\vec{v}\times\vec{B}\right)$$

$$\vec{F}=-e\left(v\,\hat{j}\times B\hat{k}\right)$$

$$\vec{F}=-eB\hat{i}$$ [1]

Thus, the direction of force on the electrons is along – x-axis. So, the electrons will move towards the centre i.e., the fixed end of the rod. This movement of electrons will effect in current and thus it will generate an emf in the rod between the fixed end and the point touching the ring.

Let θ be the angle between the rod and radius of the circle at any time t.

Then, area swept by the rod inside the circle

$$= \frac{1}{2}\pi r^2\theta$$

[1]

Induced *emf*

$$= B \times \frac{d}{dt}\left(\frac{1}{2}\pi r^2\theta\right)$$

$$= \frac{1}{2}\pi r^2 B\frac{d\theta}{dt} = \frac{1}{2}\pi r^2 B\omega$$

$$= \frac{1}{2}\pi r^2 B(2\pi v)$$

Induced *emf* = $\pi^2 r^2 Bv$ [1]

7. Let the length $RQ = x$ and RS = l.

Let the magnitude of the uniform magnetic field be B.

(i) The magnetic flux ϕ enclosed by the loop PQRS is given by, $\phi = Blx$

As, x is changing with time, the rate of change of flux $\frac{d\phi}{dt}$ will induce an emf given by:

$$E = -\frac{d\phi}{dt} = -\frac{d(Blx)}{dt} = -Bl\frac{dx}{dt} = Blv$$

$$\left[as, \frac{dx}{dt} = -v\right]$$

Current in the loop is given by,

$$I = \frac{E}{r} = \frac{Blv}{r}$$ [1]

(ii) The magnetic force on the PQ is,

$$F = BIl = B\left(\frac{Blv}{r}\right)\times l$$

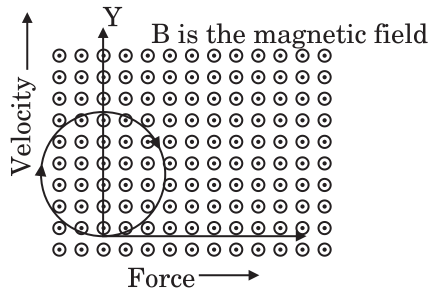
$$F = \frac{B^2l^2v}{r}$$ [1]

(iii) Power emitted to move the arm PQ is,

$$P = F \times v = \frac{B^2l^2v}{r}\times v$$

$$P = \frac{B^2l^2v^2}{r}$$ [1]

8. The direction of magnetic field is along the negative X-direction. Hence the magnetic force will act in such a way that this particle describes a circular motion as shown below. [1]



[1/2]

(ii) No, the charge does not gain kinetic energy because the force and velocity are perpendicular to each other.

$W = \vec{F}\cdot\vec{v} = Fv\cos\theta$

$\theta = 90°$ [1]

Thus, force does not bring out any change in the velocity. [1/2]

9. (a)

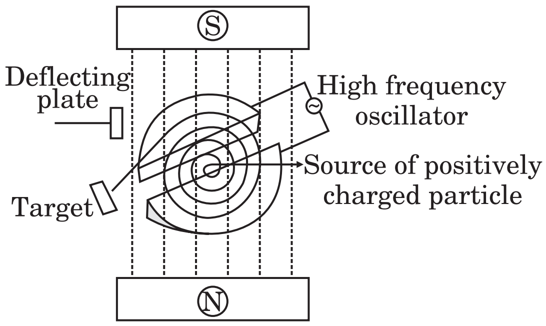


Fig.: Cyclotron

[1]

Electric field accelerate the charged particle whereas magnetic field makes its path circular.

$$K.E. = \frac{1}{2}mv^2,\ r = \frac{mv}{qB}$$

$$-\frac{1}{2}m\left(\frac{qBr}{m}\right)^2$$

$$K.E. = \frac{q^2B^2r^2}{2m}$$

(b) (i) $$f_\alpha = \frac{qB}{2\pi m} = \frac{(2e)B}{2\pi(4m_p)} = \frac{eB}{4\pi m_p}$$

$$f_p = \frac{eB}{4\pi m_p}$$

$$\therefore f_\alpha \propto f_p$$ [2]

So both can't be accelerated by same frequency.

(ii) $r = \frac{m\nu}{qB}$

$\nu = \frac{qBr}{m}$

$\nu \propto \frac{q}{m}$

$\frac{\nu_p}{\nu_\alpha} = \frac{q_p}{q_\alpha}\frac{m_\alpha}{m_p}$

$= \left(\frac{e}{2e}\right)\left(\frac{4m_p}{m_p}\right)$

$\frac{\nu_p}{v_\alpha} = 2$

$\therefore \nu_p = 2\nu_\alpha$

$\therefore \nu_p > \nu_\alpha$

[2]

10. The principle behind the working of a cyclotron is that in a uniform magnetic field, charged particle executes uniform circular motion with a frequency that is independent of its radius (which depends on the energy of the particle)

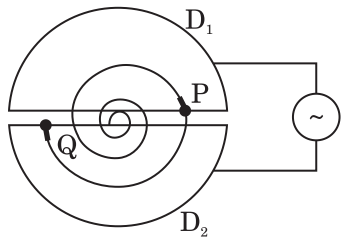


Fig.: Cyclotron [1]

A cyclotron has a uniform magnetic field spread over a circular region split in two halves by two semi-circular D's, D_1 and D_2.

Let us assume for definiteness that the particle has positive charge. Then, as it comes around in D_1 clockwise to point P, D_1 and D_2 are so connected to an alternating voltage that at this point, D_1 is at a higher potential than D_2. This accelerates the particles, so that it gains kinetic energy by $T = q(V_1 - V_2)$ where v_1 and v_2 where D_1 and D_2 are potentials of respectively. [1]

This increases the radius of circular motion since $r = \left(\frac{m\nu}{qB}\right)$ and is proportional to the square root of kinetic energy.

Since frequency of circular motion, given by $f_c = \left(\frac{qB}{2\pi m}\right)$ is independent of r, the frequency of alternating voltage is made twice fc, so that when the particle reaches point Q, $V_1 - V_2$ flip and D_2 is at higher potential than D_1, so that particle is accelerated again. This goes on till the kinetic energy is high enough and consequently the radius is large enough so that the particle is ejected out tangentially from the magnetic field with high energy.

Frequency [1]

$f_c = \left(\frac{qB}{2\pi m}\right)$ is independent of the energy or the speed of the charged particle.

Upper limit is relativistic speeds. The cyclotron is limited by relativistic effects due to which the mass of the accelerating particle increases with energy and so f_c changes after each cycle.

Also, for light charged particles, f_c is enormously high and difficult to maintain.

Also, it is not easy to maintain uniformity of the magnetic field.

To add to that, a charged particle in circular motion (even if it is uniform) is accelerating and all accelerated charges radiate electromagnetic energy, thereby losing energy. [1]

11. (a) If particle is performing circular motion due to magnetic force then

Centripetal force = Magnetic force [1/2]

$\frac{mv^2}{r} = qvB\sin 90°$

$\Rightarrow r = \frac{mv}{qB}$ [1]

So, time period $= \frac{2\pi r}{v}$

$T = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$

$T = \frac{2\pi m}{qB} \propto v^o$

× $\therefore$ Frequency $f = \frac{1}{T} = \frac{qB}{2m\pi} \propto v^o$ [1]

(b) Cyclotron is a device to accelerate ions to extremely high velocities, by accelerating them repeatedly through high voltages.

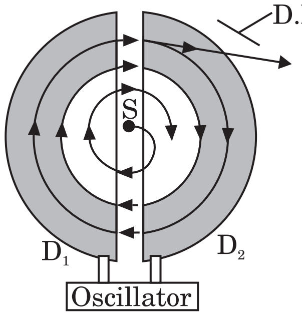


Fig.: Cyclotron [1/2]

Principle – A positive ion can acquire sufficiently large energy with a small alternating potential difference by making the ion cross the same electric field time and again by making use of a strong magnetic field. [1]

Construction- It consists of a pair of hollow metal cylindrical chambers shaped like *D*, and called the Dees; Both the Dees are placed under the circular pole pieces of a very strong electromagnet. The two Dees are connected to the terminals of a very high frequency and high voltage oscillator whose frequency is of the order of a few million cycles per second.

Working -Charged ion is passed through electric field again & again to be energized. Inside Dee's strong perpendicular magnetic field turns the particle towards gap. So radius of semi-circular path increases continuously. [1]

12.

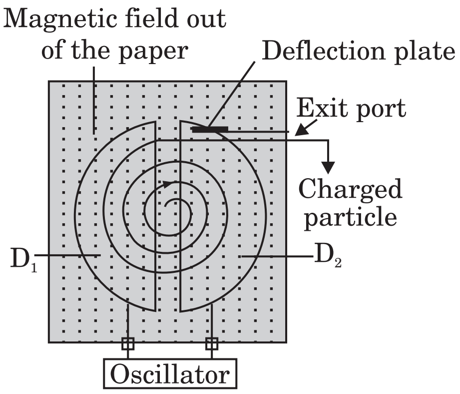


Fig.: Cyclotron [1]

Cyclotron is a machine used to accelerate charged particles or ions to high energies. It uses both electrical and magnetic fields in combination to increase the speed of the charged particles.

The particles move in two semi-circular containers D_1 and D_2, called Dees. Inside the metal box, the charged particle is shielded from external electric fields. [1]

When the particle moves from one dee to another, electric field is acted on the particle.

The sign of the electric field is changed alternately, in tune with the circular motion of the particle. Hence, the particle is always accelerated by the electric field. As the energy of the particle increases, the radius of the circular path increases.

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

[1]

Time taken for a particle for one complete revolution = T

$$T = \frac{2\pi r}{\nu}$$

$$T = \frac{2\pi m}{qB}$$

$$T = \frac{1}{\nu_c}$$

Where v, is the cyclotron frequency

Then,

$$\frac{1}{\nu_c} = \frac{2\pi m}{qB}$$

$$\nu_c = \frac{2\pi m}{qB}$$

The above expression is the expression for cyclotron frequency.

The oscillator applies an ac voltage across the Ds and this voltage must have a frequency equal to that of cyclotron frequency. [1]

13. The expression for the magnetic moment $(\vec{m})$ due to a planar square loop of side '*l*' carrying steady current '*l*' in a vector form is given as $(\vec{m}) = I\vec{A}$

Therefore, $(\vec{m}) = I(l)^2 \hat{n}$

Where, $\hat{n}$ is the unit vector along the normal to the surface of the loop. The attractive force per unit length on the loop is, [5]

Topic 3: Magnetic Force and Torque between Two Parallel Currents

Summary

Force on a current-carrying conductor in a uniform magnetic field:

- The force on a current carrying conductor of length l in a uniform magnetic field B when θ is the angle between current and magnetic field can be calculated by $F = IB\ell \sin\theta$
- Fleming's Left-Hand Rule is used to find the direction of the magnetic force which is right angled to the plane containing conductor and magnetic field.

Force between two parallel current-carrying conductors:

Two parallel conductors carrying current experiences a force. When current flows in same direction, wire B experiences magnetic field due to wire A which is: $B_1 = \frac{\mu_0 I I_1}{2\pi d}$

Force per unit length in the given wire is $\frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$

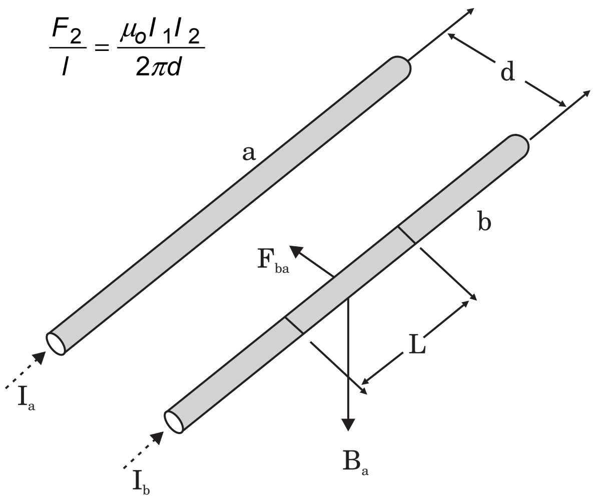


Fig.: Force between two parallel currents carrying conductores.

Torque experienced by a current loop in uniform magnetic field:

The torque experienced by a rectangular loop in uniform magnetic field B of length l, breadth b with current I lowing through it is:

$\tau = nBIA\sin\theta$

Moving coil galvanometer

- Its main use is to detect and measure small electric currents.
- The current carrying coil is suspended in a uniform magnetic field, so it produces a torque which is responsible for rotating the coil.

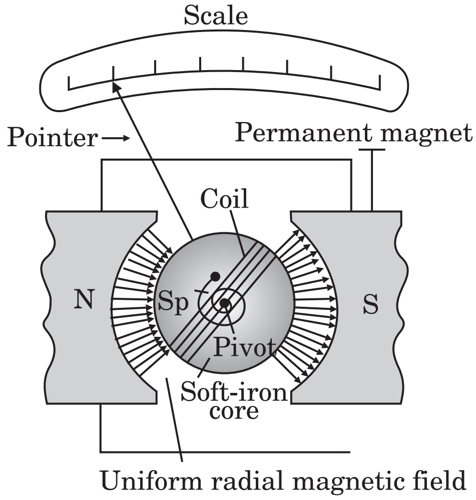


Fig.: Moving coil galvanometer

- The torque is given by $\tau = F \times b = nBIl \times b = BInA \sin\theta$

Current sensitivity of galvanometer

- When a galvanometer produces a large deflection for a small amount of current, it is said to be sensitive.
- The voltage sensitivity of galvanometer is deflection per unit voltage and is given as $\frac{\theta}{I} = \frac{nBA}{C}$

Conversion of galvanometer into ammeter

A small resistance called a Shunt resistance is attached with the galvanometer coil in parallel so that most of the current passes through the shunt resistance.

Conversion of galvanometer into voltmeter

A high resistance is connected in series with the galvanometer coil so that the galvanometer acts as a voltmeter.

PREVIOUS YEARS' EXAMINATION QUESTIONS
TOPIC 3

1 Mark Questions

1. Using the concept of force between two infinitely long parallel current carrying conductors, define one ampere of current.
[DELHI 2014]

2. Write the underlying principle of a moving coil galvanometer.
[DELHI 2016]

2 Mark Questions

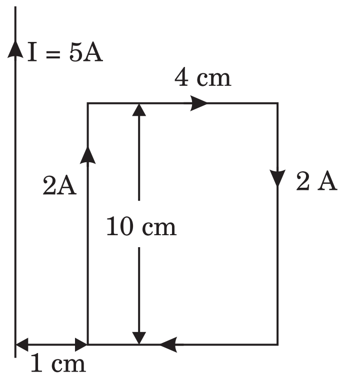
3. Two long straight parallel conductors carry steady current I_1 and I_2 separated by a distance d. If the currents are flowing in the same direction, show how the magnetic field set up in one produces an attractive force on the other. Obtain the expression for this force. Hence define one ampere.
[DELHI 2016]

4. A wire AB is carrying a steady current of $6A$ and is lying on the table. Another wire CD carrying $4A$ is held directly above AB at a height of 1mm. Find the mass per unit length of the wire CD so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in AB. [Take the value of $g = 10 ms^{-2}$]
[DELHI 2013]

5. State the principle of working of a galvanometer. A galvanometer of resistance G is converted into a voltmeter to measure up to V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure up to $\frac{V}{2}$ volts. Find the resistance, in terms of R_1 and R_2, required to be connected to convert it into a voltmeter that can read up to $2\ V$. Also find the resistance G of the galvanometer in terms of R_1 and R_2.
[DELHI 2015]

3 Mark Questions

6. A rectangular loop of wire 4 cm × 10 cm carries a steady current of $2A$. A straight long wire carrying $5A$ current is kept near the loop as shown. If the loop and the wire are coplanar, find



(i) the torque acting on the loop and
(ii) the magnitude and direction of the force on the loop due to the current carrying wire.
[DELHI 2012]

7. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, calculate the kinetic energy (in MeV) of the proton beam produced by the accelerator.
[DELHI 2015]

8. Deduce the expression for the torque τ acting on a planar loop of area Ar and carrying current I placed in a uniform magnetic field B. If the loop is free to rotate, what would be its orientation in stable equilibrium?
[DELHI 2015]

9. Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence define one ampere.

5 Mark Questions

10. (a) Draw a labeled diagram of a moving coil galvanometer. Describe briefly its principle and working.
(b) Answer the following:
(i) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?
(ii) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain, giving reason
[DELHI 2014]

11. State the underlying principle of working of a moving coil galvanometer. Write two reasons why a galvanometer cannot be used as such to measure current in a given circuit. Name any two factors on which the current sensitivity of a galvanometer depends
[DELHI 2018]

Solutions

1. One ampere is that current which if passed in each of two parallel conductors of infinite length and one meter apart in vacuum, causes each conductor to experience a force of 2×10^{-7} Newton per meter of length of conductor. [1]

2. When a current carrying coil is placed in magnetic field then it experiences a torque. [1/2

$NIAB = k\alpha$

$\Rightarrow I = \dfrac{k}{NAB}\alpha$

N ⇒ The number of turns

I ⇒ Current

A ⇒ Area of the loop

B ⇒ Magnetic field

k ⇒ Torsional constant of the wire

α ⇒ Angle of deflection [1/2]

3. Magnetic field produced on the wire (carrying current I_2) due to I_1 will be.

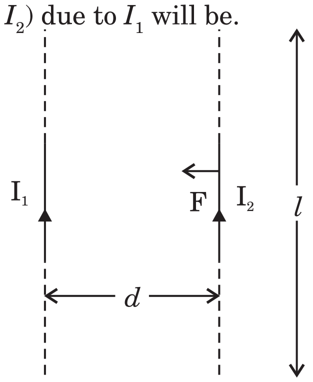


Fig.: Two long parallel conductor separated by distance d

$$B = \frac{\mu_o I_1}{2\pi d}$$

Force acting at l length is $F = I_2 lB$

$$F = \frac{\mu_o I_1 I_2 l}{2\pi d}$$ towards I_1 [1]

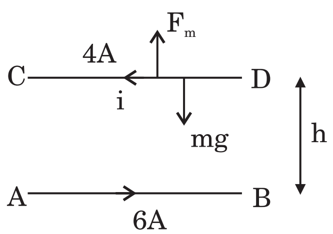
Attractive force between the wires

If $l = 1m$, $d = 1m$, $I_1 = I_2 = I$ and $F = 2 \times 10^{-7} N$

$\Rightarrow I = 1A$

So one ampere is defined as the current, which when maintained in two parallel infinite length conductors, held at a separation of one meter will produce of a force of $2 \times 10^{-7} N$ per metre of each conductor. [1]

4.



[1]

Fig.: Current carrying wires Ab & CD

($l \rightarrow$ length of CD)

for balance of CD

$f_m = mg$ [1/2]

$$\left(\frac{\mu_o}{2\pi}\frac{i_1 i_2}{h}\right)l = mg$$

$$\frac{m}{l} = \frac{2\times10^{-7}\times6\times4}{10^{-3}\times10}$$

$$\frac{m}{l} = 4.8\times10^{-4}\, kg/m$$ [1/2]

Direction of current in CD will be opposite to AB.

5. Principle: When a current-carrying coil is placed in a magnetic field, it experiences a torque. From the measurement of the deflection of the coil, the strength of the current can be computed. A high resistance is connected in series with the galvanometer to convert it into voltmeter. The value of the resistance is given by $R = \frac{V}{I_g} - R_g$ Where, $V \rightarrow$ Potential difference across the terminals of the voltmeter [1/2]

$V_g \rightarrow$ Current through the galvanometer

$G \rightarrow$ Resistance of the galvanometer

When the resistance R_1 is connected in series with the galvanometer. [1/2]

$V = I(G + R_1)$

When the resistance R_2 is connected in series with the galvanometer.

$$\frac{V}{2} = I(G + R_2)$$

$$\Rightarrow 2 = \frac{G + R_1}{G + R_2}$$

$G = R_1 - 2R_2$ [1/2]

Let R_3 be the resistance required for conversion into voltmeter of range $2V$

$\therefore\ 2V = I_g(G + R_3)$

Also, $V = I_g(G + R_1)$

$$2 = \frac{G + R_3}{G + R_1}$$

$\therefore\ R_3 = G + R_1 = R_1 - 2R_2 + 2R_1$

$R_3 = 3R_1 - 2R_2$ [1/2]

6.

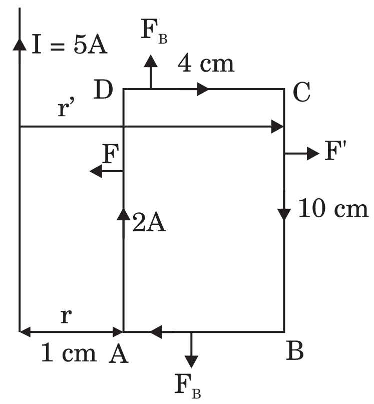


Fig.: Current carrying loop [1/2]

(i) $\vec{\tau} = \vec{M} \times \vec{B} = MB\sin\theta$

Here M and B have the same direction,

so $\theta = 90°$

$|\vec{\tau}| = MB\sin\theta = 0$

(ii) We know $\overrightarrow{F_B} = i\vec{l} \times \vec{B}$

On line AB and CD magnetic forces are equal and opposite. So they cancel out each other. Magnetic force on line AD.

$\overrightarrow{F_B} = i\vec{l} \times \vec{B}$ (Attractive)

$= ilB$ (i = 10 cm = 0.1m)

$\because B = \dfrac{\mu_o I}{2\pi r}$

$|\vec{F}| = \dfrac{\mu i I l}{2\pi r}$ (Attractive)

Magnetic force on line CB,

$\overrightarrow{F_B} = i\vec{l} \times \vec{B}$ (Repulsive)

$\Rightarrow F' = |\vec{F}| = ilB'$

$\because B = \dfrac{\mu_o I}{2\pi r'}$

$F' = \dfrac{\mu_o i I l}{2\pi r'}$ (Repulsive)

So, net force, $F_n = F - F'$

$$F_n = \frac{\mu_o i I l}{2\pi}\left[\frac{1}{r} - \frac{1}{r'}\right]$$

Given, $i = 5A$, $I = 2A$, r = 1cm = 0.01 m,

r = (1 + 4) = 5 cm and l = 10cm = 0.1m

Substituting these values in above equations, we get

$$F_n = \left(2\times10^{-7}\right)(5)(2)(0.1)\left[\frac{1}{0.01} - \frac{1}{0.05}\right]$$

$$= 2\times10\times10^{-7}\times10\left[\frac{1}{1} - \frac{1}{5}\right]$$

$= 160 \times 10^{-7}\ N$

7. Frequency of oscillators

(v) = 10 MHz = 107 Hz

Mass of proton, $m = 1.67 \times 10^{-27}\ kg$

Charge of proton = $1.6 \times 10^{-19}\ C$

Operating magnetic field is given by the relation

$$B = \frac{2\pi mv}{q} = \frac{2\times3.14\times1.67\times10^{-27}\times10^{7}}{1.6\times10^{-19}}$$

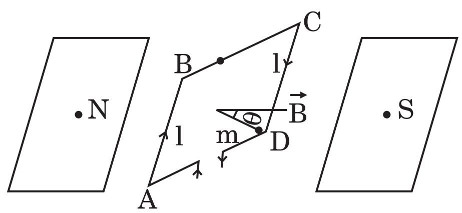
$= 0.65T$ [1]

Radius of dees = 60cm = 0.6m

$$KE = \frac{q^2B^2r^2}{2m} = \frac{\left(1.6\times10^{-19}\right)(0.65)^2(0.6)^2}{2\times1.67\times10^{-27}\times1.6\times10^{-13}}MeV$$

$= 7.28\ MeV$ [1]

8.



[1]

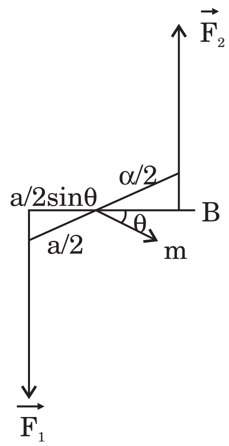


Fig. (a) & (b): Planar loop

[1/2]

(a) The area vector of the loop $ABCD$ makes an arbitrary angle θ with the magnetic field.

(b) Top view of the loop. The forces F_{1r} and F_{2r} acting on the arms AB and CD are indicated.

We consider the case when the plane of the loop, is not along the magnetic field, but makes an angle θ with it. Fig.(a) illustrates this general case. The forces on the arms BC and DA are equal, opposite, and act along the axis of the coil, which connects the centres of mass of BC and DA. Being collinear along the axis they cancel each other, resulting in no net force or torque. The forces on arms AB and CD are F_1 and F_2. They too are equal and opposite, with magnitude, $F_1 = F_2 = I\ b\ B$ But they are not collinear! This results in a couple Fig.(b) is a view of the arrangement from the AD end and it illustrates these two forces constituting a couple. The magnitude of the torque on the loop is,

$$\tau = F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta$$

$= Iab \sin\theta$

$IAB \sin\theta$ ...(i) [1]

As $\theta \to 0$, the perpendicular distance between the forces of the couple also approaches zero. This makes the forces collinear and the net force and torque zero. The torques in above equation can be expressed as vector product of the magnetic moment of the coil and the magnetic field. We define the magnetic moment of the current loop as, $m = I\ A$ where the direction of the area vector A is given by the right-hand thumb rule and is directed into the plane of the paper in Fig.(a). Then as the angle between m and B is θ, equation (i) can be expressed by one expression

$\tau = m \times B$

we see that the torque τ_r vanishes when m_r is either parallel or antiparallel to the magnetic field B_r. This indicates a state of equilibrium as there is no torque on the coil (this also applies to any object with a magnetic moment m r). When m_r and B_r are parallel the equilibrium is a stable one. [1]

9. Two long parallel conductors a and b separated by a distance I and carrying currents I_a and I_b respectively are shown below.

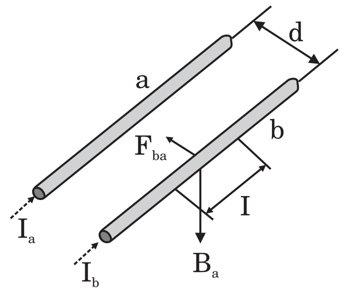


Fig.: Two parallel long conductors separated by 'd'

[1/2]

By Ampere's circuital law, we have

$$B_a = \frac{\mu_o I_o}{2\pi d}$$

Conductor b will experience a sideways force because of conductor a. Let this force be F_{ba}

$F_{ba} = I_b L B_a \quad (F = ILB)$ [1/2]

$$= \frac{\mu_o I_a I_b L}{2\mu d}$$

By symmetry,

$F_{ba} = -F_{ab}$

1 ampere is the value of that steady current which when maintained in each of the two very long, straight, parallel conductors of negligible cross section and placed one metre apart in vacuum, would produce on each of these conductors a force equal to 2×10^{-7} Newton per metre of length.

10. (a) **Principle:** When a current carrying coil is placed in magnetic field, it experiences a torque.

Construction: It consists of a narrow rectangular coil $PQRS$ consisting of a large number of turns of fine insulated copper wire wound over a frame made of light, non-magnetic metal. A soft iron cylinder known as the core is placed symmetrically within the coil and detached from it. The coil is suspended between the two cylindrical pole pieces (N and S of a strong permanent horse shoe magnet) by a thin flat phosphor bronze strip the upper end of which is connected to a movable torsion head T. The lower end of the coil is connected to a hair spring s of phosphor bronze having only a few turns. Radial magnetic field. The magnetic field in the small air gap between the cylindrical pole pieces is radial. The magnetic lines of force within the air gap are along the radii. On account of this, the plane of the coil remains always parallel to the direction of the magnetic field. [1]

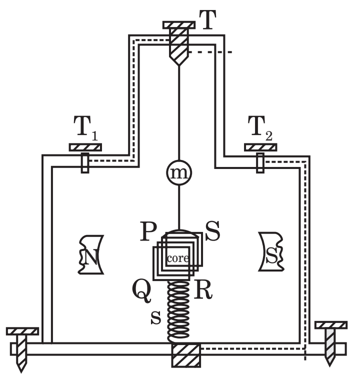


Fig.: Moving coil Galvanometer [1/2]

The magnetic field in the small air gap between the cylindrical pole-pieces is radial. On account of this, the plane of the coil remains always parallel to the direction of the magnetic field

Theory : Let I = current flowing through the coil

B = magnetic field induction

l = length of the coil;

b = breadth of the coil

N = number of turns in the coil

A $(= l \times b)$ = area of the coil

Moment of deflecting couple = $NBIl \times b$ $= NBIA$

When the coil deflects, the suspension fiber gets twisted. On account of elasticity, a restoring couple is set up in the fiber. This couple is proportional to the twist.

If α be the angular twist then Moment of restoring couple = $k\alpha$

For equilibrium of the coil, $NBIA = k\alpha$ or

$$I = \left(\frac{k}{NBA}\right)\alpha$$

or $I = K\alpha$

where $K = \left(\frac{k}{NBA}\right)$ is the galvanometer constant

Now, $I \propto \alpha$ or $\alpha \propto I$ [1]

(b) (i) By using soft iron core, magnetic field is increased so sensitivity increases and magnetic field becomes radial So angle between plane of coil & magnetic line of force is zero in all orientations of coil. [1]

(ii) Voltage sensitivity

$$= \frac{\text{Charge Sensitivity}}{\text{Resistance of Coil}}$$

[1/2]

$$VS = \frac{CS}{R_{coil}}$$

If R_{coil} = constant $V\,S \propto C\,S$

It means that VS increases if CS is increased but if resistance of coil is also increases in same ratio then VS may be constant [1]

11. The underlying principle for the working of a moving coil galvanometer is that when a current-carrying conductor is placed inside a magnetic field, it experiences a magnetic force.

The two reasons why a galvanometer cannot be used for measuring current are:

(i) The high resistance of galvanometer can disturb the original current flowing through the circuit. [2]

(ii) The high current present in the circuit can destroy the coil windings present in the galvanometer. [2]

The factors on which the current sensitivity of a galvanometer depends are:

(i) Number of turns in the coil [1/2]

(ii) Torsional spring constant [1/2]

Smart Notes ....

CHAPTER 5

Magnetism and Matter

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2015
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Current loop as a magnetic dipole and its magnetic dipole moment magnetic dipole moment of a revolving electron					
Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis		1Q (4 marks)	1Q (1marks), 1Q (3 marks)	1Q (2 marks)	
Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid,magnetic field lines					
Earth's magnetic field and magnetic elements					1Q (3 marks), 1Q (2 marks)
Para-, dia- and ferro - magnetic substances, with examples. Electromagnets and factors affecting their strengths,permanent magnets				1Q (2 marks)	

On the basis of above analysis, it can be said that from exam point of view Magnetic Dipole and Earth magnetic field are most important concepts of the chapter.

[Topic 1] Magnetic Dipole and Magnetic Field Lines

Summary

Magnetism:

- Magnetic phenomena are universal in nature. Magnetism is a physical phenomenon produced by the motion of electric charge, which results in attractive and repulsive forces between objects.
- The magnetic field of the Earth points from geographical south to the north.
- A bar magnet always points in the north-south direction when suspended freely.
- When same poles of two magnets are brought close to each other, a repulsive force is experienced. When Opposite poles of two magnets are brought close, then an attractive force is experienced.

Bar Magnet:

Iron fillings sprinkled on a glass plate kept over a short bar magnet arrange themselves in a pattern. It shows that the magnet has two poles in the same way as the positive and negative charge of an electric dipole called as the North and the South pole.

Magnetic field lines: The magnetic field lines of a bar magnet form continuous closed loops. The direction of net magnetic field at any point is determined by the tangent to the field line at that point. The magnitude of the magnetic field will be stronger for the area from which more number of field lines are passing. The magnetic field lines never intersect each other.

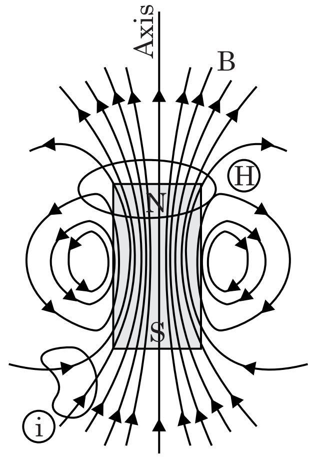


Fig: Magnetic field lines in a bar magnet

- Bar magnet as an equivalent solenoid: The magnetic field B due to bar magnet of size l and magnetic moment m which is at a distance r from the mid-point when r >> l, is given by

$$B = \frac{\mu_0 2m}{4\pi r^3} \quad \text{(Along axis)}$$

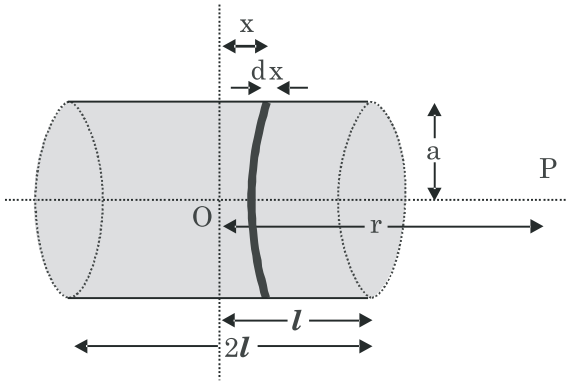


Fig: Bar magnet as an equivalent solenoid

- Dipole in a uniform magnetic field: When a bar magnet is having a dipole moment m and it is placed in uniform magnetic field B,

  The force acting on it is equal to 0.

  The torque acting on the magnet is m × B

  It has a potential energy of –m.B

Gauss's law for magnetic fields:

It states that the magnetic flux through any closed loop is equal to zero.

$$\phi_B = \sum_{all} \Delta\phi_B = \sum_{all} B.\Delta S = 0$$

PREVIOUS YEARS' EXAMINATION QUESTIONS
TOPIC 1

3 Mark Questions

1. A bar magnet of magnetic moment 6J/T is aligned at 60° with a uniform external magnetic field of 0.44T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii).

[DELHI 2018]

Solutions

1. $M = 6J/T$

$q = 60°$

$B = 0.44T$

6 × 0.44 sin 60°

$= 6 \times 0.44 \times \frac{\sqrt{3}}{2}$

$= 3\sqrt{3} \times 0.44 = 2.836$

$$dw \int_{\theta=60°}^{90°} mB \sin\theta \, d\theta = -mB[\cos 90° - \cos 60°]$$

$= 6 \times 0.44 \times \left[-\frac{1}{2}\right]$ [1]

= 3 × 0.44

= 1.32 J

(ii)

$$dw = \int_{\theta=60°}^{180°} mB \sin\theta \, d\theta = -mB[\cos 180° - \cos 60°]$$

$= -6 \times 0.44\left[-1 - \frac{1}{2}\right]$ [1]

$= -6 \times 0.44\left[-\frac{3}{2}\right]$

= 9 × 0.44 39.6J

(b) W = $m \times B$

$W = m\ B \sin\theta$

= 6 × 0.44 sin 180° [1]

$W = 0$

[Topic 2] Earth's Magnetism and Magnetic Properties of Material

SUMMARY

Earth's Magnetism:

- The earth's magnetism is of the order of 10^{-5} T. Its strength is different at different place. The pole near to geographic north pole is called the north magnetic pole and the pole near to geographic south pole is called south magnetic pole. The magnetic of the field on the earth's surface is 4×10^{-5} T.
- There are three elements of the earth's magnetic field which are used to specify the magnetic field of earth's surface – the horizontal component, the magnetic declination and the magnetic dip.
- The magnetic field of a bar magnet tilted 11° from the spin axis of Earth is in the same direction as the Earth's magnetic field.

Magnetization and magnetic field:

- The magnetization M is equal to its magnetic moment per unit volume

$$M = \frac{m_{net}}{V}$$

- The magnetic intensity H is defined as the amount of magnetic flux in a unit area perpendicular to the direction of magnetic fow.

$$H = \frac{B_0}{\mu_0}$$

- The magnetic field B in the material is given by, $B = \mu_0(H + M)$
- The degree of magnetization of a material in response to an applied magnetic field is denoted as magnetic susceptibility. It is given by

$$\chi = \frac{M}{H}$$

So, $\mu = \mu_0\mu_r$

Where $\mu_r = 1 + \chi$

Magnetic properties of materials:

Magnetic Materials are broadly classified as paramagnetic, diamagnetic and ferromagnetic materials. For paramagnetic materials χ is positive and is small, for diamagnetic materials χ is negative and lies between 0 and -1 and for ferromagnetic materials χ is positive and large.

Property	*Ferromagnetic*	*Diamagnetic*	*Paramagnetic*
Effect of magnets	*They are strongly attracted by magnets.*	*They are feebly repelled by magnets.*	*They are feebly attracted by magnets.*
Susceptibility value χ_m	*Large and positive* $\chi_m > 1000$	*Small and negative*	*Small and positive*
Permeability value	$\mu >> \mu_0$	$\mu < \mu_0$	$\mu < \mu_0$
In a uniform magnetic field	*Freely suspended rod aligns itself parallel to the field.*	*Freely suspended rod aligns itself perpendicular to the field.*	*Freely suspended rod aligns itself parallel to the field.*
Relative permeability value	*It is greater than 1000.*	*Slightly less than 1.*	*Slightly greater than 1.*
Effect of temperature	*Susceptibility decreases with temperature.*	*Susceptibility is independent of temperature.*	*Susceptibility varies inversely with temperature.*
Physical state of the material	*Solids only.*	*Solid, liquid or gas.*	*Solid, liquid or gas.*
Hysteresis effect	*Shows hysteresis*	*Does not show hysteresis.*	*Does not show hysteresis.*
Removal of magnetic field	*Magnetization retain even on removal of magnetic field.*	*Magnetization is only for the time magnetic field is applied.*	*Magnetization is only for the time magnetic field is applied.*
Examples	*Fe, Ni, Gd, Co*	*Bi, Si, Cu, Pb*	*Al, Ca, Na*

Ferromagnetic materials show the property of hysteresis.

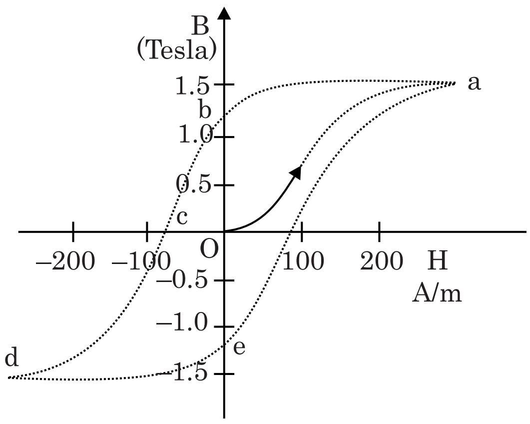


Fig: Magnetic hysteresis loop

The magnetic hysteresis loop is the B-H curve for ferromagnetic materials

Curie's law:

The intensity of magnetization I of a paramagnetic material varies directly to the strength of the external magnetic field H, called magnetizing field and is inversely proportional to absolute temperature of the material.

$\chi = \frac{C}{T}$ where C is Curie constant.

Permanent magnets:

- Permanent magnets are those substances which at room temperature retain their ferromagnetic property.
- An iron rod held in north-south direction and if it is hammered repeatedly it will become a permanent magnet.
- It can also be made by placing a ferromagnetic rod in a solenoid and passing current through it. The rod gets magnetized by the magnetic field of the solenoid.
- A material having high permeability, high coercivity, and high retentivity could be suitable for permanent magnets.

Electromagnets:

- A solenoid having a core of iron with wire wrapped around it is called an electromagnet.
- Ferromagnetic materials are used for core of electromagnets.
- Some of the applications of electromagnets are loudspeakers, electric bells, telephone diaphragms.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. The permeability of a magnetic material is 0.9983. Name the type of magnetic materials it represents.

[DELHI 2011]

2. The horizontal component of earth's magnetic field at a place is *B* and the angle of dip is 60°. What is the value of vertical component of the earth's magnetic field at equator?

[DELHI 2012]

3. What are permanent magnets? Give one example.

[DELHI 2013]

4. Which of the following substances are diamagnetic?
Bi, Al, Na, Cu, Ca and *Ni*

[DELHI 2013]

5. Which of the following substances are paramagnetic?
Bi, Al, Cu, Ca, Pb, Ni

[DELHI 2013]

2 Mark Questions

6. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip down in 60° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.4 *G*. Determine the magnitude of the earth's magnetic field at the place.

[DELHI 2011]

7. The susceptibility of a magnetic material is -2.6×10^{-5}. Identify the type of magnetic material and state its two properties.

[DELHI 2013]

8. Show diagrammatically the behaviour of magnetic field lines in the presence of (i) paramagnetic and (ii) diamagnetic substances. How does one explain this distinguishing feature? **[DELHI 2014]**

9. Out of the two magnetic materials, '*A*' has relative permeability slightly greater than unity while '*B*' has less than unity. Identify the nature of the materials '*A*' and '*B*'. Will their susceptibilities be positive or negative?

[DELHI 2014]

10. Write two properties of a material suitable for making

(a) a permanent magnet, and

(b) an electromagnet.

[DELHI 2017]

11. (i) Write two characteristics of a material used for making permanent magnets.

(ii) Why is the core of an electromagnet made of ferromagnetic materials?

Or

Draw magnetic field lines when a (i) diamagnetic, (ii) paramagnetic substance is placed in external magnetic field. Which magnetic property distinguishes this behaviour of the field due to the substances?

[DELHI 2018]

12. (a) An iron ring of relative permeability μ_r, has windings of insulated copper wire of n turns per metre. When the current in the windings is I, find the expression for the magnetic field in the ring.

(b) The susceptibility of a magnetic material is0.9853.Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field.

[DELHI 2018]

5 Mark Questions

13. (a) A small compass needle of magnetic moment τ_n is free to turn about an axis perpendicular to the direction of uniform magnetic field '*B*'. The moment of inertia of the needle about the axis is I. The needle is slightly disturbed from its stable position and then released. Prove that it executes simple harmonic motion. Hence deduce the expression for its time period.

(b) A compass needle, free to turn in a vertical plane orients itself with its axis vertical at a certain place on the earth. Find out the values of (i) horizontal component of earth s magnetic field and (ii) angle of dip at the place.

[DELHI 2013]

Solutions

1. It represents a diamagnetic substance since its permeability (0.9983) is less than 1. [1]

2. On the equator, the values of both angle of dip (δ) and the vertical component of earth's magnetic field is zero. So, $B_v = 0$ [1]

3. The magnets which have high retentivity and high coercivity are known as permanent magnets. For example: Steel [1]

4. *Bi* and *Cu* are diamagnetic substances.Since their net magnetic moment is zero. [1]

5. Paramagnetic substances are Aluminium (*Al*) and Calcium (*Ca*).They have unpaired electrons. [1]

6. Horizontal component of earth's magnetic field, $BH = 0.4G$

Angle made by the needle with the horizontal plane = Angle of dip $\delta = 60°$ [1]

Earth's magnetic field strength = B

We can relate B and B_H as:

$B_H = B \cos\theta$

$$\therefore B = \frac{B_H}{\cos\delta} = \frac{0.4}{0.5} = 0.8G$$

Hence, the strength of earth's magnetic field at the given location is $0.8G$. [1]

7. Diamagnetic materials have negative susceptibility. So, the given material is diamagnetic. Properties of magnetic material are as mentioned below: [1]

i. They do not obey Curie's law. [1/2]

ii. They are feebly repelled by a magnet. [1/2]

8.

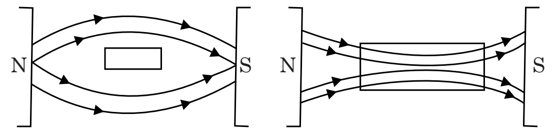


Fig.: (Diamagnetic Material) **Fig.:** (Paramagnetic Material)

[1]

Magnetic Permeability of paramagnetic's is more than air so it allows more lines to pass through it while permeability of diamagnetic is less than air so it does not allow lines to pass through it.

$$r = \frac{n^2}{z}(0.53)\overset{o}{\text{A}}$$

[1]

9. For a paramagnetic material, the relative permeability lies between $1 < \mu_r < 1 + \varepsilon$ and its susceptibility lies between $0 < \chi < \varepsilon$

Hence, '*A*' is a paramagnetic material and its susceptibility is positive. This is because its relative permeability is slightly greater than unity. For a diamagnetic material, the relative permeability lies between $0 \le \mu_r < 1$

and its susceptibility lies between $-1 < \chi < 0$. Hence, '*B*' is a diamagnetic material and its susceptibility is negative. This is because its relative permeability is less than unity. Here μ_r, and χ refer to the relative permeability and susceptibility [1+1]

10. (a) Permanent magnet [1]

1. High coercivity.
2. High retentivity [1]

(b) Electromagnet [1]

1. High permeability
2. Low coercivity.

11. (i) The material used for making permanent magnets should have the following characteristics:

(a) High retentivity: It ensures that the magnet remains strong even after removal of the magnetising field. [1/2]

(b) High coercivity: It ensures that the magnetism of the material does not get easily lost. [1/2]

Apart from these two criteria, the material should have high permeability.

(ii) The core of an electromagnet should have high permeability and low retentivity. The high permeability of the core of an electromagnet ensures that the electromagnet is strong. On the other hand, low retentivity of the core ensures that the magnetism of the core material gets lost as soon as the current is switched off. Ferromagnetic materials have both high permeability and low retentivity. Hence, ferromagnetic materials are the most suitable for making the core of an electromagnet. [1]

Or

(i) The magnetic field lines, when a diamagnetic material is placed in an external magnetic field, can be diagrammatically represented as:

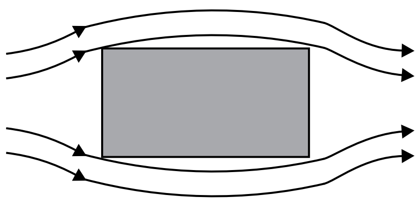


Fig: Magnetic field lines, when a diamagnetic material is placed in an external magnetic field

[1]

(ii) The magnetic field lines, when a paramagnetic material is placed in an external magnetic field, can be diagrammatically represented as:

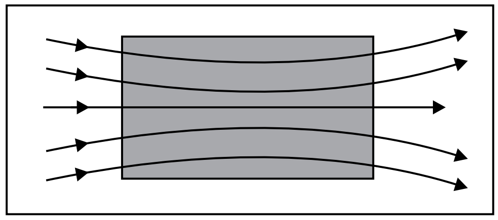
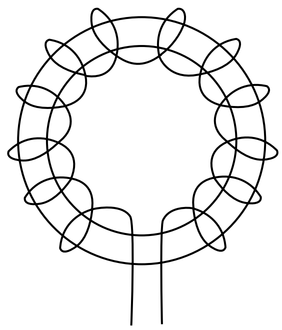


Fig: The magnetic field lines, when a paramagnetic material is placed in an external magnetic field

Diamagnetic and paramagnetic materials are distinguished by the magnetic property called magnetic susceptibility. For diamagnetic materials, magnetic susceptibility is negative, whereas for paramagnetic materials, magnetic susceptibility is slightly positive. [1]

12. (a) There will be two magnetic field one due to current and another due to magnetics.h



$$B = B_o + B_m$$

$$B = \mu_0 nI + \mu_0 M$$

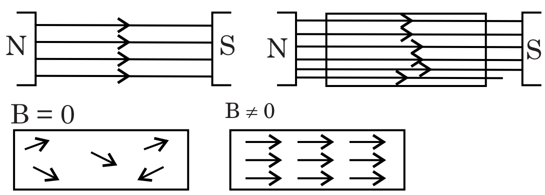
$$B = \mu_0 H + \mu_0 \mu_r H$$

$$B = \mu_0 (1 + \mu_r) H$$

$$B = \mu_0 \mu_r H$$

[1]

(b) The susceptibility of this material is between 0 and 1 so it's a paramagnetic material.



[1]

13. (a) Consider a rectangular loop-**ABCD** carrying current I.

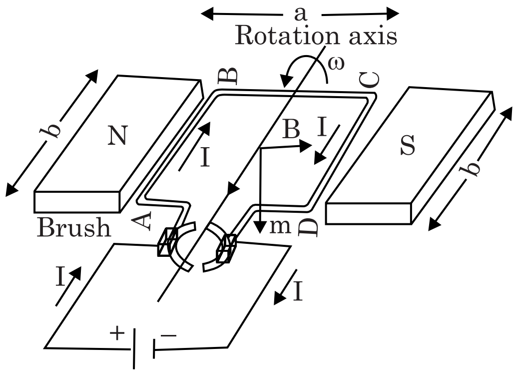


Fig: A small compass

[1]

Case I: The rectangular loop is placed such that the uniform magnetic field B is in the plane of loop. [1]

No force is exerted by the magnetic field on the arms AD and BC. Magnetic field exerts a force F_1 on arm AB.

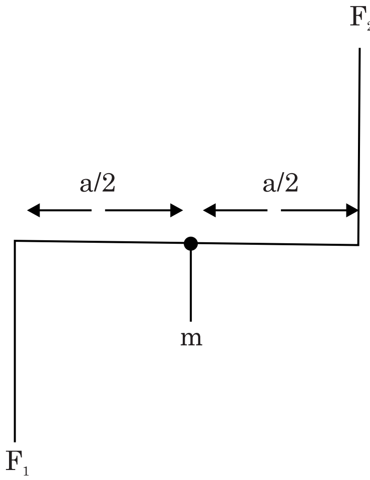
$\therefore F_1 = IbB$

Magnetic field exerts a force F_2 on arm CD

$F_2 = IbB = F_1$

Net force on the loop is zero.

The torque on the loop rotates the loop in anti-clockwise direction.



Torque,

$$\tau = F_1 \frac{a}{2} + F_2 \frac{a}{2}$$

$$= IbB \frac{a}{2} + IbB \frac{a}{2}$$

$= l\,(ab)\,B$

$\tau = BIA$

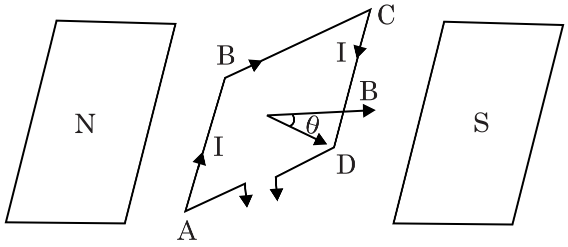
If there is V such turns the torque will be $nIAB$

Where, b $\rightarrow$ Breadth of the rectangular coil

$a \rightarrow$ Length of the rectangular coil

$A = ab \rightarrow$ Area of the coil.

Case II: Plane of the loop is not along the magnetic field, but makes angle with it.



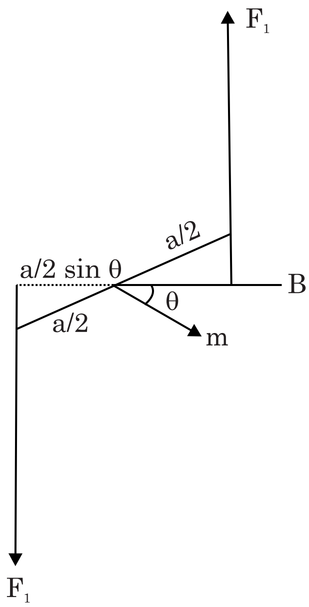
[1]

Angle between the field and the normal is θ. Forces on BC and DA are equal and opposite and they cancel each as they are collinear.

Force on AB is F_1 and force on CD is F_2

$F_1 = F_2 = IbB$

Magnitude of torque on the loop as in the figure:



[1]

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$

$\tau = labB \sin\theta$

$\tau = lAB \sin\theta$

Where, $A = ab$

(b) We know, Lorentz force, $F = Bqv \sin\theta$

Where, θ = angle between velocity of particle and magnetic field = 90°

So, Lorentz force, $F = Bqv$

Thus, the particles will move in circular path.

$$Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} m_p$$

Let m_p = mass of the proton, m_d = mass of deuteron, v_p = velocity of proton and v_d = velocity of deuteron

The charge of proton and deuteron are equal.

Given that, $m_p v_p = m_d v_d$ [1]

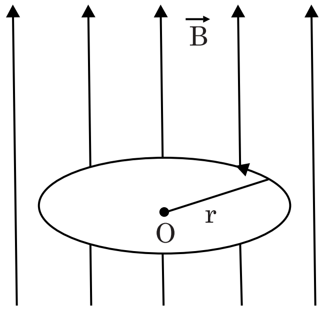
$$r_p = \frac{m_p v_p}{Bq} \quad ...(1)$$

$$r_d = \frac{m_d v_d}{Bq} \quad ...(2)$$

As (1) and (2) are equal, so $r_p = r_d = r$

Thus, the trajectory of both the particles will be same.

Or



(a) The torque on the needle is $t = m \times B$

In magnitude, $t = m \times B \sin\theta$

Here τ is restoring torque and θ is the angle between m and B.

Therefore, in equilibrium,

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

Negative sign with $mB \sin\theta$ implies that restoring torque is in opposition to deflecting torque. For small values of in radians, we approximate $\sin\theta = \theta$ and get

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

Or, $$\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$$

This represents a simple harmonic motion. The square of the angular frequency is $\omega^2 = mB/I$ and the time period is,

$$T = 2\pi\sqrt{\frac{I}{mB}}$$

(b) (i) As, Horizontal component of earth's magnetic field, $B_H = B\cos\delta$

Putting $\delta = 90°$, $B_H = 0$

(ii) For a compass needle align vertical at a certain place, angle of dip, $\delta = 90°$

Smart Notes ....

CHAPTER 6

Electromagnetic Induction

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Electromagnetic induction					
Faraday's laws, induced emf and current			1Q (1 mark)	1Q (1 mark)	
Lenz's Law, Eddy currents. Self and mutual induction			1Q (3 marks)	1Q (1 mark)	

On the basis of above analysis, it can be said that from exam point of view Electromagnetic induction, Self and Mutual Inductance are most important concepts of the chapter.

[Topic 1] Electromagnetic Induction Laws

Summary

- Electromagnetic Induction is the one in which by which electric current is generated with the help of a magnetic field.
- ***The Experiments of Faraday and Henry***

  The observations from the experiments of Faraday and Henry concluded that it is the relative motion between the magnet and the coil that is responsible for generation or induction of the electric current in the coil.
- ***Magnetic Flux***

  It is the amount of field lines cutting through a surface area A defined by unit area vector. The magnetic flux that passes through a plane of area A and has a uniform magnetic field B, is given by, ϕ_B = B.A = BA cosθ where θ is the angle between magnetic field B and Area A. Magnetic flux is a scalar quantity and its SI unit is weber.

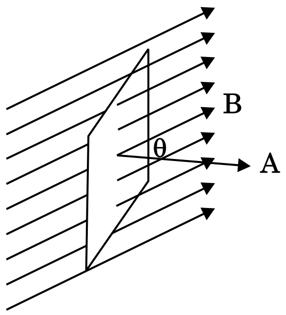


Fig. Field lines in a magnetic field

Faraday's Law of Induction

- Faraday's First Law: Whenever a conductor is placed in a varying magnetic field, there is an induced emf and if the conductor circuit is closed, there is an induced current.
- Faraday's Second Law: This law of electromagnetic induction states that the magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit. Mathematically, the induced emf is given by $\varepsilon = \frac{-d\,\phi_B}{dt}$, the negative sign indicates direction of the induced emf and hence the direction in a closed loop.

Lenz's law and Conservation of Energy

The Lenz's law states that the polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

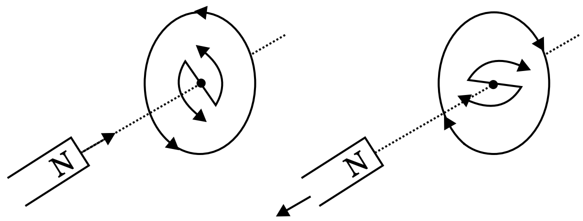


Fig. Lenz's law

Motional Electromotive Force

The relationship between induced emf and a wire moving at a constant speed v is given by ε = Blv

Energy Consideration: A Quantitative Study

- 'r' is the resistance of the movable arm PQ of the rectangular conductor. Assume that remaining arms QR, RS, SP have negligible resistance compared to r. In the presence of magnetic field there will be a force on the arm AB. This force I(l × B) is outwards directed in a direction opposite to the velocity of rod.
- Magnitude of force is $F = IlB = \frac{B^2 l^2 v}{r}$.
- Magnitude to push arm PQ $= Fv = \frac{B^2 l^2 v^2}{r}$

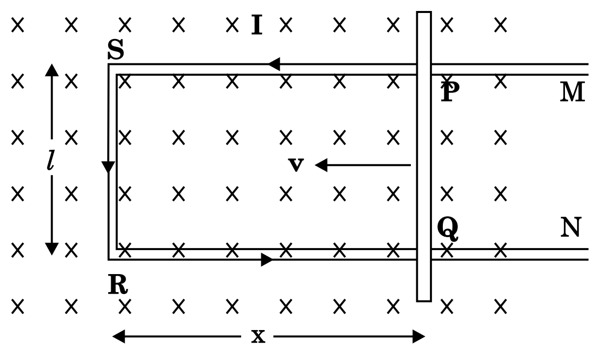


Fig. Energy Consideration in a Magnetic field

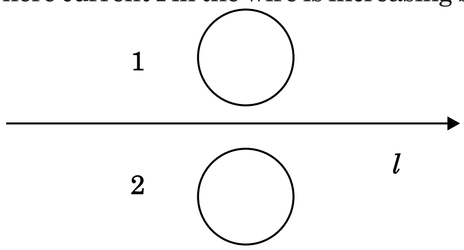
PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. Predict the directions of induced currents in metal rings 1 and 2 lying in the same plane where current I in the wire is increasing steadily.



[ALL INDIA 2012]

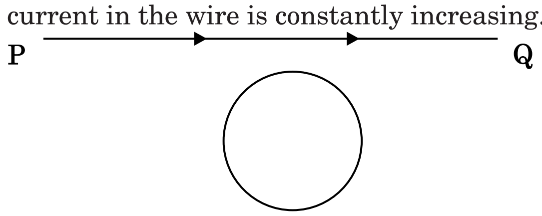
2. The motion of copper plates is damped when it is allowed to oscillate between the two poles of a magnet. If slots are cut in the plate, how will the damping be affected?

[ALL INDIA 2013]

3. Two spherical bobs, one metallic and the other of glass, of the same size are allowed to fall freely from the same height above the ground. Which of the two would reach earlier and why?

[DELHI 2014]

4. A conducting loop is held below a current carrying wire *PQ* as shown. Predict the direction of the induced current in the loop when the current in the wire is constantly increasing.



[ALL INDIA 2014]

5. A planar loop of rectangular shape is moved within the region of a uniform magnetic field acting perpendicular to its plane. What is the direction and magnitude of the current induced in it?

[ALL INDIA 2015]

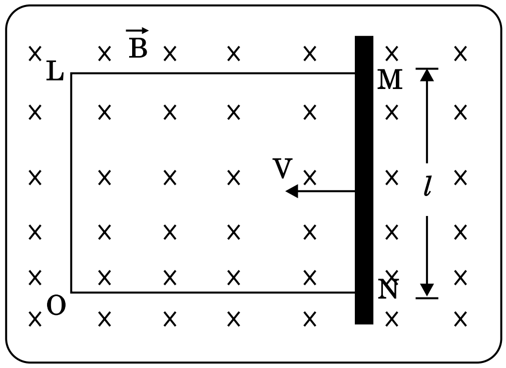
2 Mark Questions

6. A metallic rod of '*L*' length is rotated with angular frequency of ω with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius *L* about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring.

[DELHI 2012]

3 Mark Questions

7. A rectangular conductor *LMNO* is placed in a uniform magnetic field of 0.5 *T*. The field is directed perpendicular to the plane of the conductor. When the arm *MN* of length of 20 cm is moved towards left with a velocity of 10 m/s, calculate the emf induced in the arm. Given the resistance of the arm to be 5Ω (assuming that other arms are of negligible resistance) find the value of the current in the arm.



Or

A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the place is 0.4 *G* and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased?

[ALL INDIA 2011]

8. While travelling back to his residence in the car, Dr. Pathak was caught up in a thunderstorm. It became very dark. He stopped driving the car and waited for thunderstorm to stop. Suddenly he noticed a child walking alone on the road. He asked the boy to come inside the car till the thunderstorm stopped. Dr. Pathak dropped the boy at his residence. The boy insisted that Dr. Pathak should meet his parents. The parents expressed their gratitude to Dr. Pathak for his concern for safety of the child.

Answer the following questions based on the above information:

(a) Why is it safer to sit inside a car during a thunderstorm?

(b) Which two values are displayed by Dr. Pathak in his action?

(c) Which values are reflected in parent's response to Dr. Pathak?

(d) Give an example of similar action on your part in the part from everyday life.

[DELHI 2013]

9. (a) State Faraday's law of electromagnetic induction.

(b) Explain, with the help of a suitable example, how we can show that Lenz's law is a consequence of the principle of conservation of energy.

(c) Use the expression for Lorentz force acting on the charge carriers of a conductor to obtain the expression for the induced emf across the conductor of length l moving with velocity v through a magnetic field B acting perpendicular to its length.

[ALL INDIA 2015]

5 Mark Questions

10. (a) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it.

(b) The current flowing through an inductor of self-inductance L is continuously increasing. Plot a graph showing the variation.

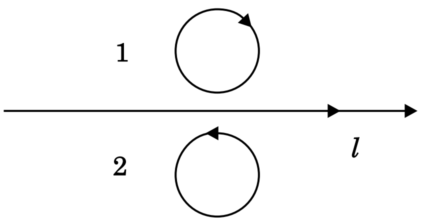
(i) Magnetic flux versus the current

(ii) Induced emf versus $\frac{di}{dt}$

(iii) Magnetic potential energy stored versus the current. **[DELHI 2015]**

Solutions

1. Using Lenz law, we can predict the direction of induced current in both the rings. Induced current oppose the cause of increase of magnetic flux. So, [½]



[½]

It will be clockwise in ring 1 and anticlockwise in ring 2.

2. When slots are made in the plate, path length of induced current increases hence resistance increased so eddy current minimized and that's why it is less damped. [1]

3. Glass bob will reach the ground earlier than the metallic bob. As the metallic bob falls, it intercepts earth's magnetic field and induced currents are set up in it which oppose its downward motion. But no such currents are induced in the glass. [1]

4. When current in wire is increased, inward flux with loop increases. According to Lenz Law, loop induces outward magnetic flux so anti clockwise current is induced in loop. [1]

5. The magnetic flux linked with a circuit is not changing with time so there will be no induced current in the loop. [1]

6. The induced emf $= \frac{d\phi_B}{dt}$

$$e = \frac{d}{dt}(BA)$$

$$= B\frac{dA}{dt}$$

$\therefore \phi_B = BA \cos\phi$

$\because \phi = 0°$ [½]

Where $\frac{dA}{dt}$ = Rate of change of area of loop formed by the sector OPQ. Let θ be the angle between the rod and the radius of the circle at P at time t.

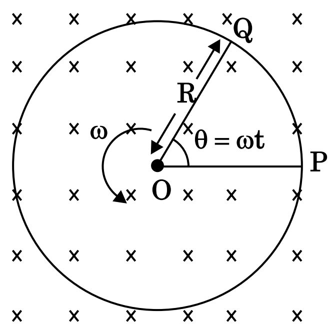
The area of the sector OPQ

$$= \pi R^2 \times \frac{\theta}{2\pi} = \frac{1}{2}R^2\theta$$ [½]

Where R = Radius of the circle

Hence $e = B \times \frac{d}{dt}\left(\frac{1}{2}R^2\theta\right) = \frac{1}{2}BR^2\frac{d\theta}{dt}$

$$e = \frac{B\omega R^2}{2}$$ [½]



[½]

7. $e = vB = 10 \times 0.5 \times 0.2 = 1$ volt

$$i = \frac{\varepsilon}{R} = \frac{1}{5}$$

$i = 0.2A$ [1 + 1 + 1]

Or

$B_H = B_e \cos\theta = 0.4 \times 10^{-4} \cos 60° = 2 \times 10^{-5}T$

$$\varepsilon = \frac{B_H \omega R^2}{2}$$

$$= \frac{2\times10^{-5}\times4\pi\,(0.5)^2}{2}$$

$$\omega = 2\pi\times\frac{120}{60} = 4\pi$$

[1 + 1 + 1]

On increasing the number of spokes, emf will remain same because they form parallel combination.

8. (a) It is safer to be inside a car during thunderstorm because the car acts like a Faraday cage. [1]

(b) Awareness and humanity [½]

(c) Gratitude and obliged [½]

(d) Once I came across a situation where a puppy was struck in the middle of a busy road during rain and was not able to cross due to heavy flow, so I quickly rushed and helped him. [1]

9. (a) Faraday gave laws for relating induced emf to the flux. These are given as under:

(i) Whenever there is a change of magnetic flux through a circuit, there will be an induced emf and this will last as long as the change persists.

(ii) The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

Mathematically, the induced emf is given by

$$\varepsilon = -\frac{d\phi_B}{dt}$$

[1]

(b) Lenz's law states that the polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it. The negative sign shown in equation $\varepsilon = -\frac{d\phi_B}{dt}$ represents this effect.

Conservation of energy:

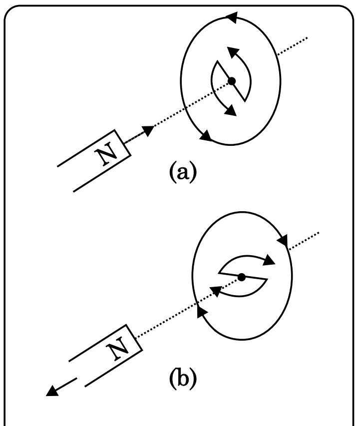
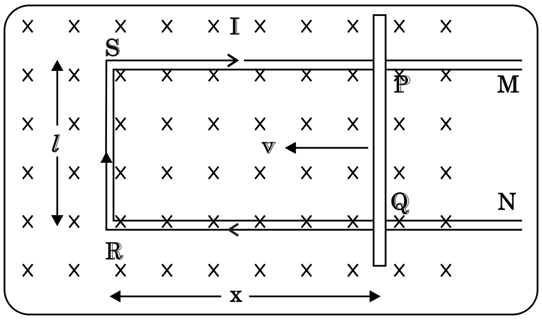


Illustration of Lenz's law.

Fig. Illustration of Lenz's Law

Suppose that the induced current was in the direction opposite to the one depicted in Fig.(a). In that case, the South-pole due to the induced current will face the approaching North-pole of the magnet. The bar magnet will then be attracted towards the coil at an ever increasing acceleration. A gentle push on the magnet will initiate the process and its velocity and kinetic energy will continuously increase without expending any energy. If this can happen, one could construct a perpetual-motion machine by a suitable arrangement. This violates the law of conservation of energy and hence cannot happen. Now consider the correct case shown in Fig.(a). In this situation, the bar magnet experiences a repulsive force due to the induced current. Therefore, a person has to do work in moving the magnet. Energy spent by the person is dissipated by Joule heating produced by the induced current. [1]

(c)



The arm PQ is moved to the left side, thus decreasing the area of the rectangular loop. This movement induces a current I as shown.

Let us consider a straight conductor moving in a uniform and time independent magnetic field. Figure shows a rectangular conductor $PQRS$ in which the conductor PQ is free to move. The rod PQ is moved towards the left with a constant velocity v as shown in the figure. Assume that there is no loss of energy due to friction. $PQRS$ forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field B which is perpendicular to the plane of this system. If the length $RQ = x$ and RS = l, the magnetic flux B enclosed by the loop $PQRS$ will be $\phi B = Blx$ Since x is changing with time, the rate of change of flux φ_B will induce an emf given by:

$$\varepsilon = -Bl\frac{dx}{dt} = Blv$$

where we have used $\frac{dx}{dt} = -v$ which is the speed of the conductor PQ. The induced emf

Blv is called motional emf. Thus, we are able to produce induced emf by moving a conductor instead of varying the magnetic field, that is, by changing the magnetic flux enclosed by the circuit. It is also possible to explain the motional emf expression by invoking the Lorentz force acting on the free charge carriers of conductor *PQ*. Consider any arbitrary charge *q* in the conductor *PQ*. When the rod moves with speed v, the charge will also be moving with speed v in the magnetic field *B*. The Lorentz force on this charge is *qvB* in magnitude, and its direction is towards *Q*. All charges experience the same force, in magnitude and direction, irrespective of their position in the rod *PQ*. The work done in moving the charge from *P* to *Q* is,

$W = qvBl$ [1]

Since emf is the work done per unit charge,

$\varepsilon = \dfrac{W}{q}$

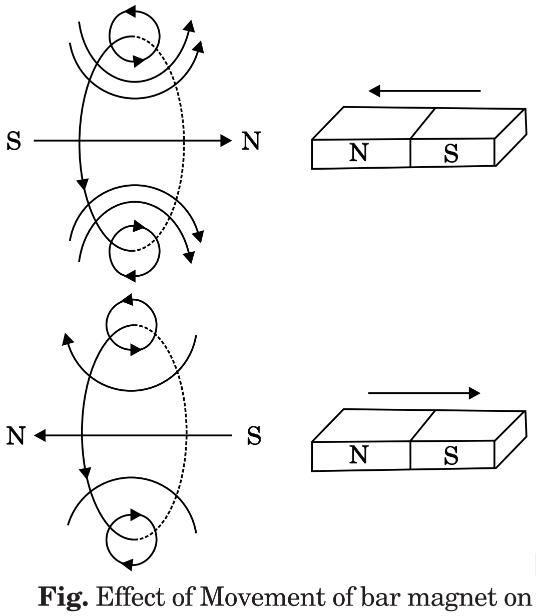
$= Blv$

This equation gives emf induced across the rod *PQ*

The total force on the charge at *P* is given by

$F = q(E + v \times B)$

10. (a) Lenz law: According to Lenz s law, the polarity of the induced emf is such that it opposes a change in magnetic flux responsible for its production



[1]

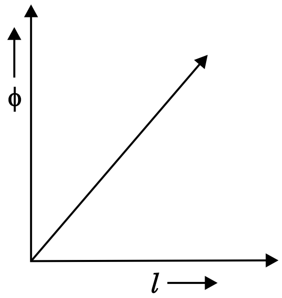
Fig. Effect of Movement of bar magnet on Magnetic flux.

When the north pole of a bar magnet is pushed towards the coil, the amount of magnetic flux linked with the coil, increase. Current is reduced in the coil from a direction such that it opposes the increase in magnetic flux. This is possible only when the current induced in the coil is in anti-clockwise direction, with respect to server. The magnetic moment M associated with this induced emf has north polarity, towards the north pole of the approaching bar magnet. Similarly, when the north pole of the bar magnet is moved away from the coil, the magnetic flux linked with the coil decreases. To counter this decrease in magnetic flux, current is induced in the coil in clockwise direction so that its south pole faces the receding north pole of the bat magnet. This would result in an attractive force which opposes the motion of the magnet and the corresponding decrease in magnetic flux. [1]

(b) (i) Since $\phi = LI$

Where, $I \rightarrow$ strength of current through the coil at any time

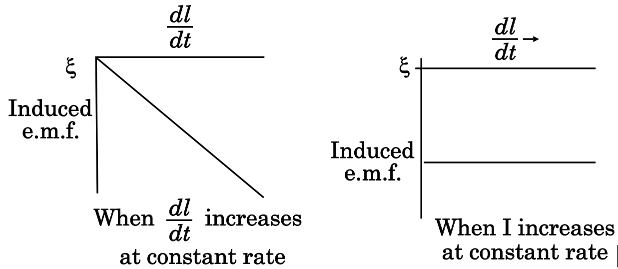
$\phi \rightarrow$ Amount of magnetic flux linked with all turns of the coil at that time and, $L \rightarrow$ Constant of proportionally called coefficient of self-induction.[1]



(ii) Induced emf, $e = \dfrac{-d\phi}{dt} = -\dfrac{d(LI)}{dt}$

i.e. $e = -L\left(\dfrac{dl}{dt}\right)$

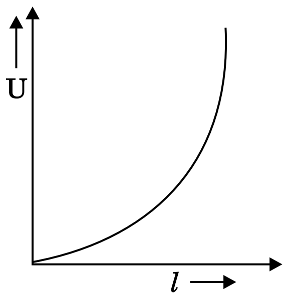
Or



[1]

(iii) Since magnetic potential energy is given by

$U = \dfrac{1}{2}LI^2$



[1]

[Topic 2] Eddy currents, self and mutual inductance

Summary

Eddy Currents

- When bulk pieces of conductors are subjected to changing magnetic flux then induced currents are produced in them which are called as eddy currents.
- Eddy currents create a significant drag known as magnetic damping.
- The applications of eddy currents are in magnetic braking in trains, electromagnetic damping, electric power meters and induction furnace.

Inductance

- Flux change produced by another coil in the close proximity of a coil or flux exchange produced by the same coil induces electric current.
- The inductance in series is given by $L_s = L_1 + L_2 + L_3 + ...$
- The inductance in parallel is given by $\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ...$

Mutual- Inductance

- When the emf is induced into the adjacent coil situated within the same magnetic field, the emf is said to be induced magnetically or by mutual induction.
- Mutual inductance of a pair of coils, solenoids etc. depends on their relative orientation as well as their separation.

$$\varepsilon_1 = -M\frac{dI_2}{dt}$$

- Mutual Inductance of two coils is given by $M = \frac{\mu_0 \mu_r N_p N_s A_s}{l_p}$ where A_p, A_s are the cross sectional areas of primary and secondary coil in m^2, I is the coil current and N_s, N_p are the number of turns of secondary and primary coils respectively.

Self – Inductance

- The production of induced emf in a circuit when the current changes in the same circuit is called self-induction.
- The induced emf is given by $\varepsilon = -L\frac{dI}{dt}$, where is the coefficient of self-induction.
- The direction of induced emf is given by Lenz's Law.

AC Generator

- The electromagnetic induction has its applications in an AC generator, where mechanical energy is converted to electrical energy.

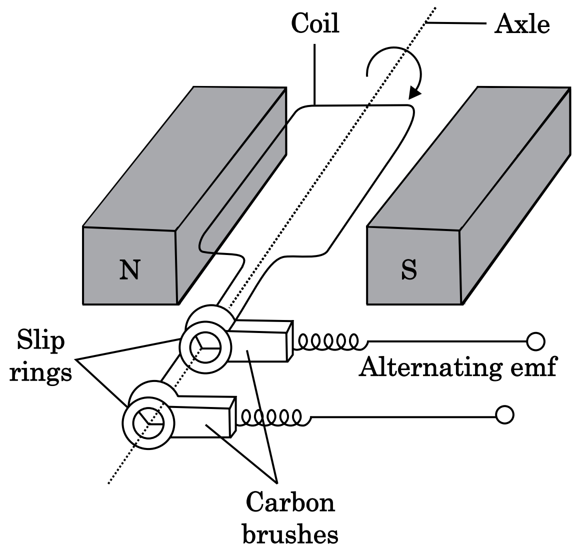


Fig. A.C. Generator

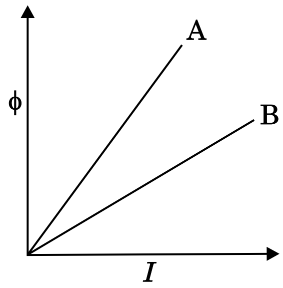
- The motional emf is of a coil with N turns and area A, rotated at v revolutions per second in a uniform magnetic field B is given as, $\varepsilon = -NBA\frac{d}{dt}(\cos\omega t)$

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. A plot of magnetic flux (ϕ) versus current (I) is shown in the figure for two inductors A and B. Which of the two has larger value of self inductance?



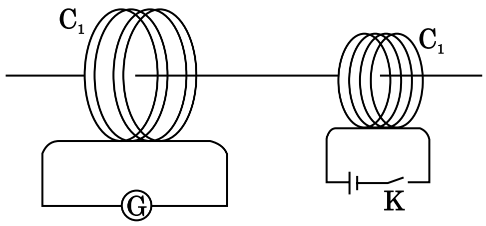
[DELHI 2018]

2 Mark Questions

2. A current is in induced in coil C_1 due to the motion of current carrying coil C_2.

(a) Write any two ways by which a large deflection can be obtained in the galvanometer G,

(b) Suggest an alternative device to demonstrate the induced current in place of a galvanometer.



[ALL INDIA 2011]

3. Define self-inductance of a coil. Show that magnetic energy required to build up the current I in a coil of self-inductance L is given by $\frac{1}{2}LI^2$. **[ALL INDIA 2012]**

4. How does the mutual inductance of a pair of coils change when

(i) distance between the coils is decreased and

(ii) number of turns in the coils is decreased?

[ALL INDIA 2013]

5. (i) Define mutual inductance.

(ii) A pair of adjacent coils has a mutual inductance of 1.3 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

[ALL INDIA 2016]

6. (a) Define self inductance. Write its S.I. units.

(b) Derive an expression for self inductance of a long solenoid of length I, cross-sectional area A having N number of turns.

[DELHI 2017]

3 Mark Questions

7. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other.

Or

Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf.

[ALL INDIA 2017]

8. State the principle of an ac generator and explain its working with the help of a labeled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area, rotating with a constant angular speed in a magnetic field B, directed perpendicular to the axis of rotation.

[ALL INDIA 2018]

5 Mark Questions

9. (a) Draw a schematic sketch of an ac generator describing its basic elements. State briefly its working principle. Show a plot of variation of

(i) Magnetic flux and

(ii) Alternating emf versus time generated by a loop of wire rotating in a magnetic field.

(b) Why is choke coil needed in the use of fluorescent tubes with ac mains?

[DELHI 2014]

Solutions

1. Inductor A Self inductance is given by $L = N\Phi/I$, where I is current . From the plot it is clear that A has a smaller value of I for given ϕ hence larger value of self inductance. [1]

2. (a) Two ways by which a large deflection can be obtained in the galvanometer are :

(i) By increasing the relative motion between the coils. [1]

(ii) By inserting an iron rod into the coils along their axes. [½]

(b) An LED. (Light Emitting Diode) [½]

3. Self-inductance is the inherent inductance of a circuit, given by the ratio of the electromotive force produced in the circuit by self-induction to the rate of change of current producing it. It is also called coefficient of self-induction.

Suppose I = Current flowing in the coil at any time

ϕ = Amount of magnetic flux linked

It is found that $\phi \propto I$

$\phi = LI$

Where, L is the constant of proportionality and is called coefficient of self induction.

SI unit of self-inductance is Henry.

Let at $t = 0$ the current in the inductor is zero. So at any instant t, the current in the inductor is I and the rate of growth of I is $\frac{dI}{dt}$.

Then, the induced emf is $e = L \times \frac{dI}{dt}$ [1]

If the source is sending a constant current I through the inductor for a small time dt, then small amount of work done by the source is given by

$$dW = eIdt = \left(\frac{LdI}{dt}\right)Idt = LIdI$$

The total amount of work done by the source of e.m.f., till the current increases from its initial value $I = 0$ to its final value I is given by

$$W = \int_0^I LIdI = L\int_0^I IdI = L\left[\frac{I^2}{2}\right]$$

$$= \frac{1}{2}LI^2$$

[1]

This work done by the source of emf is used in building up current from zero to I_o is stored in the inductor in energy form. Therefore, energy stored in the inductor is

$$U = \frac{1}{2}LI^2$$

4. (i) Mutual inductance increased on decreasing distance. [1]

(ii) Mutual inductance decreased on decreasing the number of turns. [1]

5. (i) Mutual induction is the phenomenon of production of induced emf in one coil due to change of current in the neighboring coil. The coil in which the current changes is called primary coil and the coil in which emf is induced is called the secondary coil. [1]

(ii) $M = 1.5\ H$

$I_i = OA$

$I_b = 20A$

$dI = 20A,\ \Delta t = 20s$

$$e = \frac{-MdI}{dt}$$

[1]

$$e = -1.5 \times \frac{20}{0.5}$$

$e = -60V$

So the flux linked with the other coil is given by

$\Delta\phi = e\Delta t = -60 \times 0.5$

$= -30Wb$ [1]

6. The phenomenon in which emf is induced in a single isolated coil due to change of flux through the coil by means of varying the current through the same coil is called self inductance. S.I. unit of inductance is Henry. [1]

(b) Magnetic field B inside a solenoid carrying a current i is $\mu_o ni$

$B = \mu_o ni$

Let n be the number of turns per unit length.

Where, $N\varphi_B = nlBA$

N is total number of turns [½]

l is the length of the solenoid

Inductance, $L = \frac{N\varphi_B}{i}$

Substituting, we obtain

$$L = \frac{nlBA}{i}$$

[1]

Substituting the value of B. we obtain

$$L = \frac{nl\mu_o niA}{i}$$

$$L = n^2 l\mu_o A$$

Inductance L of a solenoid is: [½]

$L = n^2\, l\mu_o A$

7. **Mutual inductance:** The phenomenon according to which an opposing emf is produced as result of change in current or magnetic flux linked with a neighboring coil. Mutual inductance of two long wareial solenoids:

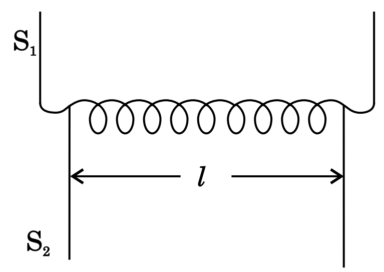


Fig. Mutual Inductance in a coil

Let n_1 be the no. of turns per unit length of S_1, n_2 be the number of turns per unit length of S_2. I_1 be current passed through S_1, ϕ_{21} be the flux linked with S_2 due to charge flowing in S_1. [1]

$\phi_{21} \propto S_1$

$\phi_{21} = M_{21}\ I_1$where M_{21} coefficient of mutual induction of two solenoid. When current is passed through , an emf is induced in solenoid S_2. Magnetic field produced inside S_1 on passing current I_1, $B_1 = \mu_o\ n_1 I_1$

Magnetic flux linked with each turn of the solenoid S_2 will be equal to B_1 times the area of cross section of solenoid S_1. So, magnetic flux linked with each turn of the solenoid $S_2 = B_1 A$ Therefore, total magnetic flux linked with solenoid S_2 will be

$$\phi_{21} = B_1 A \times n_2 l = \mu_0 n_1 l_1 \times A \times n_2 l$$

$$\phi_{21} = \mu_0 n_1 n_2 A I_1 l$$

$$M_{21} = \mu_0 n_1 n_2 Al \text{............(i)}$$

[1]

Similarly, the mutual inductance between the two solenoids, when current is passed through S_2 and induced emf is produced in solenoid S_1 and is given by

$M_{12} = \mu_o\ n_1 n_2 A I_1 l$

$M_{12} = M_{12} = M =$ (say)

Hence coefficient of mutual induction between the two long solenoid $M_{12} = \mu_o\ n_1 n_2 Al$

We can write equation (i) as

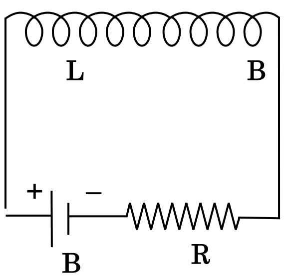
$$M = \mu_0 \left(\frac{N_1}{l}\right)\left(\frac{N_2}{l}\right)\pi r_1^2 \times l$$

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

[1]

OR

Self-inductance: Self-inductance is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.



The induced emf is also called back emf when the current in a coil is switched on. The self-induction opposes the growth of the current and when the current is switched off, the self-induction opposes the decay of the current so, self-induction is also called the inertia of electricity. [1]

(ii) Self-inductance of long solenoid: A long solenoid is one whose length is very large as compared to its area of cross section.

Magnetic field (B) at any point inside B =

$$B = \frac{\mu_0 NI}{l} \text{.............(i)}$$

Magnetic flux through each two of the solenoid

$$\phi = B \times \text{Area of each turn } \phi = \left(\mu_0 \frac{N}{l} I\right) A$$

Where A = area of each turn of the solenoid

Total magnetic flux linked with the solenoid = flux through each turn total no. of turns[1]

$N_\phi = \mu_o IA \times N$ ....(ii)

If L is coefficient of self-inductance of the solenoid then

$N_\phi = LI$ ....(iii)

from (ii) & (iii) we get

$$LI = \mu_0 \frac{N}{l} I \times N$$

$$\text{or } L = \frac{\mu_0 N^2 A}{l} \text{..........(iv)}$$

The magnitude of emf is given by

Let $e = L\dfrac{dE}{dt}$

Multiplying (I) to the both sides we get

$eIdt = LIdt$ ....(v)

But $I = \dfrac{dq}{dt}$ or $Idt = dq$

Also, work done

(dw) = voltage (e) × charge (dq)

Or $dq = e \times dq = eldt$ .... (vi)

So, from (v) & (vi)

$dw = LIdI$ .... (vii)

Total work done in increasing current from zero to I_0, we have By integrating both sides of equation (vii) we get

$$\int_0^w dw = \int_0^{I_0} LIdI$$

$w = \frac{1}{2}LI_0^2$

This work done through inductor is stored as the potential energy (u) in the magnetic field of inductor

$u = \frac{1}{2}LI_0^2$

[1]

8. Principle-Electromagnetic Induction

AC Generator: The phenomenon of electromagnetic induction has been technologically exploited in many ways. An exceptionally important application is the generation of alternating currents (*ac*). The modern ac generator with a typical output capacity of 100 *MW* is a highly evolved machine. In this section, we shall describe the basic principles behind this machine. The Yugoslav inventor Nicola Tesla is credited with the development of the machine. As was pointed out in, one method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area. As the coil rotates in a magnetic field B, the effective area of the loop (the face perpendicular to the field) is $A\cos\theta$,where θ is the angle between *A* and *B*. This method of producing a flux change is the principle of operation of a simple ac generator. An ac generator converts mechanical energy into electrical energy.

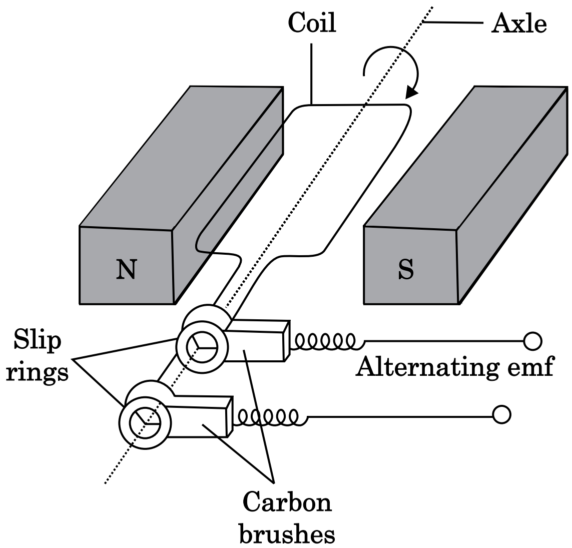


Fig. An A. C. Generator

The basic elements of an ac generator are shown in figure. It consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings and brushes.

When the coil is rotated with a constant angular speed ω the θ angle between the magnetic field vector *B* and the area vector *A* of the coil at any instant is ω = θt (assuming θ = 0° at *t* = 0).

As a result, the effective area of the coil exposed to the magnetic field lines changes with time, and from equation the flux at any time t is

$\phi_B = BA\cos\theta = BA\cos\omega t$

From Faraday's law, the induced emf for the rotating coil of *N* turns is then,

$$e = -N\frac{d\phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t)$$

Thus, the instantaneous value of the emf is

$\therefore\ e = NBA\omega\sin\omega t$ -------- (1)

Where $NBA\omega$ is the maximum value of the emf, which occurs when $\sin\omega t = \pm 1$. If we denote $NBA\omega$ as

$\phi = \phi_o \sin\omega t$ ------------ (2)

Since the value of the sine function varies between +1 and – 1, the sign, or polarity of the emf changes with time. Note from figure that the emf has has its extreme value when θ = 90° or θ = 270°, as the change of flux is greatest at these points.

The direction of the current changes periodically and therefore the current is called alternating current (a.c.)

Since $\phi = 2\pi v$, equation (2) can be written as

$\phi = \phi_o \sin 2\pi v t$ ------------------- (3)

Where ϕ is the frequency of revolution of the generator's coil.

Note that equation 2 and 3 give the instantaneous value of the emf and θ varies between $+\theta_o$ varies between $-\theta_o$ periodically. We shall learn how to determine the time-averaged value for the alternating voltage and current. [1 + 1]

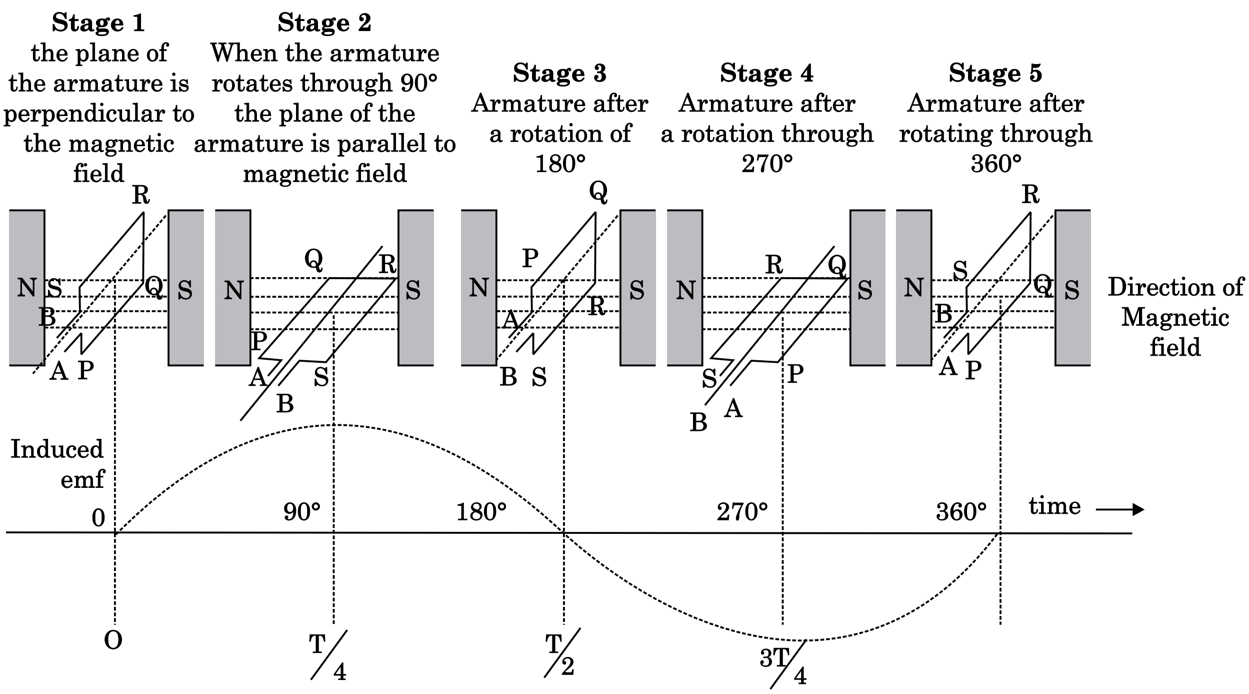


Fig. Operating/Working of an a.c. generator {1}

9. (a) Principle is "Based on the phenomenon of electromagnetic induction.

Construction:

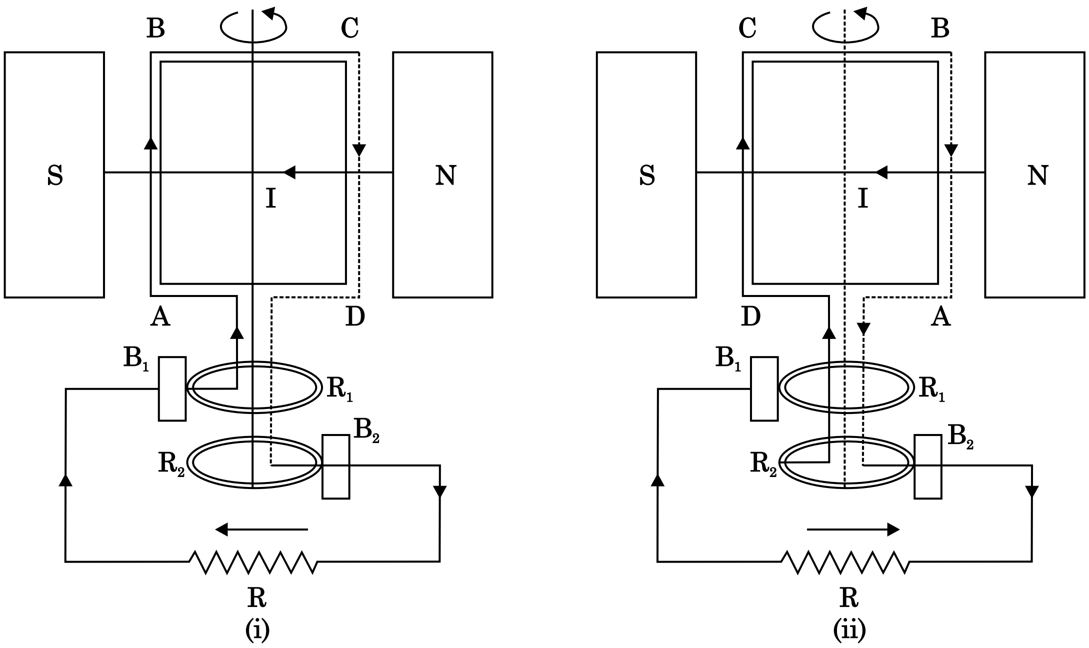


Fig. An A.C. Generator

[1]

Main parts of an ac generator:

- Armature: The rectangular coil *ABCD*
- Field Magnets Two pole pieces of a strong electromagnet
- Slip Rings. The ends of the coil *ABCD* are connected to two hollow metallic rings R_1 and R_2.
- Brushes: B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in tight contact with R_1 and R_2, respectively. [1]

Working: As the armature coil is rotated in the magnetic field, angle θ between the field and the normal to the coil changes continuously. Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. According to Flemings right hand rule, current is induced from A to B in AB and from C to D in CD. In the external circuit, current flows from B_2 to B_1. To calculate the magnitude of emf induced:

Suppose,

$A \rightarrow$ Area of each turn of the coil

$N \rightarrow$ Number of turns in the coil

$\vec{B} \rightarrow$ Strength of the magnetic field

$\theta \rightarrow$ Angle which normal to the coil makes with at any instant t [1]

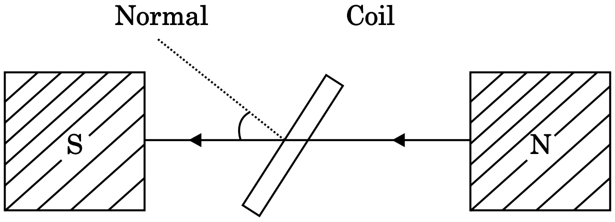


Fig. A.C. Generator

Hence, magnetic flux linked with the coil in this position is given by,

$\phi = N(\vec{B} \cdot \vec{A})$

$NBA \cos\theta = NBA \cos \omega t$ (1)

Where, "ω" is angular velocity of the coil.

Graph between magnetic flux and time, according to equation (1) is given below:

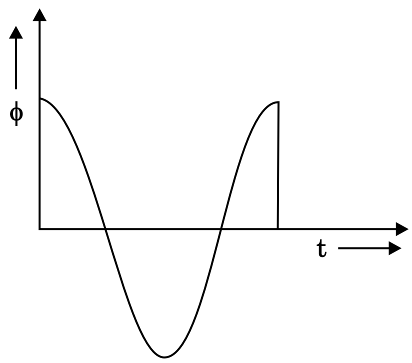


Fig. Graph Between Magnetic flux & time

As the coil rotates, angle θ changes. Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. At this instant, if e is the emf induced in the coil, then [1]

$$e = -\frac{d\theta}{dt} = -\frac{d}{dt}(NAB\cos\omega t)$$

$$= -NAB\frac{d}{dt}(\cos\omega t)$$

$= NBA(-\sin\omega t)\ \omega$

$\mathrm{e} = NBA\ \omega\sin\ \omega t$

The graph between alternating emf versus time is shown below

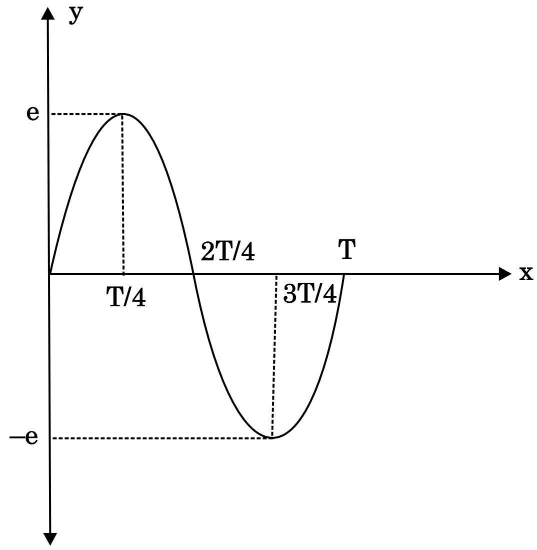


Fig. Graph Between Alternating emf versus time

(b) A choke coil is an electrical appliance used for controlling current in an a.c. circuit. Therefore, if we use a resistance R for the same purpose, a lot of energy would be wasted in the form of heat etc. [1]

Smart Notes ....

CHAPTER 7

Alternating Current

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Alternating currents, peak and RMS value of alternating current/ voltage					
Reactance and Impedance				1Q (5 marks)	
LC oscillations (qualitative treatment only); LCR series circuit,Resonance	1Q (1 mark), 1Q (5 marks)	1Q (5 marks)	1Q (3 marks)		1Q (5 marks)
Power in AC circuits, power factor wattless current. AC generator and transformer	1Q (5 marks)		1Q (5 marks), 1Q (5 marks)		1Q (3 marks)

On the basis of above analysis, it can be said that from exam point of view AC Source, AC Circuit, AC Generator, L-C-R Circuit and Phasor Diagram are most important concepts of the chapter.

[Topic 1] Introduction to Alternating Current

Summary

Alternating Current

The electric main supply that varies like a sine function with time is called alternating voltage and the current drawn by it in the circuit is called Alternating current. Alternating current is the current which varies on two factors i.e. magnitude and the direction periodically and alternatively.

Mathematically alternating current can be expressed as:

$$I = I_0 \sin\omega t$$

Where I_0, is the peak value of alternating current.

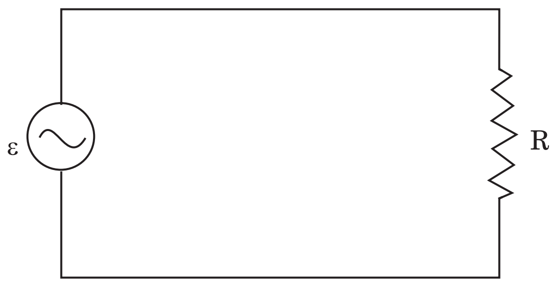


Fig.: Alternating Current in an electrical circuit.

RMS value of Alternating Current

The value of alternating current over a complete cycle which would generate same amount of heat in a given resistors that is generated by steady current in the same resistor and in the same time during a complete cycle.

$$I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707 I_o$$

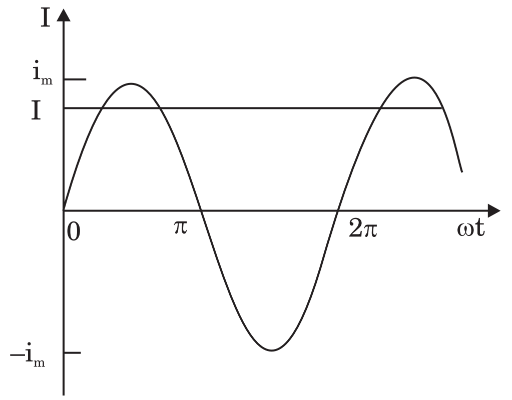


Fig.: Variation of Current with respect to wt.

Mean value of Alternating Current

The value of alternating current that would give same amount of charge in to a circuit at half cycle that is sent for steady current in the same duration.

$$I_{avg} = \frac{2I_o}{\pi} = 0.637 I_o$$

Alternating Voltage

Alternating voltage is the voltage which varies on two factors i.e. magnitude and the directions periodically and alternatively.

Alternating Voltage is expressed mathematically as,

$$V = V_o \sin \omega t$$

$$V_{rms} = \frac{V_o}{\sqrt{2}} = 0.707 V_o \text{ or } V_{rms} = 70.7\% \text{ of } V_o$$

$$V_{avg} = \frac{2V_o}{\pi} = 0.637 V_o \text{ or } V_{avg} = 63.7\% \text{ of } V_o$$

The alternating current and alternating voltage is illustrated in the following diagram:

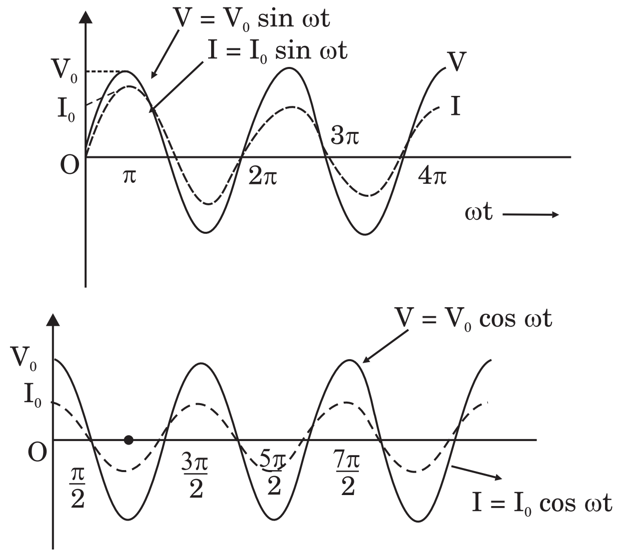


Fig.: Variation of V_0, I_0 w.r.t wt.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. Define the term 'wattless current'.

[ALL INDIA 2011]

2. Why is the use of a.c. voltage preferred over d.c. voltage? Give two reasons.

[ALL INDIA 2014]

3. Define capacitive reactance. Write its S.I. units.

[DELHI 2015]

4. Define 'quality factor 'of resonance in series LCR circuit. What is its SI unit?

[DELHI 2016]

2 Mark Questions

5. When an ideal capacitor is charged by a *dc* battery, no current flows. However, when an ac source is used the current flows continuously. How does one explain this based on the concept of displacement current?

[DELHI 2012]

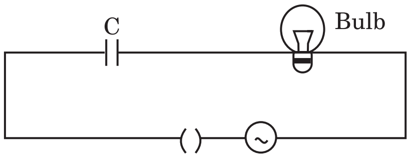
6. (a) For a given a.c, $i = i_m sin\ \omega t$ show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2}i_m^2 R$.

(b) A light bulb is rated at 125 W for a 250 V a.c. supply. Calculate the resistance of the bulb.

[ALL INDIA 2013]

7. (i) When an AC source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero.

(ii) A bulb is connected in series with a variable capacitor and an $A.C.$ source as shown. What happens to the brightness of the bulb when the key is plugged in and capacitance of the capacitor is gradually reduced .



[ALL INDIA 2016]

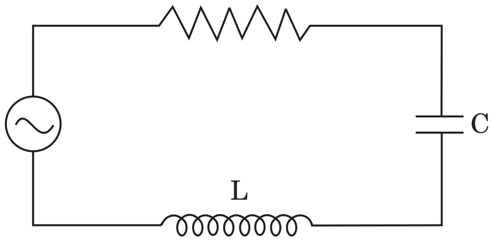
3 Mark Questions

8. The figure shows a series LCR circuit with L = 75 H, C = 80 μF, R = 40Ω connected to a variable frequency 240 V source, calculate

(i) the angular frequency of the source which drives the circuit at resonance,

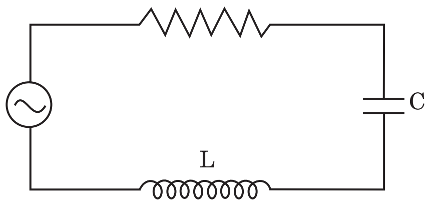
(ii) the current at the resonating frequency,

(iii) the rms potential drop across the inductor at resonance.



[ALL INDIA 2012]

9. The figure shows a series LCR circuit with L = 10.0H, C = 40 μF, R = 60Ω connected to a variable frequency 240 V source. Calculate:



(i) the angular frequency of the source which drives the circuit at resonance,

(ii) the current at the resonating frequency,

(iii) the rms potential drop across the inductor at resonance. **[DELHI 2012]**

10. In a series LCR circuit connected to an ac source of variable frequency and voltage $v = v_m \sin \omega t$, draw a plot showing the variation of current (I) with angular frequency (ω) for two different values of resistance R_1 and R_2 ($R_1 > R_2$). Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance.

[DELHI 2015]

11. An inductor L of inductance X_L is connected in series with a bulb B and an ac source. How would brightness of the bulb change when (i) number of turns in the inductor is reduced, (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your answer in each case.

[DELHI 2015]

4 Mark Questions

12. A group of students while coming from the school noticed a box marked "Danger H.T. 2200 V" at a substation in the main street. They did not understand the utility of such a high voltage, while they argued the supply was only 220 V. They asked their teacher this question the next day. The, teacher thought it to be an important question and therefore, explained to the whole class.

Answer the following questions:

(i) What device is used to bring the high voltage down to low voltage of a.c. current and what is the principle of its working?

(ii) Is it possible to use this device for bringing down the high dc voltage to the low voltage? Explain.

(iii) Write the values displayed by the students and the teacher.

[DELHI 2015]

13. (a) Using phasor diagram, derive the expression for the current flowing in an ideal inductor connected to an a.c. source of voltage, $v = v_0 \sin \omega t$. Hence plot graphs showing variation of (i) applied voltage and (ii) the current as a function of ωt.

(b) Derive an expression for the average power dissipated in a series LCR circuit.

[ALL INDIA 2015]

14. The teachers of Geeta's school took the students on a study trip to a power generating station, located nearly 200km away from the city. The teacher explained that electrical energy is transmitted over such a long distance to the city, in the form of alternating current (ac) raised to a high voltage. At the receiving end in the city, the voltage is reduced to operate the devices. As a result, the power loss is reduced. Geeta listened to the teacher and asked questions about how the ac is converted to a higher or lower voltage.

(a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.

(b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current.

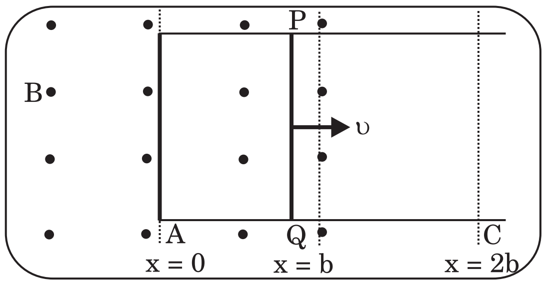
(c) Write two values each shown by the teachers and Geeta.

[ALL INDIA 2018]

5 Mark Questions

15. (a) When a bar magnet is pushed towards (or away) from the coil connected to a galvanometer, the pointer in the galvanometer deflects. Identify the phenomenon causing this deflection and write the factors on which the amount and direction of the deflection depends. State the laws describing this phenomenon.

(b) Sketch the change in flux, emf and force when a conducting rod PQ of resistance R and length l moves freely to and fro between A and C with speed υ on a rectangular conductor placed in uniform magnetic field as shown in the figure.



Or

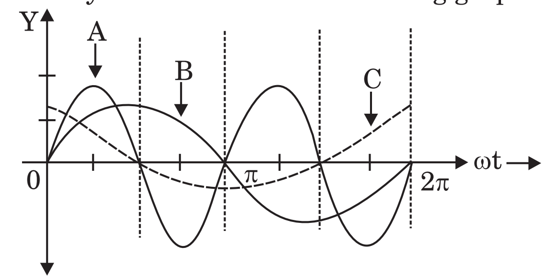
In a series LCR circuit connected to an ac source of voltage transformation ratio $v = v_m \sin \omega t$, use phasor diagram to derive an expression for the current in the circuit. hence obtaine the expression for the power dissipated in the circuit. Show that power dissipated at resonance is maximum.

[ALL INDIA 2016]

16. (i) An a. c. source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called? (ii) In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_1. Calculate $\frac{P_1}{P_2}$.

[DELHI 2016]

17. A device '*X*' is connected to an ac source $v = v_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph:



(a) Identify the device '*X*'.

(b) Which of the curves *A*, *B* and *C* represent the voltage, current and the power consumed in the circuit? Justify your answer.

(c) How does its impedance vary with frequency of the ac source? Show graphically.

(d) Obtain an expression for the current in the circuit and its phase relation with ac voltage.

Or

(a) Draw a labelled diagram of an ac generator. Obtain the expression for the emf induced in the rotating coil of *N* turns each of cross-sectional area *A*, in the presence of a magnetic field *B*

(b) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angles to the horizontal component of the Earth's magnetic field, 0.3 × 10 Wbm^{-2}. Find the instantaneous value of the emf induced in the rod.

[ALL INDIA 2017]

18. (a) Derive an expression for the average power consumed in a series LCR circuit connected to a.c. source in which the phase difference between the voltage and the current in the circuit is 0.

(b) Define the quality factor in an a.c. circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends.

Or

(a) Derive the relationship between the peak and the rms value of current in an a.c. circuit.

(b) Describe briefly, with the help of labelled diagram, working of a step-up transformer. A step-up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy? Explain.

[DELHI 2017]

19. A device *X* is connected across an ac source of voltage $V = V_0 \sin \omega t$. The current through *X* is given as $I = I_0 \cos\omega t$

(a) Identify the device *X* and write the expression for its reactance.

(b) Draw graphs showing variation of voltage and current with time over one cycle ac for *X*.

(c) How does the reactance of the device *X* vary with frequency of the ac? Show this variation graphically.

(d) Draw the phasor diagram for the device *X*.

[ALL INDIA 2018]

Solutions

1. An *AC* circuit containing only capacitor or inductor will have zero power dissipation even though the current is flowing through it. Such current is called wattless current. [1]

2. (i) AC generator are simpler & cheaper than DC generator as commutator is not used in AC generator. [½]

(ii) AC can be stepped up or down using transformer so its transmission is cheaper and efficient. [½]

3. Capacitor reactance is the resistance offered by a capacitor to the flow of a.c.

It is given by, $X_C = \dfrac{1}{\omega C}$ [½]

Where, $\omega = 2\pi f$

$f \rightarrow$ Frequency of the source

$$X_C = \frac{1}{2\pi f C}$$ [½]

$\omega \rightarrow$ Angular frequency of the source

$C \rightarrow$ Capacitance of the capacitor

The SI unit of capacitor reactance is ohm (Ω).

4. The *Q* factor of series resonance circuit is defined as the ratio of the voltage developed across the inductor or capacitor at resonance to the impressed voltage, which is the voltage across *R*. [½]

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is dimensionless hence, it has no units. [½]

5. When an ideal capacitor is charged by dc battery, charge flows till the capacitor gets fully charged. When an ac source is connected then conduction current $i_c = \frac{dq}{dt}$ flows in the connecting wire.

Due to charging current, charge deposited on the plates of the capacitor changes with time. Changing charge causes electric field between the plates of capacitor to be varying, giving rise to displacement current $i_d = \varepsilon_o \frac{d\phi_c}{dt}$. [As displacement current is proportional to the rate of flux variation] [1]

Between the plates, electric field

$$E = \frac{\sigma}{\varepsilon_o} = \frac{q}{A\varepsilon_o}$$

Electric flux, $\phi_c = E_A = \frac{q}{A\varepsilon_o} A$

So, $i_d = \frac{\varepsilon_o d\phi_c}{dt} = \frac{d}{dt}\left(\frac{qA}{A\varepsilon_o}\right) = \frac{dq}{dt} = i_c$

Displacement current brings continuity in the flow of current between the plates of the capacitor. [1]

6. $P = i^2 R = \left(i_m^2 \sin^2 \omega t\right) R$

(a) $P = \left(i_m^2 R\right)\sin^2 \omega t = \frac{i_m^2 R}{2}$ [1]

(b) $R = \frac{V^2}{P} = \frac{(250)^2}{125}$ [1]

R = 500 Ω

7. (i) Power dissipation in AC circuit is $P = V_{rms} I_{rms} cos\varphi$ where $cos\varphi = \frac{R}{Z}$

for an ideal capacitor $R = 0$

So cos $\phi = 0$, So $P = 0$

Hence power dissipated is minimum. [1]

(ii) When AC source is connected, the capacitor offers capacitive reactance $XC = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C, XC increases. Therefore, glow of the bulb reduces. [1]

8. Given, $L = 5.\ H$, $C = 80\ \mu F$, $R = 40\Omega$ $V = 240V$

(i) Resonant angular frequenc

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}}$$

$$= \frac{1}{\sqrt{400 \times 10^{-6}}}$$

$$= \frac{1}{20 \times 10^{-3}} = \frac{1000}{20}$$ [1]

$\omega = 50 \text{rads}^{-1}$

(ii) At resonant frequency, we know that the inductive reactance cancels out the capacitive reactance.

Impedance, $Z = R = 40\Omega$

The current at resonant frequency

$$I_{rms} = \frac{V}{R} = \frac{240}{40} = 6A$$ [1]

(iii) For rms potential drop across inductor

$V_L = I_{rms} \times X_L$

$= I_{rms} \times \omega L$

$= 6 \times 50 \times 5$

$= V_L = 1500V$ [1]

9. (i) Resonant angular frequency: Given $L = 10H$ and C = 40μF = × 10^{-6} F

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}}$$

$$= \frac{1}{\sqrt{400 \times 10^{-6}}} = \frac{1}{20 \times 10^{-3}}$$

$$= \frac{1000}{20}$$

$\omega_o = 50 \text{rads}^{-1}$ [1]

(ii) At resonant frequency, we know that the inductive reactance cancels out the capacitive reactance.

The impedance = Z = 60Ω the value of resistance.

The current amplitude at resonant frequency,

$$I_o = \frac{E_o}{Z} = \frac{\sqrt{2}E_v}{R} = \frac{\sqrt{2} \times 240}{60} = \frac{339.60}{60}$$

$I_o = 5.66A$ [1]

(iii) The R.M.S value of current

$$I_v = \frac{I_o}{\sqrt{2}} = \frac{5.66}{\sqrt{2}}$$

$I_v = 4A$

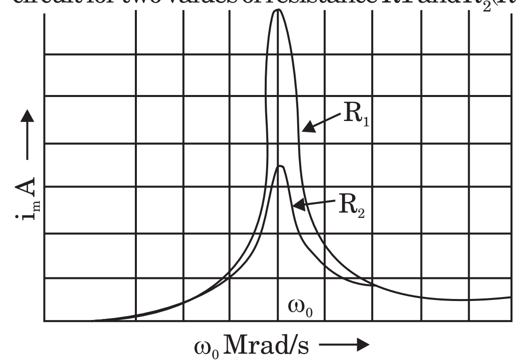
For $R.M.S$ potential drop across inductor

$V_L = I_v \times X_L$

$= I_v \times \omega L = 4 \times 50 \times 10$

$V_L = 2000V$ [1]

10. Figure shows the variation of w_o M rad/sh in a *LCR* circuit for two values of resistance R1 and $R_2 (R_1 > R_2)$,



The condition for resonance in the *LCR* circuit is, $\omega_o = \frac{1}{\sqrt{LC}}$ [1]

We can observe that the current amplitude is maximum at the resonant frequency ω_o. Since, $i_m = \frac{V_m}{R}$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1.

Quality factor or simply the Q-factor of a resonant *LCR* circuit is defined as the ratio of voltage drop across the capacitor (or inductor) to that of applied voltage.

It is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ [1]

The *Q* factor determines the sharpness of the resonance curve and if the resonance is less sharp, the maximum current decreases and also the circuit is close to the resonance for a larger range of frequencies and the regulation of the circuit will not be good. So, less sharp the resonance, less is the selectivity of the circuit while higher is the *Q*, sharper is the resonance curve and lesser will be the loss in energy of the circuit.

When $X_L = X_C$ or V_C, the LCR circuit is said to be in resonance condition. [1]

11.

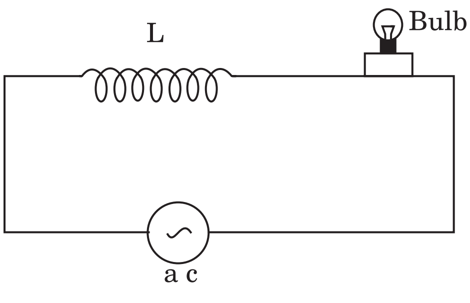


Fig.: Inductor Connected in an Electrical Circuit.

(i) When the number of turns in the inductor is reduced, its reactance X_L decreases. The current in the circuit increases and hence brightness of the bulb increases. [1]

(ii) When an iron rod is inserted in the inductor, the self-inductance increases. Consequently, the inductive reactance $X_L = \omega L$ increases. This decreases the current in the circuit and the bulb glows dimmer. [1]

(iii) With capacitor of reactance $X_C = X_L$, the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$, becomes minimum, the current in circuit becomes maximum. Hence the bulb glows with maximum brightness. [1]

12. (i) The device that is used to bring high voltage down to low voltage of an a.c. current is a transformer. It works on the principle of mutual induction of two windings or circuits. When current in one circuit changes, emf is induced in the neighbouring circuit. [1]

(ii) The transformer cannot convert d.c. voltages because it works on the principle of mutual induction. When the current linked with the primary coil changes the magnetic flux linked with the secondary coil also changes. This change in flux induces emf in the secondary coil. If we apply a direct current to the primary coil the current will remain constant. Thus, there is no mutual induction and hence no emf is induced. [1 + 1]

(iii) The value of gaining knowledge and curiosity about learning new things is being displayed by the students. The value of providing good education and undertaking the doubts of students has been displayed by the teacher. [1]

13. (a)

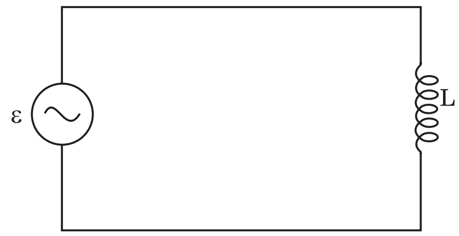


Fig.: An ac source connected to an inductor

Figure (a) shows an ac source connected to an inductor. Usually, inductors have appreciable resistance in their windings, but we shall assume that this inductor has negligible resistance. Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be $v = v_m \sin \omega t$. Using the Kirchhoff's loop rule, $\Sigma\varepsilon(t) = 0$, and since there is no resistor in the circuit,

$$v - L\frac{di}{dt} = 0 \ldots \text{(i)}$$

where the second term is the self-induced Faraday emf in the inductor; and L is the self-inductance of the inductor. The negative sign follows from Lenz's law.

From equation (i) we have

$$\frac{di}{dt} = Lv + Lv_m \sin\omega t \ldots \text{(ii)}$$

$$\frac{di}{dt} - Lv = Lv_m sin\omega t$$

Equation (ii) implies that the equation for i(t), the current as a function of time, must be such that its slope $\frac{di}{dt}$ is a sinusoidally varying quantity, with the same phase as the source voltage and an amplitude given by vm/L. To obtain the current, we integrate $\frac{di}{dt}$ with respect to time:

$$\int \frac{di}{dt}\, dt = \frac{v_m}{L}\int \sin(\omega t)dt$$

$$i = -\frac{v_m}{\omega L}\cos(\omega t) + \text{constant}$$ [1]

The integration constant has the dimension of current and is time independent. Since the source has an emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero, so that no constant or time-independent component of the current exists. Therefore, the integration constant is zero.

Using $-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$, we have

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where, $i_m = \frac{v_m}{\omega L}$ is the amplitude of the current.

The quantity ωL is analogous to the resistance and is called inductive reactance, denoted by X_L:

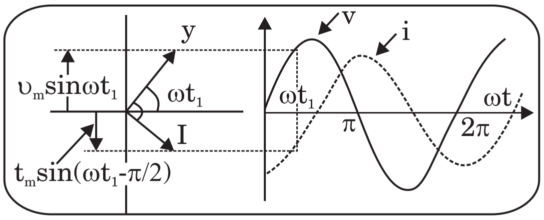
$X_L = \omega L$

The amplitude of the current is, then

$$i_m = \frac{v_m}{X_L}$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω).

The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit. The inductive reactance is directly proportional to the inductance and to the frequency of the current. [1]



(b) (c)

Fig.: (b) A phasor diagram for the circuit in fig. (a)

Fig.: (c) Graph of V and I versus.

(b) We have seen that a voltage $v = v_m \sin \omega t$ applied to a series RLC circuit drives a current in the circuit given by $i = i_m \sin(\omega t + \varphi)$where

$$i_m = \frac{v_m}{z} \text{ and } \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Therefore, the instantaneous power p supplied by the source is

$$p = vi = (v_m \sin\omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$ [1 + 1]

The average power over a cycle is given by the average of the two terms in R.H.S. of above equation. It is only the second term which is time-dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore,

$$P = \frac{v_m i_m}{2}\cos\phi = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$= VI \cos \phi$

This can also be written as

$P = I^2 Z \cos \phi$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them. The quantity $\cos\phi$ is called the power factor.

14. (a) Step up or step down transformer, Eddy current losses. [1]

(b) With higher voltage, power losses are less, so voltage can be increased by step up transformer and transformer works on A/c only. [1 + 1]

(c) Both are interested towards technical knowledge and both are having sufficient ideas about power transmission. [1]

15. (a) When a bar magnet pushed towards or away from coil, magnetic flux passing through coil change with time and cause induced emf hence induced current according to faraday's law of induction. Induced emf in the coil is given as

$$\varepsilon = \frac{Nd\phi}{dt}$$ [½]

Induced emf and hence current depends on

(i) no. of turns in the coil

(ii) motion of magnet [½]

Direction of current depends on the motion of magnet whether moving towards coil or away from the coil.

Faraday's Laws of Electromagnetic Induction

(i) Whenever there is a change in magnetic flux linked with a coil, an emf is induced in the coil. The induced emf is proportional to the rate of change of magnetic flux linked with the coil

i.e. $\varepsilon \propto \frac{\Delta\phi}{\Delta t}$ [½]

(ii) emf induced in the coil opposes the change in flux, i.e.,

$$\varepsilon \propto \frac{\Delta\phi}{\Delta t}$$

where k is a constant of proportionality Negative sign represents opposition to change in flux. In SI system ϕ is in weber, t in second, ε in volt, when $k = 1$ $\varepsilon \propto \frac{\Delta\phi}{\Delta t}$

If the coil has N-turns, then $\varepsilon = -k\frac{\Delta\phi}{\Delta t}$ [½]

(b) Case I When PQ moves forward.

(i) For $0 \le x < b$

Magnetic field, B exists in the region.

$\therefore$ Area of loop $PQRS = lx$

$\therefore$ Magnetic flux linked with loop $PQRS$,

$$\phi = BA = \frac{B}{x}$$

$$\phi = \frac{B}{x} \text{ ---------- (i) } [b > x \ge 0]$$ [½]

(ii) For, $2b \ge x \ge b$

B = 0

$\therefore$ Flux linked with loop $PQRS$ is uniform and given by

$$\varphi' = \frac{B}{b}(x = b)$$

Forward journey

Thus, for $b > x \ge 0$

flux $\varphi = BLx$

$\Rightarrow \varphi \propto = BLx$

For $2b \ge x \ge b$ Flux, $\varphi = Bbl$ [Constant]

Return journey

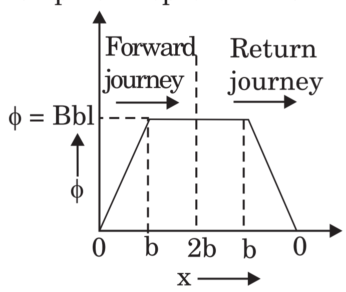
For $b \le x \le 2b$,

ϕ = constant = Bbl

For $0 \le x \le$ b,

$\varphi = BLx$ [Decreasing] [½]

Graphical representation



[½]

Case II For $b > x \ge 0$

As, $\varphi = BLx$

$$\Rightarrow \frac{d\phi}{dt} = B_1\frac{dx}{dt} = Bv_1\left[v = \frac{dx}{dt}\right]$$

Induced emf $e = \frac{d\phi}{dt_1} = -vB_1$

For $2b \ge x \ge b$,

As, $\varphi = Bbl$

$$\frac{d\phi'}{dt_1} = 0$$

$\Rightarrow e = 0$

Forward journey

For $b > x \ge 0$

$e = -vBl$

For $2b \ge x \ge b$, Backward journey

For $b >$ x ≥ 0

$e = -vBl$

For $2b \ge x \ge b$, $e = 0$ [½]

Variation of induced emf

Or

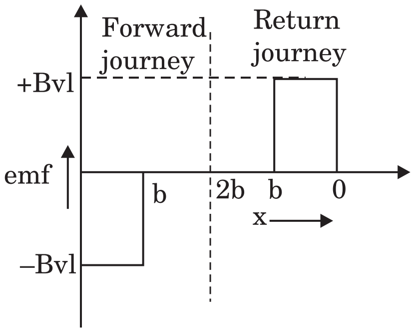


Figure shows a series LCR circuit connected to an ac source ε. As usual, we take the voltage of the source to be $v = v_m \sin\omega t$.

Assuming, $X_L > X_C \Rightarrow V_L = V_C$

So, Net voltage $V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$

where, V_L, $V_{C\ and}\ V_R$ are PD across L, C and R respectively.

But, $V_R = IR$, $V_L = IX_L$, $V_C = IX$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V = \sqrt{R^2 + (IX_L - IX_C)^2}$$

$$\text{Impedance} = Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

[½]

So $I = I_m \sin(\omega t + \varphi)$, where ϕ is the phase difference between voltage and current source.

Power dissipated in AC circuit:

We have seen that a voltage $v = v_m \sin\omega t$ applied to a series RLC circuit drives a current in the circuit given by $i = i_m \sin(\omega t + \varphi)$ where

$$i_m = \frac{v_m}{Z} \text{ and } \tan\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Therefore, the instantaneous power p supplied by the source is

$$p = vi = (v_m \sin\omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2}\cos\phi - \cos[2\omega t + \phi]$$

The average power over a cycle is given by the average of the two terms in $R.H.S$ of above equation. It is only the second term which is time-dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore,

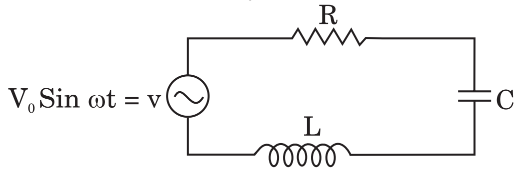
$$P = \frac{v_m i_m}{2}\cos\phi = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi = VI\cos\phi$$

This can also be written as

$P = I^2 Z \cos\phi$ [½]

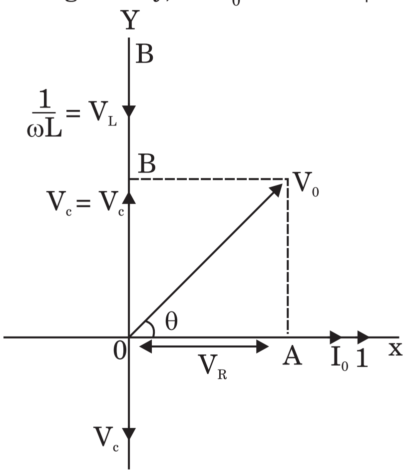
Power dissipated at resonance in LCR circuit: At resonance, $X_L - X_C = 0$. Therefore, $\cos\phi = 1$ and $P = I^2 Z = I^2 R$. That is, maximum power is dissipated in a circuit (through R) at resonance.

16. (i) Let a, series LCR circuit is connected to an ac source V (Fig). We take the voltage of the source to be $V = V_0 \sin\omega t$.



[1]

The AC current in each element is the same at any time, having the same amplitude and phase. It is given by, $I = I_0 \sin(\omega t + \phi)$



[1]

Fig.: Phasor diagram for LCR circuit.

Let V_L, V_R, V_C and V represent the voltage across the inductor, resistor, capacitor and the source respectively.

$V_C > V_L$

$$\therefore V_o^2 = V_R^2 + (V_C - V_L)^2$$

$$V_o^2 = (I_o R)^2 + (I_o X_C - I_o X_L)^2$$

$$V_o^2 = I_o^2\left[R^2 + (X_C - X_L)^2\right]$$

[1]

$$\text{And } I_o = \frac{V_o}{\sqrt{R^2 + (X_C - X_L)^2}} \Rightarrow I_o = \frac{V_o}{Z}$$

$$\text{Where, } Z = \sqrt{R^2 + (X_C - X_L)^2}$$

It is called the impedance in an AC circuit.

Condition: The current will be in phase with the voltage at resonance condition.

At resonance condition.

$X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}},\ 2\pi = \frac{1}{\sqrt{LC}}$$

[1]

(ii) As $\cos\phi = \frac{R}{Z}$

In L.R. circuit

$P_1 = \cos\phi$

$$P_1 = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{2R^2}} \quad (\because X_l = R)$$

$$P_1 = \frac{1}{\sqrt{2}}$$

In LCR when, $X_l = X_C$

$$P_2 = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$P_2 = \frac{R}{R} = 1 \quad (\because X_L = X_C)$$

$$\therefore \frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

[1]

17. (a) Since current leads the voltage by a plane angle of $\frac{\Pi}{2}$ device X is a capacitor. [1]

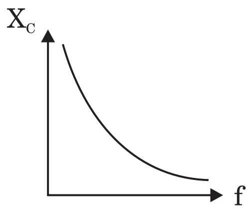
(b) Curve A shows power consumption over a full cycle, curve 'B' shows voltage and curve 'C' show current as in a perfect capacitor the current leads the voltage by a plane angle of $\frac{\pi}{2}$ [1]

(c) $z = x_c = \frac{1}{\omega_c}$

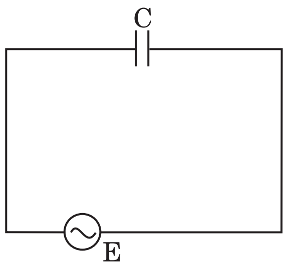
$$x_c = \frac{1}{2\pi f_c}$$

$$x_c \propto \frac{1}{f}$$

[1]



(d) AC through capacitor: Let on consider a capacitor with capacitance C be connected to an AC source with an emf having instantaneous value



$E = E_o \sin \omega t$.... (i)

Due to this emf, charge will be produced and it will charge the plates of capacitor with positive and negative charge. If potential difference across the plates of capacitor is V then

$$V = \frac{C}{q} \text{ or } q = CV$$

The instantaneous value of current in the circuit

$$I = \frac{dq}{dt} = \frac{d(CE)}{dt} (\therefore V = E) = \frac{d}{dt}(CE_0 sin\omega t)$$

$\therefore E = E_o \sin \omega t = CE_o \sin \omega t \times \omega$

$$= \frac{E_0}{1/\omega_c} \cos\omega t \quad \left[\therefore \cos\omega t = \sin\left(\frac{\pi}{2} + \omega t\right)\right]$$

$$I = \frac{E_0}{1/\omega_c} \sin\left(\frac{\pi}{2} + \omega t\right) \text{....(ii)}$$

[1]

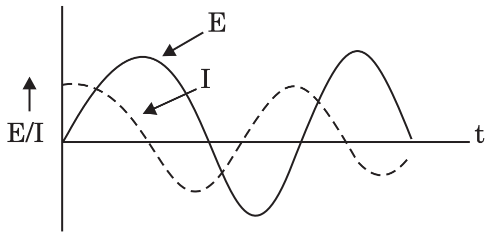
I will be maximum when

$\sin\left(\frac{\pi}{2} + \omega t\right) = 1$ so that $I = I_0$

where, peak value of current $I_0 = \frac{E_0}{1/\omega_c}$

$$I = I_0 \sin\left(\frac{\pi}{2} + \omega t\right) \text{.... (iii)}$$

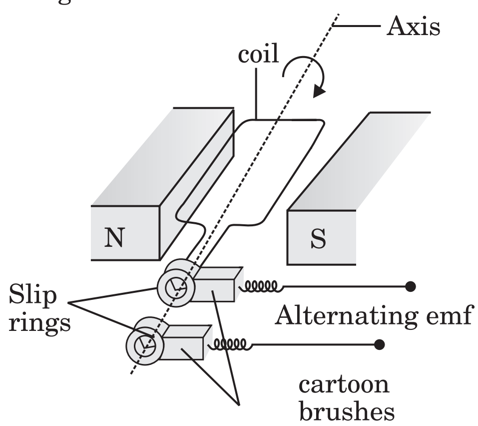
From (i) & (iii) it is clear current leads the voltage by a phase angle of $\frac{\pi}{2}(a_0)$ of $\frac{\pi}{2}$.



[1]

Or

(a) AC generator :-



[½]

Let at any instant total magnetic flux linked with the armature will is given G

(Where, $\theta = \omega t$, is the angle made by area vector of coil with magnetic field)

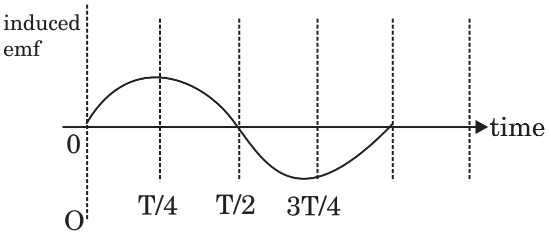
$\phi = NBA \cos\theta = NBA \cos \omega t$ [½]

$$-\frac{d\phi}{dt} = -NBA\omega \sin\omega t$$

By Faraday's law of EMI, $e = -\frac{d\phi}{dt}$ [½]

Induced emf is given by $NBA\ \omega \sin \omega t$

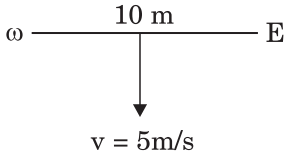
$e = e_o \sin \omega t$ where, $e_o = NBA\ \omega$ peak value of induced emf [½]



[½]

The mechanical energy spent in rotating the coil in magnetic field appears in form of electrical energy. [½]

(b)



[½]

Given velocity of straight rod v = 5m/s horizontal component of the earth's magnetic field

$B = 0.30 \times 10^{-4}\ wb/m_2$, length of wire l =10$m$

So the emf induced in the wire is given by

$e = Blv \sin\theta\ (\theta = 90°)$

$e = 0.30 \times 10^{-4} \times 10 \times 5$ [½]

($\therefore$ Wire is falling at right angle to the earth's horizontal magnetic field component

$e = 1.5 \times 10^{-3} v$ [½]

Air from west to east (According to Fleming right hand rule) [½]

18. (a) Power in ac circuit

Voltage v in an ac circuit is:

$v = v_m \sin \omega t$ which drives through the circuit a current i

$i = i_m \sin(\omega + \varphi)$ where $i_m = \frac{v_m}{Z}$ and

$$\varphi = \tan^{-1}\left[\frac{X_c - X_L}{R}\right]$$ [1]

Power $p = v = v_m \sin\omega t\left[i_m \sin(\omega t + \varphi)\right]$

$$= \frac{v_m i_m}{2}\left[\cos\varphi - \cos(2\omega t + \varphi)\right]$$

Calculating the average power, it is observed that the average of the term $\cos[2\omega t + \varphi)$ is equal to zero.

Thus,

Average power,

$$p = \frac{v_m i_m}{2}\cos\varphi = VI\cos\varphi$$ [1½]

(b) The ratio $\frac{\omega_L}{R}$ is called the quality factor or Q-factor.

$$Q = \frac{\omega_L}{R}$$ [1]

The quality factor has high value in receiving circuits in order to get a sharp gain for the desired channel frequency. The quality factor depends on the values of the following:

Inductance

Resistance

Capacitance [1½]

Or

(a) The instantaneous power dissipated in the resistor is $P = i^2 R = i_m^2 \sin^2 \omega t R$

The average value of p over a cycle is:

$$P = i^2 R = i_m^2 \sin^2 \omega t R$$

i_m^2 and R are constants. Therefore,

$$P = i_m^2 R\left(\sin^2 \omega t\right)$$ [1]

By trigonometric identity,

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

The average value of $\cos 2\omega t$ is zero.

We have:

$$\sin^2 \omega t = \frac{1}{2}(1 - 0)$$

The rms value in the ac power is expressed in the same form as dc power root mean square or effective current and is denoted by I_{rms}. Peak current is i_m therefore,

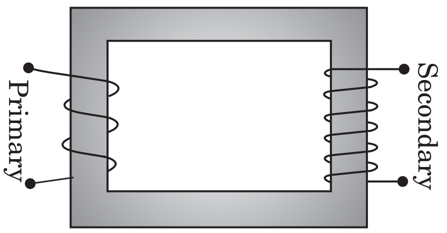
$$I = \frac{i_m}{\sqrt{2}} = 0.707 i_m$$

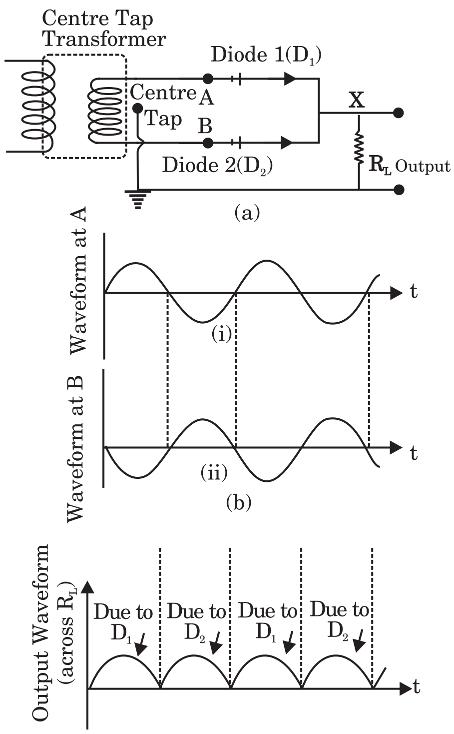
$$I^2 R = \frac{i_m^2}{2} R$$

$$I = \frac{i_m}{\sqrt{2}}$$

[1½]

(b) [½]





[1]

In a transformer with N_s secondary turns and N_p primary turns, induced emf or voltage E_s is:

$$E_s = -N_s \frac{d\varphi}{dt}$$

back emf $E_p = -N_p \dfrac{d\varphi}{dt}$ ---------- (i)

$E_p = V_p$
$E_s = V_s$

Thus, $E_s = -N_s \dfrac{d\varphi}{dt}$ ....(ii)

Dividing (i) and (ii), we obtain

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

If the transformer is 100% efficient, then

$i_p v_p = i_s v_s$ = Power (p)

Thus, combining the above equations,

$$\frac{i_p}{i_s} = \frac{v_s}{v_p} = \frac{N_s}{N_p}$$

$$v_s = \left(\frac{N_s}{N_p}\right)v_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$

[½]

If $N_s > N_p$, then the transformer is said to be step-up transformer because the voltage is stepped up in the secondary coil. No, the transformer does not violate the principal of conservation of energies. This can be easily observed by the following equation:

$V_p I_p = V_s I_s = P$

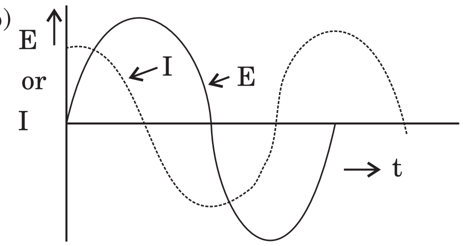
Power consumed in both the coils is the same as even if the voltage increases or current increases, their product at any instant remains the same. [1]

19. (a) X - Capacitor

$$X_c = \frac{1}{2\pi fC}$$

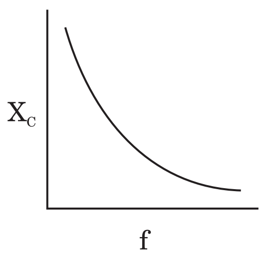
[1]

(b)



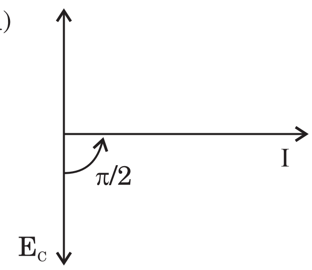
[2]

(c) $X_c \propto \dfrac{1}{f}$



[1]

(d)



[1]

[Topic 2] AC Devices

Summary

Inductive Reactance (X_L)

When the current flows in the circuit, the inductor opposes its motion, this opposing nature of the inductor is termed as Inductive Reactance.

Mathematically it can be expressed as:

$X_L = \omega L = 2\pi fL$

Where L is self-inductance.

Instantaneous power supplied to an inductor

$p_l = -\frac{i_m v_m}{2} \sin(2\omega t)$

Average power supplied to an inductor over one complete cycle is zero.

In case of inductor, the current lags the voltage by $\frac{\pi}{2}$

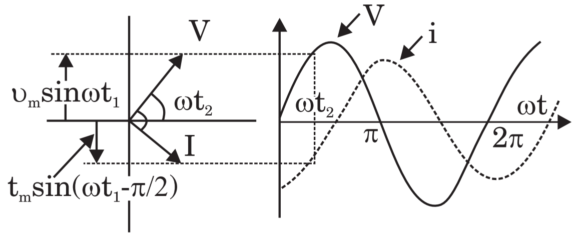


Fig.: Phasor Diagram of inductor

Inductive reactance can be graphically expressed as follows:

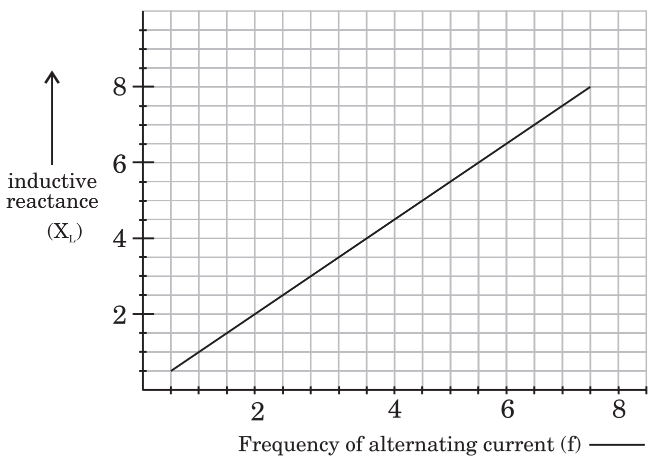


Fig.: Inductive Reactance Vs f

Capacitive Reactance (X_C)

When the current flows in the circuit, the capacitor opposes its motion, this opposing nature of the capacitor is termed as capacitive Reactance.

Mathematically it can be expressed as:

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Where C is capacitance.

Instantaneous power supplied to the capacitor is

$p_c = \frac{i_m v_m}{2} \sin(2\omega t)$

In case of capacitor, the current leads the voltage by $\frac{\pi}{2}$

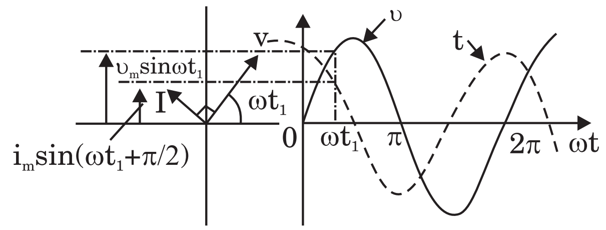


Fig.: Phasor Diagram of Capacitor

Capacitive reactance can be graphically expressed as follows:

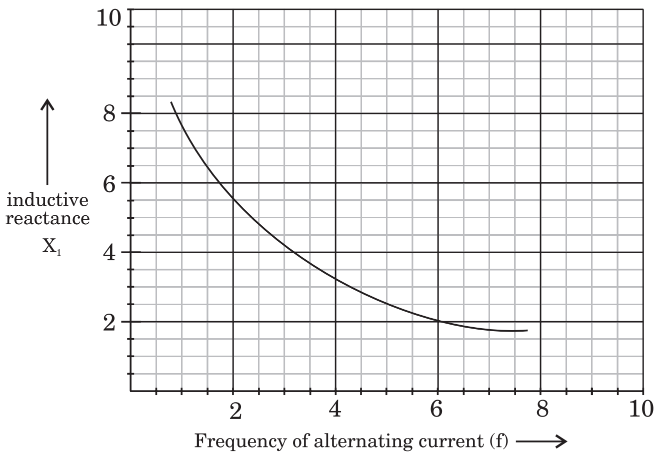


Fig.: Capacitive Reactance Vs f

- For a **series** LCR circuit driven by voltage $v = v_m \sin(\omega t)$, the current is given by $i = i_m \sin(\omega t + \phi)$.

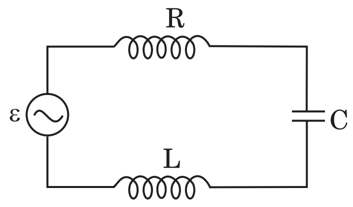


Fig.: LCR Circuit

Where $i_m = \dfrac{v_m}{\sqrt{R^2 + (X_c - X_L)^2}}$

And $\phi = tan^{-1} \dfrac{X_C - X_L}{R}$

$Z = \sqrt{R^2 + (X_C - X_L)^2}$ is called the impedance of the circuit.

Power

In an alternating circuit, the voltage and the current both keep on changing with respect to time. Hence the rate at which the electric energy is transferred in a circuit is called as it's power. The SI unit is Watt.

- Electric Power: The product of direct current flowing through a circuit and the voltage across the circuit.
  P = IV
- Instantaneous Power: The product of current and voltage as a function of time.
  $P_{inst} = E_{inst} \times I_{inst}$
- Average Power: Average of instantaneous power can be called as average power.
  Mathematically it is expressed as,
  $P_{avg} = V_{rms} I_{rms} \cos\Phi$
  where $\cos\Phi$ is power factor.
- Power factor: It is the ratio of true power to the apparent power.
- The phenomenon of ***resonance*** is an interesting characteristic of a series LCR circuit. The amplitude of the current is maximum at the resonant frequency and the circuit thus exhibits resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}}$.
  The quality factor Q is defined by $Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 CR}$
  and it tells about the sharpness of the resonance.
- ***LC Oscillations:*** When an inductor is connected to an initially charged capacitor, the charge on the capacitor and the current in the circuit exhibit the phenomenon of electrical oscillations. When the circuit has no ac source and no resistor then the charge q of the capacitor is given by $\dfrac{d^2q}{dt^2} + \dfrac{1}{LC}q = 0$
  Where $\dfrac{1}{\sqrt{LC}} = \omega_0$ is the frequency of free oscillation.

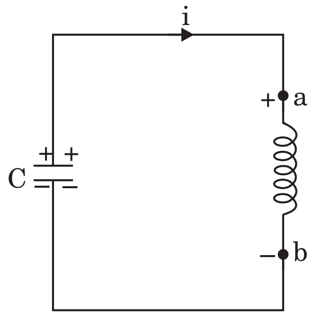


Fig.: LC Oscillalions

- ***Idle Current:*** If the average power consumed in an alternating current circuit is zero because of the current flowing through it, this current is called as Idle Current.
- Pure Inductor circuit and pure capacitor circuit are the two circuits whose average power consumed is zero as the phase difference is 90°.
- In generators and motors, the roles of input and output are reversed. In a motor, electric energy is the input and mechanical energy is the output. In a generator, mechanical energy is the input and electric energy is the output. Both devices simply transform energy from one form to another.

Transformer

- They convert an alternating voltage from one to another of greater or smaller value by using the principle of mutual induction.

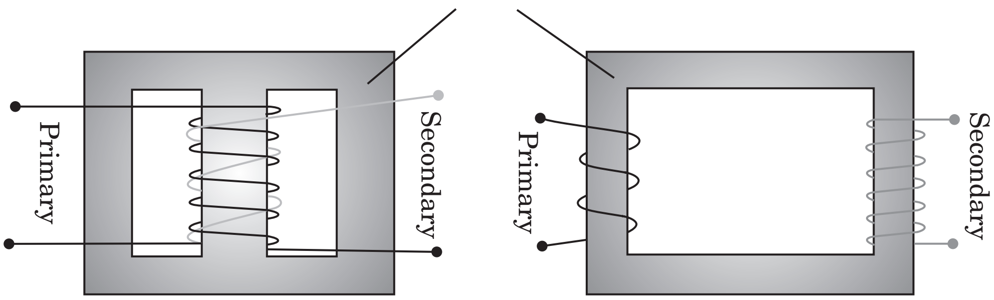


Fig.: Transformer Showing Primary & Secondary coils.

- A step-up transformer changes a low voltage in to high voltage.
- A step-down transformer changes high voltage to low voltage.
- The primary and secondary voltage and currents are given by

$$V_S = \left(\frac{N_S}{N_p}\right)V_p \quad \text{and} \quad I_s = \left(\frac{N_p}{N_s}\right)I_p$$

- Efficiency of the transformer is the ratio of the output power to the input power. It is usually for a real one.

$$\eta = \frac{E_s I_s}{E_p I_p}$$

- ***Energy losses*** in transformers may be due to Flux leakage, resistance of windings, Eddy currents and Hysteresis.
- The choice of whether the description of an oscillatory motion is by means of sine or cosine or by their linear combinations is unimportant, since changing the zero-time position transforms one to the other.

PREVIOUS YEARS'
EXAMINATION QUESTIONS
TOPIC 2
5 Mark Questions

1. (i) With the help of a labelled diagram, describe briefly the underlying principle and working of a step up transformer.

(ii) Write any two sources of energy loss in a transformer.

(iii) A step up transformer converts a low input voltage into a high output voltage.

Does it violate law of conservation of energy? Explain.

2. (i) Write the function of a transformer. State its principle of working with the help of a diagram. Mention various energy losses in this device.

(ii) The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are respectively 220V and 1100 W Calculate:

(a) Number of turns in secondary.
(b) Current in primary.
(c) Voltage across secondary.
(d) Current in secondary.
(e) Power in secondary.

[CBSE 2016]

Solutions

1. (i) Principle underlying the working of transformer: The principle is of Mutual Inductance. When a changing source of voltage is introduced across a coil (which is physically coupled to another coil), the changing current through it induces an EMF across the second coil.

A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core, either one on top of the other, or on separate limbs of the core.

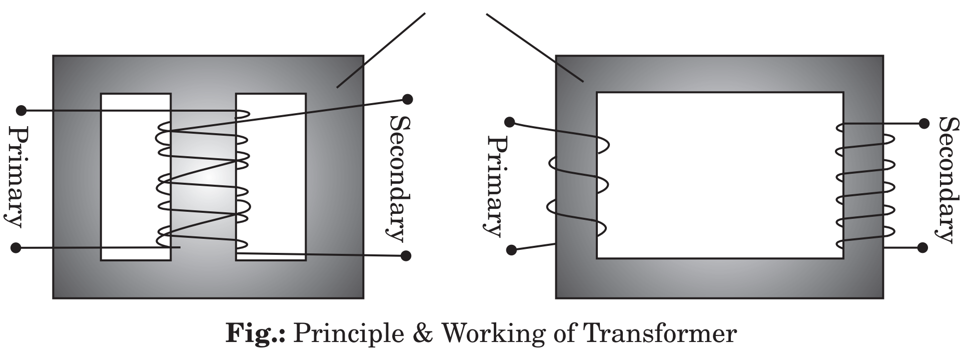


Fig.: Principle & Working of Transformer

One of the coils is called the primary coil, and has N_2 turns. The other coil, the secondary coil, has N_p turns. The relative numbers depend on whether the voltage needs to be stepped up or stepped down.

By definition, the voltage to be transformed is introduced across the primary coil. When the alternating voltage is applied across the primary, the resulting alternating current through it produces a changing magnetic field, whose flux through the secondary coil changes.

From Faraday's law, this changing flux induces an EMF across the secondary, whose magnitude depends on the amount of coupling of the two coils, numerically measured as mutual inductance. The more the coupling or association of the two coils, the more is mutual inductance, and therefore the induced EMF.

If ϕ is the flux through each turn of the core, then through N turns around the core,the total flux is $N\phi$

So, the EMF induced in the secondary coil is

$$E_s = -N_s \frac{d\phi}{dt}$$

Similarly, there will also be an *EMF* induced in the primary coil itself, due to self-inductance, given by

$$E_p = -N_p \frac{d\phi}{dt}$$

If the voltage applied across the primary is V_p, then if its resistance is R, the current through it will be $I_p = \frac{V_p - E_p}{R}$

However, assuming negligible resistance, since we cannot have an infinite current through the coil, then

$$E_p \approx V_p$$

If the secondary is an open circuit, no current is drawn from it then, voltage across it will be [1]

$$V_s = E_s = -N_s \frac{d\phi}{dt}$$

From equation it is clear that

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

If the transformer is 100% efficient, that is, all the input power is transferred to the secondary without any leakage or losses, then

$$I_p V_p = I_s V_s$$

This implies that

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

It is clear from that if $N_s > N_p$, the voltage will be stepped up, and if $N_s < N_p$ it will be stepped down.

However, in a step down transformer, there will be a greater current in the secondary as compared to the primary and vice-versa. [1 + 1]

(ii) The possible sources of power losses in practical transformers can be

(1) Flux Leakage: Not all flux of the primary can be associated with the secondary.

There is always some flux which due to lack of absolute coupling, can leak. To avoid this, the coils are wound over each other again and again.

(2) Resistance of windings: The transformer coil wires cannot have absolutely zero resistance, so some Joule loss is inevitable.

(3) Core eddy currents: Since the core is a very good conductor itself, currents are induced in it due to changing magnetic fields, called eddy currents. These also result in losses.

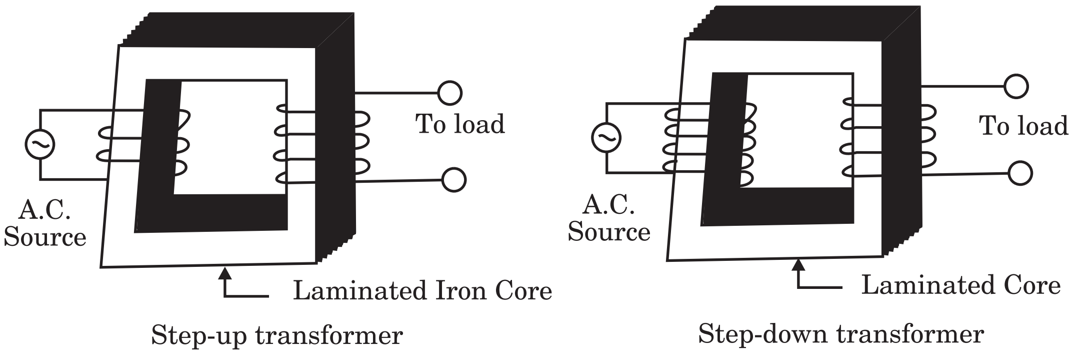
(4) Hysteresis: Some part of energy is frozen into the core permanently in the form of a residual magnetic field due to its ferromagnetic character. [1]

(iii) No, it does not violate the energy conservation. When low voltage is converted to high volatge, the current is lowered, thereby conserving the total energy dissipated across the primary & secondary coil. [1]

2. (i) A transformer is an electrical device for converting an alternating current at low voltage into that at high voltage or vice-versa.

1. If it increases the input ac voltage, it is called step up transformer.
2. If it decreases the input ac voltage, it is called step down transformer. [½]

Principle: It works on the principle of mutual induction i.e., When a changing current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil. [½]



[½]

Fig.: Step up transformer & Step down transformer

Working Theory: As the AC flows through the primary, it generate an alternating Magnetic flux in the core which passes through the secondary coil.

Let N_1 No. of turns in primary coils

N_2 = No. of turns in secondary coils

This changing flux sets up an induced emf in the secondary, also a self-induced emf in the primary.

If there is no leakage of magnetic flux, then flux linked with each turn of the primary will be equal to that linked with each of the secondary. According to Faraday's law of induction.

Induced emf in the primary coil, $\varepsilon_1 = -N_1 \frac{d\phi}{dt}$

Induced emf in the secondary coil, $\varepsilon_2 = -N_2 \frac{d\phi}{dt}$

Where, $\frac{d\phi}{dt}$ = rate of change magnetic flux associated with each turn.

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}$$ [½]

Energy Losses in transformer.

- **Copper loss:** Some energy is lost due to the heating of copper wires used in the primary and secondary windings. This power loss ($P = 1^2R$) can be minimized by using thick copper wires of low resistance.
- **Eddy current loss:** The alternating magnetic flux induces eddy current in the iron core which leads to some energy loss in the form of heat. This loss can be reduced by using laminated iron core. [½]
- **Hysteresis loss:** The alternating current carries the iron core through cycles of magnetization and demagnetization. Work done in each of these cycles is lost as heat. This is called hysteresis loss and can be magnetized by using core material having narrow hysteresis loop.
- **Flux leakage:** The magnetic flux produced by the primary may not fully pass through the secondary. Some of the flux may leak into air. This loss can be minimized by winding the primary and secondary coils over one another. [½]

(ii) Given, $N_1 = 100$

$K = 100$

$V_1 = 220V$

$P_1 = 1100\ W$

(a) As, $K = \frac{N_2}{N_1}$

$N_2 = KN_1 = 100 \times 100$

$N_2 = 10000$ [½]

(b) $P_1 = V_1 I_1$

$$I_1 = \frac{P_1}{V_1} = \frac{1100}{220}$$

$I_1 = 5V$ [½]

(c) $\frac{V_2}{V_1} = K$

$V_1 = 5V_1$; $V_2 = 100 \times 200$

$V_2 = 5V$ [½]

(d) $\frac{I_1}{I_2} = K$

$I_2 = 0.05A$

$P_2 = V_2 I_2$

$$P_2 = 22000 \times \frac{5}{100}$$

$P_2 = 1100W$ [½]

Smart Notes ....

Smart Notes ....

CHAPTER 8

Electromagnetic Waves

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Basic idea of displacement current, Electromagnetic waves, their characteristics, their transverse nature		1Q (3 marks)	1Q (1 mark)	1Q (1 mark), 1Q (1 mark)	
Electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays) including elementary facts about their use	1Q (1 mark)				1Q (1 mark), 1Q (1 mark), 1Q (2mark)

On the basis of above analysis, it can be said that from exam point of view Characteristics and Nature of Electromagnetic Spectrum, Applications of Different Parts of Electromagnetic and Electromagnetic Wave are most important concepts of the chapter.

[Topic 1] Electromagnetic Waves, its Types & Properties

Summary

Displacement Current

- It is defined as the rate of change of electric displacement.
- It is given by $I_d = \varepsilon_0 \frac{d\phi_\varepsilon}{dt}$ where ε_0 is the permittivity of the free space and ϕ_s is the amount of electric flux.

Properties of EM Waves

- The electric and magnetic fields E_x and B_y are always perpendicular to each other, and also to the direction z of propagation. E_x and B_y are given by:

$$E_x = E_0 \sin(kz\text{-}\omega t)$$

$$B_y = B_0 \sin(kz\text{-}\omega t)$$

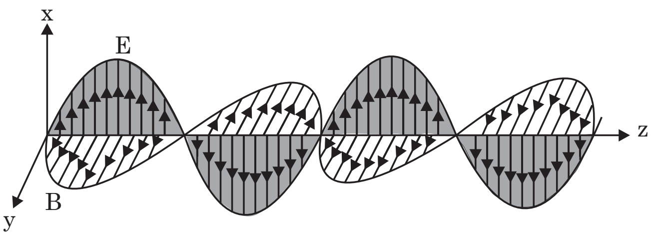


Fig. Electromagnetic Waves

Where,

"k" is the magnitude of the wave vector (or propagation vector) and can be calculated as;

$$k = \frac{2\pi}{\lambda}$$

- ω is the angular frequency,
- "k" is direction describes the direction of propagation of the wave. The speed of propagation of the wave is $\frac{\omega}{k}$.
- The frequency of EM waves can be from 0 to ∞.

Ampere Circuital Law is given by: $\oint B \cdot dl = \mu_0 i(t)$

The four Maxwell's equations are given as:

- Gauss's law of electricity: $\oint E.dA = \frac{Q}{\varepsilon_0}$
- Gauss's law of magnetism: $\oint B.dA = 0$
- Faraday's law: $\oint E.dl = \frac{-d\phi_B}{dt}$
- Ampere-Maxwell law: $\oint B.dl = \mu_0 I_c + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$

An electric charge oscillating harmonically with a frequency, produces electromagnetic waves of the same frequency . The frequency of the electromagnetic wave naturally equals the frequency ofoscillation of the charge.

An electric dipole is a basic source of electromagnetic waves.

From Maxwell's equations it can be seen that the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as $B_0 = \frac{E_0}{c}$

Properties of EM Waves

- Oscillations of electric and magnetic fields sustain in free space, or vacuum. So, the electromagnetic waves can travel in vacuum.
- An electromagnetic wave carries momentum and energy. Since an electromagnetic wave carries momentum, it also exerts pressure, called radiation pressure.
- Let the total energy transferred to a surface in time t is U, so the magnitude of the total momentum of an electromagnetic wave delivered to the surface (for complete absorption) is, $P = \frac{U}{c}$
- The energy of electromagnetic waves is shared equally by the electric and magnetic fields.

Types of EM Waves

- Radio waves are produced by the accelerated motion of charges in conducting wires. They are used in radio and television communication systems.The radio waves generally lie in the frequency range from 500 kHz to about 1000 MHz
- Microwaves have frequency in the range of gigahertz and are used in aircraft navigation.
- Infrared waves are also referred to as heat waves as they are produced by hot bodies and molecules.

- Visible rays can be detected by the human eye. They lie between frequency range of about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700 –400 nm.
- Ultraviolet radiation or the UV radiation is produced by special lamps and very hot bodies.
- X-rays lie beyond the UV region and are used as a diagnostic tool in medicine and for treating various kinds of cancer.
- Gamma rays are emitted by radioactive nuclei and also are produced in nuclear reactions and are used in destroying the cancer cells.

The properties of different types of EM Waves are:

Type	***Wavelength range***	***Production***	***Detection***
Radio	*> 0.1 m*	*Rapid acceleration and decelerations of electrons in aerials*	*Receiver's aerials*
Microwave	*0.1 m to 1 mm*	*Klystron valve or magnetron valve*	*Point contact diodes*
Infra-red	*1 mm to 700 nm*	*Vibration of atoms and molecules*	*Thermopiles, Bolometer, Infrared photographic film*
Light	*700 nm to 400 nm*	*Electrons in atoms emit light when they move from one energy level to a lower energy level*	*The eye, Photocells, Photographic film*
Ultraviolet	*400 nm to 1 nm*	*Inner shell electrons in atoms moving from one energy level to a lower level*	*Photocells, Photographic film*
X-rays	*1 nm to 10^{-3} nm*	*X-ray tubes or inner shell electrons*	*Photographic film, Geiger tubes Ionisation chamber*
Gamma rays	*$<10^{-3}$ nm*	*Radioactive decay of the nucleus*	*Photographic film, Geiger tubes Ionisation chamber*

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the direction of electric and magnetic field vectors? **[ALL INDIA 2011]**
2. Name of physical quantity which remains same for microwaves of wavelength 1 mm and *UV* radiations of 1600 *A*° in vacuum. **[ALL INDIA 2012]**
3. State De-Broglie hypothesis. **[ALL INDIA 2012]**
4. Which of the following waves can be polarized (i) Heat waves (ii) Sound waves? Give reason to support your answer. **[ALL INDIA 2013]**
5. Write the expression for the de Broglie wavelength associated with a charged particle having charge '*q*' and mass '*m*', when it is accelerated by a potential *V*. **[ALL INDIA 2013]**
6. Welders wear special goggles or face masks with glass windows to protect their eyes from electromagnetic radiations. Name the radiations and write the range of their frequency. **[ALL INDIA 2013]**
7. Why are microwaves considered suitable for radar systems, used in aircraft navigation? **[DELHI 2016]**
8. Write the following radiations in ascending order with respect to their frequencies: X-rays. Microwaves. *UV* rays and radio waves. **[DELHI 2017]**
9. Do electromagnetic waves carry energy and momentum? **[DELHI 2017]**

10. Name the phenomenon which shows the quantum nature of electromagnetic radiation. **[ALL INDIA 2017]**

11. Identify the electromagnetic waves whose wavelength vary as
(a) 10^{-12} m < 1 < 10^{-8} m
(b) 10^{-3} m < 1 < 10^{-1} m
Write one use of each. **[ALL INDIA 2017]**

12. Which part of electromagnetic spectrum is used in radar systems? **[DELHI 2018]**

2 Mark Questions

13. Name the electromagnetic radiation's used for(a) water purification, and(b)eye surgery. **[ALL INDIA 2018]**

14. How are infrared waves produced? Why are these referred to as 'heat waves'? Write their one important use. **[ALL INDIA 2011]**

15. (a) An EM wave is, travelling in a medium with a certain velocity. Draw a sketch showing the propagation of the EM wave, indicating the direction of the oscillating electric and magnetic fields.

(b) How are the magnitudes of the electric and magnetic fields related to velocity of the EM wave ? **[DELHI 2017]**

16. How does a charge q oscillating at certain frequency produce electromagnetic waves? Sketch a schematic diagram depicting electric and magnetic fields for an electromagnetic wave propagating along the Z-direction. **[DELHI 2017]**

17. (a) Why are infra-red waves often called heat waves? Explain.

(b) What do you understand by the statement, "Electromagnetic waves transport momentum"? **[ALL INDIA 2018]**

3 Mark Questions

18. Write Einstein's photoelectric equation and point out any two characteristic properties of photons on which this equation is based.
Briefly explain the three observed features which can be explained by this equation. **[ALL INDIA 2013]**

19. Write Einstein's photoelectric equation. State clearly how this equation is obtained using the photon picture of electromagnetic radiation. Write the three salient features observed in photoelectric effect which can be explained using this equation. **[ALL INDIA 2012]**

20. When Sunita, a class XII student, came to know that her parents are planning to rent out the top floor of their house to a mobile company she protested. She tried hard to convince her parents that this move would be a health hazard. Ultimately her parents agreed:

(1) In what way can the setting up of transmission tower by a mobile company in a residential colony prove to be injurious to health?

(2) By objecting to this move of her parents, what value did Sunita display?

(3) Estimate the range of e.m. waves which can be transmitted by an antenna of height 20 m. (Given radius of the earth = 6400 km) **[DELHI 2014]**

21. Answer the following:

(a) Name the em waves which are used for the treatment of certain forms of cancer. Write their frequency range.

(b) Welders wear special glass goggles while working. Why? Explain.

(c) Why infrared waves are often called as heat waves? Give their one application. **[DELHI 2014]**

22. Answer the following:

(a) Name the em waves which are used for the treatment of certain forms of cancer. Write their frequency range.

(b) Thin ozone layer on top of stratosphere is crucial for human survival. Why?

(c) Why is the amount of the momentum transferred by the em waves incident on the surface so small? **[DELHI 2014]**

23. Answer the following:

(a) Name the em waves which are suitable for radar systems used in aircraft navigation. Write the range of frequency of these waves.

(b) If the earth did not have atmosphere, would its average surface temperature be higher or lower than what it is now? Explain.

(c) An em wave exerts pressure on the surface on which it is incident. Justify.

[DELHI 2014]

24. Name the parts of the electromagnetic spectrum which is

(a) Suitable for radar systems used in aircraft navigation

(b) Used to treat muscular strain

(c) Used as a diagnostic tool in medicine

Write in brief, how these waves can be produced.

[DELHI 2015]

25. Define the terms "stopping potential' and 'threshold frequency' in relation to photoelectric effect. How does one determine these physical quantities using Einstein's equation?

[ALL INDIA 2015]

26. How are electromagnetic waves produced? What is the source of the energy carried by a propagating electromagnetic wave? Identify the electromagnetic radiations used

(i) In remote switches of household electronic device; and

(ii) as diagnostic tool in medicine

[ALL INDIA 2015]

27. How are e.m. waves produced by oscillating charges?

Draw a sketch of linearly polarized e.m. waves propagating in the Z-direction. Indicate the directions of the oscillating electric and magnetic fields.

OR

Write Maxwell's generalization of Ampere's Circuital Law. Show that in the process of charging a capacitor, the current produced within the plates of the capacitor is $i = \varepsilon_o \frac{d\phi_E}{dt}$

where ϕ_E is the electric flux produced during charging of the capacitor plates.

[DELHI 2016]

28. State two important properties of photon which are used to write Einstein's photoelectric equation. Define

(i) Stopping potential and

(ii) Threshold frequency, using Einstein's equation and drawing necessary plot between relevant quantities.

[DELHI 2016]

29. How are electromagnetic waves produce? What is the source of energy of these waves? Write mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis. Write any two important properties of electromagnetic waves.

[ALL INDIA 2016]

Solutions

1. The electromagnetic wave travels in a vacuum along the z-direction. The electric field (E) and the magnetic field (B) are in the x-y plane. They are mutually perpendicular. [1]

2. Both microwaves and UV rays are a part of the electromagnetic spectrum. Thus, the physical quantity that remains same for both types of radiation will be their speeds, equal to c. [1]

3. It states that, "Moving object sometimes acts as a wave and sometimes as a particle" or a wave is associated with the moving particle, which controls the particle in every respect. This wave associated with the moving particle is called matter wave or de Broglie wave. Its wave length is given by:

$$\lambda = \frac{h}{m\nu}$$ [1]

Where, h = planck's constant

m = mass of the object

v = velocity of the object

4. Heat waves can be polarized as they are transverse waves whereas sound waves cannot be polarized as they are longitudinal waves. [½]

Transverse waves can oscillate in the direction perpendicular to the direction of its transmission but longitudinal waves oscillate only along the direction of its transmission. So, longitudinal waves cannot be polarized. [½]

5. $\lambda = \frac{h}{\sqrt{2mqV}}$ [1]

6. Ultraviolet rays frequency range

($7.5 \times 10^{14} - 5 \times 10^{15}$ Hz) [1]

7. Microwaves of frequency 1 GHz to 300 GHz bounces from even the smallest aircraft so that they are suitable to avoid getting bombed. Microwaves can penetrate through clouds also. [1]

8. The given radiations can be arranged in ascending order with respect to their frequencies as: Radio waves < Microwaves < UV rays < X-rays [1]

9. Yes, electromagnetic waves carry energy (E) and momentum (P) is given by

$$E = \frac{hc}{\lambda}$$ [½]

$P = mC$ [½]

Here C is speed of EM(Electromagnetic) wave in vaccum λ is wavelength of EM wave.

10. "Photoelectric effect" shows the quantum nature of electromagnetic radiation. [1]

11. (a) X-ray

Uses- These are used in surgery to detect fracture, damaged organs, stones in the body, etc. [1]

(b) Gamma rays [1]

Uses- These are used in radio therapy for the treatment of tumor and cancer.

12. The microwave range [1]

13. (a) UV rays [1]

(b) Laser [1]

14. Infrared waves are produced by hot bodies or due to vibrations of atoms and molecules. They are called heat waves because they cause heating effect or increase the temperature. [1]

Use: Infrared lamps,play important role in maintaining warmth through greenhouse effect. [1]

15. (a) Given,

Velocity, $v = v\vec{i}$ and electric field, E along y-axis and magnetic field, B along z-axis, [½]

The propagation of EM wave is shown below:

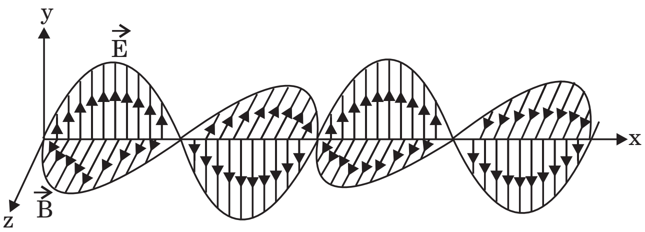


Fig.: Electromagnetic Waves [½]

(b) Speed of EM wave can be given as the ratio of magnitude of electric field (E_o) to the magnitude of magnetic field (B_o), $c = \frac{E_o}{B_o}$ [1]

16. An oscillating charge is an example of accelerated charge. We know from Maxwell's theory that accelerated charge radiates electromagnetic waves. These electromagnetic waves are produced because oscillating charge produces oscillating magnetic field which in turn produces an oscillating electric field. This process goes on, giving rise to an electromagnetic wave. [1.5]

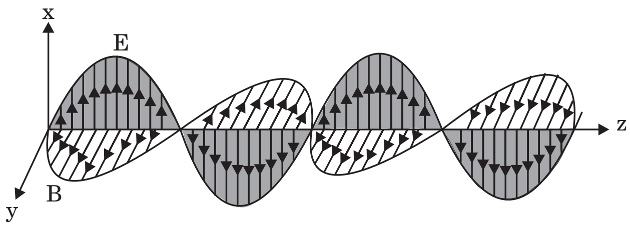


Fig.: Electromagnetic Waves [½]

17. (a) Infrared produce heat or emitted from hot bodies. [1]

(b) When electromagnetic waves hit body the mass is lost but the momentum is conserved .i.e., transferred from EMW to body. Therefore, EMW transports momentum.[1]

18. If energy of photon = E (= hv)

work function of metallic surface = ϕ = (hv_0)

kinetic energy of emitted electron $= \frac{1}{2}mv^2$

$$E = \varphi + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = h(\nu - \nu_o)$$

[½]

Einstein's equation

Two characteristics of photons

(i) This equation is based on particle nature of light [½]

(ii) Total energy of photon is transferred completely to single electron. [½]

(iii) If incident frequency $\nu <$ threshold frequency (ν_o), then kinetic energy $\frac{1}{2}mv^2$ will be negative which is not possible because kinetic energy cannot be negative. This shows that photoelectric emission is not possible if frequency (ν) of incident light is less than the threshold frequency (ν_o) of the metal. [1

(iv) One photon can emit only one electron from the metal surface, so the number of photoelectrons emitted per second is directly proportional to the intensity of incident light which depends upon number of photons present in the incident light. [½]

(v) From equation it is clear that kinetic energy, $\frac{1}{2}mv^2$ increases with the increase in the frequency(ν) of the incident light. [½]

19. Einstein's Photoelectric Equation,

$h\nu, = K_{max} + \phi$

Where, h = Planck's constant

v = frequency

ϕ = Work function

$$K_{\max} = \frac{1}{2}mv^2_{\max} = eV_o$$

(Where v_{max} = maximum velocity of emitted photoelectron and V_o = Stopping potential)

According to Planck's quantum theory, light radiations consist of small packets of energy. Einstein postulated that a photon of energy 'hv' is absorbed by the electron of the metal surface, then the energy equal to ϕ is used to liberate electron from the surface and rest of the energy i.e. $hv - \phi$ becomes the kinetic energy. [1]

Energy of photon is given by, $E = hv$ (where h = Planck's constant and v = frequency of light)

The minimum energy required by the electron of a material to escape out of it, is work function ϕ. The additional energy acquired by the electron appears as the maximum kinetic energy K_{max} of the electron. i.e. $K_{max} = hv - \phi$

Or $hv = K_{max} + \phi$ [Einstein's Photoelectron Equation] [1]

$K_{max} = eV_o$

Salient features observed in photoelectric effect:

The stopping potential and hence the maximum kinetic energy of emitted electrons varies linearly with the frequency of incident radiation.

There exists a minimum cut off frequency vo, for which the stopping potential is zero.

Photoelectric emission is instantaneous. [1]

20. (1) A transmitting tower makes use of electromagnetic waves such as microwaves, exposure to which can cause severe health hazards like, giddiness, headache, tumour and cancer. Also, the transmitting antenna operates on a very high power, so the risk of someone getting severely burnt in a residential area increases. [1]

(2) By objecting to this move of her parents, Sunita has displayed awareness towards the health and environment of her society. [1]

(3) Range of the transmitting antenna.

$d = \sqrt{2hR}$

Here, h is the height of the transmitting antenna and R is the radius of the Earth.

R = 6400 km = 64×10^5 m

$d = \sqrt{2 \times 20 \times 64 \times 10^5}$

d = 16000m [1]

21. (a) Gamma rays are used for the treatment of certain forms of cancer. Their frequency ranges. [1]

(b) Welders wear special glass goggles while working so that they can protect their eyes from harmful electromagnetic radiation.[1]

(c) Infrared waves are often called as heat waves because they induce resonance in molecules and increase internal energy in a substance. [½]

Infrared waves are used in burglar alarms, security lights and remote controls for television and DVD players. [½]

22. (a) X-ray, Gamma(y) rays are used for the treatment of certain forms of cancer. Their frequency range is 1018m to 1022m. [1]

(b) The thin ozone layer on top of stratosphere absorb most of the harmful ultraviolet rays coming from the Sun towards the Earth. They include *UVA*, *UVB* and *UVC* radiations, which can destroy the life system on the Earth. Hence, this layer is crucial for human survival. [1]

(c) Momentum transferred = Energy Speed of light $= hvc = 10^{22}$ (for $v - 1020$ Hz)

Thus, the amount of the momentum transferred by the em waves incident on the surface is very small. [1]

23. (a) Microwaves are suitable for radar systems used in aircrafts navigation. The range of frequency for these waves is 109 Hz to 1012 Hz. [1]

(b) In the absence of atmosphere, there would be no greenhouse effect on the surface of the Earth. As a result, the temperature of the Earth would decrease rapidly, making it difficult for human survival. [1]

(c) An em wave carries a linear momentum with it. The linear momentum carried by a portion of wave having energy U is given by $p = UC$ [½]

Thus, if the wave incident on a material surface is completely absorbed, it delivers energy U and momentum $p = UC$ to the surface. If the wave is totally reflected, the momentum delivered is p = $2UC$ because the momentum of the wave changes from p to $-p$. Therefore, it follows that em waves incident on a surface exert a force and hence a pressure on the surface. [½]

24. (a) Microwaves are suitable for radar systems that are used in aircraft navigation.

These rays are produced by special vacuum tubes, namely -klystrons, magnetrons and Gunn diodes. [1]

(b) Infrared waves are used to treat muscular strain. [1]

These rays are produced by hot bodies and molecules.

(c) X-rays are used as a diagnostic tool in medicine. [1]

These rays are produced when high energy electrons are stopped suddenly on a metal of high atomic number.

25. Stopping potential: For a particular frequency of incident radiation, the minimum negative (retarding) potential V_o given to the anode plate for which the photocurrent stops or becomes zero is called the cut-off or stopping potential.

Threshold frequency: There exists a certain minimum cut-off frequency V_o for which the stopping potential is zero and below V_o the electron emission is not possible. This cut-off frequency is known as threshold frequency, V_o which is different for different metal. In photoelectric effect, an electron absorbs a quantum of energy (hν) of radiation. If this quantum of energy absorbed by electron exceeds the minimum energy required to come out of the metal surface by electron, the kinetic energy of the emitted electron is

$K = h\nu - \phi$... (1) [1.5]

where, ϕ is the minimum energy for electron to come out of the metal, and is different for different electrons in the metal. The maximum kinetic energy of photo electrons is given by ...

$K_{max} = h\nu - \phi_o$ (2)

where, ϕ_o – work function or least value of ϕ equation (2) is known as Einstein's photoelectric equation.

Explanation of photoelectric effect with the help of Einstein's photoelectric equation

(i) According to equation (2), k_{max} depends linearly on ν, and is independent of intensity of radiation. This happens because, here, photoelectric effect arises from the absorption of a single quantum of radiation by a single electron. The intensity of radiation (that is proportional to the number of energy quanta per unit area per unit time) is irrelevant to this basic process. [½]

(ii) Since k_{max} must be non-negative, equation (2) implies that photoelectric emission is possible only if $h\upsilon > \phi_o$ or $\upsilon > \upsilon_o$, where $\upsilon_o = \frac{\phi_o}{h}$

Thus, there exists a threshold frequency $\upsilon_o = \frac{\phi_o}{h}$ exists, below which photoelectric emission is not possible, and is independent of intensity. [½]

(iii) As intensity of radiation is proportional to the number of energy quanta per unit area per unit time. The greater the number of energy quanta available, the greater is the number of electrons absorbing the energy quanta and therefore, the number of electrons coming out of the metal (for $\upsilon > \upsilon_o$) is more and so is photoelectric current. [½]

26. According to Maxwell's theory accelerated charges radiate electromagnetic waves. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other, so to speak, as the wave propagates through the space. The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source – the accelerated charge. The electromagnetic radiation used are: [1 + 1]

(1) Infrared rays

(2) X-rays [1]

27. These waves are constituted by varying or oscillating electric and magnetic fields. The electric and magnetic fields are perpendicular to each other and are also perpendicular to the direction of propagation of the wave E is the envelope of electric intensity vector and B is the envelope of magnetic intensity vector. [1 + 1]

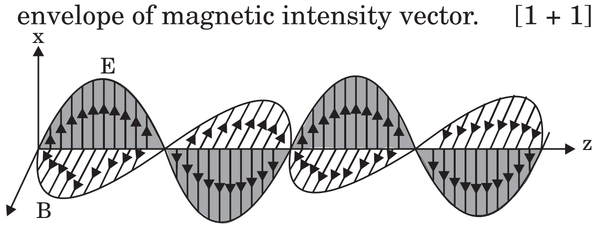


Fig.: Electromagnetic Waves [1]

OR

Correction in Amperes Circuital law (Modified Ampere's law) : Maxwell removed the problem of current continuity and inconsistency observed in Ampere's Circuital law by introducing the concept of displacement current, Displacement current arises due to change in electric flux with time and is given by $i_d = \varepsilon_o \frac{d\phi_E}{dt}$ [1]

Electric flux through the loop $\phi_E = EA$

$$\phi_E = \frac{\sigma}{\varepsilon_o} A = \frac{Q}{A}\frac{A}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

(Q = charge on either plates)

$$\phi_E = \frac{Q}{\varepsilon_o}$$

$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_o}\frac{d\phi}{dt}$$

$$\varepsilon_o \frac{d\phi_E}{dt} = \frac{dQ}{dt}$$ [1]

$\frac{dQ}{dt}$ is called conduction current which is equal to $\varepsilon_o \frac{d\phi_E}{dt}$, which is displacement current.

Hence, $i_c = i_d$

Generalization of Ampere's circuital law is:

$$\oint \vec{B}.d\vec{l} = \mu_o (i_c = i_d)$$

Conduction current is because of How of charges but displacement current is not because of How of charges but because of change in electric flux. [1]

28. Characteristic properties of photons:

(i) Energy of photon is directly proportional to the frequency (or inversely proportional to the wavelength). [1]

(ii) In photon-electron collision, total energy and momentum of the system of two constituents remains constant. [1]

(iii) In the interaction of photons with the free electrons, the entire energy of photon is absorbed. Cut off or stopping potential is that minimum value of negative potential at anode which just stops the photo electric current. For a given material, there is a minimum frequency of light frequency is called as threshold frequency. By Einstein's photo electric equation

$$KE_{max} = \frac{hc}{\lambda} - \varphi_o = h\nu - h\nu_o \text{ where } \varphi_0 = h\nu_0$$

$$eV_0 = h\nu - h\nu_0$$

$$V_o = \frac{h}{e}\nu - \frac{h}{e}\nu_o$$ [1]

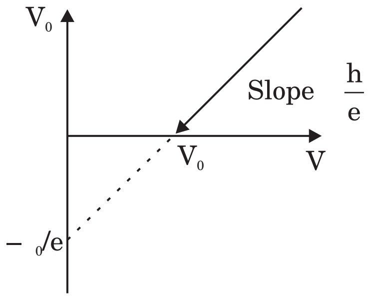


Fig.: Graph of V_0 V_s v

29. Electromagnetic waves are produced by accelerated or oscillating charges and do not need any medium for propagation.

The source of energy of these waves is energy of oscillating charge particle.

The mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis are:

$$E_x = E_o \sin(kz - \omega t)$$ [1]

$$B_y = B_o \sin(kz - \omega t)$$

Properties of electromagnetic waves:

1. The direction of E, B and direction of propagation of waves are mutually perpendicular to one another.
2. The amplitude ratio of E and B gives velocity of light.
3. Electromagnetic waves are not deflected by electric and magnetic fields. [1 + 1]

Smart Notes ....

CHAPTER 9

Ray Optics and Optical Instruments

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Reflection of light, spherical mirrors, mirror formula					1Q (3 marks)
Refraction of light, total internal reflection and its applications Optical fiber	1Q (5 marks)			1Q (3 marks)	
Refraction at spherical surfaces					
Lenses; thin lens formula, lens maker's formula; magnification, power of a lens, combination of thin lenses in contact		1Q (3 marks)	1Q (4 marks)		1Q (5 marks)
Refraction and dispersion of light through a prism		1Q (3 marks)	1Q (3 marks)	1Q (1 mark), 1Q (5 marks)	1Q (3 marks)
Scattering of light - blue color of sky and reddish appearance of the sun at sunrise and sunset		1Q (1 mark)			
Optical instruments: Microscopes and astronomical telescopes (reflecting and refracting) and their magnifying powers.	1Q (3 marks)				

On the basis of above analysis, it can be said that from exam point of view Biconvex lens, Dispersion of Prism, Reflecting Index of Glass, Prism Formula, Lens Maker's Formula and reflecting telescope are most important concepts of the chapter.

[Topic 1] Reflection, refraction and dispersion of light

Summary

The speed of light in vacuum is given by c = 3 × $10^8 ms^{-1}$, which is the highest speed that can be attained in nature.

A light wave travels along a straight line from one point to another. This path is called a ray of light, and bundle of such rays together form a beam of light.

Laws of reflection states that

- The angle of reflection (i.e., the angle between reflected ray and the normal to the reflecting surface or the mirror) equals the angle of incidence (angle between incident ray and the normal), i.e. $\angle i = \angle r$
- The incident ray, the normal to the mirror at the point of incidence and the reflected ray, they all lie in the same plane.

Snell's law for refraction is given by $\frac{\sin i}{\sin r} = n$,

where the angle of incidence, angle of refraction and refractive index of the medium is given by i, r and n respectively.

The angle of incidence at which a ray travelling from a denser to rarer medium makes an angle of refraction of 90° is a ***critical angle*** and is denoted by i_c.

Cartesian sign convention:

- Positive sign is used for distances measured in the same direction as the incident light, whereas negative sign is used for those measured in the direction opposite to the direction of incident light.
- All distances are measured from the pole of the mirror or the optical centre of the lens. The heights measured upwards with respect to x-axis and normal to the principal axis of the mirror/lens is taken as positive.
- The heights measured downwards with respect to x-axis are taken as negative.

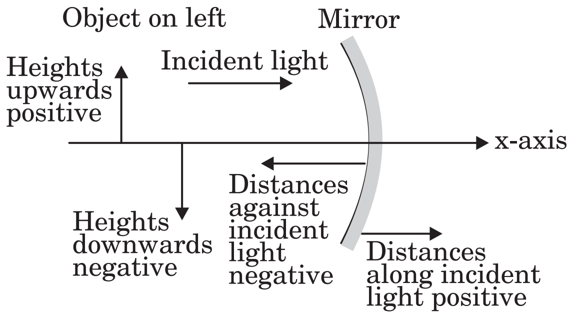


Fig.: Cartesian sign convention

If the distance of the object and the image is given by u and v, respectively and f is the focal length of the mirror. Then the ***mirror formula*** is given by,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Focal length f for a concave mirroris negative and is positive for a convex mirror.

The magnification produced by a mirror is given by $m = \frac{h'}{h} = -\frac{v}{u}$ where h′ is the height of the image and h is the height of the object.

Total Internal Reflection:

- When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called internal reflection when all light is reflected back, it is called total internal reflection.

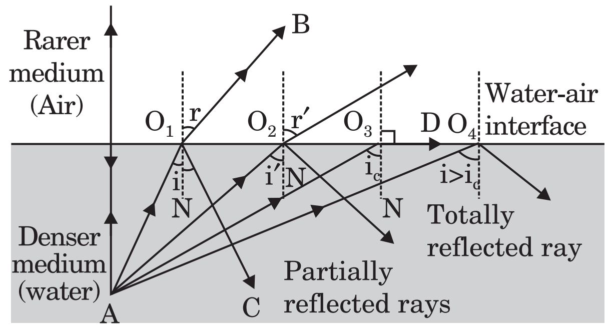


Fig.: Total Internal Reflection

- The applications of total internal reflection include mirage, diamond, prism and optical fibers.

Refraction through glass slab:

The emergent ray through a glass slab is parallel to the incident ray but it is laterally displaced.

Also, $\angle$Angle of incidence = $\angle$Angle of emergence

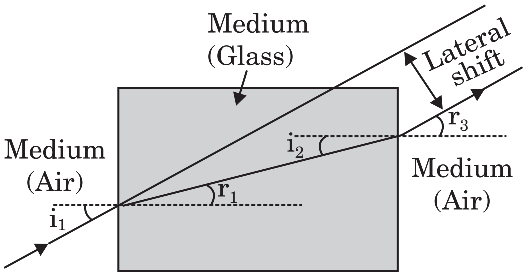


Fig.: Reflection through glass slab

Refraction at spherical surfaces

If the rays are incident from a medium of refractive index n_1 to another of refractive index n_2, then

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

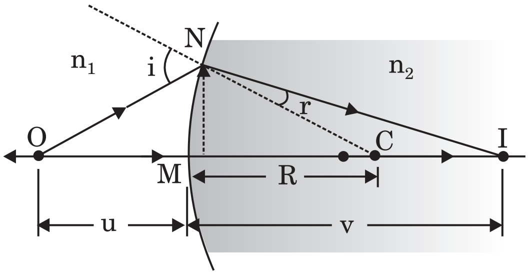


Fig.: Refraction at spherical surface

For a prism of the angle A, of refractive index n_2 placed in a medium of refractive index n_1 and D_m being the angle of minimum deviation.

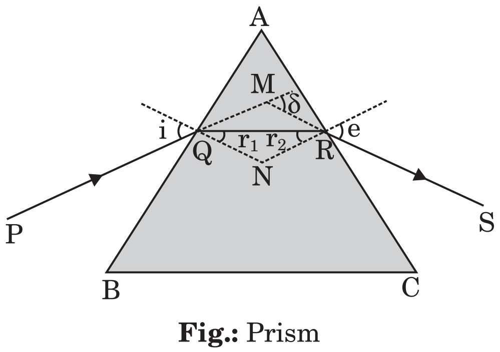


Fig.: Prism

$$n_{21}=\frac{n_2}{n_1}=\frac{sin\left[(A+D_m)/2\right]}{sin(A/2)}$$

- If the distance of the object and the image is given by u and v, respectively and f is the focal length of the lens. So, the ***lens formula*** is,

  $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

  Focal length f is positive for a converging lens and is negative for a diverging lens.
- The magnification produced by a mirror is given by $m=\frac{h'}{h}=\frac{v}{u}$ where h′ is the height of the image and h is the height of the object.
- The ***power*** (P) of a lens is given by, $P=\frac{1}{f}$.

  Where f is the focal length of the lens and the SI unit of power is dioptre (D): 1 D = 1 m^{-1}
- The effective focal length of a combination of thin lenses of focal length f_1, f_2, f_3 ..... is given by

  $$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+..........$$

  And the effective power of the same combination is given by

  $P = P_1 + P_2 + P_3$ ......

Dispersion:

- Splitting of light into its constituent colors is known as ***dispersion*** of light.
- When a white light is incident on a prism, the white light is split into seven components, violet, indigo, blue, green, yellow, orange and red (given by the acronym VIBGYOR)

Some natural phenomenon due to sunlight are rainbow and scattering of light.

The Eye: It has a convex lens of focal length about 2.5 cm. This focal length can be varied somewhat by the help of ciliary muscle so that the image is always formed on the retina. This ability of the eye of adjusting the muscle to form a clear image is called ***accommodation***.

In a defective eye, if the image is focused before the retina, it is called myopia. For correction of myopia, a diverging corrective lens is needed.

In a defective eye, if the image is focused beyond the retina, it is called hypermetropia. For correction of hypermetropia, a converging corrective lens is needed.

Astigmatism: A refractive error in which the vision is blurred at all distances, is corrected by using cylindrical lenses.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. When monochromatic light travels from one medium to another its wavelength changes but frequency remains the same. Explain.

[ALL INDIA 2011]

2. Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid?

[ALL INDIA 2012]

3. Write the relationship between angle of incidence '*i*', angle of prism '*A*' and angle of minimum deviations Δ_m for a triangular prism.

[DELHI 2013]

4. A convex lens is placed in contact with a plane mirror. A point object at a distance of 20 cm on the axis of this combination has its image coinciding with itself. What is the focal length of the lens? **[DELHI 2014]**

5. A biconcave lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens? Give reason.

[ALL INDIA 2014]

6. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? **[DELHI 2015]**

7. When light travels from an optically denser medium to a rarer medium, how does the critical angle of incidence depend on the colour of light?

[ALL INDIA 2015]

8. Why can't we see clearly through fog? Name the phenomenon responsible for it.

[ALL INDIA 2016]

9. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason.

[ALL INDIA 2017]

2 Mark Questions

10. A ray of light, incident on an equilateral prism $\left(\mu_g = \sqrt{3}\right)$ moves parallel to the base line of the prism inside it. Find the angle of incidence for this ray.

[ALL INDIA 2012]

11. A convex lens of focal length 25 m is placed co axially in contact with a concave lens of focal length 20 cm. Determine the power of the combination. Will the system be converging or diverging in nature?

[DELHI 2013]

12. (a) Write the necessary conditions for the phenomenon of total internal reflection to occur.

(b) Write the relation between the refractive index and critical angle for a given pair of optical media.

[DELHI 2013]

13. A convex lens of focal length f_1 is kept in contact with a concave lens of focal length f_2. Find the focal length of the combination.

[DELHI 2013]

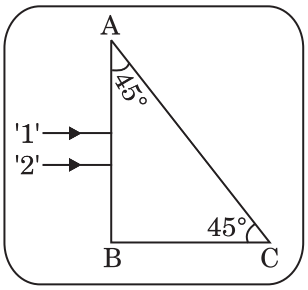
14. A convex lens of focal length 30 cm is placed coaxially in contact with a concave lens of focal length 40 cm. Determine the power of the combination. Will the system be converging or diverging in nature?

[ALL INDIA 2013]

15. A convex lens of focal length 20 cm is placed co axially in contact with a concave lens of focal length 25 cm. Determine the power of the combination. Will the system be converging or diverging in nature?

[DELHI 2013]

16. Two monochromatic rays of light are incident normally on the face *AB* of an isosceles right-angled prism *ABC*. The refractive indices of the glass prism for the two rays '1' and '2' are respectively 1.38 and 1.52. Trace the path of these rays after entering through the prism.

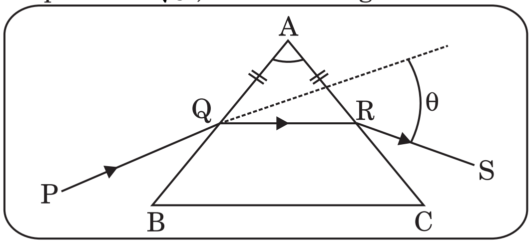


[ALL INDIA 2014]

17. Use the mirror equation to show that an object placed between *f* and 2*f* of a concave mirror produces a real image beyond 2*f*.

[DELHI 2015]

18. A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS such that $AQ = AR$. If the angle of prism $A = 60^\circ$ and refractive index of material of prism is $\sqrt{3}$, calculate angle θ.



[ALL INDIA 2016]

3 Mark Questions

19. A convex lens of focal length 20 cm is placed coaxially with a convex mirror of radius of curvature 20 cm. The two are kept 15 cm apart. A point object is placed 40 cm in front of the convex lens. Find the position of the image formed by this combination. Draw the ray diagram showing the image formation. **[ALL INDIA 2014]**

20. One day Chetan's mother developed a severe stomach ache all of a sudden. She was rushed to the doctor who suggested for an immediate endoscopy test and gave an estimate of expenditure for the same. Chetan immediately contacted his class teacher and shared the information with her. The class teacher arranged for the money and rushed to the hospital. On realizing that Chetan belonged to a below average income group family, even the doctor offered concession for test fee. The test was conducted successfully.

Answer the following questions based on the above information

(a) Which principle in optics is made use of in endoscopy?

(b) Briefly explain the values reflected in the action taken by the teacher.

(c) In what way do you appreciate the response of the doctor on the given situation?

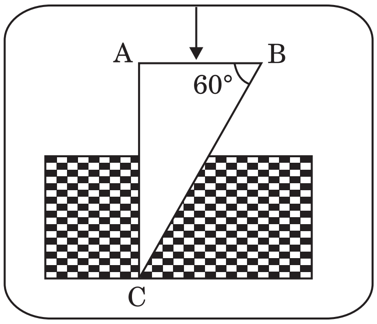
[ALL INDIA 2013]

21. (a) A mobile phone lies along the principal axis of a concave mirror. Show, with the help of a suitable diagram, the formation of its image. Explain why magnification is not uniform.

(b) Suppose the lower half of the concave mirrors reflecting surface is covered with an opaque material. What effect this will have on the image of the object? Explain.

[DELHI 2014]

22. (a) A ray of light is incident normally on the face AB of a right angled glass prism of refractive index. ${}_a\mu_g = 1.5$. The prism is partly immersed in a liquid of unknown refractive index. Find the value of refractive index of the liquid so that the ray grazes along the face BC after refraction through the prism.



(b) Trace the path of the rays if it were incident normally on the face AC.

[ALL INDIA 2015]

23. (a) Calculate the distance of an object of height h from a concave mirror of radius of curvature 20 cm, so as to obtain a real image of magnification 2. Find the location of image.

(b) Using mirror formula, explain why a convex mirror always produces a virtual image.

[DELHI 2016]

24. (i) A screen is placed at a distance of 100 cm from an object. The image of the object is formed on the screen by a convex lens for two different location of the lens separated by 20 cm. Calculate the focal length of the lens used.

(ii) A converging lens is kept coaxially in contact with a diverging lens-both the lenses being of equal focal length. What is the focal length of the combination?

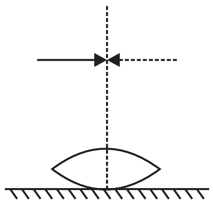
[ALL INDIA 2016]

25. (a) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If $\mu_w = 1.33$, find the wavelength, frequency and speed of the refracted light.

(b) A double convex lens is made of a glass of refractive index 1.55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm.

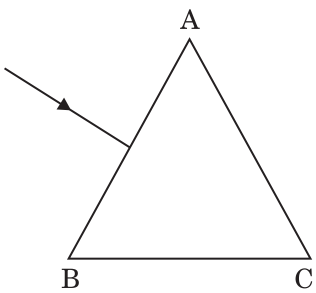
[ALL INDIA 2017]

26. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5,is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x. On removing the liquid layer and repeating the experiment, the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of x and y.



[ALL INDIA 2018]

27. The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index $\frac{3}{2}$, placed in water of refractive index $\frac{4}{3}$. Will this ray suffer total internal reflection on striking the face AC? Justify your answer.



[ALL INDIA 2018]

5 Mark Questions

28. (a) Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism. Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation.

(b) Explain briefly how the phenomenon of total internal reflection is used in fibre optics.

OR

(a) Obtain Lens Maker formula using the expression

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

Here the ray of light propagates from a rarer medium of refractive index (n_1) to a denser medium of refractive index (n_2) is incident on the convex side of spherical refracting surface of radius of curvature R.

(b) Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image found.

[ALL INDIA 2011]

29. (a) A point-object is placed on the principal axis of convex spherical surface of radius of curvature R, which separates the two media of refractive indices n_1 and n_2 ($n_2 > n_1$). Draw the ray diagram and deduce the relation between the distance of the object (u), distance of the image (v) and the radius of curvature (R for refraction to take place at the convex spherical surface from rarer to denser medium.

(b) Use the above relation to obtain the condition on the position of the object and the radius of curvature in terms of n_1 and n_2 when the real image is formed.

[ALL INDIA 2015]

30. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.

(b) Obtain the mirror formula and write the expression for the linear magnification.

(c) Explain two advantages of a reflecting telescope over a refracting telescope.

[ALL INDIA 2018]

Solutions

1. Refraction occurs when the energy of an incoming light wave matches the natural vibration frequency of the electrons in a material. The light wave penetrates deeply into the material, and causes small vibrations in the electrons. The electrons pass these vibrations on to the atoms in the material, and they send out light waves of the same frequency as the incoming wave.[1]

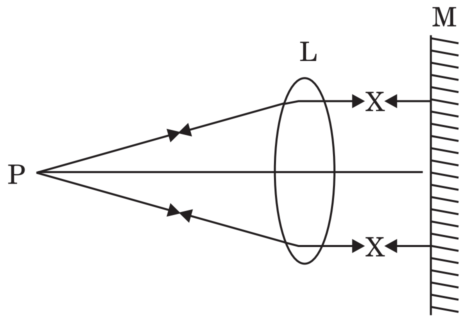
2. A biconvex lens will act like a plane sheet of glass if it is immersed in a liquid having the same index of refraction as itself. In this case the focal length. [1]

3. The relation between the angle of incidence i, angle of prism A, and the angle of minimum deviation Δ_m, for a triangular prism is given by

$$i = \frac{(A + \Delta_m)}{2}$$

[1]

4. The figure shows a convex lens L placed in contact with a plane mirror M. P is the point object, kept in front of this combination at a distance of 20 cm, from it. [½]



[½]

Fig.: Convex lens with a mirror

As the image coincides with itself, the rays from the object, after refraction from lens, should fall normally on the mirror M, so that they retrace their path. For this, the rays from P, after refraction from the lens must from a parallel beam perpendicular to M. For clarity, M has been shown at a small distance from L (in diagram). As the rays from P, form a parallel beam after refraction, P must be at the focus of the lens. Hence the focal length of the lens is 20 cm.

5. Biconcave lens will change its nature as refractive index of outside medium is greater than that of lens material so it will behave as converging lens. [1]

6. Since $(\mu_g)_{lens} < (\mu_m)_{surroundings}$ [1]

It behaves like a converging lens.

7. The critical angle for a given pair of medium is given by $\sin i_c = \frac{1}{\mu}$ where μ is refractive index of denser medium. The refractive index of a medium depends on the wavelength given by the relation $\mu = A + \frac{B}{\lambda^2}$, and each colour of light is associated with specific wavelength, so as the wavelength of light increases critical angle increases. [1]

8. Due to scattering phenomenon, we can't see clearly through fog. [1]

9. $\lambda_{red} > \lambda_{violet}$

we know $\lambda \propto \frac{1}{\mu}$

So,

$\mu_{red} > \mu_{violet}$ [½]

For minimum deviation

$$\mu = \frac{Sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

So, $(\delta_m)_{violet} > (\delta_m)_{Red}$ [½]

10. It is given that the prism is equilateral in shape. So, all the angles are equal to 60°.

Thus, the angle of prism, A = 60° The angle of refraction in case of a prism, $r = \frac{A}{2} = 30°$ [1]

Using Snell's law,

$\mu_a \sin i = \mu_g \sin r$

Here, $\mu_a \rightarrow$ refreactive index of air, $n_1 = 1$

$\mu_g \rightarrow$ refreactive index of air, $n_2 = \sqrt{3}$

i = angle of incidence

Thus,

$$\sin i = \left(\frac{\mu_g}{\mu_a}\right)\sin r = \left(\frac{\mu_g}{\mu_a}\right)\sin 30°$$

$$\sin i = \frac{\sqrt{3}}{2} = \sin 60°$$

So, the angle of incidence, $i = 60°$ [1]

11. We have focal length of convex lens,

f_1 = +25cm = + 0.25

Focal length of the concave lens,

f_2 = +25cm = + 0.25

Equivalent focal length,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{25} + \frac{1}{-20} = -\frac{1}{100}$$

∴ F = – 100 cm [1]

Power of convex lens,

$$P_1 = \frac{1}{f_1} = \frac{1}{0.25}$$

Power of concave lens,

$$P_2 = \frac{1}{f_2} = \frac{1}{-0.20}$$

Power of the combination,

$P = P_1 + P_2$

$$P = \frac{1}{0.25} + \frac{1}{-0.20} = \frac{100}{25} - \frac{100}{20}$$

$$= \frac{400-500}{100} = -\frac{100}{100} = -1$$

The focal length of the combination = –1m = –100cm As the focal length is negative, the system will be diverging in nature. [1]

12. (a) Necessary conditions for total internal reflection to occur are:

(1) The incident ray on the interface should travel in optically denser medium.

(2) The angle of incidence should be greater than the critical angle for the given pair of optical media. [1]

(b) $^{a}\mu_{b} = \dfrac{1}{\sin C}$

Where a and b are the rarer and denser media respectively. C is the critical angle for the given pair of optical media. [1]

13. $\dfrac{1}{f} = \dfrac{1}{+f_1} + \dfrac{1}{-f_2}$

$\dfrac{1}{f} = \dfrac{f_2 - f_1}{f_1 f_2}$

$f = \dfrac{f_1 f_2}{f_2 - f_1}$ [2]

14. We have,

Focal length of convex lens,

$f_1 = +30\text{cm} = +0.30\text{m}$

Focal length of concave lens,

$f_2 = -40\text{cm} = -0.40\text{m}$

Equivalent focal length,

$\Rightarrow \dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{1}{30} + \dfrac{1}{-40}$

$= \dfrac{40-30}{1200} = \dfrac{1}{120}$

$\therefore F = 120\text{cm} = 1.2\text{m}$ [1]

Power of the combination lens,

$P = \dfrac{1}{F} = \dfrac{1}{1.2} = 0.83D$

The focal length of combination

= 1.2 m = 120 m

As the focal length is positive the system will be converging in nature. [1]

15. We have,

Focal length of convex lens,

$f_1 = +20\text{cm} = +0.20\text{m}$

Focal length of concave lens,

$f_2 = -25\text{cm} = -0.25\text{m}$

Equivalent focal length,

$\therefore F$ 100cm

Power of convex lens,

$P_1 = \dfrac{1}{f_1} = \dfrac{1}{0.20}$

Power of concave lens,

$P_2 = \dfrac{1}{f_2} = \dfrac{1}{-0.25}$ [1]

Power of the combination lens,

$P = P_1 + P_2$

$= \dfrac{1}{0.20} + \dfrac{1}{-0.25}$

$= \dfrac{100}{20} + \dfrac{100}{-25}$

$= \dfrac{500-400}{100}$

$= \dfrac{100}{100} = 1D$

The focal length of the combination = 1 m = 100 cm. As the focal length is positive, the system will be converging in nature. [1]

16. $\sin i = \sin 45° = \dfrac{1}{\sqrt{2}} = \dfrac{1}{1.414}$

$\sin i = \dfrac{1}{1.414} > \dfrac{1}{1.52}$

$\therefore \sin i > \sin i_c$ for ray (2)

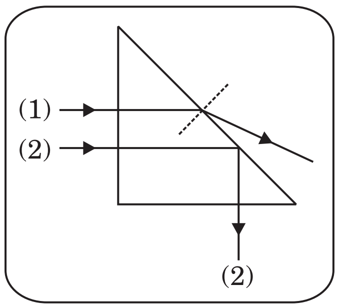
$\Rightarrow i > i_c$ for ray (2) [1]

So, ray (2) will suffer T.I.R.

As $\dfrac{1}{1.414} < \dfrac{1}{1.38}$

$\Rightarrow i > i_c$ (1)

So, ray (1) got refracted. [½]



[½]

Fig.: Trace of ray path through a prism.

17. Mirror equation is $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where, u is distance of object from the mirror v is the distance of image from the mirror and f is the focal length of the mirror.

For a concave mirror f is negative i.e. $f < 0$, $v < 0$

For a real object (on the left of mirror)

$\therefore 2f < v < f$ [1]

$\Rightarrow \frac{1}{2f} > \frac{1}{u} > \frac{1}{v} - \frac{1}{f} > \frac{1}{f} - \frac{1}{f}$

$\Rightarrow \frac{1}{2f} > \frac{1}{v} > 0$

$\because \frac{1}{v} > \frac{1}{f} > -\frac{1}{v}$

$\Rightarrow \frac{1}{2f} > \frac{1}{v} < 0$

$\Rightarrow v < 0$ Also, $v > 2f$

Image implies that v is negative and greater than $2f$. This means that the image lies beyond $2f$ and it is real. [1]

18. Given $AQ = AR$ hence in ΔAQR

$\angle Q = \angle R = \varphi$ (Let)

$A + 2\varphi = 180°$ (for ΔAQR)

$\Rightarrow \varphi = 60 \Rightarrow \angle r_1 = \angle r_2 = 830°$

This is the condition of minimum deviation

So using relation

$$\nu = \left(\frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}} \right)$$

[1]

Given $\nu = \sqrt{3}$ & $A = 60°$

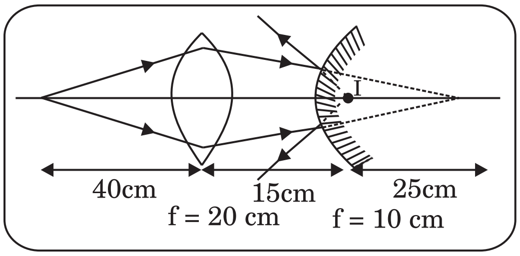
$$\nu = \left(\frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}} \right)$$

$$\sin\left(\frac{60+\delta_{min}}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\left(\frac{60+\delta_{min}}{2}\right) = 60$$

$\delta_{min} = 60°$ [1]

19.



[1]

'O' is at 2f of lens so it will form image at $2f$ i.e. 40 cm from lens so position of object for mirror is at (40 – 15) cm = 25cm behind the mirror For mirror [1]

$f = +10$ cm

$u = +25$ cm

$u = ?$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \frac{1}{25} = \frac{1}{+10}$

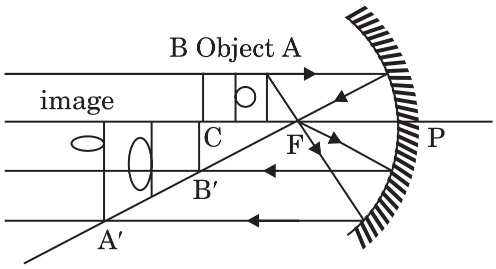
$\therefore v = +\frac{50}{3} cm$ *i.e.* $\frac{50}{3}$ behind the mirror [1]

20. (a) Total internal reflection.

(b) Teacher plays a good role to teach a moral in a perfect way to student how to support for humanity by presenting him as an example for students. [1]

(c) As Chetan belongs to below average income group family, keeping this in his/her mind, doctor offered concession for the test fee and served his oath for humanity as a doctor in a best possible way. [1]

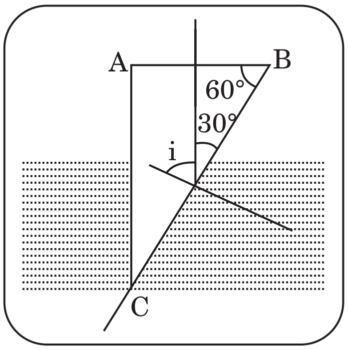
21. (a)



[1]

The image of the mobile phone formed by the concave mirror is shown in the above figure. The part of the mobile phone that is at C will form an image of the same size only at C. In the figure, we can see that $B'C = BC$. The part of the mobile phone that lies between C and F will form enlarged image beyond C as shown in the figure. It can be observed that the magnification of each part of the mobile phone cannot be uniform on account of different locations. That is why the image formed is not uniform. [1]

(b) As the laws of reflection are true for all points of the mirror, the height of the whole image will be produced. However, as the area of the reflecting surface has been reduced, the image intensity will be reduced. In other words, the image produced will be less bright. [1]



22. (a)

Given the refractive index of prism $\mu_g = 1.5$ the ray will grazes along the face BC when the angle of incidence i is equal to the critical angle for the glass and liquid interface here $i = 60°$ (from the fig.) [½]

and $sin\ i = \frac{\mu_l}{\mu_g}$

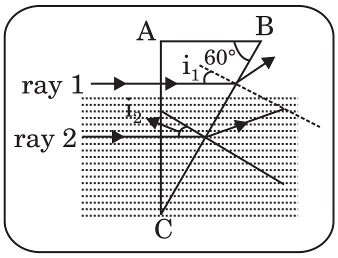
$\mu_i = \mu_g \sin i$

$\mu_l = 1.5 \sin 60°$

$\mu_l = 1.3$

So the refractive index of the liquid is 1.3 [½]

(b)



[½]

The angle of incidence for ray 1 and ray 2 on face BC is equal to 30°

For ray 1

The critical angle for glass-air interface is

$$\sin i_c = \frac{\mu_a}{\mu_g} = \frac{1}{1.5} = 0.66$$

$i_c = 41°$

Now since the angle of incidence is smaller than the critical angle refraction will take place as shown in figure

For ray 2

The critical angle for glass-liquid interface is

$$\sin i_c = \frac{\mu_l}{\mu_g} = \frac{1.3}{1.5} = 0.866$$

$i_c = 60°$ [½]

Now since the angle of incidence is smaller than the critical angle refraction will take place as shown in figure [½]

23. (a) Given, Height of object = h_o

Radius of curvature = 20cm

Magnification, $m = 2$

Object distance, u =?

Image distance, υ =?

Magnification,

$$M = \left|\frac{\upsilon}{u}\right| = \frac{h_i}{h_o}$$ [1]

$$2 = \frac{\upsilon}{u} \Rightarrow \upsilon = 2u \quad \text{(i)}$$

Using mirror formula,

$$\frac{1}{\upsilon} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-2u} - \frac{1}{u} = \frac{-2}{20}$$

$$\frac{+3}{2u} = \frac{+2}{20}$$

$$u = \frac{60}{4} = 15cm$$

Putting in (i) v = 1 × 15cm = 30 cm in front of the mirror.

$$\frac{h_i}{h_o} = \left|\frac{\upsilon}{u}\right|$$

$$\frac{h_i}{h_o} = \left|\frac{2u}{u}\right|$$ [1]

Height of image, $h_1 = 2h_o$

For convex mirror, f = +ve (always)

(b) Mirror formula,

$$\frac{1}{\upsilon} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{\upsilon} = \frac{1}{f} - \frac{1}{u}$$

As u = -ve(for real object)

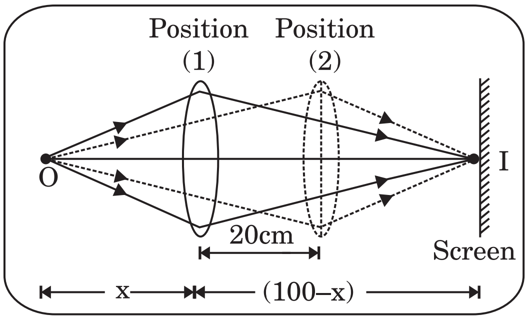
$$\frac{1}{\upsilon} = \frac{1}{f} - \left(\frac{1}{-u}\right)$$

$$\frac{1}{\upsilon} = \frac{1}{f} + \frac{1}{u}$$

υ = +ve [1]

Hence, it will form virtual image.

24. (i)



[1]

Lens at position (1)

Object distance, $u = -x$

Image distance, $u = (10 - x)$

Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{100-x} + \frac{1}{x} \text{ -------------- (i)}$$

Lens at position (2)

Object distance, $u = -(x + 20)$

Image distance, $v = -(20 - x)$ [1]

Using lens formula

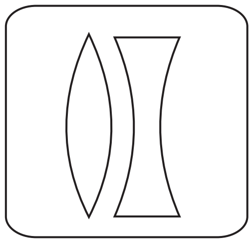
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{80-x} + \frac{1}{x+20} \text{ ---------- (ii)}$$

Solving equation (1) & (2)

f = 24 cm

(ii)



(both are of equal focal length = f)

equivalent power of combination

$P_{eqL} = P_1 = P_2$

$$P_{eqL} = \left(\frac{1}{+f} + \frac{1}{-f}\right)$$

$$P_{eqL} = 0 \Rightarrow f_{eqL} = \infty$$

[1]

Focal length of the combination is infinite.

25. (a) Given: Monochromatic light of wavelength

λ_1 = 589 nm = 589 × 10^{-9} m

Speed of light in air,

$v_1 = 3 \times 10^8 ms^{-1}$

Refractive index of water,

$\mu_w = \mu_2 = 1.33$

Refractive index of air, [½]

$\mu_a = \mu_1 = 1$

Find: wavelength of reflected light (λ_2) = ?

Frequency of refracted light (f_2) = ?

Speed of refracted light (v_2) = ?

We know, $\frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} = \frac{\mu_w}{\mu_a}$ [½]

$$\frac{589 \times 10^{-9}}{1.33} = 442.85 \times 10^{-9}$$

= 442.85 nm

For speed

$$\nu_2, \frac{\mu_w}{\mu_a} = \frac{\mu_2}{\mu_1} = \frac{\nu_1}{\nu_2}$$

$$\frac{1.33}{1} = \frac{3 \times 10^8}{\nu_2}$$

= 2.25 × 10^8 m/s

For frequency, $v_2 = f\lambda_2$

$$f = \frac{v_2}{\lambda_2} = \frac{2.25 \times 10^8}{442.85 \times 10^{-9}}$$

[½]

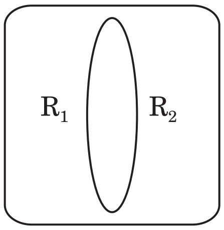
f = 5.08 × 10^{14} m/s

(b) Given the refractive index of glass with respect to air $a_{\mu g} = 1.55$

$a_{\mu g} = 1.55$

For double lens

$R_1 = R, R_2 = -R$



[½]

$\therefore$ both faces have same radius of curvature

[for double convex lens, one radius is taken as positive and other negative] focal length of lens,

f = + *20 cm*

Using lens formula

$$\frac{1}{f} = (a_{\mu g} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

in this formula put for a

$$\Rightarrow \frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$ [½]

$$\Rightarrow \frac{1}{20} = 0.55 \times \frac{2}{R}$$

$R = 0.55 \times 2 \times 20 = 22\text{cm}$

Thus the required radius of curvature is 22 cm [½]

26. f = focal length liquid + lens

f_1 = focal length of lens only

f_2 =focal length of liquid mirror

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$$\frac{1}{f_2} = \frac{1}{x} - \frac{1}{y}$$

$$f_2 = \frac{xy}{y - x}$$ [1]

Now, $\frac{1}{f_1} = (\mu - 1)\left(\frac{2}{R}\right)$ (Convex lens)

$$\frac{1}{y} = (1.5 - 1)\frac{2}{R}$$

$$\frac{1}{y} = \frac{0.5 \times 2}{R}$$

$R = y$

Now for liquid [1]

$$\frac{1}{f_2} = (\mu' - 1)\left(\frac{1}{R}\right)$$

$$-\frac{(y - x)}{xy} = (\mu' - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

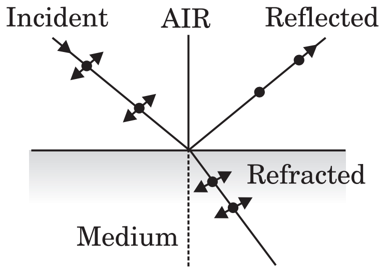
$$-\frac{(y - x)}{xy} = \frac{(\mu' - 1)}{y}$$

$$\therefore \frac{(x - y)}{x} = (\mu' - 1)$$

$$1 + 1 - \frac{y}{x} = \mu'$$

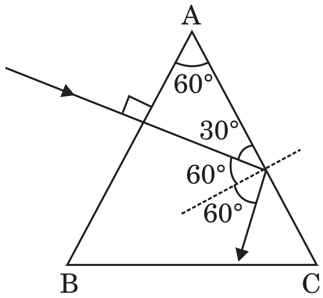
$$2 - \frac{y}{x} = \mu'$$ [1]

27. (a)



[1]

(b) No it will not suffer total internal reflection



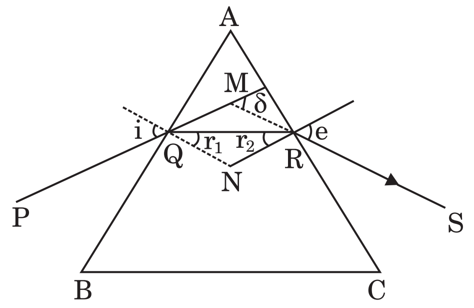
[1]

$$\mu_v = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

$$i = \sin^{-1}\mu_v = \sin^{-1}\left(\frac{8}{9}\right) = 62.73°$$

So if I is less than 60° then TIR will not happen. [1]

28. (a)



[1]

Fig.: Ray diagram to depict refraction of monochromatic ray through a glass prisms.

From $\Delta MQR, (i - r_1) + (e - r_2) = \delta$

So $(i + e) - (r_1 + r_2) = \delta$

From $\Delta RQN, r_1 + r_2 + \angle QNR = 180°$

Also, $A + \angle QNR = 180°$

Thus $A = r_1 + r_2$

So $i = e - A = \delta$

At minimum deviation, $i = e, r_1 = r$ and $\delta = \delta_m$

$$\Rightarrow i = \frac{A + \delta_m}{2} \text{ and } r = \frac{A}{2}$$

Also $\mu = \dfrac{\sin i}{\sin r}$ [1]

Hence $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$ [1]

(b)

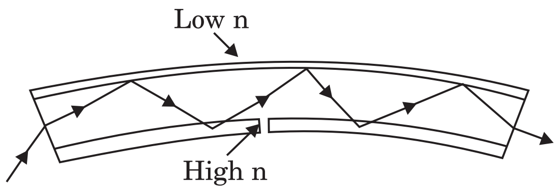


Fig.: Total internal reflection in fibre optics

Each optical fibre consists of a core and cladding, refractive index of the material of the core is higher than that of cladding. When a signal, in the form of light, is directed into the optical fibre, at an angle greater than the critical angle, it undergoes repeated total internal reflections along the length of the fibre and comes out of it at the other end with almost negligible loss of intensity. [1]

OR

(a)

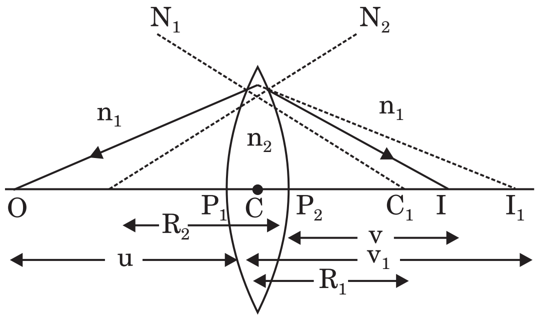


Fig.: Lens Maker Formula

For refraction at the first surface

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R_1} \text{ ------------- (1)}$$

For the second surface, I_1 acts as a virtual object (located in the denser medium) So, for refraction at this surface, we have

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{(n_1 - n_2)}{R_2} \text{ ---------- (2)}$$

Adding (1) and (2)

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{(n_2 - n_1)}{R_1} + \frac{(n_1 - n_2)}{R_2}$$

$$\frac{n_1}{v} - \frac{n_2}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

From the above two equations,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

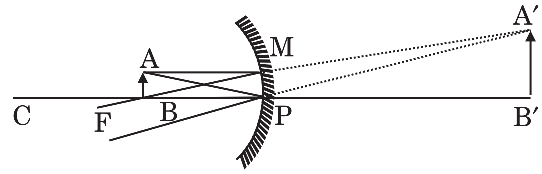
[1]

The point, where image of an object, located at infinity is formed, is called the focus F, of the lens and the distance *f* gives its focal length.

So for $u = \infty, v = +f$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(b)



ΔABP is similar to $\Delta A'B'P$

So, $\dfrac{A'B'}{AB} = \dfrac{B'P}{BP}$

Now, $A'B' = I$

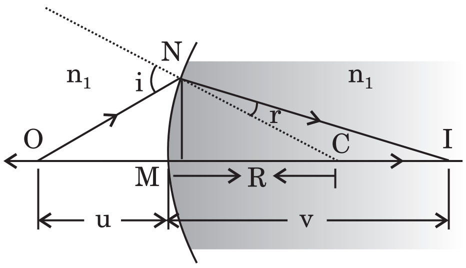
$AB = O$

$B'P' = v$

$BP = -u$

Magnification, $m = \frac{I}{O} = -\frac{v}{u}$ [2]

29. (a)



Refraction at spherical surface separating two media.

Figure shows the geometry of formation of image I of an object O on the principal axis of a spherical surface with centre of curvature C, and radius of curvature R. The rays are incident from a medium of refractive index n_1, to another of refractive index n_2. As before, we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. In particular, NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

We have, for small angles, [2]

$$\tan \angle NOM = \frac{MN}{OM}$$

$$\tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

Now, for ΔNOC, i is the exterior angle. Therefore $i = \angle NOM + \angle NCM$,

$$i = \frac{MN}{OM} + \frac{MN}{MC} \quad \text{-------- (1)}$$ [½]

Similarly,

$r = \angle NCM + \angle NIM$

i.e, $r = \frac{MN}{MC} - \frac{MN}{MI}$ -------- (2)

Now, by Snell's law

$n_1 \sin i = n_2 \sin r$

Or for small angles

$n_1 i = n_2 i$

Substituting i and r from Eqs.(1) and (2), we get

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad \text{---------- (3)}$$

Here, OM, MI and MC represent magnitudes of distances. Applying the Cartesian sign convention

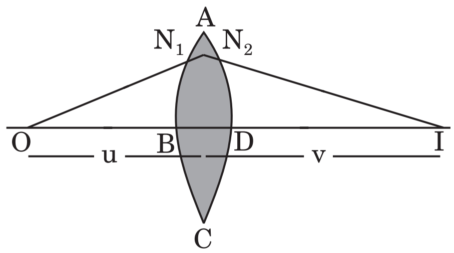
$OM = -u$, $MI = +v$, $MC + R$

Substituting these in Eq. (3), we get

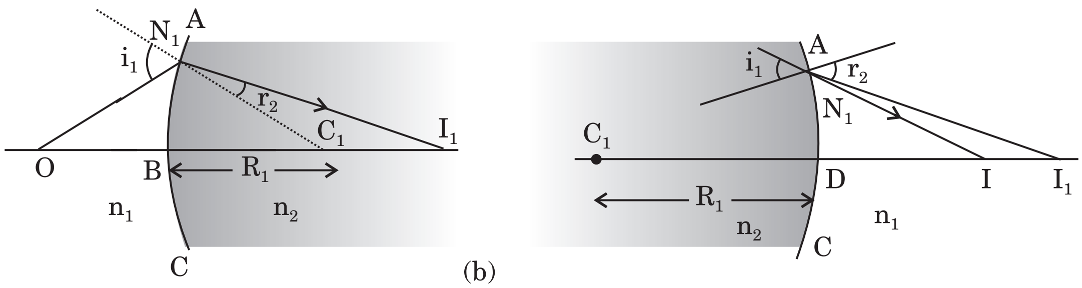
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \text{------------ (4)}$$

Equation (4) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface. [½]

(b)



(a)



(b)

(a) The position of object, and the image formed by a double convex lens

(b) Refraction at the first spherical surface and

(c) Refraction at the second spherical surface.

Figure (a) shows the geometry of image formation by a double convex lens. The image formation can be seen in terms of two steps: (i) The first refracting surface forms the image I_1 of the object O [Fig. (b)]. The image I_1 acts as a virtual object for the second surface that forms the image at I [Fig. (c)]. Applying

Equation $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ to the first interface ABC, [1]

We get,

$$\frac{n_1}{OB} + \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} \quad ..(1)$$

A similar procedure applied to the second interface ADC gives,

$$-\frac{n_2}{D_1} + \frac{n_1}{D_1} = \frac{n_2 - n_1}{DC_2} \quad ..(2)$$

For a thin lens, $BI_1 = DI_1$

Adding eqs.(1) and (2), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \quad ..(3)$$

Suppose the object is at infinity, i.e, $OB \to \infty$ and $DF = f$, Eq.(3) gives

$$\frac{n_1}{f} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \quad ...(4)$$

The point where image of an object placed at infinity is formed is called the focus F, of the lens and the distance f gives its focal length. By the sign convention,

$BC_1 = +R_1$,

$DC_1 = -R_2$

So, eq 4 can be written as

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ..(5)$$

$$\left(\therefore n_{21} = \frac{n_2}{n_2}\right)$$

Equation (5) is known as the lens maker's formula. It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature. Note that the formula is true for a concave lens also. In that case R_1 is negative, R_2 is positive and therefore, f is negative.

From Eqs. (3) and (4), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = \frac{n_1}{f} \quad ..(6)$$

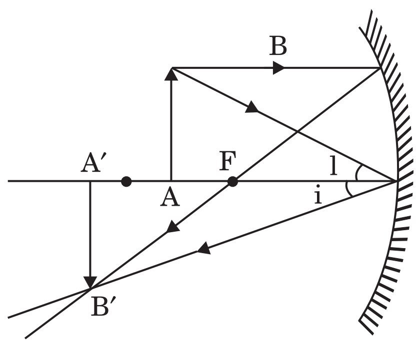
Again, in the thin lens approximation, B and D are both close to the optical centre of the lens. Applying the sign convention,

$BO = -u$, $DI = +v$, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ..(7)$$

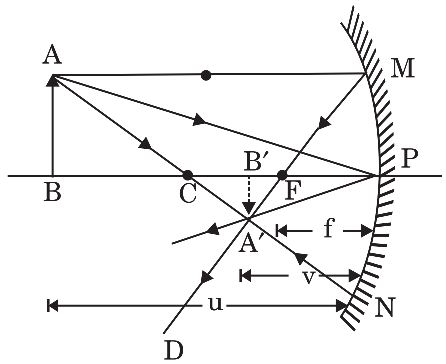
Equation (7) is the familiar thin lens formula.

30. (a)



[1]

(b)



[½]

Ray diagram for image formation by a concave mirror

Figure shows the ray diagram considering three rays. It shows the image $A'B'$(in this case, real) of an object AB formed by a concave mirror. It does not mean that only three rays emanate from the point A. An infinite number of rays emanate from any source, in all directions. Thus, point A' is image point of A if every ray originating at point A and falling on the concave mirror after reflection passes through the point A. [½]

We now derive the mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f). From figure, the two right-angled triangles $A'B'F$ and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP.) Therefore,

$$\frac{B'A'}{PM} = \frac{B'F'}{FP}$$

Or $\frac{B'A'}{BA} = \frac{B'F'}{FP}$ $(\because PM = AB)$ ----------- (i)

Since $\angle APB = \angle A'PB'$, the right angles triangle $A'B'P$ and ABP are also similar. Therefore, [½]

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \text{ ---------- (ii)}$$

Comparing equation (i) and (ii), we get

$$\frac{B'F}{FP} = \frac{B'P - FP}{FP} = \frac{B'P}{BP} \text{ ------------ (iii)}$$

Equation (iii) is a relation involving magnitude of distances. We now apply the sign convention. We note that light travels from the object to the mirror MPN. Hence this is taken as the positive direction. To reach the object AB, image $A'B'$as well as the focus F from the pole P, we have to travel opposite to the direction of incident light. Hence, all the three will have negative signs. Thus,

$B'P = -v$, $FP = -f$, $BP = -u$ [½]

Using these in equations, we get

$$\frac{-v + f}{-f} = \frac{-v}{-u}$$

Or $\frac{v - f}{f} = \frac{v}{u}$

This relation is known as the mirror equation. The size of the image relative to the size of the object is another important quantity to consider. We define linear magnification (m) as the ratio of the height of the image h' to the height of the object(h): [½]

$$m = \frac{h'}{h}$$

h and h' will be taken positive or negative in accordance with the accepted sign convention. In triangles $A'B'P$ and ABP, we have,

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

With the sign convention, this becomes

$$\frac{-h'}{h} = \frac{-v}{-u}$$

So that, $m = \frac{h'}{h} = -\frac{v}{u}$ [½]

(c)

1. Spherical and chromatic abbreviation eliminated.

2. Objective lenses are large and expensive in refracting telescope, whereas reflecting telescope is economical. [1]

[Topic 2] Optical Instrument

Summary

Simple microscope

- A simple magnifier or microscope is a converging lens of small focal length.

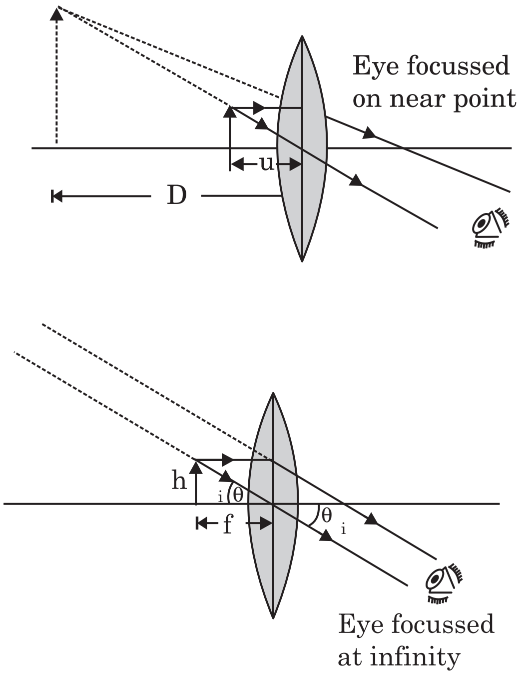


Fig.: Magnifier

- The magnifying power (m) is given by $m = 1 + \left(\frac{D}{f}\right)$, where D = 25 cm is the least distance of distinct vision and f is the focal length of the convex lens. If the image is at infinity, magnifying power (m) is given by, $m = \frac{D}{f}$.

- For a ***compound microscope***, the magnifying power m is given by $m = m_e \times m_0$ where, $m_e = 1 + \left(\frac{D}{f}\right)$ is the magnification due to the eyepiece and m_o is the magnification produced by the objective. Also, $m = \frac{L}{f_o} \times \frac{D}{f_e}$, where f_0 and f_e are the focal length of the objective and the eyepiece, respectively, and L is the distance between their focal points.

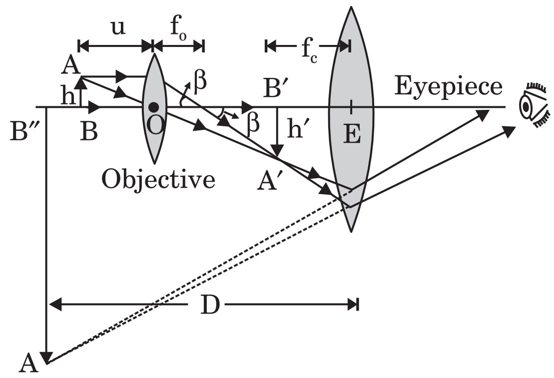


Fig.: Compound microscope

Telescope

- The telescope is used to provide angular magnification of distant objects. It also has an objective and an eyepiece
- Magnifying power (m) of a telescope is the ratio of the angle β subtended at the eye by the image to the angle α subtended at the eye by the object.

$m = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$, f_0 and f_e are the focal length of the objective and the eyepiece, respectively.

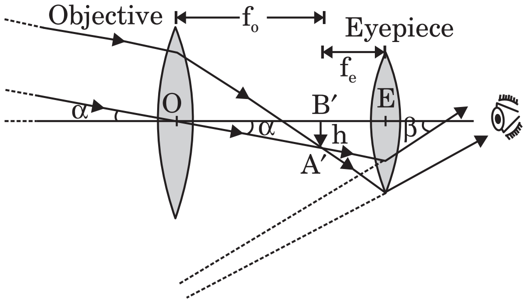


Fig.: Telescope

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

2 Mark Questions

1. Two convex lenses of same focal length but of aperture A_1 and A_2 where $A_1 < A_2$ are used as the objective lenses in two astronomical telescopes having identical eyepieces. What is the ratio of their resolving power? Which telescope will you prefer and why? Give reason.

[ALL INDIA 2011]

2. (i) A giant refracting telescope has an objective lens of focal length 15 m. If an eye-piece of focal length 1.0 cm is used, what is the angular magnification of the telescope? (ii) If this telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^8 m and the radius of lunar orbit is 3.8×10^8 m.

[DELHI 2015]

3 Mark Questions

3. Draw a ray diagram showing the image formation by a compound microscope. Hence obtain expression for total magnification when the image is formed at infinity.

[DELHI 2013]

4. Draw a labeled ray diagram of a refracting telescope. Define its magnifying power and write the expression for it.

Write two important limitations of a refractive telescope over a reflecting type telescope.

[DELHI 2013]

5. (a) Draw a labelled ray diagram showing the formation of a final image by a compound microscope at least distance of distinct vision.

(b) The total magnification produced by a compound microscope is 20. The magnification produced by the eye piece is 5. The microscope is focussed on a certain object. The distance between the objective and eye-piece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal length of the objective and the eye-piece. **[DELHI 2014]**

6. Draw a schematic ray diagram of reflecting telescope showing how rays coming from a distant object are received at the eyepiece. Write its two important advantages over a refracting telescope.

[DELHI 2016]

7. (a) Draw a ray diagram depicting the formation of the image by an astronomical telescope in normal adjustment

(b) You are given the following three lenses. Which two lenses will you use as an eyepiece and as an objective to construct an astronomical telescope? Give reason.

Lenses	Power	Aperture
L_1	3	8
L_2	6	1
L_3	10	1

[ALL INDIA 2017]

5 Mark Questions

8. Define magnifying power of a telescope. Write its expression.

A small telescope has an objective lens of focal length 150 cm and an eye piece of focal length 5 cm. if this telescope is used to view a 100 m high tower 3 km away, find the height of the final image when it is formed 25 cm away from the eye piece.

Or

How is the working of a telescope different from that of microscope?

The focal lengths of the objective and eye piece of a microscope are 1.25 cm and 5 cm respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 in normal adjustment.

[ALL INDIA 2012]

9. (a) Draw a labelled ray diagram showing the formation of image by a compound microscope in normal adjustment. Derive the expression for its magnifying power.

(b) How does the resolving power of a microscope change when

(i) the diameter of the objective lens is decreased.

(ii) the wavelength of the incident light is increased ?

Justify your answer in each case.

[ALL INDIA 2015]

Solutions

1. Resolving power $= \dfrac{a}{1.22\lambda}$

$$\Rightarrow \frac{(R.P)_1}{(R.P)_2} = \frac{A_1}{A_2}$$ [1]

The telescope with the objective of aperture A_1 should be preferred for viewing as because $A_1 > A_2$, this would:

(i) Give a better resolution

(ii) Have a higher light gathering power of telescope. [1]

2. (i) Let, f_o → Focal length of the objective lens = 15m = 1500cm

f_e → Focal length of the eye lens = .0m

Angular magnification of the giant refracting telescope is given by, [1]

$$m_o = \left|\frac{f_o}{f_e}\right|$$

(ii) Diameter of the image of the moon formed by the objective lens, $d = \alpha f_o$

$$\Rightarrow d = \frac{Diameter\ of\ the\ moon}{Radius\ 0f\ the\ lunar\ orbit} \times f_o$$

$$\Rightarrow d = \frac{3.48 \times 10^8}{3.8 \times 10^8} \times 1500 \times 10^{-2}$$

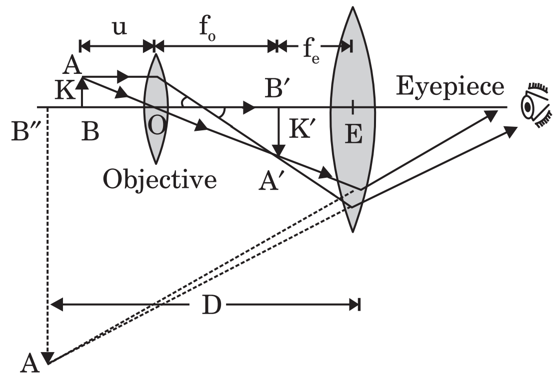
d = 1373.68 cm = 13.73m [1]

3. A compound microscope consists of two convex lenses parallel separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.

Magnifying power, when final image is at infinity: The magnification produced by the compound microscope is the product of the magnifications produced by the eyepiece and objective.

$\therefore M = M_e \times M_o$ [1]

Where, M_e and M_o are the magnifying powers of the eyepiece and objective respectively.



[1]

Fig.: Ray Diagram showing image formation by a compound microscope.

If u_o is the distance of the object from the objective and v_o is the distance of the image from the objective, then the magnifying power of the objective is

$$M_o = \frac{h'}{h} = \frac{L}{f_o}\left(\text{Using}, \tan\beta = \left(\frac{h}{f_o}\right) = \left(\frac{h'}{L}\right)\right)$$

Where, h, h are object and image heights respectively and f_o is the focal length of the objective.

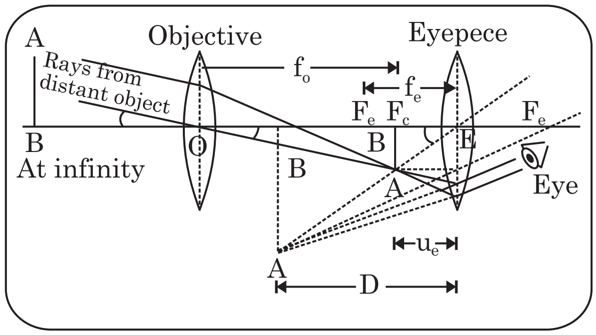
L is the tube length i.e. the distance between the second focal point of the objective and the first focal point of the eyepiece.

When the final image is at infinity, $M_e = \dfrac{D}{f_e}$

Magnifying power of compound microscope,

$$M = M_o \times M_e = \frac{L}{f_o} \times \frac{D}{f_e}$$ [1]

4.



[1]

Fig.: Astronominal Telescope

Magnifying power of an astronomical telescope is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object at the eye. If α and β, be the angles subtended by the object and image with the eye respectively, then

$$\text{M.P.} = \frac{\beta}{\alpha}$$

(a) For distinct vision

$$\text{M.P.} = \frac{-f_o}{f_c}\left(1+\frac{f_e}{D}\right)$$

(b) For normal vision, $u_e = f_e$

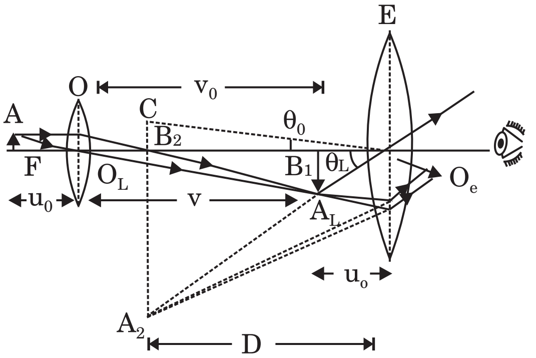
$$\text{M.P.} = -\left(\frac{f_o}{f_c}\right)$$ [1]

Limitations of refracting telescope:

(i) The refracting telescope suffers from spherical & chromatic aberrations.

(ii) Objective lens of refracting telescope of very large apertureis very difficult to manufacture. [1]

5. (a)



[1]

Fig.: Ray Diagram in a Compound Microscope

(b) For the least distance of clear vision, the total magnification is given by:

$$m = \frac{L}{f_o}\left(1+\frac{D}{f_e}\right) = m_o m_e$$

Where, L is the separation between the eyepiece and the objective

f_o is the focal length of the objective

f_e is the focal length of the eyepiece

D is the least distance for clear vision

Also, the given distance is for the eyepiece:

$$m_e = 5 = 1+\frac{D}{f_e}$$

$$\Rightarrow 5 = 1+\frac{20}{f_e}$$

$\Rightarrow f_e = = 5$ cm [1]

Substituting the value of m and m_e in equation (1), we get:

$m = m_o m_e$

$20 = m_o \times 5$

$m_o = 4$

Now, we have $|u_e| + |\upsilon_o| = 14$

(i.e. distance between objective and eye piece)

$u + v_o = 14$

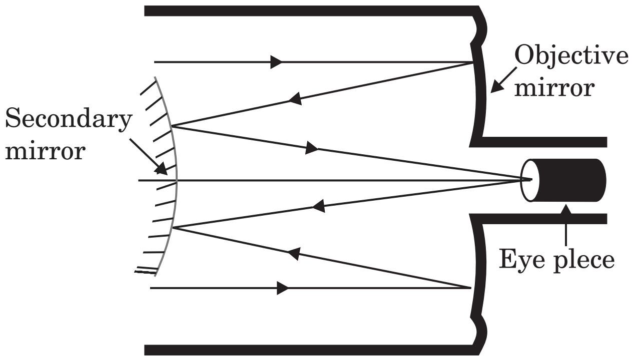
$v_o = 10$cm

Now, $m_o = 1 - \dfrac{\upsilon_o}{f_o}$

$-4 = 1 - \dfrac{10}{f_o}$

$f_o = 2$cm [1]

6. Reflecting Telescope: The reflecting telescope makes us of a concave mirror as objective. The rays of light coming from distant object are incident on the objective (parabolic reflective). After reflection the rays of light meet at a point where another convex mirror is placed. This mirror focusses light inside the telescope tube. The final image is seen through the eye piece. The images produced by the reflecting telescope are very bright and its resolving power is high. [1]



[1]

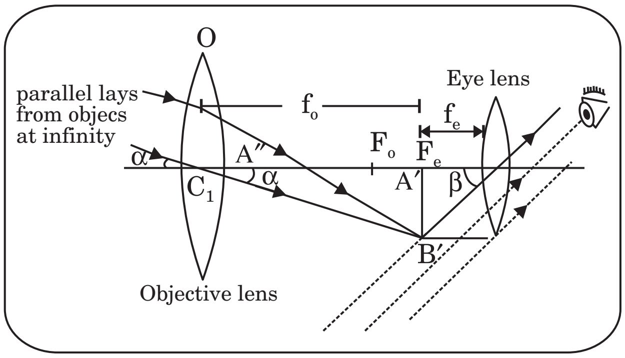
Fig.: Reflecting Telescope

Advantages:

(i) The resolving power (the ability to observe two objects distinctly) is high, due to the large diameter of the objective.

(ii) There is no chromatic aberration as the objective is a mirror. [1]

7. Ray diagram of Astronomical telescope :



[1]

Fig.: Ray diagram of astronomical telescope

(b) We use L_1 as an objective lens because it has higher aperture. That is 8 cm. So that it has high resolving power and lens L_3 use as an eye piece because it has high power. So that magnification is more. [2]

8. Magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the image formed at the least distance of distinct vision to the angle subtended at the eye by the object lying at the infinity, when seen directly.

Magnifying power, $M = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$

Where, f_o = Focal length of the objective = 150 cm

f_e = Focal length of the eye-piece = 5 cm

D = Least distance of distinct vision = 25 cm

$$M = -\frac{150}{5}\times\left(1+\frac{5}{25}\right) = -36$$

$$M = \frac{\beta}{\alpha}$$

$$M = \frac{\tan\beta}{\tan\alpha}$$ (As angles α and β are small)

$$\tan\alpha = \frac{\text{Height of the object}}{\text{Distance of object from objective}}$$ [2]

$$= \frac{H}{u} = \frac{100}{3000} = \frac{1}{30}$$

$$M = \frac{\tan\beta}{\frac{1}{30}}$$

$$\tan\beta = -\frac{36}{30}$$

$$\tan\beta = \frac{\text{Height of image}}{\text{Distance of image formation}} = \frac{H'}{D}$$

Thus, $H' = \frac{-36\times 25}{30} = -30\,\text{cm}$

Negative sign indicates that image formed will be inverted. [3]

Or

A microscope is used to look into smaller details i.e. structures of cells etc. on the other hand, a telescope is used to see larger objects that are too much far like stars, moon etc.

Telescope mainly focuses on collecting the light into the objective lens, which should thus be large, while the microscope already has a focus and the rest is blurred out. There is a big difference in their magnification factor.

For telescope the angular magnification is given by, $M = -\frac{f_o}{f_e}$

Where f_o = is the focal length of the objective lens and f_e is focal length of the eye-piece. For microscope the angular magnification is given by, [2]

$M = \left(1+\frac{D}{f_o}\right)$; When image is formed at distance of least distinct

$M = \frac{D}{f_o}$; When image is formed at infinity

Where D is the distance of least distinct vision and f_o is the focal length of objective lens.
Magnifying power of compound microscope

Given: f_o = 1.25, f_e = 5 cm and M = – 30

(magnifying power is negative)

We know that, $M = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$

Where, v_o = distance of image from the objective [2]

u_o = distance of object from the objective

D = distance of least distinct vision

$$-30 = \frac{v_o}{u_o}\left(1+\frac{25}{5}\right)$$

$v_o = -5\,u_o$

Using lens formula,

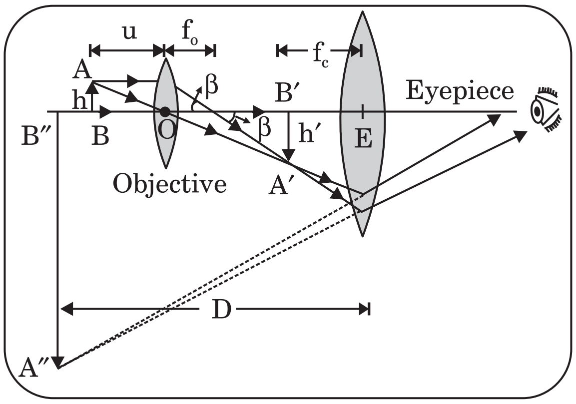
$$\frac{1}{f_o} = -\frac{1}{u_o}+\frac{1}{v_o}$$

$$\frac{1}{1.25} = -\frac{1}{u_o} - \frac{1}{5u_o}$$

$U_o = -1.5$ cm

Thus, the difference of object from objective is 1.5 cm. [1]

9. (a)



[1]

Fig.: Compound microscope

Ray diagram for the formation of image by a compound microscope.

A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope. A schematic diagram of a compound microscope is shown in Fig.. The lens nearest the object, called the objective, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the eyepiece, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object. We now obtain the magnification due to a compound microscope. The ray diagram of Figure shows that the (linear) magnification due to the objective, namely $\frac{h'}{h}$, equals

$$m_o = \frac{h'}{h} = \frac{L}{f_o}$$

[1]

Where we have used the result

$$\tan\beta = \left(\frac{h}{f_o}\right) = \left(\frac{h'}{L}\right)$$

Here h' is the size of the first image, the object size being h and f_o being the focal length of the objective. The first image is formed near the focal point of the eyepiece. The distance L, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e) is called the tube length of the compound microscope. As the first inverted image is near the focal point of the eyepiece, we use the result for the simple microscope to obtain the (angular) magnification me due to it equation $m = \left(1 + \frac{D}{f}\right)$, when the final image is formed at the near point, is [1]

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

When the final image is formed at infinity, the angular magnification due to the eyepiece is

$$m_e = \left(\frac{D}{f_e}\right)$$

Thus, the total magnification, when the image is formed at infinity, is

$$m = m_o m_e = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_e}\right)$$

[1]

Clearly, to achieve a large magnification of a small object (hence the name microscope), the objective and eyepiece should have small focal lengths.

(b) The resolving power of a microscope is given by the relation

$$RP = \frac{1}{d_{\min}} = \frac{2n\sin\beta}{1.22\lambda}$$

(i) When the diameter of the objective lens is decreased β decreases so resolving power decreases.

(ii) When the wavelength of the incident light is increased resolving power decreases. [1]

Smart Notes ....

Smart Notes ....

CHAPTER 10

Wave Optics

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Wave front and Huygen's principle		1Q (3 marks)		1Q (5 marks)	1Q (2 marks)
Reflection and refraction of plane wave at a plane surface using wave fronts					
Proof of laws of reflection and refraction using Huygens's principle. Interference					
Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light	1Q (5 marks)	1Q (5 marks)	1Q (5 marks)	1Q (2 marks)	1Q (3 marks), 1Q (3 marks)
Diffraction due to a single slit; width of central maximum					
Resolving power of microscopes and astronomical telescopes			1Q (3 marks)	1Q (3 marks)	
Polarisation, plane polarised light, Brewster's law; uses of plane polarised light and Polaroids	1Q (2 marks)	1Q (5 marks)	1Q (5 marks)		

On the basis of above analysis, it can be said that from exam point of view Young's Double Slit Experiment, Wavefront and Huygen's Principle are most important concepts of the chapter.

[Topic 1] Huygens Principle

Summary

- The effects which depend on wave nature of light are included under wave optics. Interference and diffraction of light shows that light behaves as wave and not as a stream of particles.
- ***Huygens principle:*** It states that each point of the wavefront is the source of a secondary disturbance. Also, the wavelets emanating from these points spread out in all directions with the speed of the wave which are referred to as secondary wavelets and if we draw a common tangent to all these spheres, a new position of the wavefront is obtained at a later time.
- When a wave gets refracted into a denser medium the wavelength and the speed of propagation decrease but the frequency remains the same.

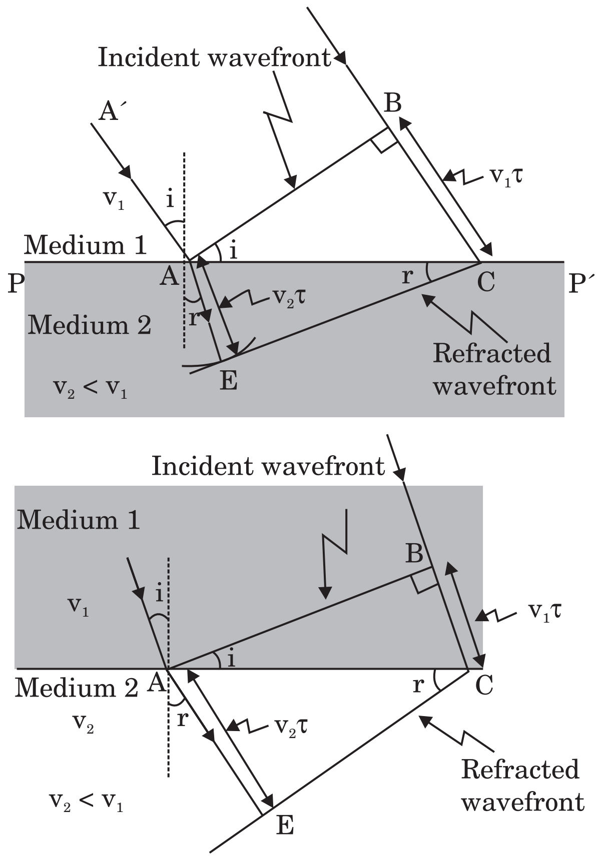


Fig.: Huggen's Principle

$n_1 \sin i = n_2 \sin r$ is the ***Snell's law*** of refraction.

- Doppler Effect is defined as the change in wavelength or frequency of a wave in relation to observer who is moving relative to the wave source. The Doppler shift is expressed as:

$$\frac{\Delta \nu}{\nu} = -\frac{\upsilon_{radial}}{c}$$

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. In a transistor, doping level in base is increased slightly. How will it affect
(1) Collector current and
(2) Base current? **[ALL INDIA 2011]**

2. What type of wave front will emerge from a (i) point source, and (ii) distant light source? **[DELHI 2017]**

2 Mark Questions

3. Define a wavefront. Using 'Huygens' principle, draw the shape of a refracted wavefront, when a plane wavefront is incident on a convex lens. **[ALL INDIA 2015]**

3 Mark Questions

4. Use Huygen's principle to verify the laws of refraction. **[ALL INDIA 2011]**

4 Mark Questions

5. Use huygen's principle to explain the formation of diffraction pattern due to a single slit illuminated by a monochromatic source of light. When the width of the slit is made double the original width, how would this affect the size and intensity of the central diffraction band? **[ALL INDIA 2012]**

5 Mark Questions

6. (a) Define wave front. Use Huygens' principle to verify the laws of refraction.
(b) How is linearly polarised light obtained by the process of scattering of light? Find the Brewster angle for air – glass interface, when the refractive index of glass = 1.5 **[ALL INDIA 2017]**

7. (a) Define a wavefront. Using Huygens' principle, verify the laws of reflection at a plane surface.

(b) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band? Explain.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain why.

[ALL INDIA 2018]

Solutions

1. If doping level of base is increased, the resistivity of the base will reduce.

(1) The collector current (I_C) will decrease. Consequently the current gain of the transistor, $\frac{I_C}{I_B}$ will decrease. [½]

(2) As a result the base current (I_B) will increase. Since, $I_E = I_B + I_C$ [½]

2. (i) For point source, wave front will be spherical. [½]

(ii) For a distant light source, the wave fronts will be plane wave fronts. [½]

3. The locus of points, which oscillate in phase is called a wavefront; thus a wavefront is defined as a surface of constant phase. [1]

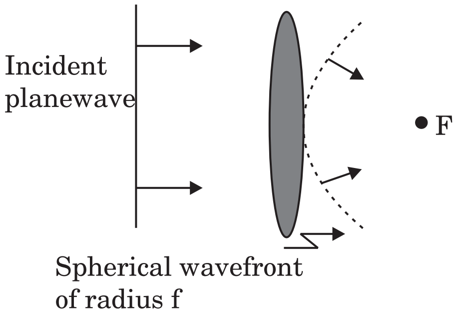


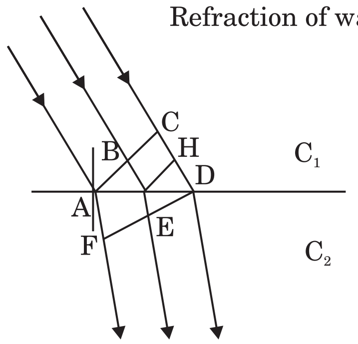
Fig.: Reflected wavefront when a plane wave front is incident on convex lens

Given above, we consider a plane wave incident on a thin convex lens, the emerging wave front is spherical and converges to the point F which is known as the focus. [1]

4. Laws of refraction: Proof using Huygen's principle

If AC is the incident wavefront and FD is refracted wavefront then we can show that refraction of wavefront obeys Snell's law.

Refraction of wavefront



[1]

Fig.: Law of refraction

Let the time taken by to reach from C to D be t and now in same time point A would be at F hence we can write,

$CD = C_1 t$ and $AF = C_2 t$

Also the angle CAD= angle i

[Angle of incidence for wavefront]

Similarly, angle ADF= angle r [1]

Now, $\sin i = \sin\angle CAD = \frac{CD}{AC} = C_1\frac{t}{AC}$

And, $\sin r = \sin\angle FDA = \frac{AF}{FD} = C_2\frac{t}{FD}$

We get the ratio $\frac{\sin i}{\sin r} = \frac{c_1}{c_2} = {}_1\alpha^2$ replace ${}_1\mu_2$

This verifies the law of refraction or Snell's law. [1]

5. Consider a parallel beam of monochromatic light is incident normally in a slit of width b (as shown in figure). According to Huygens's principle every point of slit acts as a source of secondary wavelets spreading in all directions. Screen is placed at a larger distance.

Consider a particular point P on the screen receives waves from all the secondary sources. All these waves start from different point of the slit and interface at point P to give resultant intensity. [1]

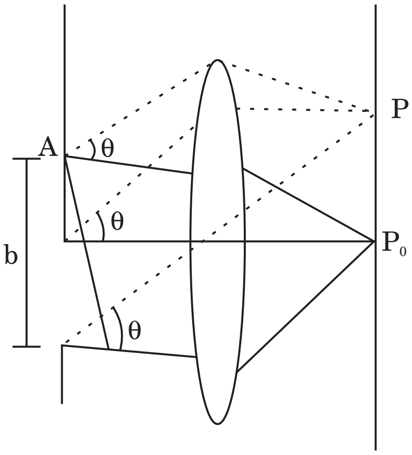


Fig.: Single Slit Experiment

[1]

Point P_o is at bisector plane of the slit. At P_o, all waves are traveling in equal optical path. So are in phase the waves thus interfere constructively with each other and maximum intensity is observed. As we move from P_o, the wave arrives with different phase and intensity is changed. Intensity at point P is given by: [1]

$$I = I_o \frac{\sin^2 \alpha}{\alpha}$$

Where, $\alpha = \frac{\pi}{2} b \sin\theta$

For central maxima α = 0; thus, $I = I_o$

When the width of slit is doubled the original width intensity will get four times of its original value. Width of central maxima is given by,

$$\beta = \frac{2D\lambda}{b}$$

Where, D = Distance between screen and slit

λ = wavelength of the light,

b = size of slit

So, with the increase in size of the slit the width of central maxima decreases. Hence, double the size of slit would result in half the width of the central maxima. [1]

6. (a) Wave front: where light is emitted from a source, then the particles present around it begins to vibrate, the locus at all such particles which are vibrating in the same phase is termed as wave front.

Laws of refraction: Suppose when distribution from point P on incident wave front reaches point P on the refracted wave front the disturbance from point Q reaches the point Q or the refracting surface XY. Since, A 'Q' P' represents the refracted wave front the time takes by light to travel from a point on incident wave front to the corresponding point on refracted wave front would always be the same. Now, time taken by light to go from r to Q' will be [1]

$$t = \frac{Qk}{c} + \frac{kQ'}{v} \quad \dots \text{(i)}$$

(Where c and v are the velocities of light in two medium)

In right angled ΔAQk, $\Delta QAk = I$,

$Qk = Ak \sin i$ … (ii)

In right angle$\Delta P'Q'$,$\Delta QP'Q'k = r$,

KQ'= KP' sin r … (iii)

Substituting eq. (ii) and (iii) in eq. (i), we get

$$t = \frac{Ak \sin i}{c} + \frac{KP' \sin r}{\nu}$$

or $$t = \frac{Ak \sin i}{c} + \frac{(AP' - Ak)\sin r}{\nu} \quad \dots \text{(iv)}$$

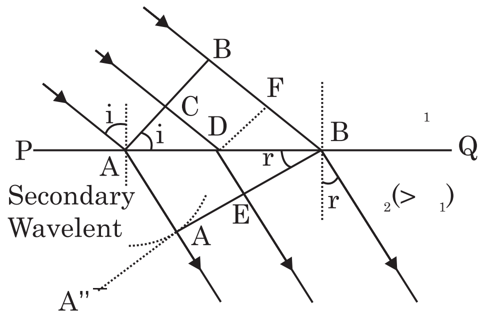
The rays from different points or the president wave front will take the same time to reach the corresponding points on the refracted wave front i.e., given by equation (iv) is independent of Ak. will happens so, if

$$\frac{\sin i}{c} - \frac{\sin r}{v} = 0$$

$$= \frac{\sin i}{\sin r} = \frac{c}{v}$$ [1]

However, $\frac{c}{v} = n$

This is the shell's law for refraction of light.



[1]

Fig.: Huygen's Principle

(b) **Polarization by scattering:** Polarization also occurs when light is scattered while travelling though of medium. When light strikes the atoms of a material if will often set the electrons of those atoms into vibrations the vibrating electrons then produce their own electromagnetic wave that is radiated outward in all directions. These vibrating electrons produce another electron magnetic wave that is once more radiated outward in all directions. This absorption and refraction of light waves causes the light to be scattered about the medium.

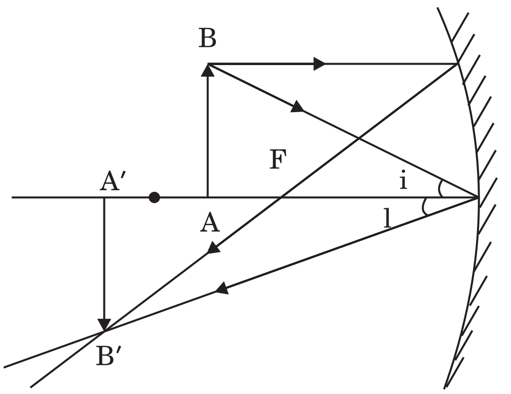
This process of scattering contributes to the blueness of our sky. This scattered light is partially moralized. [2]

Here, refractive index $m = 1.5$ $i_B = ?$

From Brewster's law $\tan i_B = m = 1.5$

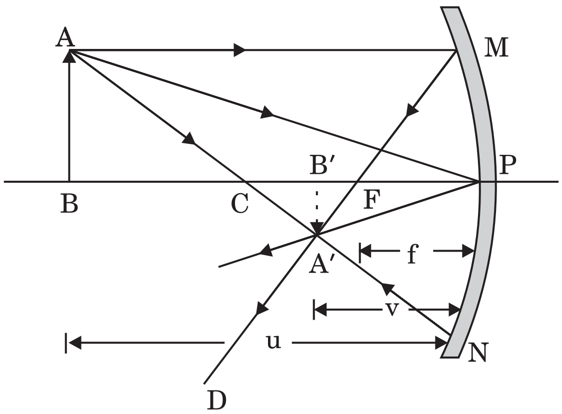
$i_B = \tan^{-1}(1.5) = 56.3°$

7. (a)



[½]

(b)



[½]

Figure shows the ray diagram considering three rays. It shows the image *A'*B '(in this case, real) of an object *AB* formed by a concave mirror. It does not mean that only three rays emanate from the point *A*. An infinite number of rays emanate from any source, in all directions. Thus, point *A'* is image point of *A* if every ray originating at point *A* and falling on the concave mirror after reflection passes through the point *A*. [1]

We now derive the mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f). From Figure, the two right-angled triangles *A'B'F* and *MPF* are similar. (For par axial rays, *MP* can be considered to be a straight line perpendicular to *CP*.) Therefore,

$$\frac{B'A'}{PM} = \frac{B'F'}{FP}$$

Or $\frac{B'A'}{BA} = \frac{B'F'}{FP} \left(\because PM = AB\right)$ ----------- (i)

Since $\angle APB = \angle A'PB'$, the right angles triangle *A'B'P* and *ABP* are also similar. Therefore,

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \text{ ---------------- (ii)}$$

Comparing equation (i) and (ii), we get [½]

$$\frac{B'F}{FP} = \frac{B'P - FP}{FP} = \frac{B'P}{BP} \text{ --------------- (iii)}$$

Equation (iii) is involving magnitude of distances. We now apply the sign convention. We note that light travels from the object to the mirror *MPN*. Hence this is taken as the positive direction. To reach the object *AB*, image *A'B'* as well as the focus *F* from the pole *P*, we have to travel opposite to the direction of incident light. Hence, all the three will have negative signs. Thus,

$B'P = -\nu$, $FP = -f$, $BP = -u$

Using these in equations, we get

$$\frac{-\nu + f}{-f} = \frac{-\nu}{-u}$$

Or $\frac{\nu - f}{f} = \frac{\nu}{u}$ [½]

This relation is known as the mirror equation. The size of the image relative to the size of the object is another important quantity to consider. We define linear magnification (m) as the ratio of the height of the image 'h' to the height of the object (h):

$$m = \frac{h'}{h}$$

h and h' will be taken positive or negative in accordance with the accepted sign convention. In triangles *A' B' P* and *ABP*, we have,

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$ [1]

With the sign convention, this becomes

$$\frac{-h'}{h} = \frac{-\nu}{-u}$$

So that, $m = \frac{h'}{h} = -\frac{\nu}{u}$

(c)

1. Spherical and chromatic abbreviation eliminated.
2. Objective lenses are large and expensive in refracting telescope, whereas reflecting telescope is economical. [1]

[Topic 2] Interference of Light

Summary

- ***Superposition principle*** states that at a particular point in the medium, the resultant displacement produced by a number of waves is the vector sum of the displacements produced by each of the waves.
- The resultant displacement at a point from two coherent sources will be equal to the sum of the individual displacement at that point.

  $y = 2a \cos \omega t$

  Resultant intensity is four times the intensity produced by each source.

  $I = 4I_0$ and $I_0 \propto a^2$
- ***Constructive interference:*** It is observed in cases when two coherent sources are vibrating in phase having path difference for a point P as $S_1P - S_2P = n\lambda$ ($n = 0, 1, 2, ...$) and resultant intensity is $4I_0$
- ***Destructive interference:*** It is observed in cases when two coherent sources are vibrating in phase having path difference for a point P as

  $S_1P - S_2P = \left(n + \frac{1}{2}\right)\lambda \quad (n = 0, 1, 2, ...)$ and

  resultant intensity is zero.
- ***Young's double slit*** of length d gives equally spaced fringes which are at angular separation $\frac{\lambda}{d}$. The midway-point of the slits, the central bright fringe and the source, all lie in a straight line. But this fringe gets destroyed by an extended source, if the angle subtended is more than $\frac{\lambda}{d}$ at the slits.

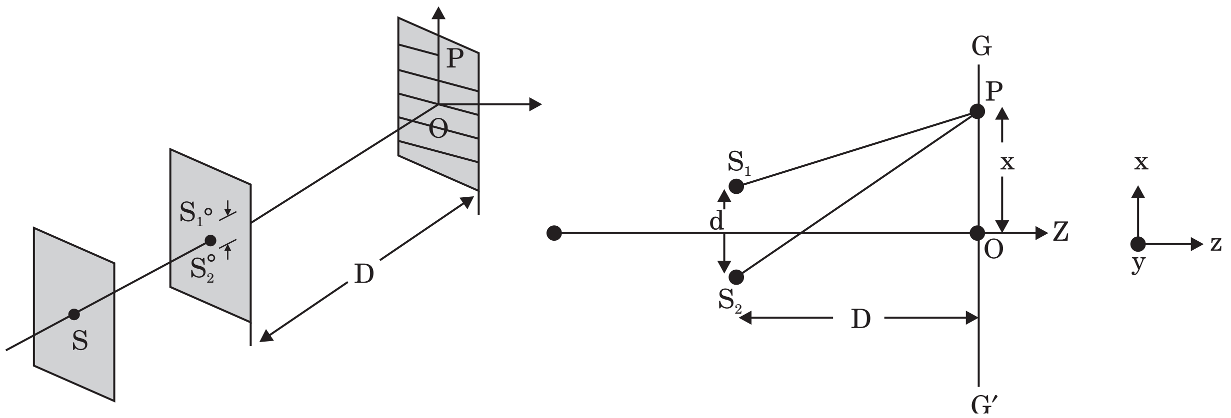


Fig.: Young's Double Slit Experiment

- Path difference, $y = \frac{n\lambda D}{d}$
- Fringe width: Distance between two consecutive bright and dark fringes represented by $\frac{\lambda D}{d}$

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. When monochromatic light travels from one medium to another its wavelength changes but frequency remains the same, Explain.

[DELHI 2012]

2 Mark Questions

2. A parallel beam of light of 450 nm falls on a narrow slit and resulting diffraction pattern is observed on a screen 1.5 m away. It is observed that the first minimum is at a distance of 3 mm from the centre of the screen. Calculate the width of the slit.

[ALL INDIA 2013]

3. For a single slit of width a, the first minimum of the diffraction pattern of a monochromatic light of wavelength k occurs at an angle of k/a. At the same angle of k/a, we get a maximum for two narrow slits separated by a distance 'a'. Explain.

[DELHI 2014]

4. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns.

[ALL INDIA 2017]

4 Mark Questions

5. Describe Young's double slit experiment to produce interference pattern due to a monochromatic source of light. Deduce the expression for the fringe width.

[ALL INDIA 2011]

6. (a) Why are coherent sources necessary to produce a sustained interference pattern?
   (b) In Young's double slit experiment using monochromatic light wavelength 'λ', the intensity of light at a point on the screen where the path differences is 'λ', is K units. Find out the intensity of light at a point where path difference is $\frac{\lambda}{3}$.

[ALL INDIA 2012]

7. (a) Write two characteristic features distinguishing the diffraction pattern from the interference pattern fringes obtained in Young's double slit experiment.
   (b) Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place due to a single slit of aperture 1×10^{-4} m. The distance between the slit and the screen is 1.8 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.

[DELHI 2013]

8. (a) In what way is diffraction from each slit related to the interference pattern in a double slit experiment.
   (b) Two wavelengths of sodium light 590 nm and 596 nm are used, in turn to study the diffraction taking place at a single slit of aperture 2×10^{-4}m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.

[DELHI 2013]

9. Answer the following questions:
   (a) In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1°. Find the spacing between the two slits.
   (b) Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected?

[DELHI 2015]

10. In Young's double slit experiment, the two slits are separated by a distance of 1.5 mm and the screen is placed 1 m away from the plane of the slits. A beam of light consisting of two wavelengths 650 nm and 520 nm is used to obtain interference fringes. Find :
    (a) the distance of the third bright fringe for λ = 520 nm on the screen from the central maximum.
    (b) the least distance from the central maximum where the bright fringes due to both the wavelengths coincide.

[ALL INDIA 2015]

11. In a single slit diffraction experiment, when tiny circular obstacle is placed in path of light from a distance source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why? State two points of difference between the interference patterns obtained in Young's double slit experiment and the diffraction pattern due to a single slit. **[DELHI 2017]**

12. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringes in the interference pattern.

(b) What kind of fringes do you except to observe if white light is used instead of monochromatic light? **[ALL INDIA 2018]**

5 Mark Questions

13. (i) 'Two independent mono-chromatic sources of light cannot produce a sustained interference pattern. Give reason.

(ii) Light wave each of amplitude 'a' and frequency 'ω', emanating from two coherent light sources superpose at a point. If the displacements due to these waves is given by $y_1 = a \cos \omega t$ and $y_2 = a \cos (\omega t + \phi)$ where ϕ is the phase difference between the two, obtain the expression for the resultant intensity at the point. **[DELHI 2014]**

14. (a) In Young's double slit experiment, describe briefly how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence obtain the expression for the fringe width.

(b) The ratio of the intensities at minima to the maxima in the Young's double slit experiment is 9 : 25. Find the ratio of the widths of the two slits.

[ALL INDIA 2014]

15. (i) In Young's double slit experiment, deduce the condition for (a) constructive, and (b) destructive interference at a point on the screen. Draw a graph showing variation of intensity in the interference pattern against position y on the screen.

(ii) Compare the interference pattern observed in Young's double slit experiment with single slit diffraction pattern, pointing out three distinguishing features.

[DELHI 2016]

16. (a) Derive an expression for path difference

in Young's double slit experiment and obtains the conditions for constructive and destructive interference at a point on the screen.

[DELHI 2016]

Solutions

1. Atoms (of the second medium) oscillate with the same (incident light) frequency and in turn, emit light of the same frequency. [1]

2. $\lambda = 450 \times 10^{-9}$ m

$D = 1.58$ m

for 1st minima

$$y = \frac{D\lambda}{a}$$ [½]

$$3 \times 10^{-3} = \frac{1.5 \times 450 \times 10^{-9}}{a}$$

$$a = \frac{1.5 \times 450 \times 10^{-9}}{3 \times 10^{-3}}$$

$a = 0.225$ m [½]

3. The path difference of two secondary wavelets is given by $n\lambda = a \sin\theta$. Since, θ is very small sin $\theta = \theta$. So, for the first order diffraction $n = 1$, the angle is $\frac{\lambda}{a}$. Now, we know that θ must be very small, because of which the diffraction pattern is minimum. [1]

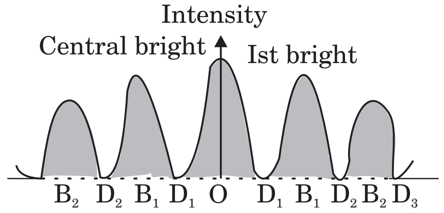
Now for interference case, for two interfering waves of intensity I_1 and I_2 we must have two slits separated by a distance. We have the resultant intensity, $I = I_1 + 2I_2 + I_1 I_2 \cos\theta$ Since, $\theta = 0$ (nearly) corresponding to angle $\frac{\lambda}{a} \cos\theta = 1$ (nearly). So,

$I = I_1 + 2I_2 + I_1 I_2 \cos\theta$

$\Rightarrow I = I_1 + 2I_2 + I_1 I_2$

We see the resultant intensity is sum of the two intensities, so there is maxima corresponding to the angle $\frac{\lambda}{a}$. This is why, at the same angle of a, we get a maximum for two narrow slits separated by a distance a. [1]

4. Intensity distribution graph for double slit experiment is shown below.



[1]

Fig.: Intensity Pattern for Double slit diffraction

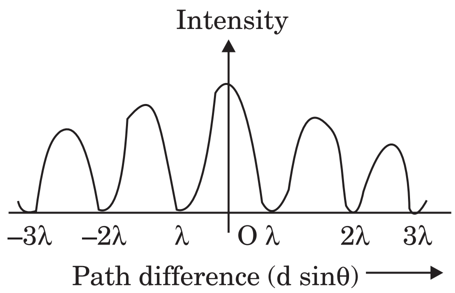


Fig.: "Intensity distribution graph for single slit diffraction" is given below.

Difference between interference pattern and the diffraction pattern [1]

Interference

1. All bright and dark fringes are of equal width.
2. All bright fringes are of same intensity.

Diffraction

1. The central bright fringe has got double width to that of width of secondary maxima and minima.
2. Central fringe is the brightest and intensity of secondary maxima decreases with increase of order of secondary maxima on either side of central maxima.

5. Young double slit experiment:

A train of plane light waves is incident on a barrier containing two narrow slits separated by a distance d. The widths of the slits are small compared with wavelength of the light used, so the interference occurs in the region where the light from s_1 overlaps that from s_2 [½]

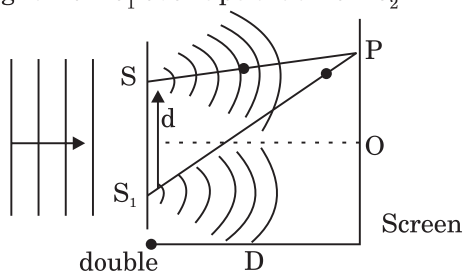


Fig.: Young double slit experiment

A series of alternately bright and dark bands can be observed on a screen placed in this region of overlap.

The variation in light intensity along the screen near the centre O shown in the figure

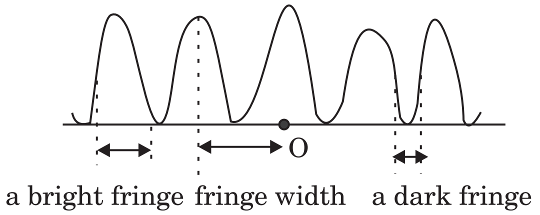
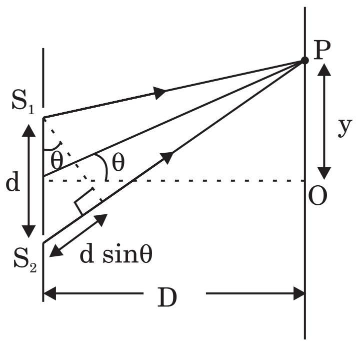


Fig.: Pattern Distribution [1]

Now consider a point P on the screen. The phase difference between the waves at P is , where

(Where ΔP_o is optical path difference, $\Delta P_o = \Delta P_{g;}$ ΔP being the geometrical path difference.)



$$= \frac{2\pi}{\lambda}\left[S_2P - S_1P\right] \text{(here } \lambda = 1 \text{ in air)}$$

As D > > d,

$S_2P - S_1P \approx \lambda d \sin\theta$

$$\sin\theta\lambda \approx \tan\theta\left(= \frac{y}{D}\right)$$

[for very small θ]

Thus, $\theta = \dfrac{2\pi}{\lambda}\left(\dfrac{dy}{D}\right)$

For consecutive interference,

θ = 2*n* λ (n = 0, 1, 2, ...)

$$\Rightarrow \frac{2\pi}{\lambda}\left(\frac{dy}{D}\right) = 2n\pi \Rightarrow y = n\lambda\frac{D}{d}$$ [1]

Similarly for destructive interference,

$$y = (2n+1)\lambda\frac{D}{2d}\left(n = 1, 2, 3......\right)$$

Fringe Width W

It is the separation of two consecutive maxima or two consecutive minima.

Near the centre *O* [where θ is very small],

$y_{n+1} - y_n$ [y_n gives the position of nth maxima on screen]

$$W = \lambda\frac{D}{d}$$ [½]

6. (a) Coherent sources produce coherent light which have wavelength restricted to a very small range. So the sources have almost same wavelength, hence produce stable and sustained interference pattern. [1]

(b) We know, phase difference $= \left(\dfrac{2\pi}{\lambda}\right) \times$ path difference

At path difference λ, phase difference

$\phi = \left(\frac{2\pi}{\lambda}\right) \times \lambda = 2\pi$ [1]

Intensity, $I = 4I_o cos^2\left(\frac{\phi}{2}\right)$

Or, $K = 4I_o cos^2\left(\frac{2\pi}{2}\right)$ [given, I = K at path difference $K = 4I_o$]

Or, $K = 4I_o$

$\Rightarrow I_o = \frac{K}{4}$ [1]

Now, at path difference $= \frac{\lambda}{3}$

$\phi' = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$

Intensity,

Or, $I' = 4I_o \cos^2 \frac{1}{2}\left(\frac{2\pi}{3}\right)$ [1]

7. (a) Two characteristic features distinguishing the diffraction pattern from the interference fringes obtained in Young's double slit experiment are: [1]

(i) The interference fringes may or may not be of the same width whereas the fringes of diffraction pattern are always of varying width. [1]

(ii) In interference the bright fringes are of same intensity whereas in diffraction pattern the intensity falls as we go to successive maxima away from the centre, on either side. [1]

(b) Wavelength of the light beam, λ_1 = 590 nm = 5.9×10^{-7} m

Wavelength of another light beam, λ_2 = 596 nm = 5.96×10^{-7} m

Distance of the slits from the screen = D = 1.8 m

Distance between the two slits =1×10^{-4} m

For the first secondary maxima,

$x_1 = \frac{3\lambda_1 D}{2a}$ and $x_2 = \frac{3\lambda_2 D}{2a}$

Spacing between the positions of first secondary maxima of two sodium lines

$= \frac{3D}{2a}(\lambda_2 - \lambda_1) = 1.62 \times 10^{-4} m$ [1]

8. (a) If the width of each slit is comparable to the wavelength of light used, the interference pattern thus obtained in the double-slit experiment is modified by diffraction from each of the two slits. [2]

(b) Given that: Wavelength of the light beam,

$\lambda_1 = 590 nm = 5.9 \times 10^{-7} m$

Wavelength of another light beam,

$\lambda_2 = 596 nm = 5.96 \times 10^{-7} m$

Distance of the slits from the screen = D = 1.5 m

Distance between the two slits, $a = 2 \times 10^{-4}$ m [1]

For the first secondary maxima,

$\sin\theta = \frac{3\lambda_1}{2a} = \frac{x_1}{D}$

$x_1 = \frac{3\lambda_1 D}{2a}$ And $x_2 = \frac{3\lambda_2 D}{2a}$

$x_1 = \frac{3 \times 590 \times 10^{-9} \times 15}{2 \times 2 \times 10^{-4}}$ And

$x_2 = \frac{3 \times 596 \times 10^{-9} \times 1.5}{2 \times 2 \times 10^{-4}}$

$x_1 = 663.75 \times 10^{-5}$ And $x_2 = 670.50 \times 10^{-5}$

Spacing between the positions of first secondary maxima of two sodium lines

$x_2 - x_1$ $6.75 \times 10^{-5} M = 0.0675$ mm [1]

9. (a) Angular width (θ) of fringe in double-slit experiment is given by $\theta = \frac{\lambda}{d}$

Where, $d \rightarrow$ spacing between the slits [1]

Given, wavelength of light A = 600nm

Angular width of fringe, θ = 0.1° = ... = 0,0018 rad

$\therefore d = \frac{\lambda}{\theta}$

$d = \frac{600 \times 10^{-9}}{18 \times 10^{-1}}$

$\Rightarrow d = 0.33 \times 10^{-3} m$ [1]

(b) The frequency and wavelength of reflected wave will not change.

The refracted wave will have same frequency, only wavelength will change.

The velocity of light in water is given by $v = \lambda f$

Where, $v \rightarrow$ Velocity of light [1]

$f \rightarrow$ Frequency of light

$\lambda \rightarrow$ Wavelength of light

As light ray in travelling from rare (air) medium to denser medium, its speed will decrease. Hence wave length (λ) will also decrease. [1]

10. (a) Third bright fringe for λ_1 = 520 nm is given by

$x_3 = \frac{3\lambda D}{d} = \frac{3 \times 520 \times 10^{-9} \times 1}{1.5 \times 10^{-3}} = 1.04 \times 10^{-3} m$

x_3 = 1.04 mm [1]

(b) Let n_1 bright band of λ_1 = 520 nm coincides with n_2 bright band of λ_2 = 650 nm

So, $\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

$n_1\lambda_1 = n_2\lambda_2$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{650}{520} = \frac{5}{4}$ [1]

So the least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

$$x = \frac{n_1\lambda_1 D}{d} = \frac{5\times 520\times 10^{-9}\times 1}{1.5\times 10^{-3}}$$ [2]

$= 1.73 \times 10^{-3}\ m$

$= 1.73\ mm$ [1]

11. A bright spot is observed when a tiny circular object is placed in path of light from a distant source in a single slit diffraction experiment because light rays flare into the shadow region of the circular object as they pass the edge of the tiny circular object. The lights from all the edges of the tiny circular object are in phase with each other. Thus, they form a bright spot at the centre of the shadow of the the tiny circular object. The two differences between the interference patterns obtained in Young's double slit experiment and the diffraction pattern due to a single slit are as follows: [2]

(i) The fringes in the interference pattern obtained from diffraction are of varying width, while in case of interference, all are of the same width.

(ii) The bright fringes in the interference pattern obtained from diffraction have a central maximum followed by fringes of decreasing intensity, whereas in case of interference, all the bright fringes are of equal width. [2]

12. We know that intensity is directly proportional to square of amplitude. [1]

$I \propto a^2$

If $I_1 = \frac{I}{2}$

If intensity reduced to 50%, amplitude will be $\frac{a}{\sqrt{2}}$ then $r = \sqrt{2}$

$$\frac{I_{max}}{I_{min}} = \frac{(r+1)^2}{(r-1)^2} = \frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)^2}$$ [1]

$$\frac{I_{max}}{I_{min}} = \left(\frac{2.414}{0.414}\right)^2 = (5.83)^2$$ [1]

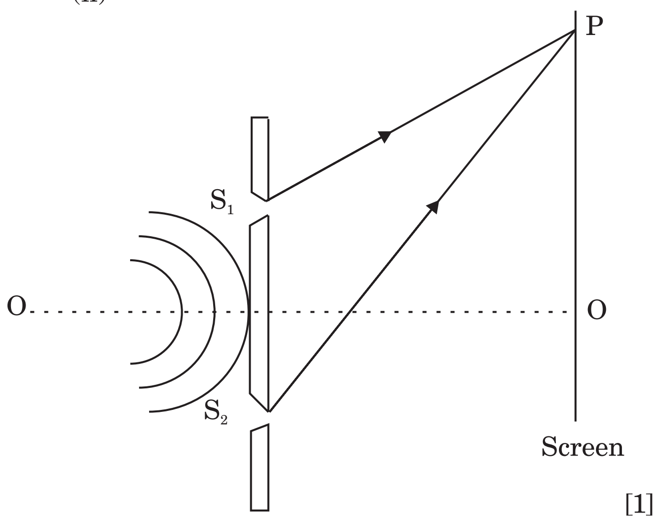
$$\frac{I_{max}}{I_{min}} = 33.98 \simeq 34$$

(b) Central fringe will be white and remaining will be coloured in VIBGYOR sequence. [1]

13. (i) The condition for the sustained interference is that both the sources must be coherent (i.e. they must have the same wavelength and the same frequency, and they must have the same phase or constant phase difference). [1]

Two sources are monochromatic if they have the same frequency and wavelength. Since they are independent, i.e. they have different phases with irregular difference, they are not coherent sources. [1]

(ii)



[1]

Let the displacement of the waves from the sources and at point P on the screen at any time be given by:

$y_1 = a\cos\omega t$ and $y_2 = a\cos(\omega t + \phi)$

Where, ϕ is the constant phase difference between the two waves.

By the superposition principle, the resultant displacement at point P is given by:

$y = y_1 + y_2$

$y = a\cos\omega t\ a\cos(\omega t + \phi)$ [1]

$$y = 2a\left[\cos\left(\frac{\omega t + \omega t + \phi}{2}\right)\cos\left(\frac{\omega t - \omega t - \phi}{2}\right)\right]$$

$$y = 2a\cos\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) \quad (1)$$

Let $2a\cos\left(\frac{\phi}{2}\right) = A$ (2)

Then equation (1) becomes:

$$y = A\cos\left(\omega t + \frac{\phi}{2}\right)$$

Now, we have:

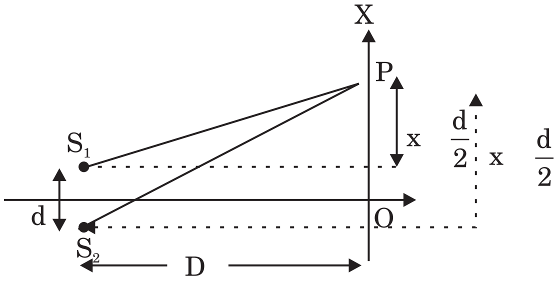
$$= A^2 = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

The intensity of light is directly proportional to the square of the amplitude of the wave. The intensity of light at point on the screen is given by:

$$I = 4a^2 \cos^2 \frac{\phi}{2}$$

[1]

14. (a) The path difference between two rays coming from holes S_1 and S_2 is $(S_2P - S_1P)$. If point P corresponds to a maximum.



[1]

Now

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$= 2xd$, where $S_1S_2 = d$ and $OP = x$

$(S_2P + S_1P)(S_2P - S_1P) = 2xd$

$$(S_2P - S_1P) = \frac{2xd}{(S_2P + S_1P)}$$

[1]

For maximum, $S_2P + S_1P = n\lambda$

Thus, $n\lambda = \dfrac{xd}{D}$

Or, $x = x_n = \dfrac{n\lambda D}{d}$ [1]

$n = 0, \pm 1, \pm 2, \pm 3$ …….. [For maximum]

Now, for minimum,

$$S_2P - S_1P = (2n-1)\frac{\lambda}{2}$$

Thus, $(2n-1)\dfrac{\lambda}{2} = \dfrac{xd}{D}$

Or, $x = x_n = (2n-1)\dfrac{\lambda D}{2d}$

$n = 0, \pm 1, \pm 2, \pm 3$…………. [For minima]

Thus, bright and dark bands appear on the screen, as shown in Figure. Such bands are called 'fringes'. These dark and bright fringes are equally spaced. [1]

Expression for fringewidth (β).

Let n^{th} order bright fringe is at a distance x_n and $(n + 1)^{th}$ order bright fringe is at x_{n+1} from O, Then

$x_n \dfrac{n\lambda D}{d}$ and $x_{n+1} = \dfrac{(n+1)\lambda D}{d}$

--------------[From eq. (2)]

Now the fringe width is

$$\beta = x_{n+1} - x_n = \frac{\lambda D}{d}$$

Thus, the expression for fringwidth is

$$\beta = \frac{\lambda D}{d}$$

(b) $\dfrac{I_{\min}}{I_{\max}} = \dfrac{9}{25}$

$$\left[\frac{\left(\sqrt{I_1} - \sqrt{I_2}\right)}{\sqrt{I_1} + \sqrt{I_2}}\right]^2 = \frac{9}{25}$$

$$\frac{\left(\sqrt{I_1} - \sqrt{I_2}\right)}{\sqrt{I_1} + \sqrt{I_2}} = \frac{3}{5}$$

$$5\sqrt{I_1} - 5\sqrt{I_2} = 3\sqrt{I_1} + 3\sqrt{I_2}$$

$$2\sqrt{I_1} = 8\sqrt{I_2}$$

$$\sqrt{\frac{I_1}{I_2}} = \frac{4}{1}$$

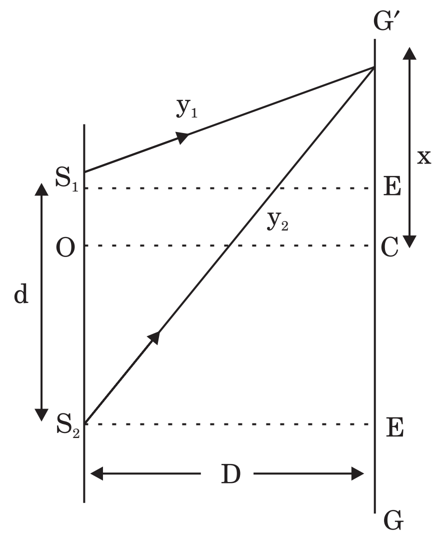
$$\frac{I_1}{I_2} = \frac{16}{1}$$

[1]

Ratio of intensity

15. (i) Let the two waves arising from the slits A and B have the amplitudes a and b and the phase difference ϕ. Such that $y_1 = a \sin \omega t$ and $y_2 = b \sin(\omega t + \phi)$.

The resultant displacement is given as:



[1]

$y = y_1 + y_2$

$y = a \sin \omega t + b (\omega t + \phi)$

$y = a\sin\omega t + b\sin\omega t\cos\phi + b\cos\omega t\sin\phi$

$y = (a + b\cos\phi)\sin\omega t + b\sin\phi\cos\omega t$

Let $a + b\cos\phi = A\cos\delta$

And $b\sin\phi = A\sin\delta$

Hence, $y = A\sin\omega t\cos\delta + A\cos\omega t\sin\delta$

$y = A\sin(\omega t + \delta)$

Where the amplitude A of the resultant Wave can be given as:

$A = \sqrt{a^2 + b^2 + 2ab\cos\phi}$ [1]

And $\tan\delta = \dfrac{b\sin\phi}{a + b\cos\phi}$

Constructive Interference: Intensity $I \propto A^2$ and for A to be maximum $\cos\phi = 1$

Or $\cos\phi = \cos 2n\pi, n = 0,1,2,3.....$

Or, $\phi = 2n\pi.$

And path difference $\Delta x = n\lambda$

$A_{max} = a + b$

(a) Destructive Interference: For I to be minima, $\cos\phi = -1$

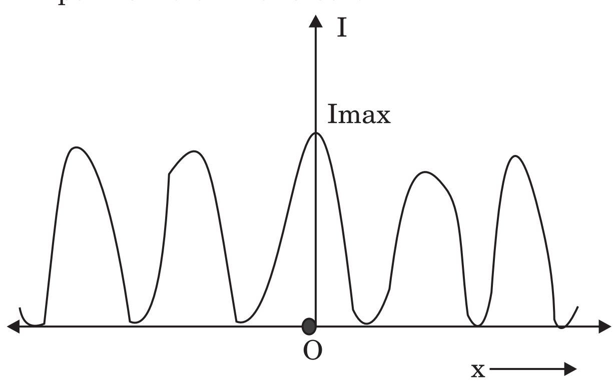
Phase difference: $\Delta\phi = (2n+1)\pi$

And path difference:

$\Delta x = (2n+1)\dfrac{\lambda}{2}$ [1]

$A_{min} = a - b$

Graph showing interference pattern against position 'x' on the screen.

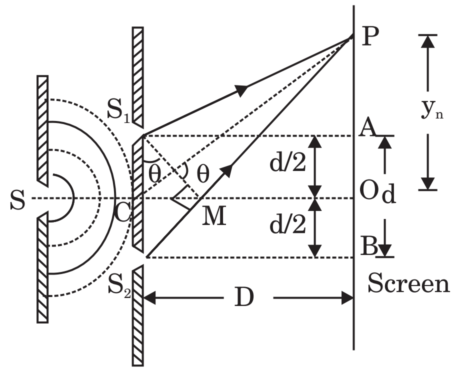


(ii) Compose of interference pattern observed in Young's double slits and the single slits diffraction:

S. No.	Interference	Diffraction
1	Interference is the result of superposition of secondary wavesstarting from two different wave fronts originating from two coherent sources.	Diffraction is the result of superposition of secondary waves starting from different part of same wave front.
2	All bright and dark fringes are of equal width.	The width of central bright fringe is twice the width of any secondary maximum.
3.	All bright fringes are of same intensity.	Intensity of bright fringes decreases as we move away from central bright fringes on either side.

[1]

16. (a) To observe interference fringe pattern, there is need to have coherent sources of light which can produce light of constant phase difference. Let two coherent sources of light, S_1 and S_2 (narrow slits) are derived from a source S. The two slits, S_1 and S_2 are equidistant from source, S. Now suppose S_1 and S_2 are separated by distance d. The slits and screen are distance D apart. [2]



[1]

Considering any arbitrary point P on the screen at a distance y_n from the centre O

The path difference between interfering waves is given by $S_2P - S_1P$ i.e. Path difference $S_2P - S_1P$ [2]

[Topic 3] Diffraction and Polarisation of Light

Summary

- ***Diffraction:*** Bending of light around corners of an obstacle into the region where shadow of obstacle is expected.

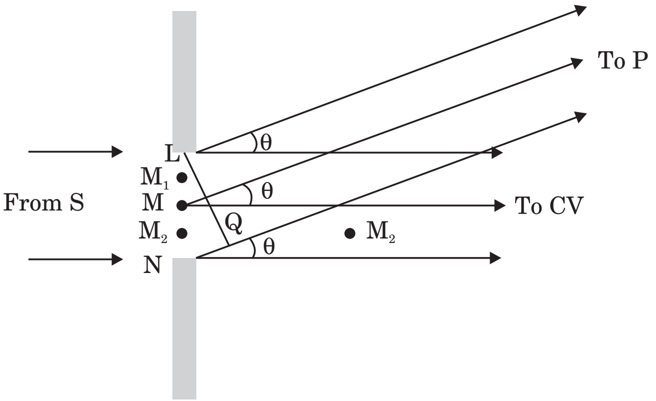


Fig.: Diffraction Phenomenon

- Light energy is redistributed in interference and diffraction. When it reduces in one region, emitting a dark fringe, it increases in another region, emitting a bright fringe. In this process the energy remains constant i.e. neither energy is gained nor lost, with the principle of conservation of light.
- The ***resolving power of the microscope*** is given by the reciprocal of the minimum separation of two points seen as distant. The resolving power can be increased by choosing a medium of higher refractive index.

$$d_{min} = \frac{1.22\lambda}{2 \sin \beta}$$

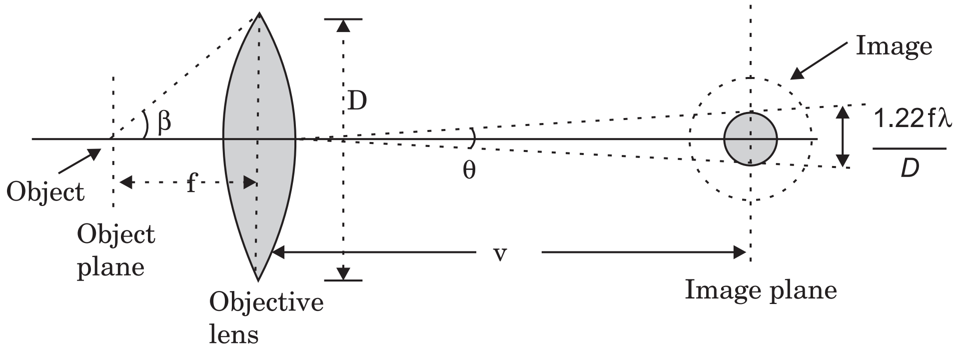


Fig.: Resolving power of the microscope

- ***Resolving power of telescope:*** For two stars to be just resolved,

$$f\,\Delta\theta \approx \frac{0.61\lambda f}{a}$$

So, $\Delta\theta \approx \frac{0.61\lambda}{a}$

Telescope will have better resolving power if a is large.

- A diffraction pattern with a central maximum is given by a single slit of width a. At angles of $\pm\frac{\lambda}{a}, \pm\frac{2\lambda}{a}$, etc., along with successively weaker secondary maxima in between, the intensity reduces to zero. The angular resolution of a telescope is limited to $\frac{\lambda}{D}$, due to diffraction where D is the diameter. Strongly overlapping images are formed when two stars are closer than this angle. Similarly, in a medium of refractive index n, a microscope objective subtending angle 2β at the focus, will just separate two objects spaced at a distance $\frac{\lambda}{(2n \sin \beta)}$, which is the resolution limit of a microscope.
- The ***Fresnel distance*** is given by the formula $Z_P = \frac{a^2}{\lambda}$, where a is the size of the aperture and λ is the wavelength.

- ***Polarized wave:*** A long string is held horizontally, the other end of which is assumed to be fixed. If the end of the string is moved up and down in a periodic manner, a wave propagating in the +x direction will be generated. Each point on the string moves on a straight line, the wave is also referred to as linearly polarised wave. The linearly polarized waves are transverse waves; i.e., the displacement of each point of the string is always at right angles to the direction of propagation of the wave.
- ***Unpolarized wave:*** When the plane of vibration of the string is changed randomly in very short intervals of time, then we have what is known as an unpolarised wave. Thus, for an unpolarised wave the displacement will be randomly changing with time though it will always be perpendicular to the direction of propagation.
- A ***Polaroid*** consists of long chain molecules aligned in a particular direction. The electric vectors along the direction of the aligned molecules get absorbed. Thus if an unpolarised light wave is incident on such a Polaroid then the light wave will get linearly polarized with the electric vector oscillating along a direction perpendicular to the aligned molecules; this direction is known as the pass-axis of the Polaroid.
- If I is the intensity of polarized light after passing through the first polariser P_1 then the intensity of the light after passing through the second polarizer P_2 will be I = Icosθ. This is called ***Malus' Law***.
- Natural light from the sun is unpolarised which means that the electric vector takes all possible random directions in the transverse plane. A polaroid transmits only one component of these vectors, which is parallel to a special axis. Therefore the light wave is called plane polarised. When this kind of light is viewed through another polaroid which is rotated through an angle 2π, we can see two maxima and minima of same intensity.
- Plane polarised light can also be producedby reflection at a special angle known as the Brewster angle and by scattering through $\frac{\pi}{2}$ in the earth's atmosphere.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 3

1 Mark Questions

1. If the angle between the pass axis of polarizer and the analyser is 45°, write the ratio of the intensities of original light and the transmitted light after passing through the analyzer. **[DELHI 2017]**

2 Mark Questions

2. (a) When a wave is propagating from a rarer to a denser medium, which characteristic of the wave does not change and why ?

(b) What is the ratio of the velocity of the wave in the two media of refractive indices μ_1 and μ_2? **[ALL INDIA 2015]**

3. Unpolarised light is passed through a Polaroid P_1. When this polarized beam passes through another Polaroid P_2 which makes an angle θ with the pass axis of P_1, then write the expression for the polarized beam passing through P_2. Draw a plot showing the variation of intensity when θ varies from 0 to 2π. **[ALL INDIA 2017]**

3 Mark Questions

4. (a) Describe briefly, with the help of suitable diagram, how the transverse nature of light can be demonstrated by the phenomenon of polarization.

(b) When unpolarized light passes from air to a transparent medium, under what condition does the reflected light get polarized?

[ALL INDIA 2011]

4 Mark Questions

5. (a) What is linearly polarized light? Describe briefly using a diagram how sunlight is polarized.

(b) Unpolarized light is incident on a Polaroid. How would the intensity of would transmitted light change when the Polaroid is rotated? **[ALL INDIA 2013]**

6. What is an unpolarized light? Explain with the help of suitable ray diagram how an unpolarized light can be polarized by reflection from a transparent medium. Write the expression for Brewster angle in terms of the refractive index of the denser medium. **[DELHI 2018]**

5 Mark Questions

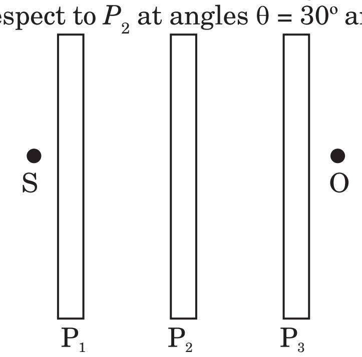
7. (a) How does one demonstrate, using a suitable diagram, that unpolarised light when passed through a Polaroid gets polarized?

(b) A beam of unpolarised light is incident on a glass-air interface. Show, using a suitable ray diagram, that light reflected from the interface is totally polarised, when μ = tan *iB*, where μ is the refractive index of glass with respect to air and *iB* is the Brewster's angle. **[ALL INDIA 2014]**

8. Describe briefly how a diffraction pattern is obtained on a screen due to a single narrow slit illuminated by a monochromatic source of light. Hence obtain the conditions for the angular width of secondary maxima and secondary minima. **[ALL INDIA 2014]**

9. (a) Why does unpolarised light from a source show a variation in intensity when viewed through a polaroid which is rotated ? Show with the help of a diagram, how unpolarised light from sun gets linearly polarised by scattering. **[DELHI 2016]**

(b) Three identical polaroid sheets P_1, P_2 and P_3 are oriented so that the pass axis of P_1, P_2 and P_3 are inclined at angles of 60º and 90º respectively with the pass axis of P_1. A monochromatic source S of unpolarized light of intensity I0 is kept in front of the polaroid sheet P_1 as shown in the figure. Determine the intensities of light as observed by the observer at O, when polaroid P_3 is rotated with respect to P_2 at angles θ = 30º and 60º.



[DELHI 2018]

Solutions

1. $I = I_m \cos^2\theta$

Where, *I* is the transmitted intensity

I_m is the maximum value of the transmitted intensity

θ is the angle between the two polarising directions

$$\frac{I}{I_m} = \cos^2\theta$$

[½]

$$\frac{I}{I_m} = \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{I}{I_m} = \frac{1}{2}$$

[½]

2. (a) Frequency of a wave does not change when the wave is propagating from a rarer to a denser medium because frequency (ν) is given by the relation [1]

$\upsilon = \frac{v}{\lambda}$, As the medium changes velocity (v) and wavelength (λ) changes such that ratio remains constant.

(b) $\frac{\nu_1}{\nu_2} = \frac{\mu_1}{\mu_2}$, Here V_1 and V_2 are the velocity of the wave in medium 1 and medium 2 and μ_1 and μ_2 are the refractive index of medium 1 and medium 2. [1]

3. According to law of Malus, when a beam of completely plane polarized light is incident on an analyzer resultant intensity of light (*I*) transmitted from the analyzer varies directly as the square of cosine of angle θ between the plane of analyzer and polarizer

i.e $I \propto \cos^2\theta$

$\Rightarrow I = I_0 \cos^2\theta$

When polarizer and analyzer are parallel

θ = 0° or 180°

So,

cos θ = + 1, –1

$I = I_0 \cos\theta = \cos 90° = 0$

$I = 0$

In unpolarised light, vibrations are probable in all direction in a plane perpendicular to the direction of propagation. [1]

Therefore, θ can have any value from 0 to 2π

$$\therefore [cos^2\theta]_{av} = \frac{1}{2}\int_0^{2\pi} cos^2\theta d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{1+\cos 2\theta}{2} d\theta$$

$$= \frac{1}{2\pi \times 2}\left[0 + \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

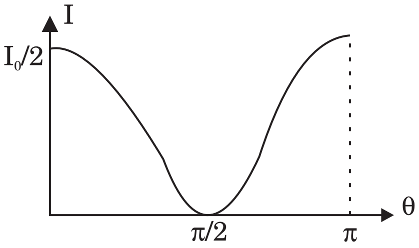
$= \frac{1}{2}$

If we use malus law then,

$I = I_0 \cos^2 \theta$

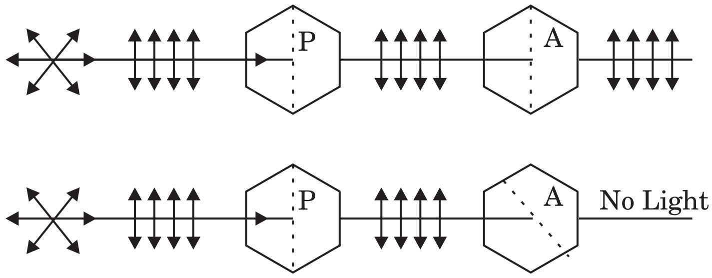
$\mathrm{I} = \mathrm{I}_0 \times \frac{1}{2} = \frac{1}{2}\mathrm{I}_0$ [1]

The required graph would have the form as shown in figure.



[1]

4. (a) When a polaroid P_1 is rotated in the path of an unpolarised light, there is no change in transmitted intensity, v. The light transmitted through polaroid P_1 is made to pass through polaroid P_2. On rotating polaroid P_2, in path of light transmitted from P_1 we notice a change in intensity of transmitted light. This shows the light transmitted from is polarized. Since light can be polarized, it has transverse nature. [1]



[1}

(b) Whenever unpolarised light is incident from air to a transparent medium at an angle of incidence equal to polarizing angle, the reflected light gets fully polarized.

According to Brewster's law

$n = \tan i_p$

Where i_p in the polarizing angle and n is refractive index of the transparent material

By Snell's law, we have

$$n = \frac{\sin i_p}{\sin r}$$

$$\tan i_p = \frac{\sin i_p}{\sin r}$$

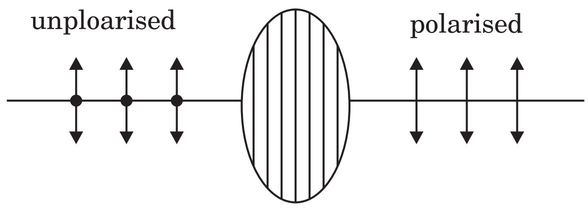
$$\therefore \cos i_p = \cos(90° - r)$$

$$\therefore \cos i_p = \sin r$$

$$\therefore i_p + r = 90°$$

[1]

5. (a) When vibrations of light wave are confined to only one direction then light is called linearly polarized. [1]

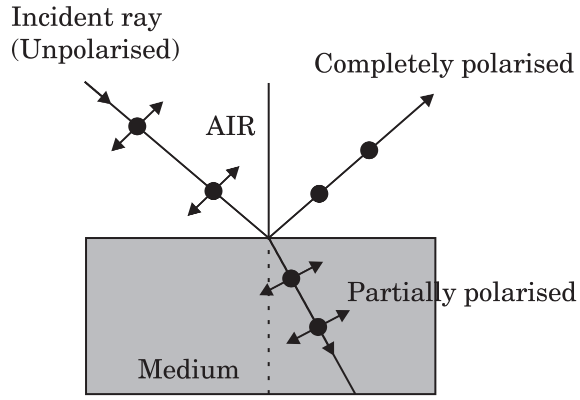


[1]

When sunlight passes through Polaroid then components parallel to axis passes in unaffected way and components perpendicular to axis are absorbed so transmitted light is polarized. [1]

(b) On rotating the Polaroid, intensity remains unchanged as half of the incident intensity.

6.

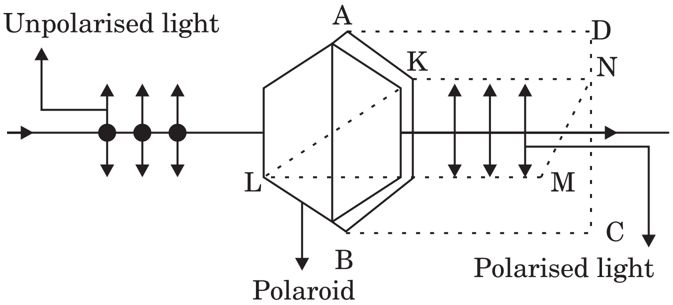


[1]

An unpolarized light is one in which the vibration of electric field vector is not restricted in one particular plane. When an unpolarized light falls on the surface, the reflected light is such that the vibration of its electric field vector is confined to one particular plane. The direction of this plane is parallel to the surface of reflection. A component of electric field vector is absent from the refracted light. Therefore, the refracted light is partially polarized. [2]

The expression for Brewster angle in terms of the refractive index of denser medium is $\tan i_B = \mu$, Where 'μ' is the refractive index of the denser medium with respect to the rarer medium. [1]

7. (a)



[1]

The phenomenon of restricting the vibration of light (electric vector) in a particular directionperpendicular to the direction of the wave propagation is called polarization of light. When unpolarised light is passed through a Polaroid, only those vibrations of light pass through the crystal, which are parallel to the axis of the crystal (*AB*). All other vibrations are absorbed and that is why intensity of the emerging light is reduced. [1]

The plane *ABCD* in which the vibrations of the polarised light are confined is called the plane of vibration. The plane *KLMN* that is perpendicular to the plane of vibration is defined as the plane of polarization.

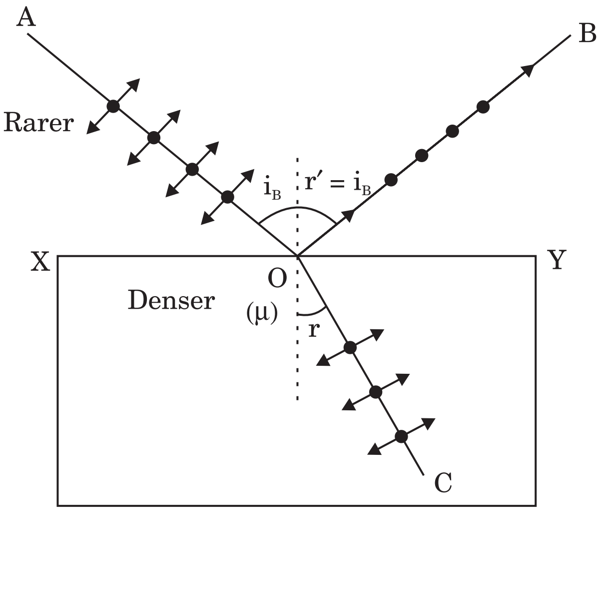
When unpolarised light is incident on the glass-air interference at Brewster angle *iB*, then reflected light is totally polarised. This is called Brewsters Law.

When light is incident at Brewster angle, the reflected component *OB* and the refracted component OC are mutually perpendicular to each other. [1]

From the figure,

We have: $\angle BOY + \angle YOC = 90^0 a$

$\left(90^o - i_B\right) + \left(90^o - r\right) = 90^o$



[1]

Where, *r* is angle of refraction

According to the Snell's law:

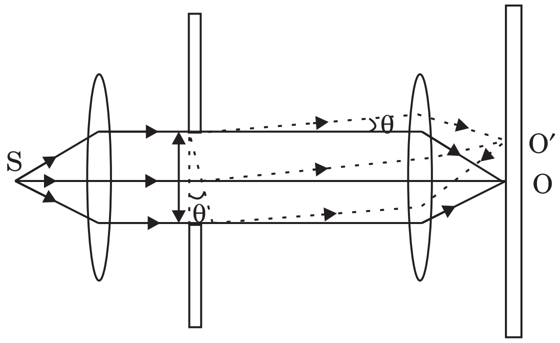
$$\propto = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin(90 - i_B)}$$

$$\propto = \frac{\sin i_B}{\cos i_B}$$

$\mu = \tan i_B$ [1]

Hence proved

8.



[1]

A point source *S* is placed at the focus of a converging lens. The source-lens arrangement provides a plane wavefront which is then diffracted. Another converging lens is introduced between the diffracting slit and the observation screen such that the screen is in the focal plane of the lens. Plane wavefront emerging from the slit at different angles are brought to focus on the screen using the second lens, as shown in the Fig. [1]

Condition for minima

Divide the slit into two equal halves AC and CB, each of size $\frac{a}{2}$. For every point M_1 in *AC*, there exists a point M_2 in *CB* such that $M_1M_2 = \frac{a}{2}$. The path difference between secondary waves from M_1 and M_2 reaching *P* is

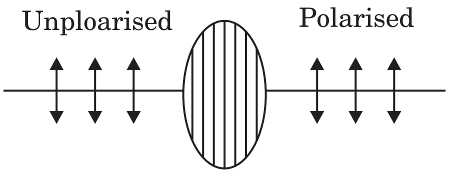
$$M_2P - M_1P = \frac{a}{2}\sin\theta$$

Point *P* on the screen would be a first minimum if this path difference is λ between the secondary waves from extreme points *A* and *B*. Thus, path difference between waves from *A* and *C* or between waves from will be $\frac{\lambda}{2}$. [1]

Hence, $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ or $a\sin\theta = \lambda$ for *P* to be first minimum. *P* is a second minimum if Path difference, a sin = 2λ Proceeding in the same manner, we can show that the intensity at P is zero if Path difference, a sin θ = nλ (condition for minima) where n = 1, 2, 3,...... Condition for secondary maxima Imagine the slit to be divided into three parts AM_2, M_1, M_2 and M_2B. Let the secondary waves reaching P from the extreme points A and B be $\frac{3\lambda}{2}$. The secondary waves reaching P from the corresponding points of the parts AM_1, M_1, M_2 will have path difference of $\frac{\lambda}{2}$ and interfere destructively. The secondary waves reaching P from points in the third part M_2B will contribute to the intensity at P. Therefore, only one-third of the slit contributes to the intensity at point P between two minima. This will be much weaker than the central maximum. This is the first secondary maximum. The condition for first secondary maximum is [1 + 1]

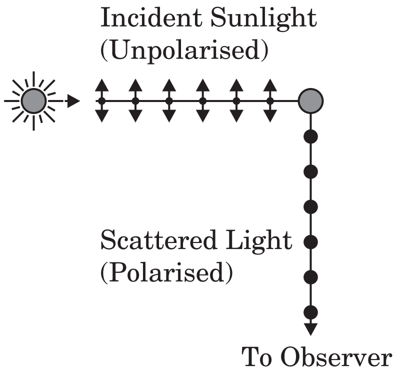
Path difference, $a\sin\theta = \frac{3\lambda}{2}$,The condition for second secondary maximum is Path difference, $a\sin\theta = \frac{5\lambda}{2}$.Proceeding in the same manner, we can show that the condition for a secondary maxima is Path difference, $a\sin\theta = (2n + 1)\left(\frac{\lambda}{2}\right)$ where n = 1, 2, 3, ... [1]

9. (a) When vibrations of light wave are confined to only one direction then light is called linearly polarised.

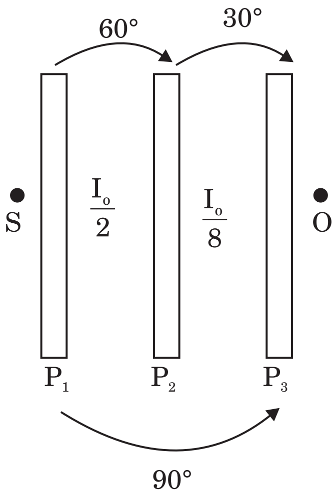


When sunlight passes through polaroid then components parallel to axis passes unaffected and components perpendicular to axis are absorbed so transmitted light is polarised. [1]

As Fig. shows, the incident sunlight is unpolarised. The dots stand for polarisation perpendicular to the plane of the figure. The double arrows show polarisation in the plane of the figure. (There is no phase relation between these two in unpolarised light). Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions. We have drawn an observer looking at 90° to the direction of the sun. Clearly, charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore represented by dots. It is polarised perpendicular to the plane of the figure. This explains the polarisation of scattered light from the sky. [1]



(b) On rotating the polaroid, intensity remains unchanged as half of the incident intensity.



Intensity of unpolarized light on P_1 is I_o

After passing through P_1 the intensity is $\left(\frac{I_o}{2}\right)$ [1]

After passing through P_2 polaroid $P = I \cos 2\theta$

where $\left(I = \frac{I_o}{2}\right)$

$$I' = \frac{I_o}{2}\cos^2 60°$$

$$I' = \frac{I_o}{8}$$

Case 1^{st} $I" = I'\cos^2\theta \left(here\ \theta = 0°\right)$

$I" = I'$

$$I = \frac{I_o}{8}$$

[1]

Case II^{nd}

P^3 rotated away (θ = 30°) from pass axis of P_2

Now angle between pass axis of P_2 and P_3 will be

ϕ = 60°

$I" = I' \cos^2 60°$ [½]

So, $I" = \frac{I_o}{8}\left(\frac{I}{2}\right)^2$

$I" = \frac{I_o}{32}$

Case IIIrd:

P_3 rotated towards pass axis of P_2 by

Now again angle between pass axis of P_2 and P_3 will be

ϕ = 30°

$I" = I' \cos^2 30°$

So, $I" = \frac{I_o}{8}\left(\frac{\sqrt{3}}{2}\right)^2$

$I" = \frac{3I_o}{32}$

Case IV^{th}

P_3 rotated away from pass axis of P_2 by θ = 60°

Now angle between pass axis of and P_2 and P_3 will be

ϕ = 90°

$I" = I' \cos^2 90°$

$I" = 0$ [½]

Smart Notes ....

Smart Notes ....

CHAPTER 11

Dual Nature of Radiation and Matter

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Dual nature of radiation, Photoelectric effect; Hertz and Lenard's observations				1Q (3 marks)	
Einstein's photoelectric equation-particle nature of light	1Q (3 marks)	1Q (3 marks)	1Q (3 marks)		
Matter waves-wave nature of particles					
de Broglie relation; Davisson-Germer experiment	1Q (1 mark)	1Q (2 marks)	1Q (2 marks)		

On the basis of above analysis, it can be said that from exam point of view Phenomenon of Photoelectric Effect, Wave Theory, Intensity of Light and Einstein's Photoelectric Equation are most important concepts of the chapter.

[Topic 1] Photoelectric Effect

Summary

- ***Work Function:*** The minimum energy which is necessary for an electron to get away from the surface of metal is called the work function of the metal which is denoted by ϕ_0. The unit for measuring work function is electron volt (eV). This minimum energy can be provided by thermionic emission, field emission or photo-electric emission.

  Thermionic emission: When a metal is heated, thermal energy is imparted to free the electrons from the surface of the metal.

  Field emission: Electrons can be pulled out of metal by applying a very strong electric field (of the order of 108 Vm^{-1}) to it, as in a tesla coil.

  Photo-electric emission: Electrons are emitted when a light of suitable frequency hits a metal surface. This can be seen in a photodiode.
- 1eV is the energy attained by an electron when it has been accelerated by a potential difference of 1, so that 1eV = 1.602×10^{-19}J.
- ***Photoelectric Effect:*** When metals are irradiated by light of suitable frequency, electrons start emitting from the metal surface. This phenomenon is known as photoelectric effect.

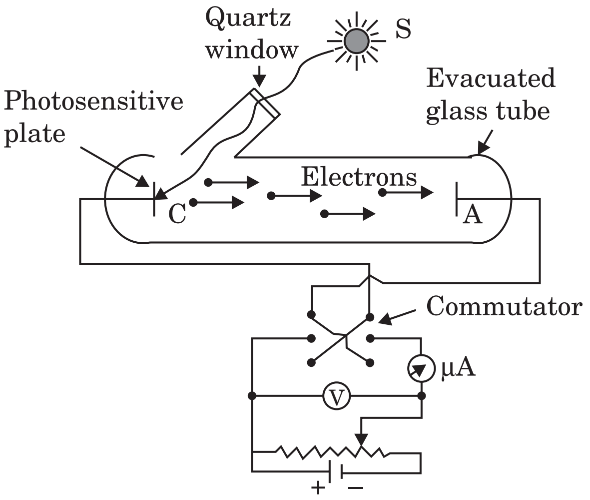


Fig.: Depiction of Photoelectric effect

- Some metals are sensible to ultraviolet light and some to visible light also. Photocurrent depends upon the intensity of light, frequency of incident light, potential difference between both the plates and the material of the plate.
- ***Stopping Potential:*** Stopping potential or cut-off potential is the minimum retarding (negative) potential for which the photoelectric current stops at a particular frequency of incident light. It is denoted by V_0.
- ***Saturation Current:*** At a certain potential difference, the photoelectric current stops increasing further. This maximum value of photocurrent is known as the saturation current.
- ***Maximum Kinetic Energy:*** The maximum kinetic energy of the photoelectric electrons is denoted by K_{max} and it depends directly on the frequency of the incident light. It is independent of the intensity of the light.

  The maximum kinetic energy $K_{max} = eV_0$
- ***Threshold Frequency:*** The minimum cut-off frequency which is required for the emission of electrons is called the threshold frequency which is denoted by ν_0. No emission is possible for the frequency lower than the cut-off frequency.
- In the photoelectric effect, the light energy is converted into the electrical energy. The photoelectric emission is a quick process having very less time lag.
- ***Effect of intensity of light on photocurrent:*** Number of photoelectrons emitted per second varies directly with the intensity of incident radiation.
- ***Effect of potential on photoelectric current:*** The stopping potential is independent of its intensity for a given frequency of the incident radiation.
- ***Effect of frequency of incident radiation on stopping potential:***

  The stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

  There exists a certain minimum cut-off frequency v_0 for which the stopping potential is zero.
- ***Einstein's Photoelectric Equation:*** Einstein proposed that light is comprised of small discrete

energy packets known as photons or quanta and energy carried by each photon is hv, where v is the frequency of light and Planck's constant. The momentum carried by each photon is $\frac{h}{\lambda}$. In photoelectric effect, emission is possible because of the absorption of a photon by an electron. The maximum kinetic energy of the emitted electron is:

$K_{max} = h\nu - \phi_0$, where ϕ_0 is the work function.

$= h(\nu - \nu_0)$

The photoelectric emission is possible only when $h\nu > \phi_0$ as K_{max} must be non-negative.

$\Rightarrow \nu > \nu_0$ where $\nu_0 = \frac{\phi_0}{h}$

- From the photoelectric equation,

$eV_0 = h\nu - \phi_0$, for $\nu \geq \nu_0$ (as $K_{max} = eV_0$)

or $V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$

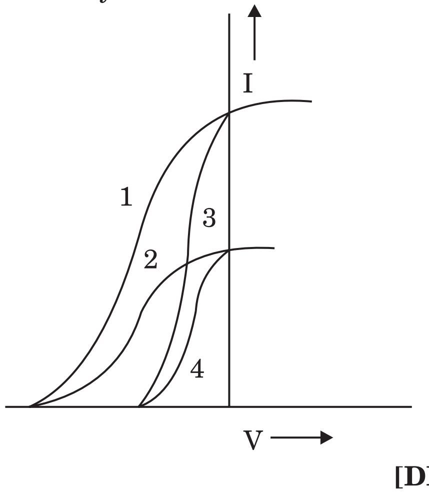
According to this result, the graph of V_0 versus ν is a straight line having the slope equal to $\left(\frac{h}{e}\right)$.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

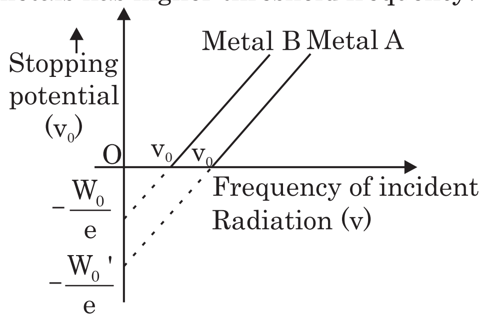
1 Mark Questions

1. The given graph shows the variation of photoelectric current (I) versus applied voltage (V) for two different photosensitive materials and for two different intensities of the incident radiations. Identify the pairs of curves that correspond to different materials but same intensity of incident radiation.



[DELHI 2013]

2. The graph shows variation of stopping potential versus frequency of incident radiation ν for two photosensitive metals A and B. Which of the two metals has higher threshold frequency?

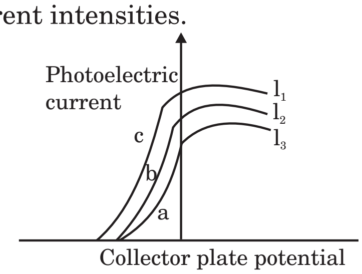


[ALL INDIA 2014]

3. Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity.

[DELHI 2014]

4. The figure shows a plot of three curves a, b, c, showing the variation of photocurrent vs collector plate potential for three different intensities I_1, I_2 and I_3 having frequencies υ_1, υ_2 and υ_3 respectively incident of a photosensitive surface. Point out the two curves for which the incident radiations have same frequency but different intensities.



[DELHI 2017]

2 Mark Questions

5. Using Bohr's postulates, obtain the expressions for (i) kinetic energy and (ii) potential energy of the electron in stationary state of hydrogen atom.

Draw the energy level diagram showing how the transitions between energy levels result in the appearance of Lyman series.

[DELHI 2013]

6. (i) Monochromatic light of frequency 6.0×10^{14} Hz is produced by a laser. The power emitted is 2.0×10^{-3} W. Estimate the number of photons emitted per second on an average by the source.

(ii) Draw a plot showing the variation of photoelectric current versus the intensity of incident radiation on a given photosensitive device.

[DELHI 2014]

7. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. **[ALL INDIA 2017]**

8. Plot a graph showing the variation of stopping potential with the frequency of incident radiation for two different photo sensitive materials having work functions W_1 and W_2 ($W_1 > W_2$). On what factors does the (i) slope and (ii) intercept of the line lines depend?

[DELHI 2018]

9. If light of wavelength 412.5nm is incident on each of the metals given below, which ones will show photoelectric emission and why?

Metal	Work function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

[ALL INDIA 2018]

3 Mark Questions

10. An electron and a photon each have a wavelength 1.00 nm. Find
(a) their momenta,
(b) the energy of the photon and
(c) the kinetic energy **[ALL INDIA 2011]**

11. (a) What is the significance of negative sign in the expression for the energy?
(b) Draw the energy level diagram showing how the line spectra corresponding to Paschen series occur due to transition between energy levels.

[DELHI 2013]

12. (a) Why photoelectric effect cannot be explained on the basis of wave nature of light? Give reasons.
(b) Write the basic features of photon picture of electro-magnetic radiation on which Einstein's photoelectric equation is based.

[DELHI 2013]

13. (a) Deduce the expression, $N = N_o e^{-\lambda t}$ for the law of radioactive decay.
(b) (i) Write symbolically the process expressing the β+ decay of ${}^{22}_{11}Na$. Also write the basic nuclear process underlying this decay.
(ii) Is the nucleus formed in the decay of the nucleus ${}^{22}_{11}Na$, an isotope or isobar?

[DELHI 2014]

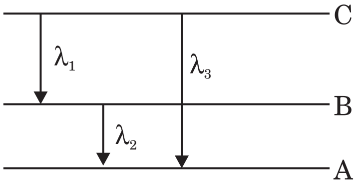
14. For the past some time, Aarti had been observing some erratic body movement, unsteadiness and lack of coordination in the activities of her sister Radha, who also used to complain of severe headache occasionally. Aarti suggested to her parents to get a medical check-up of Radha. The doctor thoroughly examined Radha and diagnosed that she has a brain tumour.
(a) What, according to you, are the values displayed by Aarti?
(b) How can radioisotopes help a doctor to diagnose brain tumour?

[ALL INDIA 2014]

15. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.
The maximum kinetic energy of the photo electrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2. Derive the expressions for the threshold wavelength λ_0 and work function for the metal surface.

[DELHI 2015]

16. (i) State Bohr's quantization condition for defining stationary orbits. How does de-Broglie hypothesis explain the stationary orbits?
(ii) Find the relation between the three wavelengths from the energy level diagram shown below.



[DELHI 2016]

17. Write three characteristic features in photoelectric effect which cannot be explained on the basis of wave theory of light, but can be explained only using Einstein's equation.

[DELHI 2016]

18. Using photon picture of light, show how Einstein's photoelectric equation can be established. Write two features of photoelectric effect which cannot be explained by wave theory.

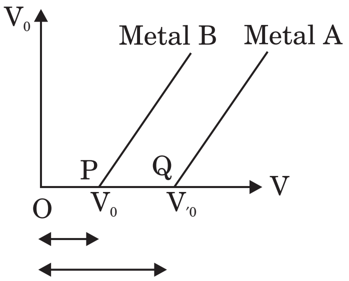
[ALL INDIA 2017]

Solutions

1. Curves 1 and 2 correspond to similar materials while curves 3 and 4 represent different materials, since the value of stopping potential for 1, 2 and 3, 4 are the same. For the given frequency of the incident radiation, the stopping potential is independent of its intensity. **[½]**

So, the pairs of curves (1 and 3) and (2 and 4) correspond to different materials but same intensity of incident radiation. **[½]**

2.



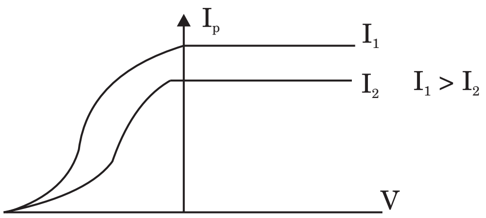
[½]

As $OP > OQ$

$\therefore \nu'_o > \nu_o$

$\therefore$ Threshold frequency of A > Threshold frequency of B **[½]**

3.



[1]

Fig.: Graph Showing plot of stopping Potential vs Frequency

4. Curves a and b have the same frequency but different intensities. **[1]**

5. According to Bohr's postulates, in a hydrogen atom, a single electron revolves around a nucleus of charge $+ke$. For an electron moving with a uniform speed in a circular orbit or a given radius, the centripetal force is provided by Coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small. **[½]**

So, $\frac{mv^2}{r} = \frac{ke^2}{r^2}$

$$mv^2 = \frac{ke^2}{r} \qquad (1)$$

Where, m = mass of the electron

r =radius of electronic orbit

v = velocity of electron.

Again, $mvr = \frac{nh}{2\pi}$

From equation (1), we get,

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r}$$

$$m\left(\frac{n^2h^2}{4\pi^2m^2r^2}\right) = \frac{ke^2}{r}$$

[½]

$$\frac{n^2h^2}{4\pi^2mr^2} = \frac{ke^2}{r}$$

$$\frac{n^2h^2}{4\pi^2mre^2} = k$$

Using equation (2), we get

$$E_k = \frac{ke^2 4\pi^2 kme^2}{2n^2h^2}$$

(ii) Potential energy,

$$E_p = \frac{-k(e)\times(e)}{r} = -\frac{ke^2}{r}$$

Using equation (2), we get

$$E_p = -ke^2 \times \frac{4\pi^2 kme^2}{n^2h^2}$$

$$E_p = -\frac{4\pi^2k^2me^4}{n^2h^2}$$

Hence, total energy of the electron in the n[th] orbit, $E = E_p + E_k$

$$E = -\frac{4\pi^2k^2me^4}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2}$$

$$E = -\frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{13.6}{n^2}eV$$

[½]

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

In H-atom, when an electron jumps from the orbit to orbit , the wavelength of the emitted radiation is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

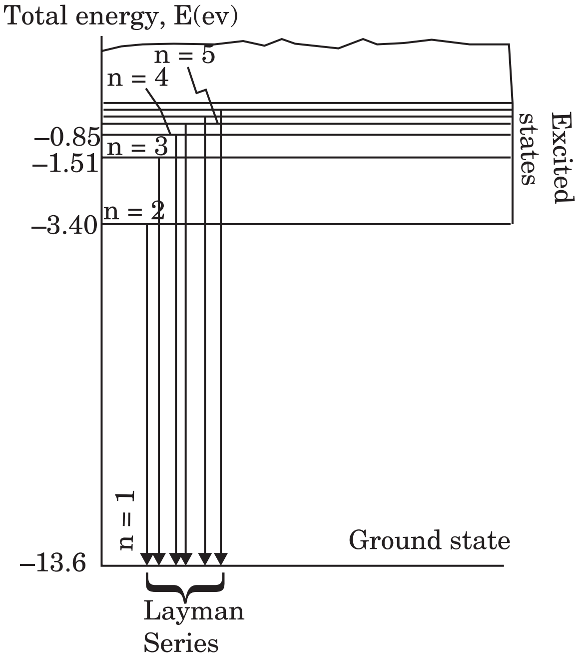
Where, $R \rightarrow$ Rydberg's constant

$= 1.09678 \times 10^{-7}$ m^{-1}

For Balmer series, $n_f = 2$ and $n_i = 3, 4, 5.....$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

Where, $n_i = 3, 4, 5.........$ **[½]**



6. (i) The energy of a proton of frequency υ is .

$$E = \hbar\upsilon = \left(6.63 \times 10^{-34}\,Js\right) \times \left(6 \times 10^{14}\,s^{-1}\right)$$

$$= 3.98 \times 10^{-19}\,J$$ **[½]**

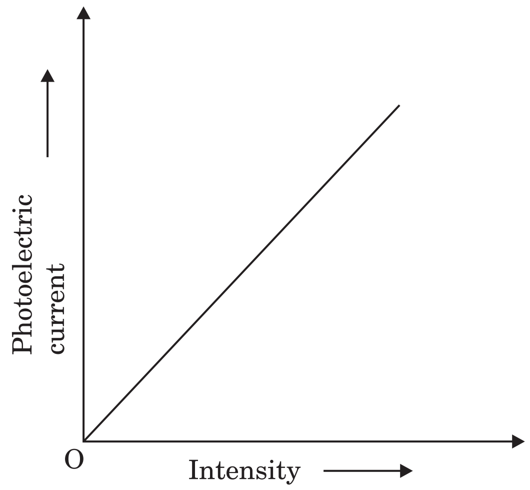
If n be the number of photons emitted by the source per second, then the power P transmitted in the beam is given by, $P = nE$

$$\therefore n = \frac{P}{E}$$

$$\Rightarrow n = \frac{2 \times 10^{-3}}{4 \times 10^{-19}}$$

$n = 5 \times 1015$ photons/sec **[½]**

(ii)



[1]

Fig.: Graph Showing variation of Photoelectric current with respect to Intensity

7. The energy of gaseous hydrogen at room temperature are

$E_1 = 13.6\ eV$

$E_2 = 3.4\ eV$

$E_3 = 1.51\ eV$

$E_4 = 0.85\ eV$

$E_3 - E_1 = -1.51\ (-13.6) = 12.09\ eV$ and

$E_4 - E_1 = -0.85\ (-13.6) = 12.75\ eV$

As, both the values does not match the given value but it is nearest to $E_4 - E_1$ upto E_4 energy level the hydrogen atoms would be excited. **[1]**

Lyman series:

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

For first member n = 2

$$\therefore \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 1.097 \times 10^7 \left[\frac{4-1}{4}\right]$$

$$\Rightarrow \lambda_1 = 1.215 \times 10^{-7}\,m$$ [½]

Balmer series:

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

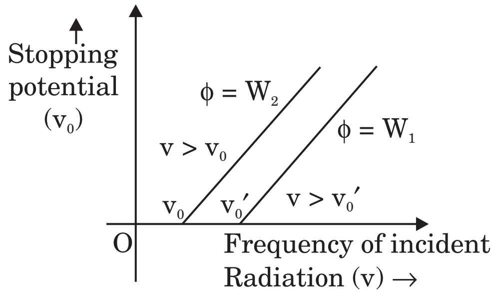
For first member $n = 3$

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{9}\right]$

$\Rightarrow \lambda_1 = 6.56 \times 10^{-7} m$ [½]

8.



[1]

Fig.: Variation of Stopping Potential with respect to frequency of incident radiation.

(i) The slope of the graph is constant and equals to $\frac{h}{e}$. Therefore, the slope does not depend on any factor. [1]

(ii) The intercept of the lines depends on the work function ϕ of the metals.

9. $E = h\nu = \frac{hc}{\lambda}$ (J)

$= \frac{hc}{\lambda e}$ (eV) [1]

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{412.5 \times 10^{-9} \times 1.6 \times 10^{-19}}$ [½]

$E = 3eV$

$Na \rightarrow$ emission $E > \lambda_o$

$K \rightarrow$ emission $E > \lambda_o$

$Ka \rightarrow$ No emission $E > \lambda_o$

$Mo \rightarrow$ No emission $E < \lambda_o$ [½]

10. $\lambda_c = \lambda_{photon} = 1.00nm = 10^{-9} m$

(a) For electron or photon, momentum

$p = p_e = p_r = \frac{h}{\lambda}$ [1]

$p = \frac{6.63 \times 10^{-34}}{10^{-9}}$

$= 6.63 \times 10^{-25}$ kg m/s

(b) Energy of photon, $E = \frac{hc}{\lambda}$

$= \left(6.63 \times 10^{-34}\right) \times \frac{3 \times 10^8}{10^{-9}}$

$\approx 19.89 \times 10^{-17} J$ [1]

(c) Kinetic energy of electron $= \frac{p^2}{2m}$

$= \frac{1}{2} \times \frac{\left(6.63 \times 10^{-34}\right)^2}{9.1 \times 10^{-31}} J$

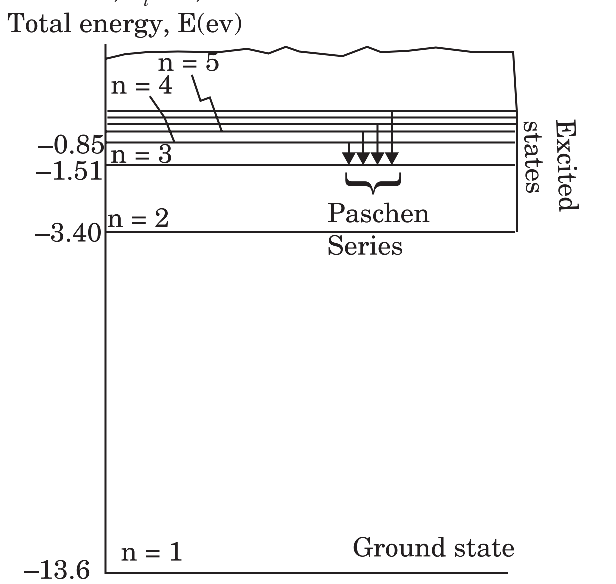
$\approx 2.42 \times 10^{-19} J$ [1]

11. (a) Negative sign indicates that revolving electron is bound to the positive nucleus. [1]

(b) For paschen series, $n = 3$ and $n_i = 4, 5$……

$\frac{1}{\lambda} = r\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$ [2]

Where, $n_i = 4, 5$………



12. (a) Wave nature of radiation cannot explain the photoelectric effect because of:

(i) The immediate ejection of photo electrons.

(ii) The presence of threshold frequency for a metal surface.

(iii) The fact -that kinetic energy of the emitted electrons is independent of the intensity of light and depends upon its frequency.

Thus, the photoelectric effect cannot be explained on the basis of wave nature of light. [1½]

(b) Photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based on particle nature of light. Its basic features are:

(i) In interaction with matter, radiation behaves as if it is made up of particles called photons.

(ii) Each photon has energy ($E = hv$), momentum $\left(p = \frac{hv}{c}\right)$, and speedc, the speed of light.

(iii) All photons of light of a particular frequency v, or wavelength λ, have the same energy $\left(E = hv = \frac{hc}{\lambda}\right)$ and momentum $\left(p = \frac{hv}{c}\right)$, may be.

(iv) By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

(v) Photons are electrically neutral and are not deflected by electric and magnetic fields.

(vi) In a photon—particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, number of photons may not be observed. [1½]

13. (a) According to the law of radioactive decay, we have:

$$\frac{\Delta N}{\Delta t} \propto N$$

Where,

$N \rightarrow$ Number of nuclei in the sample

$\Delta N \rightarrow$ Amount undergoing decay

ΔtTime

$$\frac{\Delta N}{\Delta t} = \lambda N$$

Where, λ = Decay constant or disintegration constant

$\Delta t \rightarrow$ A

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt$$

[1]

On integrating both sides, we get:

$$\int_{N_o}^{N} \frac{dN}{N} = \lambda \int_{t_o}^{t} dt$$

$$\text{In } N = \ln N_o = -\lambda(t - t_o)$$

At $t_o = 0$

$$\text{In } \frac{N}{N_o} = -\lambda t$$

$$\therefore N(t) = N_o e^{-\lambda t}$$

[1]

(b) (i) The β decay for ${}^{22}_{11}Na$ is given below:

$${}^{22}_{11}Na \longrightarrow {}^{22}_{11}Ne + {}^{0}_{1}\beta + \upsilon$$

$$\text{Or } {}^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + e^{+} + \upsilon$$

If the unstable nucleus has excess protons than required for stability, a proton converts itself into a neutron. In the process, a positron e^+ (or a β^+) and a neutrino υ are created and emitted from the nucleus.

$$p \longrightarrow n + \beta + \upsilon$$

$$\text{Or } p \longrightarrow \eta + e^{+} + \gamma$$ [½]

This process is called beta plus decay

(ii) The nucleus so formed is an isobar of ${}^{22}_{11}Na$ because the mass number is same, but the atomic numbers are different. [½]

14. (a) Aarti shows good awareness towards health and care for her sister. [1½]

(b) Certain radio isotopes are injected to body and they are absorbed by brain tumour and by detecting intensity of radiations we can measure location and severity of tumour.[2½]

15. Einstein's photoelectric equation is given by,

$$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2$$

$$K_{\text{max}} = \hbar\upsilon - \phi_o$$

$$\text{Or, } \hbar\upsilon = \hbar\upsilon_o + \frac{1}{2} m \upsilon_{\text{max}}^2$$

Where, K_{max} = Maximum kinetic energy of the photoelectron

υ_{max} = Maximum velocity of the emitted photoelectron

m =Mass of the photoelectron

υ = Frequency of the light radiation

ϕ_o = Work function

$\hbar$ = Plank's constant

If υ_o is the threshold frequency, then the work function can be written as, $\phi_o = \hbar\upsilon_o$

$$\Rightarrow K_{max} = \frac{1}{2}mv_{max}^2 = \hbar\upsilon - \hbar\upsilon_o = \hbar(\upsilon - \upsilon_o)$$ [1]

The above equations explain the following results.

(1) If $v < v_0$, then the maximum kinetic energy is negative, which is impossible. Hence, photoelectric emission does not take place for the incident radiation below the threshold frequency. Thus, the photoelectric emission can take place if $v > v_0$. [1]

(2) The maximum kinetic energy of emitted photoelectrons is directly proportional to the frequency of the incident radiation. This means that maximum kinetic energy of photo electron depends only on the frequency of incident light.

According to the photoelectric equation,

$$K_{max} = \frac{1}{2}mv_{max}^2 = \hbar\upsilon - \phi_o$$

$$K_{max} = \frac{\hbar c}{\lambda_1} - \phi_o$$

Let the maximum kinetic energy for the wavelength of the incident λ_2 be k_2.

$$k_2 = \frac{\hbar c}{\lambda_1} - \phi_o$$

$k_2 = 2k_1$

From (1) and (2), we have

$$\frac{\hbar c}{\lambda_2} - \phi_o = 2\left(\frac{\hbar c}{\lambda_1} - \phi_o\right)$$

$$\Rightarrow \phi_o = \hbar c\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \hbar\upsilon_o = \hbar c\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \frac{c}{\lambda_o} = c\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \frac{1}{\lambda_o} = \left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \lambda_o = \left(\frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}\right)$$

Work function is the energy required to eject a photoelectron from the metal.

$$W = \frac{\hbar c}{\lambda_o}$$

$$\therefore W = \frac{\hbar c(2\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$$ [1]

16. (i) **Quantization condition:** Of all possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, h being Plank's constant.

Therefore, for any permitted orbit,

$$L = m\upsilon r = \frac{n\hbar}{2\pi}; n = 1, 2, 3, ...$$

Where L, m and υ is the angular momentum, mass and speed of the electron, r is the radius of the permitted orbit and n is positive integer called principle quantum number.

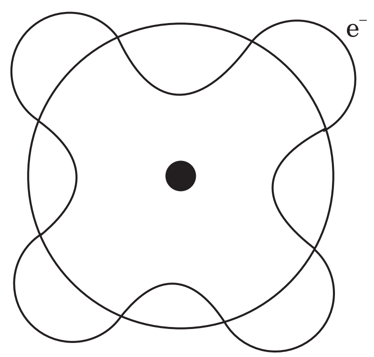
The above equation is Bohr's famous quantum condition. When an electron of mass m is confined to move on a line of length l with velocity v, the de-Broglie wavelength λ associated with electron is:

$$\lambda = \frac{\hbar}{mv} = \frac{\hbar}{p}$$ [1]

Or, Linear momentum

$$= p = \frac{\hbar}{\lambda} = \frac{\hbar}{2l/n} = \frac{n\hbar}{2l}$$

When electron revolves in a circular orbit of radius 'r' then $2l = 2\pi n$



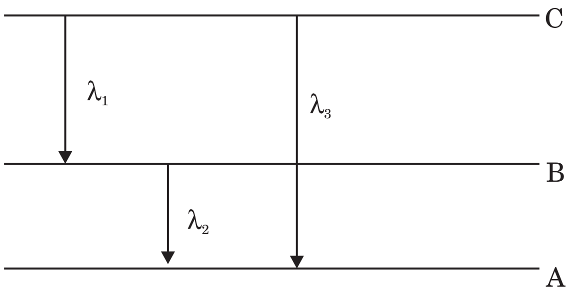
$\therefore P = \dfrac{n\hbar}{2\pi r}$ Or $P \times r = \dfrac{n\hbar}{2\pi}$

Or angular momentum $|\vec{L}| = p \times r$ is integral multiple of $\dfrac{\hbar}{2\pi}$ which is Bohr's quantization of angular momentum.

(ii) $E_{CB} = \dfrac{\hbar c}{\lambda_1}$

$E_{BA} = \dfrac{\hbar c}{\lambda_2}$

[1]



$E_{CA} = \dfrac{\hbar c}{\lambda_3}$

Where, $E_{CA} = E_{CB} + E_{BA}$ (E_{CB} = Energy gap between level B and C)

$$\frac{\hbar c}{\lambda_3} = \frac{\hbar c}{\lambda_1} + \frac{\hbar c}{\lambda_2}$$

(E_{BA} = Energy gap between level A and B)

$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

(E_{CA} = Energy gap between level A and C)

$$\lambda_3 = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$

[1]

17. (i) **Existence of threshold frequency:** According to wave theory, there should not exist any threshold frequency but Einstein's theory explains the existence of Threshold frequency. [1]

(ii) **Dependence of kinetic energy on frequency of incident light:** According to wave theory, the maximum kinetic energy of emitted electrons should depend on intensity of incident light and not on frequency whereas Einstein's equation explains that it depends on frequency and not on intensity. [1]

(iii) **Instantaneous emission of electrons:** According to wave theory there should be time lag between emission of electrons and incident of light whereas Einstein's equation explains why there is no time lag between incident of light and emission of electrons. [1]

18. Einstein photoelectric equation:

When a photon of energy hυ falls on a metal surface the energy of the photon is absorbed by the electrons and is used in following two ways.

(i) A part of energy is used to overcome the surface barrier and come out of the metal surface. This part of energy is called "work function". It is expressed as $\phi_o = h\, v_o$ [1]

The remaining part of energy is used in giving velocity v to the emitted photoelectron.

This is equal to maximum kinetic energy of photo electron $\frac{1}{2}V_{max}^2$

According to the law of conservation of energy

$$h\upsilon = \phi_o + \frac{1}{2}mv_{max}^2 = h\upsilon_o + \frac{1}{2}mv_{max}^2$$

[1]

$$\therefore \frac{1}{2}mv_{max}^2 = k_{max} = h(\nu - \nu_o) = h\nu - \phi_o$$

$$\therefore k_{max} = h\nu - \phi_o$$

This equation is called Einstein Photoelectric Equation. [1]

[Topic 2] Matter Wave

Summary

Dual Nature of matter

- ***Particle Nature of matter:***

  Radiation behaves as if it is made up of particles in interaction of radiation with matter, called photons.

  Each photon has energy E = hv and momentum $p = \frac{h\upsilon}{c}$, and speed c that is the speed of light.

- ***Wave Nature of Matter:***

  De Broglie proposed that the moving particles are associated with the waves. If a particle is having a momentum p, then the associated wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$, where v is the speed of the moving particle and its mass. The wavelength λ is known as the ***de Broglie wavelength*** and the above relation as the ***de Broglie relation***.

  The wavelength of an electron accelerated with the potential V is:

  $$\lambda = \frac{1.227}{\sqrt{V}} nm$$

- ***Heisenberg's uncertainty principle:*** This principle states that, "it is not possible to measure both the position and momentum of an electron at the same time exactly. There is always some uncertainty in the position and in momentum.

  $\Delta x \, \Delta p \approx \hbar$, where $\hbar = \frac{h}{2\pi}$

- The wave nature of electron was verified and confirmed by the electron diffraction experiments performed by Davisson and Germer, and G.P. Thomson. Many other experiments later also confirmed the wave nature of electron.

PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 2

1 Mark Questions

1. Show graphically, the variation of the de-Broglie wavelength (λ) with the potential (V) through which an electron is accelerated from rest.

[DELHI 2011]

2. State de-Broglie hypothesis.

[DELHI 2012]

2 Mark Questions

3. A proton and a deuteron are accelerated through the same accelerating potential. Which one of the two has,

(a) Greater value of de-Broglie wavelength associated with it, and

(b) Less momentum?

Give reasons to justify your Answer.

4. A proton and an α-particle have the same de Broglie wavelength. Determine the ratio of (i) their accelerating potentials (ii) their speeds.

[DELHI 2015]

5. Plot a graph showing variation of de-Broglie wavelength λ versus $\frac{1}{\sqrt{V}}$, where V is accelerating potential for two particles A and B carrying same charge but of masses m_1, m_2 ($m_1 > m_2$). Which one of the two represents a particle of smaller mass and why?

[DELHI 2016]

6. A proton and an α particle are accelerated through the same potential difference. Which one of the two has (i) greater de-Broglie wavelength, and (ii) less kinetic energy? Justify your answer.

[ALL INDIA 2016]

7. An electron is accelerated through a potential difference of 64 volts. What is the De-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond?

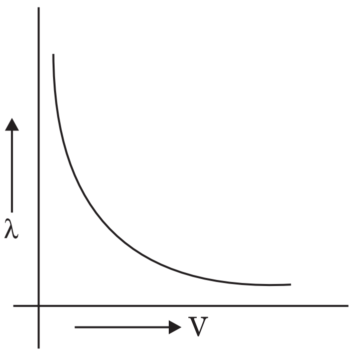
[DELHI 2018]

3 Mark Questions

8. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. Taking other factors, such as numerical aperture etc. to be same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light? **[ALL INDIA 2014]**

Solutions

1.



[1]

2. De Broglie postulated that the material particles may exhibit wave aspect. Accordingly a moving material particle behaves as wave and the wavelength associated with material particle is **[½]**

$$\lambda = \frac{h}{m\nu}$$ **[½]**

Where h = Planck's constant

m = mass of the object

ν = velocity of the object

3. (a) de-Broglie wavelength,

$\lambda \propto \frac{1}{mass}$ (For same accelerating potential)

Mass of a proton is less as compared to a deuteron. So, proton will have greater value of de-Broglie wavelength associated with it. **[1]**

(b) Momentum, $p \propto$ mass (for same accelerating potential). Mass of deuteron is more as compared to a proton. So, it will have a greater value of momentum. **[1]**

4. (i) The de-Broglie wavelength of a particle is given by, $\lambda = \frac{12.27}{\sqrt{V}}\overset{o}{A}$

Where, V is the accelerating potential of the particle. It is given that,

$\lambda_{proton} = \lambda_{alpha}$ **[½]**

$$\Rightarrow \frac{12.27}{\sqrt{V_{proton}}} = \frac{12.27}{\sqrt{V_{alpha}}}$$

Thus,

$$\frac{V_{proton}}{V_{alpha}} = 1$$ **[½]**

(ii) We can also write de-Broglie wavelength as,

$$\lambda = \frac{\hbar}{mv}$$

Where, $\hbar \rightarrow$ plank's constant, $m \rightarrow$ mass of the particle and speed of the particle.

It is given that, $\lambda_{proton} = \lambda_{alpha}$

We know, $m_{alpha} = 4m_{proton}$

$$\therefore \lambda_{alpha} = \frac{\hbar}{4m_{proton}v_{alpha}}$$ **[½]**

$$\Rightarrow \frac{\hbar}{m_{proton}v_{proton}} = \frac{\hbar}{4m_{proton}v_{alpha}}$$

$$\frac{v_{proton}}{v_{alpha}} = 4$$ **[½]**

5. $qV = \frac{1}{2}mv^2$

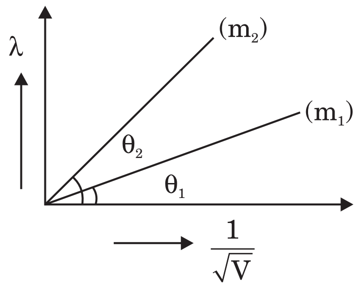
$qV = \frac{p^2}{2m}$ **[½]**

$\Rightarrow p = \sqrt{2mqV}$

$p = \frac{h}{\lambda}$

$\Rightarrow \lambda = \frac{h}{\sqrt{2mqV}}$

$\Rightarrow$ Slope $\propto \frac{1}{\sqrt{m}}$ **[1]**



[½]

6. When a charge particle is accelerated through V potential difference then its kinetic energy

$K.E. = qV$ (q = charge; V = potential difference)

Proton and α-particle accelerated through same potential difference so **[½]**

because $q_\alpha > q_{proton}\left(q_\alpha = +2e \text{ and } q_{proton} = +e\right)$
$K.E_\alpha > K.E_{proton}$

Their Debroglie wavelength

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

V = same **[½]**

$m_\alpha q_\alpha > m_p q_p \left[m_\alpha = 4\ m_p\right]$

So $\lambda_\alpha < \lambda_p$ (λ_α = Debroglie λ_p wavelength of α-particle; Debroglie wavelength of proton)

(1) Proton have greater De Broglie wavelength

(2) Proton have lesser kinetic energy **[1]**

7. De-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2meV}}$ Where, m (mass of electron) = 9.1×10^{-31} kg [½]

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 64}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{1863.68 \times 10^{-50}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{1863.68} \times 10^{-25}}$$

$$\lambda = \frac{6.626}{43.17} \times 10^{-9}$$

l = $0.15 \times 10^{-9}\ m = 1.5 \times 10^{-10}\ m$ [1]

This value of wavelength corresponds to the X-ray region of the electromagnetic spectrum. [½]

8. $\lambda = \frac{12.27 \overset{o}{A}}{\sqrt{V}}$

$$\lambda = \frac{12.27 \overset{o}{A}}{\sqrt{50000}}$$

$$\lambda = \frac{12.27 \overset{o}{A}}{\sqrt{233}}$$

$\lambda = 0.00526$ Å [1]

Resolving Power (RP) $\propto \frac{1}{\lambda}$ as wavelength of moving electron is very small as compared to that of yellow light so it has high Resolving Power than optical microscope. [2]

Smart Notes ....

CHAPTER 12

Atoms

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Alpha-particle scattering experiment					
Rutherford's model of atom			1Q (2 marks)		
Bohr model, energy levels, hydrogen spectrum	1Q (1mark)	1Q (2 marks)		1Q (2 marks)	1Q (3 marks)

From analysis of previous years' papers, it is clear that Bohr Model and Rutherford's model are most important topics of the chapter from exam point of view.

Topic 1: Rutherford's Atomic Model, Bohr's Model & Energy Level Diagram

Summary

Introduction

- Atoms in simple terms are defined as the smallest unit of matter.
- Atoms are electrically neutral because they contain same number of electrons and protons.

Plum-Pudding Model

- In 1898, J. J. Thomson proposed the first model of atom.
- He stated, there is a uniform distribution of the positive charge of the atom throughout the volume of the atom and like seeds in a watermelon, the negatively charged electrons are embedded in it. This model was picturesquely called plum pudding model of the atom.

Alpha-Particle Scattering

- Rutherford used a "Gold foil experiment"
- Rutherford only identified one of type of radiation given off by radioactive elements like polonium, uranium and named them as alpha particles.
- The alpha particles are fast moving and positively charged Helium nuclei with two protons and two neutrons.

Rutherford observed the deflection of alpha particles after passing through metal sheet and proposed his atomic model

- After passing through the metal sheet, the alpha particles strike on fluorescent screen which was coated with zinc sulphide and produced a visible flash of light
- He concluded that an atom consists of a minute positively charged body at its center called as nucleus. The nucleus, though small, contains all the protons and neutrons.

Alpha-Particle Trajectory

- The trajectory traced by an α particle depends on the impact parameter, b of collision.
- The particle near to the nucleus suffers large scattering.
- Only a small fraction of the number of incident particles rebound back indicating that the number of α-particles undergoing head on collision is small.

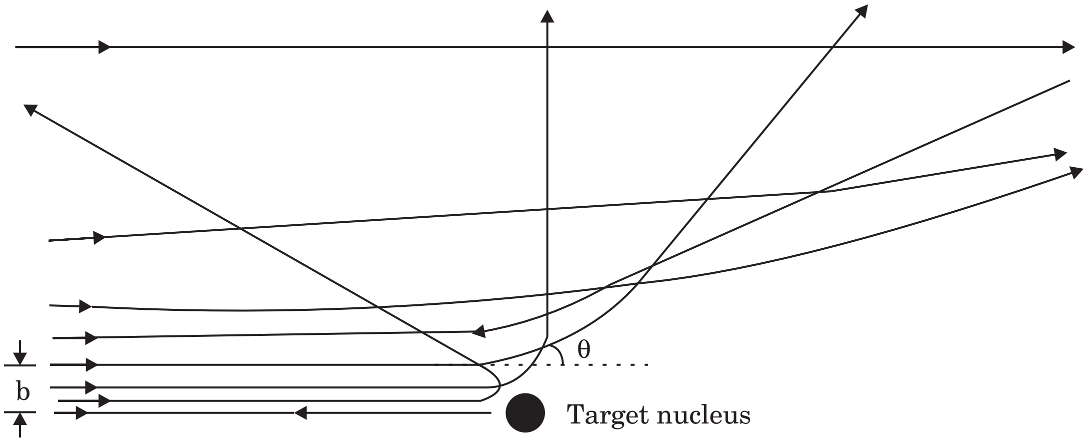


Fig.: Alpha-Particle Trajectory

Rutherford's nuclear model of Atom

- According to Rutherford's model, the entire positive charge and most of the mass of the atom is concentrated in a small volume called the nucleus with electrons revolving around the nucleus just as planets revolve around the sun.
- Rutherford scattering is a powerful way to determine an upper limit to the size of the nucleus.
- **Drawbacks of Rutherford's model:** There were two major drawbacks in Rutherford nuclear model in explaining the structure of atom:

It cannot explain the characteristic line spectra of atoms of different elements.

It contradicts the stability of matter because it speculates that atoms are unstable because the accelerated electrons revolving around the nucleus must spiral into the nucleus.

Electron Orbits

- The electrostatic force of attraction, F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits. Hence, for a dynamically states orbit in a hydrogen atom $F_e = F_c$
- The total energy of the electron is negative. It is given by $E = -\frac{e^2}{8\pi\varepsilon_0 r}$.

Atomic Spectra

- Each element has a characteristic spectrum of radiation, which it emits.
- Study of emission line spectra of a material can therefore serve as a type of "fingerprint" for identification of the gas.
- The atomic hydrogen emits a line spectrum consisting of various series as:

  Lyman series: $v = Rc\left(\frac{1}{1^2} - \frac{1}{n^2}\right) : n = 2, 3, 4, \ldots$

  Balmer series: $v = Rc\left(\frac{1}{2^2} - \frac{1}{n^2}\right) : n = 3, 4, 5, \ldots$

  Paschen series: $v = Rc\left(\frac{1}{3^2} - \frac{1}{n^2}\right) : n = 4, 5, 6, \ldots$

  Brackett series: $v = Rc\left(\frac{1}{4^2} - \frac{1}{n^2}\right) : n = 5, 6, 7, \ldots$

  Pfund series: $v = Rc\left(\frac{1}{5^2} - \frac{1}{n^2}\right) : n = 6, 7, 8, \ldots$

Bohr Model of the Hydrogen Atom

Bohr combined classical and early quantum concepts, explained the spectrum of hydrogen atom based on quantum ideas and gave his theory in the form of three postulates. These are:

- Bohr's first postulate was that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory. According to this postulate, each atom has certain definite stable states in which it can exist, and each possible state has definite total energy. These are called the stationary states of the atom.
- Bohr's second postulate defines these stable orbits. This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$ where h is the Planck's constant (= 6.6×10^{-34} Js). Thus the angular momentum (L) of the orbiting electron is quantised. That is $L = nh/2\pi$.
- Bohr's third postulate incorporated into atomic theory the early quantum concepts that had been developed by Planck and Einstein. It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is then given by

  $hv = E_i - E_f$, where E_i and E_f are the energies of the initial and final states and $E_i > E_f$.
- Bohr radius is represented by the symbol a_0, is given by $a_0 = \frac{h^2\varepsilon_0}{\pi m e^2}$.
- The total energy of the electron in the stationary states of the hydrogen atom is given by

  $E_n = -\frac{13.6}{n^2} eV$

De Broglie's Explanation of Bohr's Second Postulate of Quantisation

- De Broglie hypothesis provided an explanation for Bohr's second postulate for the quantisation of angular momentum of the orbiting electron. The quantised electron orbits and energy states are due to the wave nature of the electron and only resonant standing waves can persist.
- De Broglie's hypothesis is that electrons have a wavelength $\lambda = \frac{h}{mv}$.

Limitations of Bohr's model: Bohr's model however has many limitations.

- It is applicable only to hydrogenic (single electron) atoms.
- It cannot be extended to even two electron atoms such as helium.
- While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, the model is unable to explain the relative intensities of the frequencies in the spectrum.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. Show graphically, the variation of the de-Broglie wavelength (λ) with the potential (V) through which an electron is accelerated from rest. **[All India 2011]**

2. Write the expression for Bohr's radius in hydrogen atom. **[DELHI 2018]**

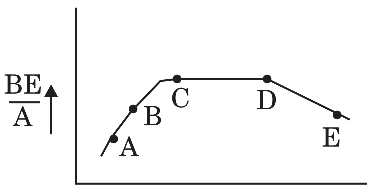
2 Mark Questions

3. Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron?

OR

Using Bohr's postulates of the atomic model derive the expression for radius of nth electron orbit. Hence obtain the expression for Bohr's radius. **[All India 2014]**

4. (a) The figure shows the plot of binding energy (BE) per nucleon as a function of mass number A. The letters A, B, C, D and E represent the positions of typical nuclei on the curve. Point out, giving reasons, the two processes (in terms of A, B, C, D and E), one of which can occur due to nuclear fission and the other due to nuclear fusion.



(b) Identify the nature of the radioactive radiations emitted in each step of the decay process given below.

$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y \rightarrow {}^{A-4}_{Z-1}W$ **[All India 2015]**

5. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV projected towards a nucleus of $Z = 80$, stops and reverses its direction. **[All India 2015]**

6. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. **[DELHI 2015]**

7. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, show how the de Broglie wavelength associated with it would be affected. **[All India 2015]**

8. When is Hα line in the emission spectrum of hydrogen atom obtained? Calculate the frequency of the photon emitted during this transition.

Or

Calculate the wavelength of radiation emitted when electron in a hydrogen atom jumps from $n = \infty$ to $n = 1$. **[All India 2016]**

3 Mark Questions

9. An electron and a photon each have a wavelength 1.00 nm. Find

(i) Their momenta.

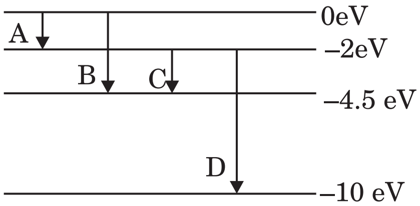
(ii) The energy of the photon and

(iii) The kinetic energy of electron.

[All India 2011]

10. The energy levels of a hypothetical atom are shown below. Which of the shown transitions will result in the emission of a photon of wavelength 275 nm?

Which of these transitions correspond to emission of radiation of (i) maximum and (ii) minimum wavelength?



[All India 2011]

11. (i) Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the nth orbital state in hydrogen atom is 'n' times the De-Broglie wavelength associated with it.

(ii) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

[All India 2012]

12. Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels.

[All India 2013]

13. A 12.5 *eV* electron beam is used to bombard gaseous hydrogen at room temperature. Up to which energy level the hydrogen atoms would be excited? Calculate the wavelength of the first member of Lyman and first member of Balmer series. **[DELHI 2014]**

14. In the study of Geiger-Marsden experiment on scattering of α-particles by a thin foil of gold, draw the trajectory of α-particles in the Coulomb field of target nucleus. Explain briefly how one gets the information on the size of the nucleus from this study.

From the relation $R = R_0 A\frac{1}{3}$, where R_0 is constant and A is the mass number of the nucleus, show that nuclear matter density is independent of A.

OR

Distinguish between nuclear fission and fusion. Show how in both these processes energy is released. Calculate the energy released in *MeV* in the deuterium-tritium fusion reaction:

$${}_1^2H + {}_1^3H \rightarrow {}_1^4H + {}_on^1$$

Using the data:

$m\left({}_1^2H\right) = 2.014102u$

$m\left({}_1^3H\right) = 3.016049u$

$m\left({}_1^4H\right) = 4.002603u$

$m_n = 1.008665u$

$u = 931.5MeV/c^2$ **[All India 2015]**

15. (a) Derive the mathematical expression for law of radioactive decay for a sample of a radioactive nucleus.

(b) How is the mean life of a given radioactive nucleus related to the decay constant?

[All India 2016]

16. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de Broglie's hypothesis explain the stability of these orbits?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the 4th level. Estimate the frequency of the photon.

[All India 2018]

4 Mark Questions

17. Asha's mother read an article in the newspaper about a disaster that took place at Chernobyl. She could not understand much from the article and asked a few questions from Asha regarding the article. Asha tried to answer her mother's questions based on what she learnt in Class XII Physics

(a) What was the installation at Chernobyl where the disaster took place?

(b) What, according to you, was the cause of this disaster?

(c) What are the values shown by Asha?

[All India 2014]

18. Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number n_i) to the lower state, (n_f). When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3$, 2, 1, identify the spectral series to which the emission lines belong.

Or

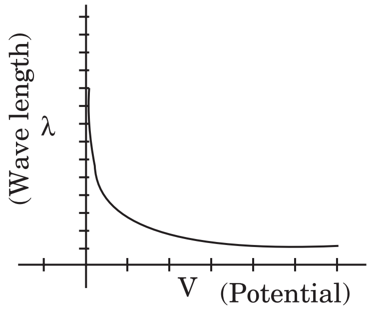
(a) Draw the plot of binding energy per nucleon $\left(\frac{BE}{A}\right)$ as a function of mass number A. Write two important conclusions that can be drawn regarding the nature of nuclear force.

(b) Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.

[All India 2014]

Solutions

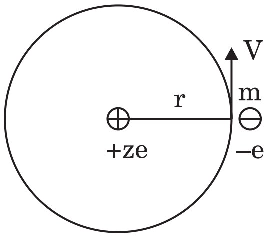
1. Since wavelength λ is inversely proportional to potential V the graphical variation of same is



[1]

2. $a_o = \frac{h^2\varepsilon_o}{\pi me^2}$ where h is planks constant, ε_o is permittivity of free space, m is rest mass of electron and e is charge on electron. **[1]**

3. Electron revolves around nucleus and required centripetal force is provided by attractive force between electron and nucleus. **[½]**



[½]

$$\frac{kze^2}{r^2} = \frac{mv^2}{r}$$

$$\therefore mv^2 = \frac{kze^2}{r}$$

$$\therefore KE = \frac{1}{2}mv^2 = \frac{kze^2}{2r}$$ **[½]**

And potential energy

$$U = -\frac{kze^2}{r}$$

$\therefore$ Total energy

$E = KE + U$ **[½]**

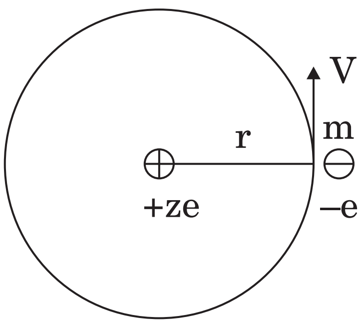
Negative sign of total energy shows that electron is bound to revolve around nucleus.

Or

Electron revolves around the nucleus and required centripetal force is provided by electrostatic force of attraction between nucleus and electron **[½]**

$$\frac{kze^2}{r^2} = \frac{mv^2}{r}$$ **[½]**

$$\therefore mv^2 = \frac{kze^2}{r} \quad \text{... (1)}$$



[½]

By Bohr's postulate

$$mvr = \frac{nh}{2\pi} \quad \text{... (2)}$$

$$\therefore v r = \frac{nh}{2\pi mr}$$ **[½]**

on putting value of v in equation (i)

$$r = \frac{n^2}{z}\left[\frac{h^2}{4\pi^2 kme^2}\right]$$

$$r = \frac{n^2}{z}(0.53)\overset{o}{\text{A}}$$

4. (a) The nuclei at *A* and *B* undergo nuclear fusion as their binding energy per nucleon is small and they are less stable so they fuse with other nuclei to become stable. The nuclei at E undergo nuclear fission as its binding energy per nucleon is less it splits into two or more lighter nuclei and become stable. **[1]**

(b) ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y \rightarrow {}^{A-4}_{Z-1}W$ **[½]**

An alpha particle $\left({}^{4}_{2}He\right)$ is emitted in the first reaction as atomic mass of *Y* is reduced by 4 and atomic number is reduced by 2. An electron is emitted $\left({}^{0}_{-1}e\right)$ in the second reaction as atomic mass of W remains the same and atomic number is increased by 1. **[½]**

5. At the distance of nearest approach

$PE = KE$ **[½]**

$$\frac{k(ze)(2e)}{r_o} = 4.5MeV = 4.5\times10^6\times1.6\times10^{-19}J$$ **[½]**

$$r_o = \frac{k(ze)(2e)}{4.5\times1.6\times10^{-13}}$$ **[½]**

$$= \frac{\left\{9\times10^9\times(80)\times2\times\left(1.6\times10^{-19}\right)^2\right\}}{4.5\times1.6\times10^{-13}}$$ **[½]**

$= 51.2 \times 10^{-15}\ m$

6. According to the Bohr's theory of hydrogen atom, the angular momentum of a revolving electron is given by,

$$mvr = \frac{n\hbar}{2\pi} \quad \text{.... (i)}$$

If an electron of mass m and velocity v is moving in a circular orbit of radius r, then the centripetal force required is given by, $F = \frac{mv^2}{r}$. **[½]**

Also, if the charge on the nucleus is Ze, then the force of electrostatic attraction between the nucleus and the electron will provide the necessary centripetal force. **[½]**

$$F = \frac{1}{4\pi\varepsilon_o}\frac{(Ze)(e)}{r^2} = \frac{KZe^2}{r^2}$$

Where, $K = \frac{1}{4\pi\varepsilon_o}$ **[½]**

$$\therefore \frac{mv^2}{r} = \frac{KZe^2}{r^3} \quad \text{... (ii)}$$

From (i), we get $v = \frac{n\hbar}{2\pi mr}$

Putting this value in (ii), we get

$$\frac{m}{r}\frac{n^2\hbar^2}{4\pi^2m^2r^2} = \frac{KZe^2}{r^2}$$

$$\Rightarrow r = \frac{n^2\hbar^2}{4\pi^2 mKZe^2} \Rightarrow r \propto n^2.$$ **[½]**

7. The velocity of a electron in a hydrogen atom is given by the relation

$$v_n = \frac{e^2}{2n\varepsilon_o n} \text{ so } v_n \propto \frac{1}{n} \text{ ---------- (i)}$$ **[½]**

$\lambda = \frac{h}{p} = \frac{h}{mv}$ and the de Broglie wavelength associated with it is **[½]**

So, $\lambda \propto \frac{1}{v_n}$ ... (ii)

using equation (i) and (ii) $\lambda \propto$ n

So when electron jump from n = 1 to n = 4 level

$$\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2} = \frac{1}{4}$$ **[½]**

$\lambda_2 = 4\lambda_1$

so wavelength increases four times. **[½]**

8. Hα is obtained when the electron makes transition from n = 3 to n = 2, level

$E_3 - E_2 = hf$ (According to Bohr)

$$-\frac{13.6}{3^2} - \frac{13.6}{2^2} = \frac{hf}{1.6\times10^{-19}}$$

$$13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{hf}{1.6\times10^{-19}}$$ **[1]**

$$13.6\left[\frac{5}{36}\right] = \frac{6.63\times10^{-34}f}{1.6\times10^{-19}}$$

$$f = \frac{0.4558\times10^{-19}}{10^{-34}}$$ **[1]**

$= 0.4558\times10^{15} = 4.558\times10^{14}$

Or

In hydrogen atom wavelength of radiation for transition $n = \infty$ to n = 1

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_L^2} - \frac{1}{n_H^2}\right]$$

$\{Here, n_L = 1, n_H = \infty \mid For\ H\ atom\ Z = 1\}$ **[1]**

$$\frac{1}{\lambda} = R(1)^2\left[\frac{1}{1^2} - \frac{1}{\infty}\right]$$

$$\frac{1}{\lambda} = R \Rightarrow \frac{1}{R} = \lambda$$

$$\lambda \approx 912\,\overset{o}{A}$$

$\{R = 1.09\times10^7 m^{-1}\}$ **[1]**

9. Wavelength of an electron λ and a photon λ_p, λ_e = $\lambda_p = \lambda = 1\ nm$

$1\times10^{-9}\ m$

Planck's constant, $h = 6.63\times10^{-34}\ Js$

(i) The momentum of an elementary particle is given by de Broglie relation:

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$ **[½]**

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

$$\therefore p = \frac{6.63\times10^{-34}}{1\times10^{-9}}$$

$p = 6.063\times10^{-25}\ kg\ ms^{-1}$ **[½]**

(ii) The energy of a photon is given by the relation:

$$E = \frac{hc}{\lambda}$$

Where,

Speed of light, $c = 3\times10^8\ m/s$

$$\therefore E = \frac{6.63\times10^{-34}\times3\times10^8}{1\times10^{-9}\times1.6\times10^{-19}}$$ **[1]**

$1243.1eV = 1.243KeV$

Therefore, the energy of the photon is $1.243keV$.

(iii) The kinetic energy (K) of an electron having momentum p, is given by the relation:

$$K = \frac{1}{2}\frac{p^2}{m}$$

Where,

m = Mass of electron = 9.1×10^{-31} kg

$p = 6.63 \times 10^{-25}$ $kg\ ms^{-1}$

$$\therefore K = \frac{1}{2} \times \frac{6.63 \times 10^{-25}}{9.1 \times 10^{-31}} - 2.415 \times 10^{-19} J$$

$= 1.51eV$ **[1]**

Hence, the kinetic energy of the electron is $1.51\ eV$.

10. For A:

Energy change: $E_1 - E_2 = 0 - (-2) = 2eV$

$$\lambda_A = \frac{12.3}{2} \times 10^{-7} = 618nm$$ **[1]**

$$\lambda_B = \frac{12.3}{4.5} \times 10^{-7} = 275nm$$

$$\lambda_C = \frac{12.3}{2.5} \times 10^{-7} = 500nm$$

$$\lambda_D = \frac{12.3}{8} \times 10^{-7} = 153nm$$ **[1]**

Maximum wavelength: emission A

Minimum wavelength: emission D **[1]**

11. (i) According to Bohr's second postulate of quantization, the electron can revolve round the nucleus only in those circular orbits in which the angular momentum of the electron is integral multiple of $\frac{h}{2\pi}$ where 'h' is Planck's constant ($= 6.62 \times 10^{-34}$ Js) **[½]**

So, if 'm' is the mass of electron and 'v' is the velocity of electron in permitted quantized orbit with radius 'r' them,

$$mv\,r = n\frac{h}{2\pi} \quad \text{... (1)}$$

Where 'n' is the principle quantum number and can take integral value i.e. n = 1, 2, 3, ...

This is Bohr's quantization condition. Now, De-Broglie wavelength is given as,

$$\lambda = \frac{h}{mv}$$ **[½]**

Where, λ = wavelength associated with electron.

v = velocity of electron

h = Planck's constant

m = mass of electron

$$\lambda = \frac{h}{mv} \quad \text{... (2)}$$

Putting value of 'v' from eqn. (2) in eqn.(1)

$$m \times \frac{h}{m\lambda} \times r = n\frac{h}{2\pi}$$ **[½]**

$$\frac{rh}{\lambda} = \frac{nh}{2\pi}$$

$2\pi r = \lambda n$

Now, circumference of the electron in the nth orbital state of Hydrogen atom with radius 'r' is $2\pi r$ **[½]**

(ii) If 'n' is the quantum number of the highest energy level involved in the transition, then the total number of possible spectral lines emitted is:

$$N = \frac{n(n-1)}{2}$$ **[½]**

Third excited state means fourth energy level i.e. $n = 4$. Here, electron makes transition from $n = 4$ to $n = 1$. So, the highest value of 'n' is 4.

Therefore, the maximum number of spectral lines would be 6. **[½]**

12. According to Bohr's postulates, in a hydrogen atom, a single electron revolves around a nucleus of charge $+e$. For an electron moving with a uniform speed in a circular orbit of a given radius, the centripetal force is provided by Coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small. So, **[½]**

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \quad (1)$$

Where, m = mass of the electron

r = radius of electronic orbit

v = velocity of electron.

Again,

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr}$$ **[½]**

From equation (i), we get, $m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r}$

$$m\left(\frac{n^2h^2}{4\pi^2m^2r^2}\right) = \frac{ke^2}{r}$$

$$\frac{n^2h^2}{4\pi^2mr^2} = \frac{ke^2}{r}$$ **[½]**

$$\frac{n^2h^2}{4\pi^2mre^2} = k \quad (2)$$

Using equation (2), we get

$$E_k = \frac{ke^2 4\pi^2kme^2}{2n^2h^2}$$

$$E_k = \frac{2\pi^2k^2me^4}{n^2h^2}$$ **[½]**

(ii) Potential energy,

$$E_p = \frac{-k(e)\times(e)}{r} = -\frac{ke^2}{r}$$

Using equation (2), we get

$$E_p = -ke^2 \times \frac{4\pi^2kme^2}{n^2h^2}$$ **[½]**

$$E_p = -\frac{4\pi^2k^2me^4}{n^2h^2}$$

Hence, total energy of the electron in the n^{th} orbit,

$E = E_p + E_k$

$$E = -\frac{4\pi^2k^2me^4}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2}$$ **[½]**

$$E = -\frac{2\pi^2k^2me^4}{n^2h^2}$$

$$E = -\frac{13.6}{n^2}eV$$

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

In H-atom, when an electron jumps from the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Where,

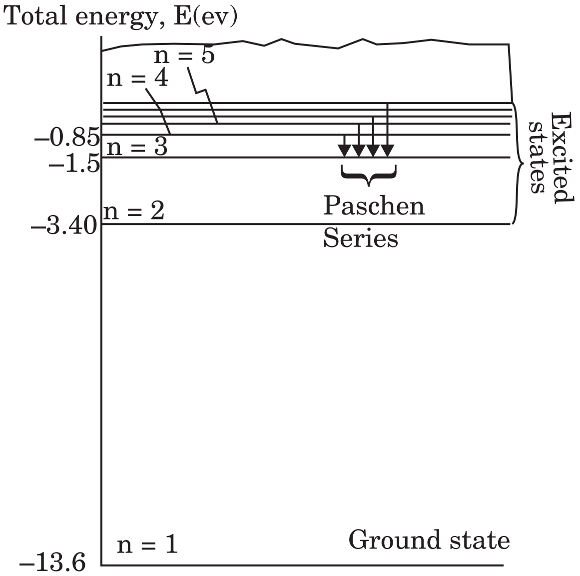
$R \to$ Rydberg's constant = $1.09678 \times 10^{-7} m^{-1}$

For Balmer series, $n_f = 2$ and $n_i = 3, 4, 5.....$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

Where, $n_i = 3, 4, 5.........$

These spectral lines lie in the visible region.



13. Energy of the electron in the n^{th} state of an atom $= -13.6z^2 n^2 eV$

Here, z is the atomic number of the atom. For hydrogen atom, z is equal to 1. Energy required to excite an atom from the initial state (n_i) to the final state

$$E_f = -13.6 \ln_f^2 + 13.6 n_i^2 eV$$ **[½]**

This energy must be equal to or less than the energy of the incident electron beam.

$$E_f = \frac{-13.6}{n_f^2} + \frac{13.6}{n_i^2}$$

$$\frac{-13.6}{n_f^2} + \frac{13.6}{n_i^2} = 12.5$$

$$\frac{-13.6}{n_f^2} + 13.6 = 12.5$$

$$13.6 - 12.5 = \frac{13.6}{n_f^2}$$ **[½]**

Energy of the electron in the ground state = 13.6eV

$$-13.6\ln_f^2 + 13.6 = 12.5$$

$$13.6 - 12.5 = 13.6\ln_f^2$$

$$n_f^2 = \frac{13.6}{1.1} = 3.5$$ **[½]**

State cannot be a fraction number $n_j = 3$

Hence, hydrogen atom would be excited up to 3rd energy level. Rydberg formula for the spectrum of the hydrogen atom is given below. **[½]**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, λ is the wavelength and R is the Rydberg constant, $R = 1.097 \times 10^7\ m^{-1}$.

For the first member of the Lyman series:

$n_1 = 1, n_2 = 2$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$ **[½]**

$$\lambda = 1215\left(\mathring{A}\right)$$

For the first member of Balmer series:

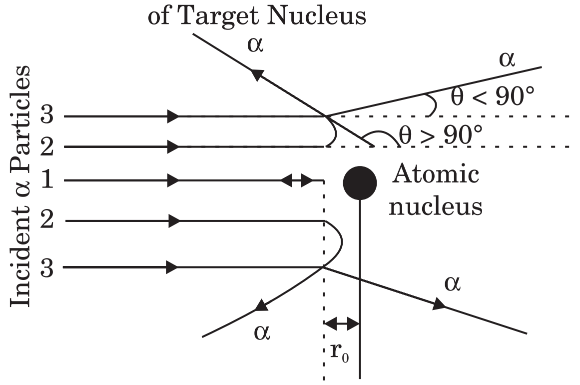
$n_1 = 1, n_2{}^2 = 23$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \lambda = 6563\left(\mathring{A}\right)$$ **[½]**

14.

Trajectory of a Particles in Coulomb Field of Target Nucleus



From this experiment, the following is observed:

(1) Most of the alpha particles pass straight through the gold foil. It means that they do not suffer any collision with gold atoms.

(2) About one alpha particle in every 8000 alpha particles deflects by more than 90. **[1]**

As most of the alpha particles go undeflected and only a few get deflected, this shows that most of the space in an atom is empty and at the centre of the atom, there exists a nucleus. By the number of the alpha particles deflected, the information regarding size of the nucleus can be known. **[½]**

If m is the average mass of a nucleon and R is the nuclear radius, then mass of nucleus = mA, where A is the mass number of the element.

Volume of the nucleus, **[½]**

$$V = \frac{4}{3}\pi R^3$$

$$\Rightarrow V = \frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3$$ **[½]**

$$\Rightarrow V = \frac{4}{3}\pi R_o^3 A$$

Density of nuclear matter,

$$\rho = \frac{mA}{V}$$

$$\rho = \frac{mA}{\frac{4}{3}\pi R_o^3 A}$$

$$\Rightarrow \rho = \frac{3m}{4\pi R_o^3}$$ **[½]**

This shows that the nuclear density is independent of A.

OR

Nuclear fission: Nuclear fission is a disintegration process, in which a heavier nucleus gets split up into two lighter nuclei, with the release of a large amount of energy. **[1]**

$${}_{92}U^{235} + {}_on^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_on^1 + Q$$

Here, the energy released per fission of ${}^{92}U_{235}$ is 200.4 MeV

Nuclear fusion: When two or more light nuclei combine to form a heavy stable nuclide, part of mass disappears and is converted into energy. This phenomenon is called nuclear fusion. **[1]**

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + V + 0.42MeV$$

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + V + 0.42MeV$$

$_1H^2 + {_1H^2} \rightarrow {_1H^3} + {_1H^1} + 4.03MeV$

${}_1^2H + {}_1^3H \rightarrow {}_2^4H + n$

$\therefore\ \Delta m = (2.104102 + 3.016049) - (4.002603 + 1.008665) = 0.01888\ 3u$

Energy released, $Q = 0.018883 \times 931.5\ MeV/e^2$

$Q = 17.589\ MeV$ **[1]**

15. (a) Radioactive decay Law: The rate of decay of radioactive nuclei is directly proportional to the number of undecayed nuclei at that time.

Derivation of formula: Suppose initially the number of atoms in radioactive element is N_0 and N the number of atoms after time t. After time t, let dN be the number of atoms which disintegrate in a short interval dt, then rate of disintegration will be $\frac{dN}{dt}$, this is also called the activity of the substance/element. According to Rutherford-Soddy law

$$\frac{dN}{dt} \propto N$$ **[½]**

Or $\frac{dN}{dt} = -\lambda N$

Where λ is a constant, called decay constant or disintegration constant of the element. Its unit is s^{-1}. Negative sign shows that the rate of disintegration decreases with increase of time. For a given element/substance λ is a constant and is different for different elements. Equation (i) may be rewritten as **[½]**

$$\frac{dN}{dt} = -\lambda dt$$

Integrating $\log_e N - \lambda t + C$ -------- (ii) where C is a constant of integration.

At $t = 0, N = N_0$

$\therefore \log_e N = 0 + C$

$\Rightarrow C = \text{loge}\, N_o$

Equation (ii) gives $\log_e N - \lambda t + \log_e N_o$

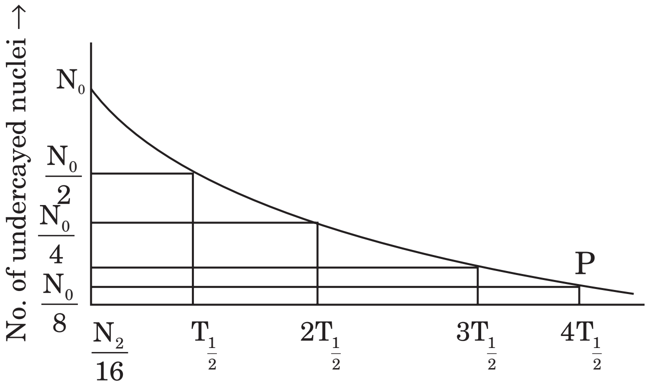
Or $\log_e N - \log_e N_o = -\lambda t$

Or $\log_e \frac{N}{N_o} = -\lambda t$

Or $\frac{N}{N_o} = e^{-\lambda t}$ **[1]**

$\therefore N = N_o e^{-\lambda t}$ ... (iii)

According to this equation, the number of undecayed atoms/nuclei of a given radioactive element decreases exponentially with time (i.e., more rapidly at first and slowly afterwards.)



[½]

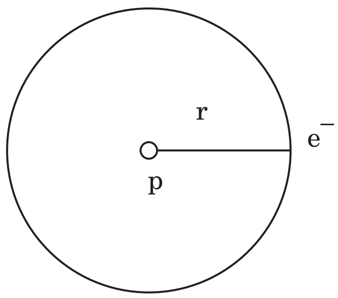
(b) Relation between mean life and half life

Mean life $\tau = \frac{1}{\lambda}$ where $\lambda \rightarrow$ decay constant

[½]

16. (a) Bohr'spostulate

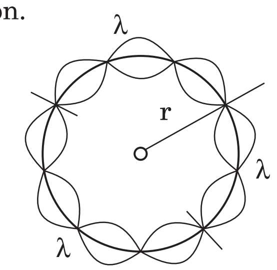
1. All the electrons revolve in circular orbit. Necessary centripetal force is provided by electrostatic force between electron and proton.



2. Electron revolve only in those orbits for which angular momentum is integral multiple of $\left(\frac{h}{2\pi}\right)$

$$L = mvr = \frac{nh}{2\pi}$$ **[1]**

3. When electron jumps from n^{th} higher orbit to p^{th} lower orbit it emits energy in form of proton.



$$E_n - E_p = h\upsilon$$

According to de Broglie hypothesis

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

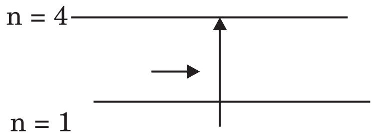
$$n\lambda = 2\pi r$$

$$n\frac{h}{mv} = 2\pi r$$

$$nh = 2\pi mvr$$

$$L = \frac{nh}{2\pi}$$ **[1]**

(b)



$$E_4 - E_1 = \frac{hv}{e}$$

$$\frac{13.6}{4} - 13.6 = \frac{6.63\times10^{-34}\times\upsilon}{1.6\times10^{-19}}$$

$$13.6\left[\frac{1}{4}-1\right] = \frac{6.63\times10^{-34}\times\upsilon}{1.6\times10^{-19}}$$

$$\frac{1.6\times10^{-19}\times3}{6.63\times10^{-34}\times4} = \upsilon$$

$$\upsilon = 2.4615\times10^{15}\,Hz$$ **[1]**

17. (a) "Oh April 1986, the world's worst nuclear accident happened at the Chernobyl. Plant near pripyat Ukraine in the soviet union. **[1]**

(b) An explosion and fire in the No. 4 reactorsent radioactivity into the atmosphere **[1]**

(c) The value displayed by the Asha is that she is caring and having helping nature towards her mother. The value displayed by Asha's mother is that she has no idea the outburst take place in Chernobyl (Ukraine) but she has the curiosity about the incident that take place on April 26, 1986, at the Chernobyl plant near Priyat, Ukraine, in the Soviyat union. **[2]**

18. According to Bohr, energy is radiated in the form of a photon when the electron of an excited hydrogen atom returns from higher energy state to the lower energy state. In other words, energy is radiated in the form of a photon when electron in hydrogen atom jumps from higher energy orbit $(n = n_i)$ where $n_i = n_f$. The energy of the emitted radiation or photon is given by

$$h\nu = E_{n_i} - E_{n_f}$$ **[1]**

We know that $E_n = \dfrac{-me^4}{8h^2\varepsilon_o^2 n^2}$

$$\therefore h\nu = \frac{me^4}{8h^2\varepsilon_o^2 n_i^2} - \frac{me^4}{8h^2\varepsilon_o^2 n_f^2}\ i.e.$$

$$h\nu = \frac{me^4}{8h^2\varepsilon_o^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\text{or}\ \ \nu = \frac{me^4}{8h^3\varepsilon_o^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$ **[1]**

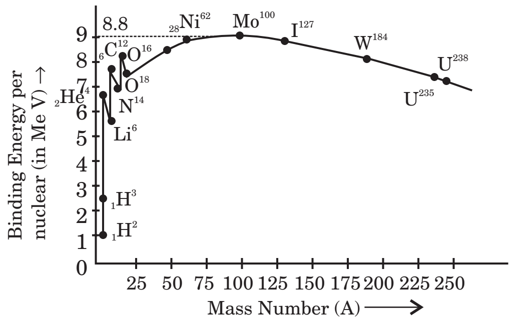
Name of series

$n = 4$ to $n = 3$ (Paschan)

$n = 4$ to $n = 2$ (Balmer)

$n = 4$ to $n = 1$ (Lyman)

Or



[1]

Conclusions :-

(i) The intermediate nuclei have large value of $\dfrac{BE}{A}$ so they are more stable.

(ii) $\dfrac{BE}{A}$ has low value for both of light and heavy nuclei so they are unstable nuclei.

(b) In nuclear fission, unstable heavy nuclei splits into two stable intermediate nuclei and in Nuclear fusion, 2 unstable light nuclei combines to form stable intermediate nuclei so in both processes energy liberates as stability increases

(c) $n \to P + \beta_{-1}^{0} + \bar{\nu}$ Neutrinos are difficult to detect as they go through all object by penetrating them. **[1]**

Smart Notes ....

Smart Notes ....

CHAPTER 13

Nuclei

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Composition and size of nucleus, atomic masses, isotopes, isobars; isotone			1Q (2 marks)		
Radioactivity alpha, beta and gamma particles/ rays and their properties; radioactive decay law.		1Q (3 marks)			
Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number	1Q (2 marks)				1Q (1 marks)
Nuclear fusion and Nuclear Fission	1Q (2 marks)			1Q (4 marks)	1Q (3 marks)

On the basis of above analysis, it can be said that from exam point of view Binding Energy, Nuclear Force, Nuclear Fission, Nuclear Fusion and Radioactive Decay are the most important concepts of the chapter.

Topic 1: Radioactivity and Decay Law

Summary

- **Nucleus:** Nucleus can be defined as the central part of an atom, made up of neutrons, protons, and other elementary particles. The nucleus has protons and neutrons inside it. They are called nucleons.
- **Mass Number:** The total number of protons and neutrons present inside the nucleus of an atom of an element is referred to as mass number (A) of the element.
- **Atomic Number:** The number of protons present in the nucleus of an atom of an element is known as atomic number (Z) of the element.
- **Nuclear Size:** The radius of the nucleus $R \propto A^{\frac{1}{3}}$
- $R = R_o A^{\frac{1}{3}}$ where $R_0 = 1.2 \times 10^{-15}$ m is an empirical constant.
- **Nuclear Density:** Nuclear density is independent of mass number and is therefore same for all nuclei.

  $$\rho = \frac{Mass\ of\ nucleus}{volume\ of\ nucleus}$$

  $\rho = \frac{3m}{4\pi R_0^3}$ where is the average mass of a nucleon.
- **Atomic Mass Unit:** Abbreviated as amu and is defined as one-twelfth of the mass of a carbon nucleus. It is also denoted by u.

  Therefore,

  $$1\ amu = \frac{1.992678 \times 10^{-26}}{12}\ kg$$

  $= 1.6 \times 10^{-27}$ kg = 931 MeV
- **Isomers:** The atoms that have the same mass number, atomic number but different radioactive properties are known isomers.
- **Isotones:** Atoms of elements that have different mass numbers, atomic numbers but same number of neutrons are known as isotones. e.g., $_1H^3$, $_2H^4$ and $_6C^{14}$, $_8O^{16}$ are isotones.
- **Isobars**: The atoms of an element having different atomic numbers but same mass numbers are known as isobars. e.g., $_1H^3$, $_2H^3$ and $_{10}Na^{22}$, $_{10}Ne^{22}$ are isobars.
- **Isotopes**: Atoms of an element that have different mass numbers but same atomic number are known as isotopes. e.g., $_1H^1$, $_1H^2$, $_1H^3$ is an example of isotopes.
- **Nuclear Force**: Nuclear force can be referred to as the force that acts inside the nucleus or between nucleons. These forces are neither electrostatic nor gravitational in nature. They have a very short range and are independent of any charge. They are a hundred times that of electrostatic force and 10^{38} times that of gravitational force.
- **Radioactivity**: Radioactivity refers to the breakdown of heavy elements into comparably lighter elements by the emission of radiations. This phenomenon was discovered by Henry Becquerel in 1896.
- **Packing Fraction (P):**

  $$P = \frac{(Exact\ nuclear\ mass) - (Mass\ number)}{Mass\ number}$$

  $$= \frac{(A - M)}{M}$$

For greater stability of the nucleus, the value of packing friction should be larger.

- ***Radioactive Decay law***

The Radioactive law states that the rate of disintegration of radioactive atoms at any instance is directly proportional to the number of radioactive atoms present in the given sample at that instant.

Rate of disintegration $\left(-\frac{dN}{dt}\right) \propto N$

$-\frac{dN}{dt} = \lambda N$, where λ is the decay constant.

The number of undecayed atoms present in the sample at any instance $N = N_0 e^{-\lambda t}$ where, N_0 is number of atoms at time t = 0 and N is number of atoms at time t.

- **Activity of a radioactive element**

The activity of a radioactive element is equal to its rate of disintegration.

Activity $R = \left(-\frac{dN}{dt}\right)$

Activity of the sample after time t, $R = R_0 e^{-\lambda t}$

Its SI unit is Becquerel (Bq). Curie and Rutherford are its other units.

1 Curie = 3.7 × 10^{10} decay/s and 1 rutherford = 10^6 decay/s

- **Half-life of a radioactive element**

Half-life (T) of a radioactive element is the time taken for the radioactivity of an isotope to fall to half its original value. The relation between disintegration constant and half-life is given by

$$T = \frac{\log_e 2}{\lambda} = \frac{0.6931}{\lambda}$$

- **Average Life or Mean Life (τ)**

Average life or mean life (τ) of a radioactive element can be defined as the ratio of total life time of all the atoms and total number of atoms present, initially in the sample.

Relation between half-life and average life τ = 1.44T

Relation between average life and decay constant $\tau = \frac{1}{\lambda}$

- **Alpha decay:** In alpha decay, a nucleus gets transformed into a different nucleus and an α particle is emitted. The general form can be expressed as:

  $^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He$ and the Q value:

  $Q = (m_X - m_Y - m_{He})c^2$

- **Beta decay:** When a nucleus undoes beta decay, it emits an electron or a positron. When an electron is emitted, it is said to be beta minus decay while in beta plus decay, a positron is emitted.
- **Gamma decay:** In gamma decay, the photons are emitted from the nuclei having MeV energy and thus the gamma rays are emitted. This is called as gamma decay.

PREVIOUS YEARS'
EXAMINATION QUESTIONS
TOPIC 1

1 Mark Questions

1. Define the activity of a given radioactive substance. Write its S.I. units.

[DELHI 2011]

2. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay?

[ALL INDIA 2014]

3. Two nuclei have mass numbers in the ratio 1: 2. What is the ratio of their nuclear densities?

[DELHI 2017]

2 Marks Questions

4. A radioactive nucleus '*A*' undergoes a series of decays according to the following scheme:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of *A* are 180 and 72 respectively.

What are these numbers for A_4?

[DELHI 2017]

3 Marks Questions

5. State the law of radioactive decay.

Plot a graph showing the number (*N*) of undecayed nuclei as a function of time (*t*) for a given radioactive sample having half life.

Depict in the plot the number of undecayed nuclei at (i) $t = 3T_{1/2}$ and (ii) $t = 5T_{1/2}$

[ALL INDIA 2011]

6. Show that the density of nucleus over a wide range of nuclei is constant independent of mass number A.

[ALL INDIA 2012]

7. Draw a plot of potential energy between a pair of nucleons as a function of their separation. Mark the regions where potential energy is (i) positive and (ii) negative

[DELHI 2013]

8. (a) In a typical nuclear reaction, e.g.

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{2}He + ^{1}_{0}n + 3.27 MeV,$$

although number of nucleons is conserved, yet energy is released. How? Explain.

(b) Show that nuclear density in a given nucleus is independent of mass number *A*.

[DELHI 2013]

9. Identify the nature of the radioactive radiations emitted in each step of the decay process given below.

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y \rightarrow {}^{A-4}_{Z-1}W$$

[ALL INDIA 2015]

10. (a) Derive the mathematical expression for law of radioactive decay for a sample of a radioactive nucleus.

(b) How is the mean life of a given radioactive nucleus related to the decay constant?

[DELHI 2016]

11. A radioactive isotope has a half-life of 10years. How long will it take for the activity to reduce to 3.125%

[ALL INDIA 2018]

12. (i) Define 'activity' of a radioactive material and write its S.I. units.

(ii) Plot a graph showing a variation of activity of a given radioactive sample with time.

(iii) The sequence of stepwise decay of a radioactive nucleus is

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^{-1}} D_2$$

If the atomic number and mass number of D_2 are 71 and 176 respectively, what are their corresponding values of D?

[DELHI 2018]

Solutions

1. Activity: Rate of disintegration of radioactive substance. i.e. number of radioactive nuclei disintegrating in unit time is called activity. SI unit: **[1]**

1 disintegration per second (*dps*) or 1 *Bq*

2. Neutrinos are difficult to detect experimentally in β decay because they are uncharged particles with almost no mass. Also, neutrinos interact very weakly with matter, so they are very difficult to detect. **[1]**

3. Nuclear density is independent of mass number. Hence, both the atoms have the same nuclear density. **[1]**

4. A has mass number as 180 and atomic number 172.

Formation of A_1 by α decay:

$$A \xrightarrow{\alpha} {}^{180-4}_{172-2}A_1$$ **[½]**

$$A \xrightarrow{\alpha} {}^{176}_{170}A_1$$

Formation of A_2 by β decay:

$$A \xrightarrow{\beta} {}^{176}_{170+1}A_2$$

$$A_1 \xrightarrow{\beta} {}^{176}_{170+1}A_2$$ **[½]**

Formation of A_3:

$$A_2 \xrightarrow{\alpha} {}^{172}_{169}A_3$$

Formation of A_4: **[½]**

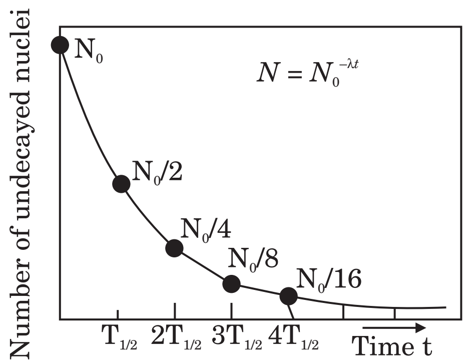
In γ-decay, mass number and atomic number remain the same. Thus,

$$A_3 \xrightarrow{\gamma} {}^{172}_{169}A_4$$

Mass number of A_4 = 172

Atomic number of A_4 = 169 **[½]**

5. According to radioactive decay law the rate of disintegration of a radioactive substance at an instant is directly proportional to the number of nuclei in the radioactive substance at that time i.e, $N = N_0 e^{-\lambda t}$, where symbols have their usual meanings. **[1]**



From graph: **[½]**

Number of un-decayed nuclei at $t = 3T_{1/2}$ is $\frac{N_o}{8}$

Number of un-decayed nuclei at $t = 5T_{1/2}$ is $\frac{N_o}{32}$

[1½]

6. To find the density of nucleus of an atom, we have an atom with mass number suppose A. (Here, we are neglecting mass of the orbital electrons)

Mass of the nucleus of the atom of the mass number A = A a.m.u = A × 1.660565 × 10^{-27} kg

Let the radius of nucleus is 'R'. **[1]**

Then, volume of nucleus $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3$$

$= \frac{4}{3}\pi R_o^3 A$

Now, we know $R_o = 1.1 \times 10^{-15}$ m

Volume of the nucleus **[1]**

$= \frac{4}{3}\pi \left(1.1 \times 10^{-15}\right)^3 \times A\ m^3$

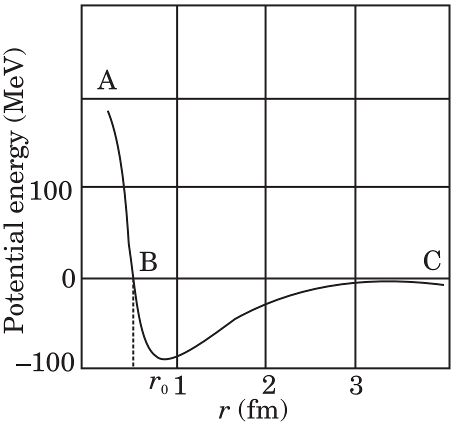
Density of the nucleus,

$$\delta = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{A \times 1.660565 \times 10^{-27}}{\frac{4}{3}\pi\left(1.1 \times 10^{-15}\right)^3 \times A}$$

$\delta = 2.97 \times 10^{17}\ m^{-3}$

Thus, we can see the density of nucleus is independent of the mass number and is constant for all nuclei.

7. The potential energy is minimum at r_o: For distance larger than ro the negative potential energy goes on decreasing and for the distances less than ro the negative potential energy decrease to zero and then becomes positive and increases abruptly. Thus, *A* to *B* is the positive potential energy region and *B* to *C* is the negative potential energy region. **[1]**



[1]

8. (a) In a nuclear reaction, the aggregate of the masses of the target nucleus ${}^2_1H$ and the bombarding particle may be greater or less than the aggregate of the masses of the product nucleus (${}^2_3He$) and the outgoing particle (${}^1_0n$) So from the law of conservation of mass-energy some energy (3.27 *MeV*) is evolved or involved in a nuclear reaction. This energy is called *Q*-value of the nuclear reaction. **[1]**

(b) Density of the nucleus

$$= \frac{mass\ of\ nucleus}{volume\ of\ nucleus}$$ **[1]**

Mass of the nucleus

= A amu = A × 1.666 × 10^{-27} kg

Volume of the nucleus,

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3$$

Where, $R = R_o A^{\frac{1}{3}}$

Thus, density $= \frac{A \times 1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi R_0^3\right)A} = \frac{1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi R_0^3\right)}$,

shows that the density is independent of mass number A. **[1]**

9. ${}^A_Z X \rightarrow {}^{A-4}_{Z-2}Y \rightarrow {}^{A-4}_{Z-1}W$ **[1]**

An alpha particle $\left({}^4_2He\right)$ is emitted in the first reaction as atomic mass of *Y* is reduced by 4 and atomic number is reduced by 2. An electron is emitted $\left({}^{\ 0}_{-1}e\right)$ in the second reaction as atomic mass of W remains the same and atomic number is increased by 1. **[1]**

10. (a) Radioactive decay Law: The rate of decay of radioactive nuclei is directly proportional to the number of un decayed nuclei at that time. **[1]**

Derivation of formula: Suppose initially the number of atoms in radioactive element is N_0 and N the number of atoms after time t. After time t, let dN be the number of atoms which disintegrate in a short interval dt, then rate of disintegration will be $\frac{dN}{dt}$, this is also called the activity of the substance/element. According to Rutherford-Soddy law

$$\frac{dN}{dt} \propto N$$

Or $\frac{dN}{dt} = -\lambda N$ **[1]**

Where λ is a constant, called decay constant or disintegration constant of the element. Its unit is s^{-1}. Negative sign shows that the rate of disintegration decreases with increase of time. For a given element/substance λ is a constant and is different for different elements. Equation

(i) may be rewritten as

$$\frac{dN}{dt} = -\lambda dt$$

Integrating $\log_e N - \lambda t + C$ -------- (ii) where C is a constant of integration.

At $t = 0, N = N_0$

$\therefore \log_e N = 0 + C$

$\Rightarrow C = \text{loge}\, N_o$

Equation (ii) gives $\log_e N - \lambda t + \log_e N_o$

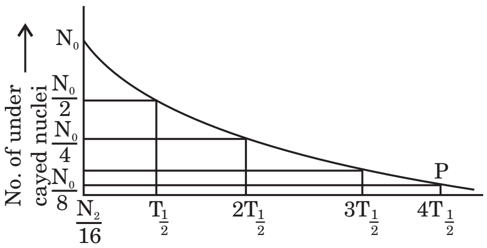
Or $\log_e N - \log_e N_o = -\lambda t$

Or $\log_e \dfrac{N}{N_o} = -\lambda t$

Or $\dfrac{N}{N_o} = e^{-\lambda t}$

$\therefore N = N_o e^{-\lambda t}$ -------- (iii)

According to this equation, the number of undecayed atoms/nuclei of a given radioactive element decreases exponentially with time (i.e., more rapidly at first and slowly afterwards.)



(b) Relation between mean life and half life

Mean life $\tau = \dfrac{1}{\lambda}$ where $\lambda \rightarrow$ decay constant

11. $A_o = 100$ **[½]**

$A_t = 3.125$

$A_t = A_o e^{-\mu}$.

$$3.125 = 100e^{\frac{0.693}{10}t}$$

$$\frac{3.125}{100} = e^{-0.0693t}$$

$$\frac{100}{3.125} = e^{0.0693t}$$ **[½]**

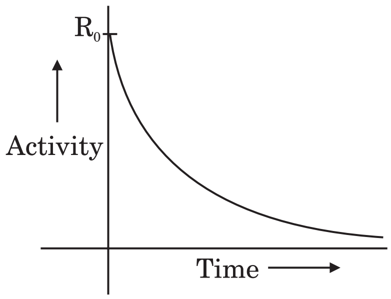
$$\log_e \frac{100}{3.125} = 0.0693t$$

$$t_2 = \frac{2.303 \log \frac{32}{10}}{0.0693}$$

$t = 50.1$ years **[½]**

12. (i) The activity of a radioactive material is defined as the decay rate of a sample containing one or more nuclides. The S.I unit of radioactivity is Becquerel (B). **[1]**

(ii)



(iii) $D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^{-1}} {}^{176}_{71}D_2$ **[½]**

Therefore,

$D \xrightarrow{\alpha} {}^{176}_{72}D_1 \xrightarrow{\beta^{-1}} {}^{176}_{71}D_2$ **[½]**

Therefore,

${}^{180}_{74}D \xrightarrow{\alpha} {}^{176}_{72}D_1 \xrightarrow{-\beta} {}^{176}_{71}D_2$ **[½]**

So, the corresponding values of atomic number and mass number for D are74 and 180. **[½]**

Topic 2: Mass Defect and Binding Energy

Summary

- **Mass Defect***:* Mass defect can be mentioned as the difference between the sum of masses of all nucleons (M) and the mass of the nucleus (m).

  Mass Defect (Δm) = M – m

  $= [Zm_p + (A - z)m_n - m_n]$
- **Nuclear Binding Energy***:* Nuclear binding energy can be referred to as the minimum energy that is required to separate the nucleons up to an infinite distance from the nucleus.

Nuclear binding energy per nucleon = Nuclear binding energy / Total number of nucleons Binding energy, $E_b = [Zm_p + (A - Z)m_n - m_n]c^2$

- **Nuclear Fission**

The process of the splitting a heavy nucleus into two or more lighter nuclei is known as nuclear fission.

- **Nuclear Reactor**

When a slow moving neutron strikes with a uranium nucleus $\left({}_{92}U^{235}\right)$, it splits into ${}_{56}Ba^{141}$ and ${}_{36}Kr^{92}$ along with three neutrons and a lot of energy.

$${}_{92}U^{235} + {}_0n^1 \to {}_{56}Ba^{141} + {}_{36}Kr^{92} \to 3\,{}_0n^1 + energy$$

- **Nuclear Chain Reaction**

Nuclear chain reactions are defined as a chain of nuclear fission reactions (splitting of atomic nuclei), and each one of them is initiated by a neutron produced in the previous fission reaction. Nuclear chain reactions are of two types:

➢ Controlled chain reaction

➢ Uncontrolled chain reaction

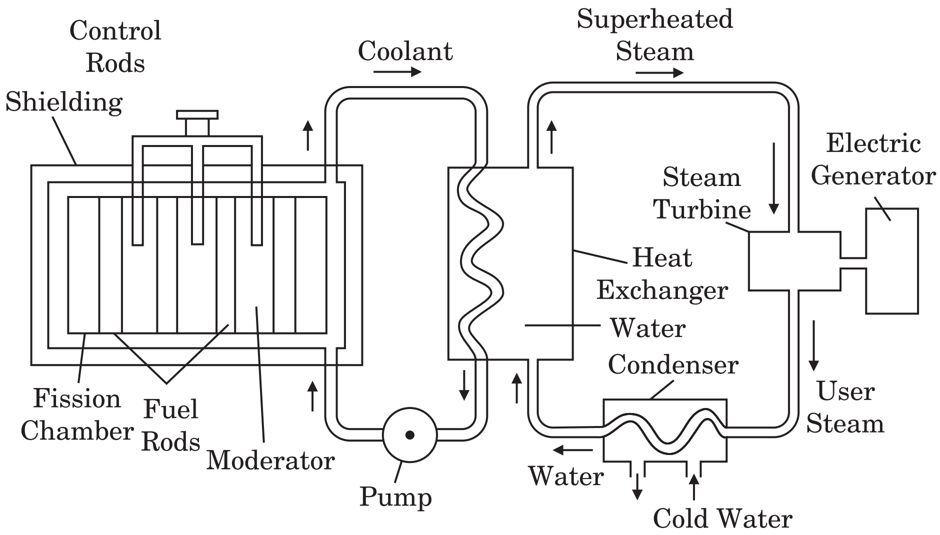


Fig.: Setup of a Nuclear Reactor

The vital parts of a nuclear reactor are the following:

➢ Fuel: Fissionable materials like ${}_{92}U^{235}$, ${}_{92}U^{238}$, ${}_{94}U^{239}$ are used as fuel.

➢ Moderator: Graphite, heavy water and beryllium oxide are used to slower down fast moving neutrons.

➢ Coolant: Liquid oxygen, cold water, etc. are used to remove heat generated in the fission process.

➢ Control rods: Cadmium or boron rods are considered as good absorber of neutrons and are therefore used to control the fission reaction.

- **Nuclear Fusion:** The process of combining two light nuclei in order to form a single large nucleus is called nuclear fusion. A large amount of energy is released in this process. The example of nuclear fusion is:

${}_1^1H + {}_1^1H \to {}_1^2H + e^+ + \nu + 0.42$ MeV, where a deuteron and a positron are formed by the combination of two protons and 0.42 MeV energy is released.

The source of energy of sun and all the stars is a nuclear fusion reaction in which hydrogen nuclei combine to form helium nuclei.

$$4\,{}_1^1H + 2e^- \to {}_2^4He + 2\nu + 6\gamma + 26.7 \text{ MeV}$$

- **Advantages of Nuclear fusion:**
  ➢ Nuclear fusion does not cause any waste as the only by product is helium.

  ➢ Nuclear fusion is very simple to control as there is no change of chain reaction.

  ➢ There is unlimited supply of fuel for nuclear fusion.

PREVIOUS YEARS' EXAMINATION QUESTIONS
TOPIC 2

1 Mark Questions

1. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two: the parent or the daughter nucleus would have higher binding energy per nucleon? **[ALL INDIA 2018]**

2. What characteristic property of nuclear force explains the constancy of binding energy per nucleon $\left(\frac{BE}{A}\right)$ in the range of mass number 'A' lying $30 < A < 170$? **[ALL INDIA 2012]**

2 Marks Questions

3. A nucleus with mass number $A = 240$ and $\frac{BE}{A} = 7.6\ MeV$ breaks into two fragments each of $A = 120$ with $\frac{BE}{A} = 8.5$ MeV Calculate the released energy.

Or, calculate the energy in fusion reaction:

$^2_1\text{H} + ^2_1\text{H} \longrightarrow ^3_2\text{He} + \text{n}$, Where BE of $^2_1\text{He} = 2.23\text{MeV}$,

And of $^3_2\text{He} = 7.73\text{MeV}$., **[DELHI 2016]**

4. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is split into two fragments Y and Z of mass numbers 110 and 130 respectively. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy Q released per fission in MeV. **[DELHI 2018]**

5. In a nuclear reaction

$$^3_2H + ^3_2H \longrightarrow ^4_2He + ^1_1H + ^1_1H + 12.86MeV,$$

though the number of nucleons is conserved on both sides of the reaction, yet the energy is released. Explain. **[DELHI 2013]**

3 Marks Questions

6. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released. Calculate the energy release in MeV in the deuterium-tritium fusion reaction:

$$^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n$$

Using the data:

$m\left(^2_1H\right) = 2.014102u$

$m\left(^3_1H\right) = 3.016049u$

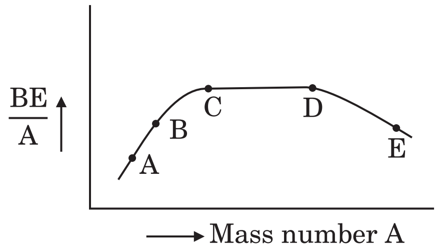
$m\left(^4_2He\right) = 4.002603u$

$m_n = 1.008665u$

$lu = 931.5MeV/c^2$

[DELHI 2014]

7. The figure shows the plot of binding energy (BE) per nucleon as a function of mass number A. The letters A, B, C, D and E represent the positions of typical nuclei on the curve. Point out, giving reasons, the two processes (in terms of A, B, C, D and E), one of which can occur due to nuclear fission and the other due to nuclear fusion.



[ALL INDIA 2015]

8. Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon $\left(\frac{BE}{A}\right)$ versus the mass number A. **[All India 2018]**

Solutions

1. In fusion, parent nuclei fuse to form daughter atom and release some amount of energy. It means daughter nuclei are more stable than parent nuclei hence Daughter nuclei $B.E.$ > Parent nuclei $B.E.$ **[1]**

2. The approximate constancy of $\frac{BE}{A}$ over the most of the range is saturation property of nuclear force. In heavy nuclei: nuclear size > range of nuclear force. So, a nuclear sense approximately a constant number of neighbours and thus, the nuclear $\frac{BE}{A}$ levels off at high A.

This is saturation of the nuclear force. **[1]**

3. Gain in binding energy for nucleon is about 0.9 *MeV*. **[½]**

Binding energy of the nucleus,

$B_1 = 7.6 \times 240 = 1824\ MeV$

Binding energy of each product nucleus,

$B_2 = 8.5 \times 120 = 1020\ MeV$

Then, the energy released as the nucleus breaks. **[½]**

$E = 2B_2 - B_1 = \times 120 = 1020 - 1824 = 216\ MeV$

Or, calculate the energy in fusion reaction:

${}_1^2\text{H} + {}_1^2\text{H} \longrightarrow {}_2^3\text{He} + \text{n}$, where BE of

${}_1^2\text{H} = 2.23\text{MeV}$, **[½]**

And of ${}_2^3\text{He} = 7.73\text{MeV}$.

$\Delta E = (7.73) - 2(2.23)$

$= 7.73 - 4.46$

$= 3.27\ MeV$ **[½]**

4. Total energy of nucleus $X = 240 \times 7.6 = 1824\ MeV$

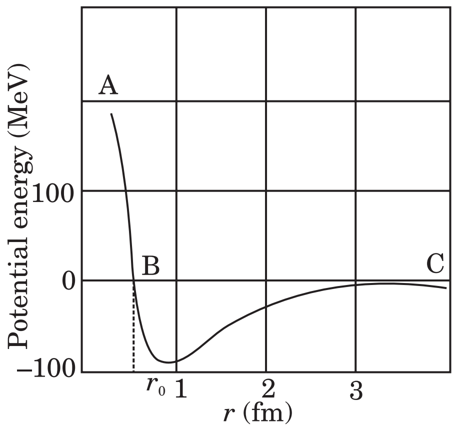
Total energy of nucleus $V = 110 \times 8.5 = 935\ MeV$

Total energy of nucleus $Z = 130 \times 8.5 = 1105\ MeV$

Therefore, energy released from fission, $Q = 935 + 1105 - 1824$ **[½ + ½ + ½ + ½]**

$Q = 216\ MeV$

5. 'In a nuclear reaction, the sum of the masses of the target nucleus ${}_2^3He$ may be greater or less the sum of the masses of tiny product nucleus ${}_4^3He$ and the ${}_1^1He$. So from the law of conservation of mass energy some energy (12.86 *MeV*) is evolved in nuclear reaction. This energy is called *Q*-value of the nuclear reaction. The binding energy of the nuclear reaction. The binding energy of the nucleus on the left side is not equal to the right side. The difference in the binding energies on two sides appears as energy released or absorbed in the nuclear reaction. **[1]**



[1]

6. Nuclear fission: Nuclear fission is a disintegration process, in which a heavier nucleus gets split up into two lighter nuclei, with the release of a large amount of energy. **[1]**

$${}_{92}U^{235} + {}_on^1 \to {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_on^1 + Q$$

Here, the energy released per fission of ${}^{92}U_{235}$ is 200.4*MeV*

Nuclear fusion: When two or more light nuclei combine to form a heavy stable nuclide, part of mass disappears and is converted into energy. This phenomenon is called nuclear fusion. **[1]**

$${}_1H^1 + {}_1H^1 \to {}_1H^2 + e^+ + V + 0.42MeV$$

$${}_1H^2 + {}_1H^2 \to {}_2H^3 + {}_on^1 + 3.27MeV$$

$${}_1H^2 + {}_1H^2 \to {}_1H^3 + {}_1H^1 + 4.03MeV$$

$${}_1^2H + {}_1^3H \to {}_2^4H + n$$

$\therefore\ \Delta m = (2.104102 + 3.016049) - (4.002603 + 1.008665) = 0.01888\ 3u$

Energy released, $Q = 0.018883 \times 931.5\ MeV/e^2$

$Q = 17.589\ MeV$ **[1]**

7. The nuclei at A and B undergo nuclear fusion as their binding energy per nucleon is small and they are less stable so they fuse with other nuclei to become stable. The nuclei at E undergo nuclear fission as its binding energy per nucleon is less it splits into two or more lighter nuclei and become stable.

8. Nuclear Fission: When heavy nucleus bombarded with neutron and it splits into smaller nucleus and energy released. **[½]**

Ex : ${}_{92}U^{235} + {}_0n^1 \to {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3\,{}_0n^1 +$ Energy

Nuclear Fusion: When 2 smaller nucleus fuse to form ea heavy nucleus again energy released.

Ex : ${}_1H^2 + {}_1H^2 \to {}_2He^4 +$ approx. 26 MeV

BE Graph: **[½]**

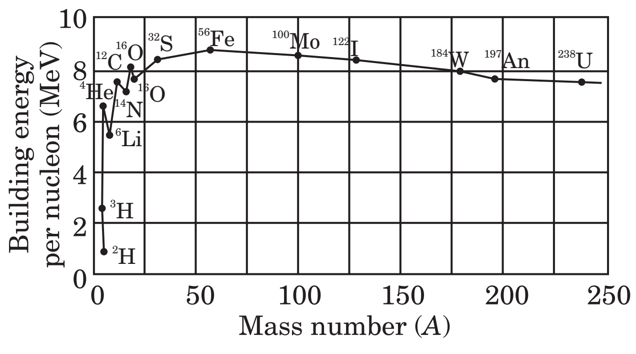


Fig.: Graph Showing Plot of Binding Energy per nucleon(MeV) Vs Mass Number(A)

[½]

Smart Notes ....

CHAPTER 14

Semiconductor Electronics

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Energy bands in conductors; semiconductors and insulators					
Semiconductor diode: I-V characteristics in forward and reverse bias; diode as a rectifier			1Q (1 mark)		
Special purpose p-n junction diodes: LED, photodiode, solar cell and Zener diode and their characteristics; Zener diode as a voltage regulator	1Q (4 marks)	1Q (3 marks)	1Q (3 marks)		1Q (1 mark), 1Q (1 mark)
Junction transistor; transistor action; characteristics of a transistor and transistor as an amplifier (common emitter configuration);	1Q (3 marks)		1Q (3 marks)	1Q (3 marks)	1Q (3 marks)
Basic idea of analog and digital signals; Logic gates (OR, AND, NOT, NAND and NOR)		1Q (3 marks)			

On the basis of above analysis, it can be said that from exam point of view, Logic Gates, p-n Junction and circuit based questions are most important concepts of the chapter.

[Topic 1] Semiconductor, diode and its applications

Summary

- The materials which are present in solid state and their conductivity lies between insulator and conductor are called as semiconductors. Semiconductors are either pure substance like silicon, germanium or they can also be formed by addition of impurities which form a compound like gallium arsenide, cadmium selenide, etc.
- Semiconductors have resistivity in the range of metals and insulators. Insulators have resistivity in the range of $10^{11} - 10^{19}$ Ωm and metals have resistivity in the range of 10^{-2} to 10^{-8} Ωm while semiconductors have resistivity in the range of $10^{-5} - 10^{6}$ Ωm
- Semiconductors can be elemental (without doping) as well as compound (by doping).
- ***Intrinsic semiconductors:*** These are pure semiconductors where the conductivity is due to electrons moving from valence band to conduction band. Their conductivity is called intrinsic conductivity. In intrinsic conductors, $n_e = n_h$
- ***Extrinsic semiconductors:*** When Impurity is added to pure semiconductor to increase its conductivity, is called as extrinsic semiconductor.

  It can be divided into two types, i.e. p-type semiconductors and n-type semiconductors.

  In p-type semiconductors, number of holes are greater than number of electrons.

  $n_h >> n_e$

  In n-type semiconductors, number of electrons are greater than number of holes.

  $n_e >> n_h$

  Trivalent atoms (B, Al, etc.) called acceptor atoms are used for doping p-type semiconductors while pentavalent atoms (As, Sb, etc.) called donor atoms are used for doping n-type semiconductors.
- ***Energy bands:*** Valence electrons of an atom are shared by different number of atoms in the crystal which causes splitting of energy levels. These energy levels are called energy bands. The energy band which contains valence electrons is called as valence band. It always has some electrons and can never be empty.
  - The energy band which contains conduction electrons is called as conduction band. It can be empty or have some electrons which take part in flow of current.
  - The band which lies between conduction band and valence band is called as forbidden band. The minimum amount of energy required to transfer electrons from valence band to conduction band is called as band gap.
  - Metals do not have any band gap and $E_g \approx 0$

    while band gap in insulators is greater than 3 eV and the band gap for semiconductors lies between 0.2 eV and 3 eV.

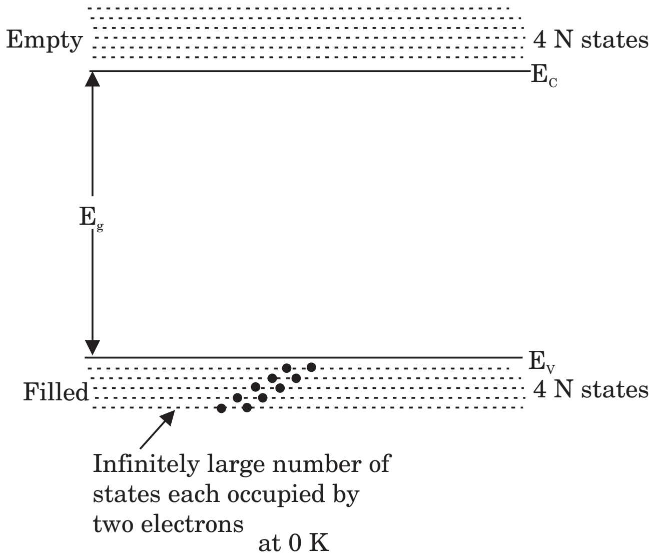


Fig.: The energy band positions in a semiconductor at 0 K. The upper band is conduction band and the lower band is called valence bond.

- ***p-n junction:*** p-type semiconductor when brought in contact with n-type semiconductor forms a p-n junction.

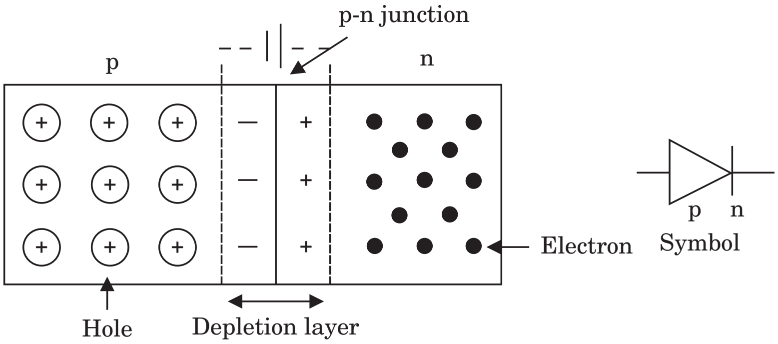


Fig.: p–n junction depicting Depletion layer

When there are no charge carriers, a region is created at the p-n junction called as depletion layer.

- Forward Biasing: When the p-side is connected to the positive terminal and n-side is connected to the negative terminal of a battery, it is called forward biasing. Majority charge carriers cause forward current flow in this biasing and the width of the depletion layer decreases.
- Reverse Biasing: When the n-side is connected to the positive terminal and p-side is connected to the negative terminal of the battery then it is called reverse biasing. Minority charge carriers cause reverse current flow in this biasing and width of the depletion layer increases.

- ***Junction diode as rectifier:*** By applying alternating voltage across a diode the current flows in only that part if the diode is forward biased and by using this property diode could be used to design a circuit which can be used as a rectifier.

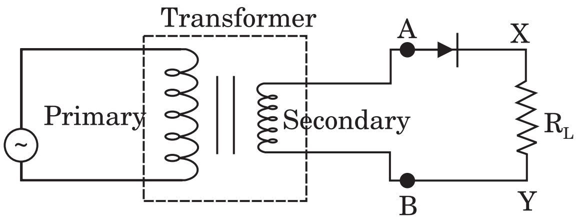


Fig.: Half-wave rectifier circuit using diode

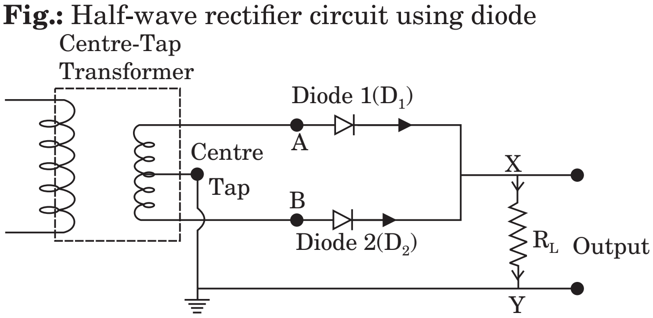


Fig.: Full-wave rectifier circuit using diode

- ***Diodes:*** Ac voltage can be restricted to one direction using diodes. Some examples of p-n junction diodes are zener diode, light-emitting diode, photo-diode, etc.

- In ***zener diodes***, when it is reversed biased the current increases after a certain voltage and the voltage is called breakdown voltage. This property of zener diodes is used in regulating voltage.

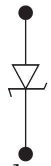


Fig.: Zener diode

- In ***photodiodes***, photons are excited which result in change of reverse saturation current to measure light intensity.

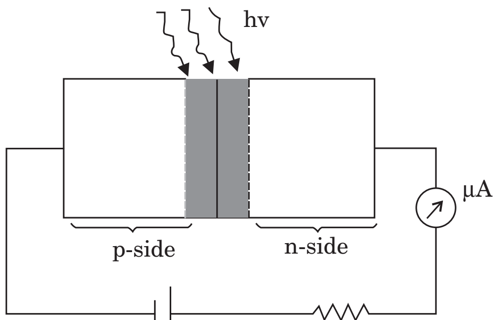


Fig.: An illuminated photodiode

- In ***light emitting diodes,*** electrons are excited by a biasd voltage resulting in generation of light.
- In ***solar cells,*** emf is generated when solar radiation falls on the p-n junction. It works on the principle of photovoltaic effect.

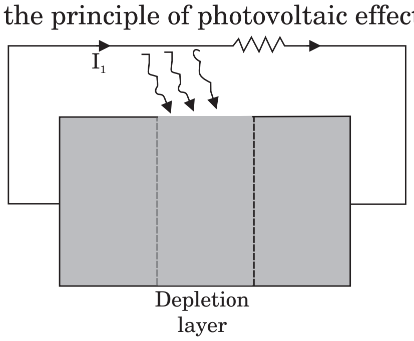


Fig.: Typical p-n junction solar cell

PREVIOUS YEARS'
EXAMINATION QUESTIONS
TOPIC 1

2 Mark Questions

1. Name the semiconductor device that can be used to regulate an unregulated *dc* power supply. With the help of I-V characteristics of this device, explain its working principle.
[DELHI 2011]

2. The current in the forward bias is known to be more (~ *mA*) than the current in the reverse bias (~ μA). What is the reason, then, to operate the photodiode in reverse bias?
[ALL INDIA 2012]

3. Distinguish between 'intrinsic' and 'extrinsic' semiconductors.
[DELHI 2015]

4. Explain, with the help of a circuit diagram, the working of a p-n junction diode as a half-wave rectifier.
[ALL INDIA 2014]

5. Draw the circuit diagram of an illuminated photodiode in reverse bias. How is photodiode used to measure light intensity?
[DELHI 2018]

3 Mark Questions

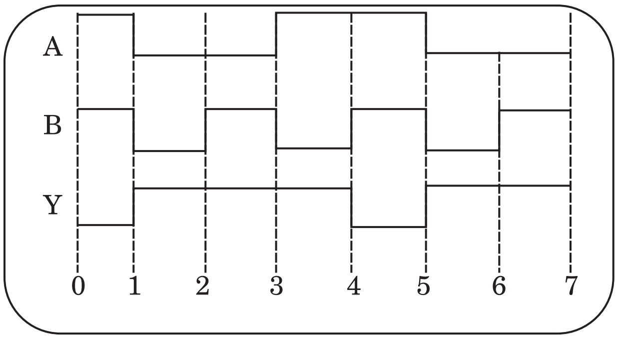
6. Draw V–I characteristics of a p-n junction diode. Answer the following questions, giving reasons:
(i) Why is the current under reverse bias almost independent of the applied potential up to a critical voltage ?
(ii) Why does the reverse current show a sudden increase at the critical voltage Name any semiconductor device which operates under the reverse bias in the breakdown region
[ALL INDIA 2013]

7. Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams.
[DELHI 2014]

8. With what considerations in view, a photo diode is fabricated? State it's working with the help of a suitable diagram. Even though the current in the forward bias is known to be more than in the reverse bias, yet the photo diode works in reverse bias. What is the reason?
[DELHI 2015]

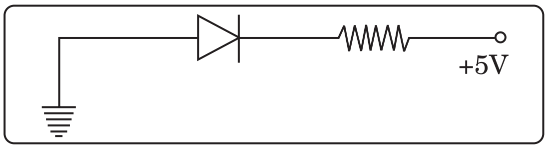
9. With the help of a circuit diagram, explain the working of a junction diode as a full wave rectifier. Draw its input and output waveforms. Which characteristic property makes the junction diode suitable for rectification?
[ALL INDIA 2015]

10. (i) Distinguish between a conductor and a semiconductor on the basis of energy band diagram.
(ii) The following figure shows the input waveforms (A, B) and the output waveform (Y) of a gate. Identify the gate, write its truth table and draw its logic symbol.



[DELHI 2016]

11. (a) In the following diagram, is the junction diode forward biased or reverse biased ?



(b) Draw the circuit diagram of a full wave rectifier and state how it works.
[ALL INDIA 2017]

12. (a) A student wants to use two p-n junction diodes to convert alternating current into direct current. Draw the labelled circuit diagrams he would use and explain how it works.
(b) Give the truth table and circuit symbol for NAND gate.
[ALL INDIA 2018]

4 Mark Questions

13. Meeta's father was driving her to the school. At the traffic signal she noticed that each traffic light was made of many tiny lights instead of a single bulb. When Meeta asked this question to her father, he explained the reason for this.

Answer the following questions based on above information:

(i) What were the values displayed by Meeta and her father?

(ii) What answer did Meeta's father give?

(iii) What are the tiny lights in traffic signals called and how do these operate?

[DELHI 2016]

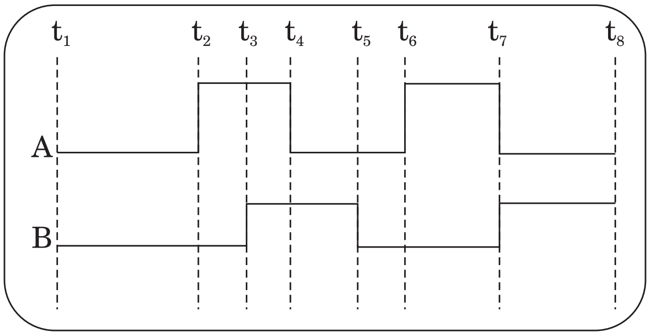
5 Mark Questions

14. (a) Draw the circuit diagram of a full wave rectifier using p-n junction diode. Explain its working and show the output, input waveforms.

(b) Show the output waveforms (Y) for the following inputs A and B of

(i) OR gate

(ii) NAND gate



[ALL INDIA 2012]

15. (a) State briefly the processes involved in the formation of p-n junction explaining clearly how the depletion region is formed.

(b) Using the necessary circuit diagram, show how the V-I characteristics of a p-n junction are obtained in

(i) Forward biasing

(ii) Reverse biasing

How these characteristics are made use of in rectification? **[DELHI 2014]**

16. How is a zener diode fabricated so as to make it a special purpose diode? Draw I-V characteristics of zener diode and explain the significance of breakdown voltage. Explain briefly, with the help of a circuit diagram, how a p-n junction diode works as a half wave rectifier.

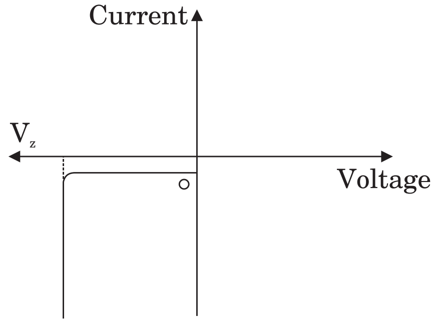
[DELHI 2017]

17. (a) With the help of circuit diagram explain the working principle of a transistor amplifier as an oscillator.

(b) Distinguish between a conductor, a semiconductor, a semiconductor and an insulator on the basis of energy band diagrams. **[DELHI 2018]**

Solutions

1. A Zener diode is a specially designed diode which is operated in reverse breakdown region continuously with any damage. When Zener diode is operated in the reverse break down region, the voltage across it remains practically constant (V_z) for a large change in reverse current. Therefore, for any increase/decrease of the input voltage there is a increase/decrease of the voltage drop across series resistance (R_s) without any change in the voltage across Zener diode [½]



[½]

2. The photodiode always work under reverse biasing conditions although the current produced is less. This is because in reverse bias, the width of the depletion layer increases which reduces the capacitance across the junction, there by increasing response time. The sensitivity of a photodiode is thus very high, a property that is certainly desired. [1]

3.

Intrinsic semiconductor	Extrinsic semiconductor
It is pure semi-conducting material with no impurity atoms added to it.	It is prepared by doping a small quantity of impurity atoms to the pure semiconductor.
The number of free electrons in the conduction band and the number of holes in valence band is exactly equal.	The number of free electrons and holes is never equal. There is an excess of electrons in n-type semiconductors and an excess of holes in p-type semiconductors.
Its electrical conductivity is a function of temperature alone.	Its electrical conductivity depends upon the temperature and the amount of impurity added in them.

[1]

4. Half-wave rectifier

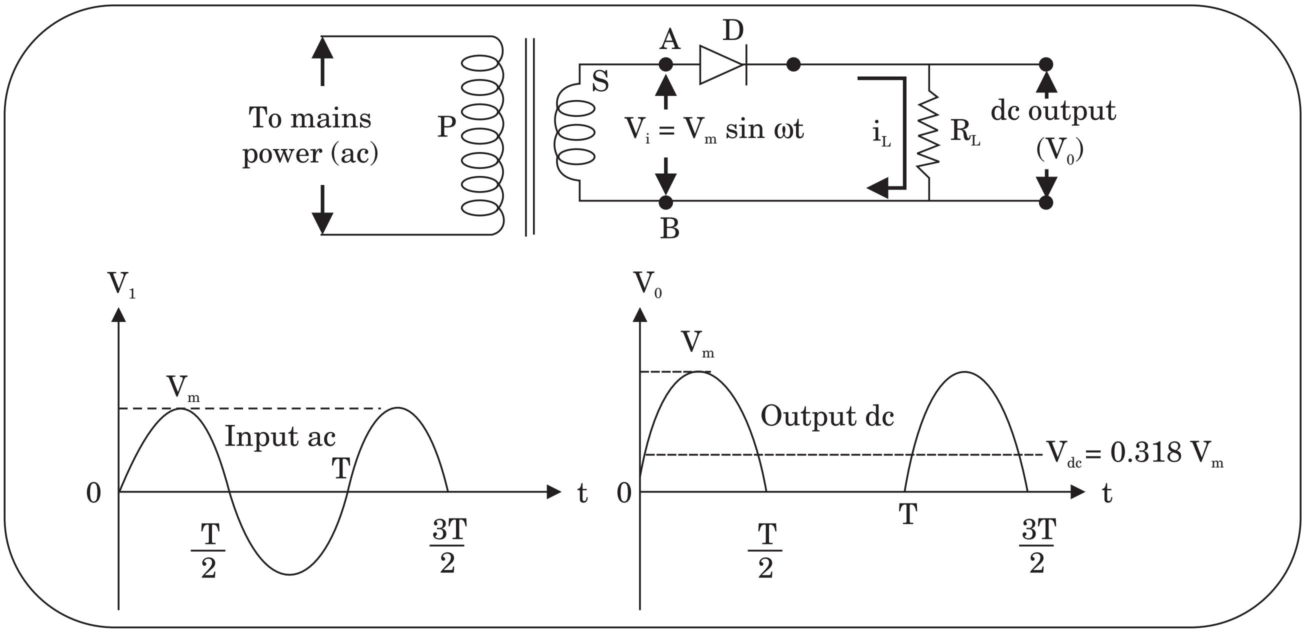
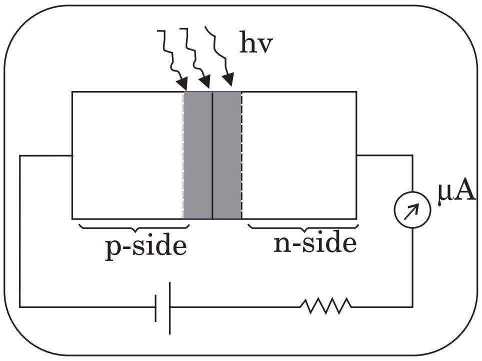


Fig.: Circuit & Voltage characteristics of half wave rectifier. [½]

During the positive half cycle of the input voltage, end *A* of the secondary is positive and end *B* is negative. This polarity makes the diode *D* to be forward biased. The diode conducts and a current i_L flows through the load resistance R_L. Since a forward biased diode offers a very low resistance, the voltage drop across the diode is also very small. Therefore, the voltage across R_L is almost equal to the voltage viat every instant of time. During the negative half cycle of the input voltage end *A* of the secondary is negative and end B is positive. Thus, the diode is in reverse bias. The diode does not conduct. No current flows through R_L. Hence, no voltage is developed across R_L. All the input voltage appears across the diode itself. This explains the output waveform. [½]

5. The circuit diagram of an illuminated photodiode in reverse bias can be represented as below:



[½]

Fig.: An illuminated photodiode

Greater the intensity of light, the greater is the number of photons falling per second per unit area. Thus, the greater the intensity of light, the greater is the number of electron-hole pairs produced at the junction. The photocurrent is, thus, directly proportional to the intensity of light. This can be used for measuring the intensity of incident light.

6. [1]

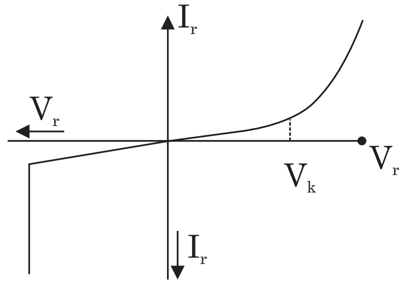


Fig.: V-I Characteristics of p–n junction diode.

(i) When p-n junction is reverse biased, the majority carriers in p and n region are repelled away from the junction. [½] There is small current due to the minority carriers. This current attains its maximum or saturation value immediately and is independent of the applied reverse voltage.

(ii) As the reverse voltage is increased to a certain value, called break down voltage, large amount of covalent bonds in *p* and n regions are broken. As a result of this, large electron-hole pairs are produced which diffuse through the junction and hence there is a sudden rise in the reverse current. Once break down voltage is reached, the high reverse current may damage the ordinary junction diode. Device is zener diode. [½]

7. The band model

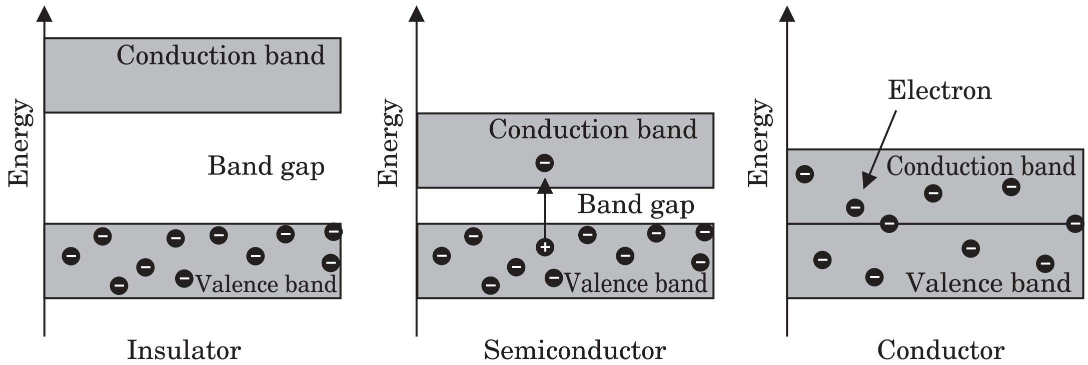


Fig.: Energy band gaps in insulators, Semiconductor & conductor [1]

	Conductors	Semiconductors	Insulators
(i)	Conduction band is completely filled	Conduction and valance bands are partially filled	Conduction band is empty & valance band is completely filled
(ii)	Conduction and valance bands overlap	Forbidden energy gap is of the order of 1 *eV*	Forbidden energy gap is very large i.e more than 5 *eV*.

[1]

8. A photo diode is used to observe the change in current with change in the light intensity under reverse bias condition.

In fabrication of photo diode, material chosen should have band gap —1.5 *eV* or lower so that solar conversion efficiency is better. This is the reason to choose Si or GaAs material.

Working: It is a p-n junction fabricated with a transparent window to allow light photons to fall on it. These photons generate electron hole pairs upon absorption. If the junction is reverse biased using an electrical circuit, these electron hole pair moves in opposite directions so as to produce current in the circuit. This current is very small and is detected by the micro ammeter placed in the circuit. [½]

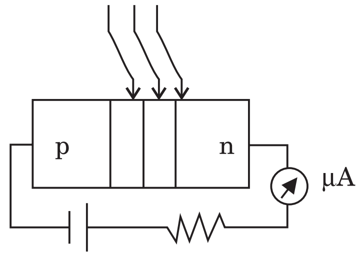
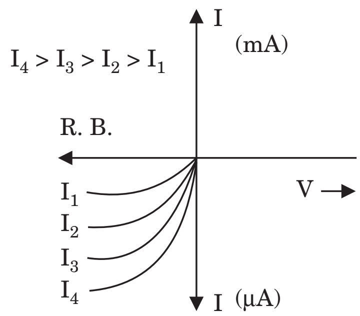


Fig.: An illuminated photodiode



[½ + ½]

Fig.: I–V character of photodiode for different illumination intensities

A photo diode is preferably operated in reverse bias condition. Consider an n- type semiconductor. Its majority carrier (electron) density is much larger than the minority hole density i.e. $n << p$. When illuminated with light, both types of carriers increase equally in number.

$n' = n + \Delta n; p' p + \Delta p$

Now, $n >> p$ and $\Delta n = \Delta p$

$$\frac{\Delta n}{n} << \frac{\Delta p}{p}$$

That is, the fractional increase in majority carries is much less than the fractional increase in minority carriers. Consequently, the fractional change due to the photo-effects on the minority carrier dominated reverse bias current is more easily measurable than the fractional change in the majority carrier dominated forward bias current. Hence, photo diodes are preferablly used in the reverse bias condition for measuring light intensity. [½]

9.

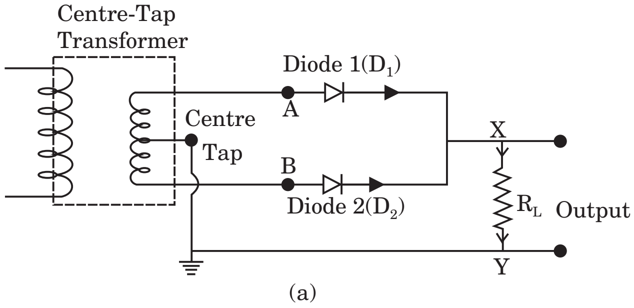


Fig.: Fill wave rectifier circuit

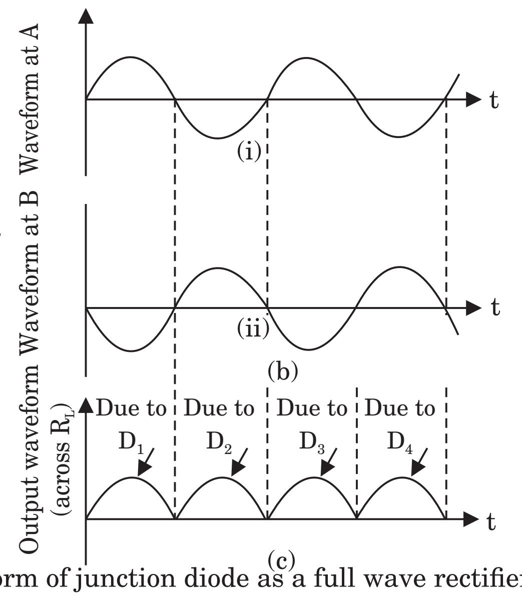
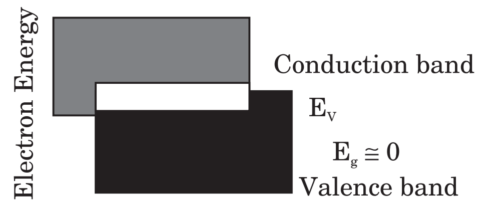


Fig.: Circuit diagram, input wave form & output wave form of junction diode as a full wave rectifier. [1]

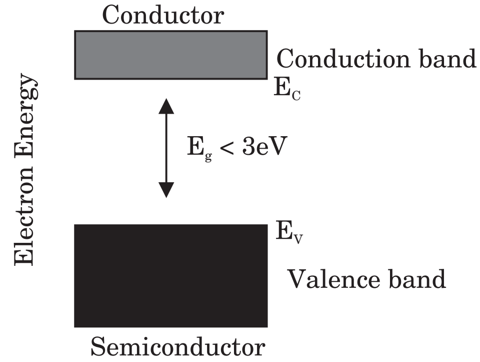
(a) A Full-wave rectifier circuit; enter (b) Input wave forms given to the diode D_1 at A and to the diode D_2 at B; (c) Output waveform across the load R_L connected in the full-wave rectifier circuit.

The circuit using two diodes, shown in Fig (a), gives output rectified voltage corresponding to both the positive as well as negative half of the ac cycle. Hence, it is known as full-wave rectifier. Here the p-side of the two diodes are connected to the ends of the secondary of the transformer. The n-side of the diodes are connected together and the output is taken between this common point of diodes and the midpoint of the secondary of the transformer. So for a full-wave rectifier the secondary of the transformer is provided with a centre tapping and so it is called centre-tap transformer. As can be seen from Fig.(c) the voltage rectified by each diode is only half the total secondary voltage. Each diode rectifies only for half the cycle, but the two do so for alternate cycles. Thus, the output between their common terminals and the centre tap of the transformer becomes a full-wave rectifier output. Suppose the input voltage to A with respect to the centre tap at any instant is positive. It is clear that, at that instant, voltage at B being out of phase will be negative as shown in Fig.(b). So, diode D_1 gets forward biased and conducts (while D_2 being reverse biased is not conducting). Hence, during this positive half cycle we get an output current (and a output voltage across the load resistor R_L) as shown in Fig.(c). In the course of the ac cycle when the voltage at A becomes negative with respect to centre tap, the voltage at B would be positive. In this part of the cycle diode D_1 would not conduct but diode D_2 would, giving an output current and output voltage (across R_L) during the negative half cycle of the input ac. Thus, we get output voltage during both the positive as well as the negative half of the cycle. The diode under forward biased offers negligible resistance so it will conduct while under reverse biased it offers very high resistance so it will not conduct. Therefore it is a unidirectional device which conducts only in one direction. This characteristic property makes the junction diode suitable for rectification. [1]

10.



[½]



Distinguishing features

Fig.: Distinguish between conducter, semiconductive & Insulator.

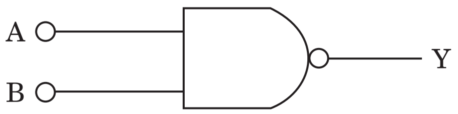
(a) **In conductors:** Valence band and conduction band overlap each other. In semiconductors: Valence band and conduction band are separated by a small energy gap. [½]

(b) **In conductors:** Large number of free electrons are available in conduction band. In semiconductors: A very small number of electrons are available for electrical conduction. [½]

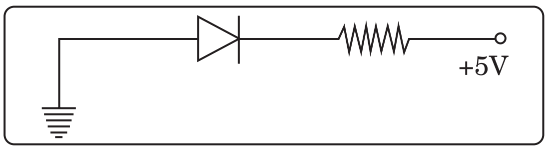
(ii) Gate From the given output waveform, it is clear that output is zero only when both inputs are 1, so the gate is NAND gate

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Logic Symbol: $Y = \overline{A \cdot B}$ [½]

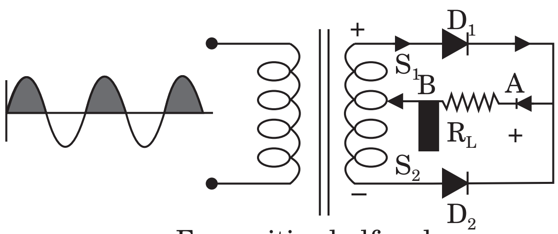


11. (a)

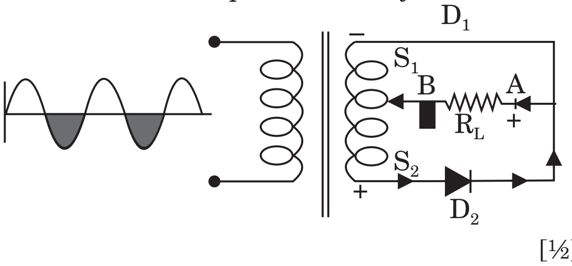


Voltage at *P* side is less than voltage at *N* side of the diode so it is in "Reverse bias". [½]

(b) Full wave rectifier:



For positive half cycle



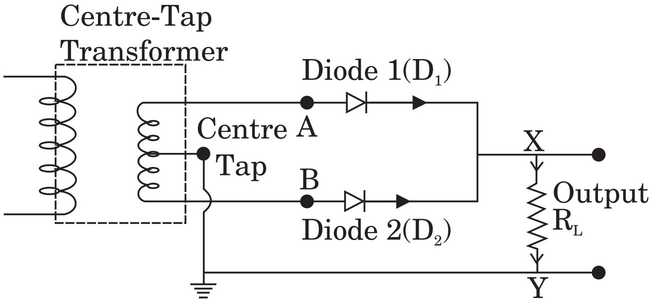
[½]

The input and output wavefront have been given in diagram. In general full wave rectifier is used to convert *AC* into *DC*. [½]

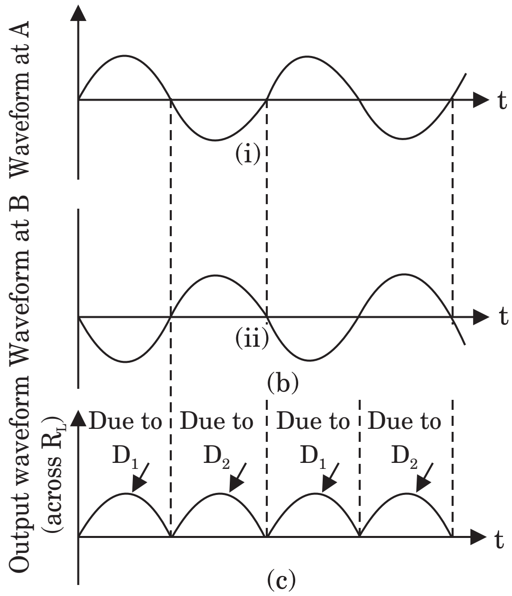
Working: During its positive half cycle of the input *AC* and diode D_1 is forward biased and is reverse biased. The forward current flows through diode D_1.

During the negative half cycle of the input *AC* the diode D_1 is reverse biased and diode D_2 is forward biased. Thus current flows through diode Thus we find that during both the valves, current flows in the same direction. [½]

12. (a) Full wave rectifier:

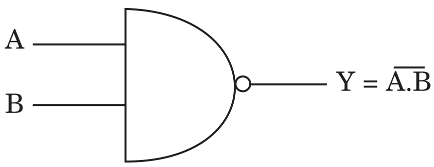


[½]



Wave: During positive half cycle diode D_1 is forward bias and during negative half cycle diode D_2 is forward bias so due to conduction of diode *D* positive half will appear and due to conduction of D_2 negative half cycle will appear. [½]

(b) NAND gate:



Input		Output
A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

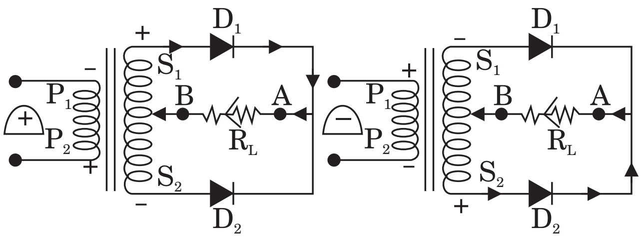
[½]

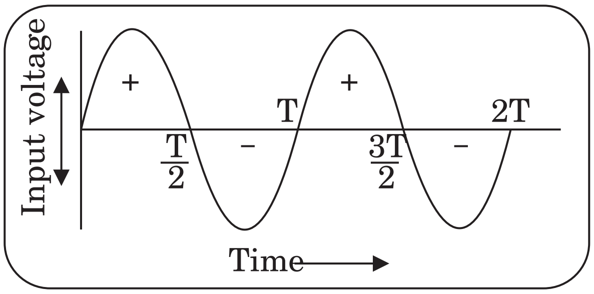
13. (i) Awareness for energy conservation, power saving and knowledge about traffic rules. [1]

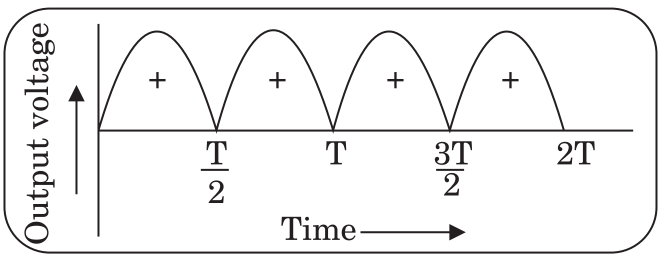
(ii) Meeta's father said that these are LED lights which consume less power and high reliability. [1]

(iii) The tiny lights in traffic signals are Light Emitting Diode. These are operated by connecting the P-N junction diode in forward biased condition. [2]

14. (a)







[2]

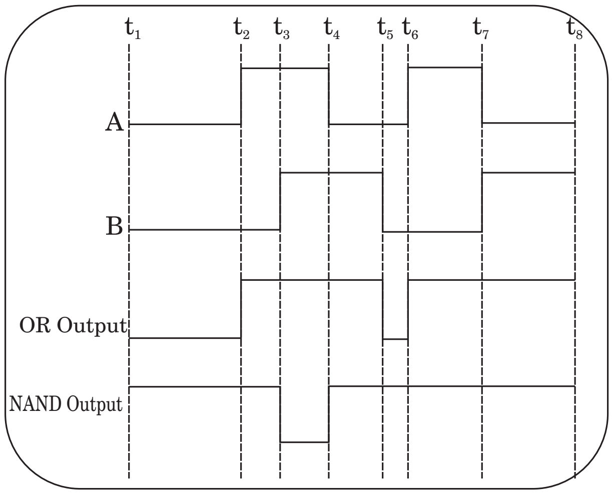
Full Wave Rectifier:

When the diode rectifies the whole of the *AC* wave, it is called full wave rectifier.

The figure shows the arrangement for using diode as full wave rectifier. The alternating input signal is fed to the primary P_1P_2 of a transformer. The output signal appears across the load resistance R_L. [1]

During the positive half of the input signal, suppose P_1 and P_2 are negative and positive respectively. This would mean that S_1 and S_2 are positive and negative respectively. Therefore, the diode D_1 is forward biased and D_2 is reverse biased. The flow of current in the load resistance R_L is from A to B. [1]

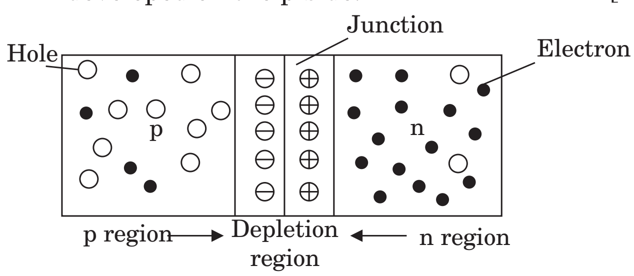
Output waveforms (Y)



[1]

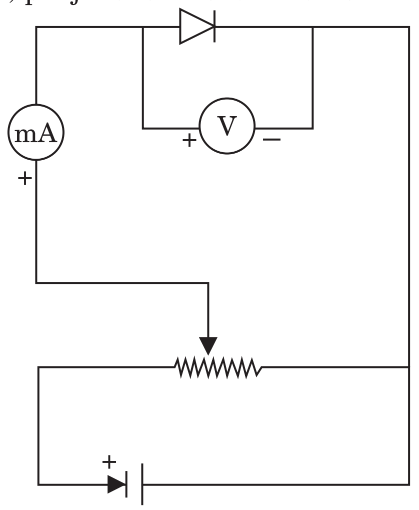
15. (a) As we know that n-type semi-conductor has more concentration of electrons than that of a hole and p-type semi-conductor has more concentration of holes than an electron. Due to the difference in concentration of charge carriers in the two regions of p-n junction, the holes diffuse from p-side to n-side and electrons diffuse from n-side to p-side.

When an electron diffuses from n to p, it leaves behind an ionized donor on n-side. The ionised donor (+ve charge) is immobile as it is bonded by the surrounding atoms. Therefore, a layer of positive charge is developed on the n-side of the junction. Similarly, a layer of negative charge is developed on the p-side. [1]



Hence, a space-charge region is formed on both side of the junction, which has immobile ions and is devoid of any charge carrier, called as depletion layer or depletion region. [1]

(b) (i) p-n junction diode under forward bias

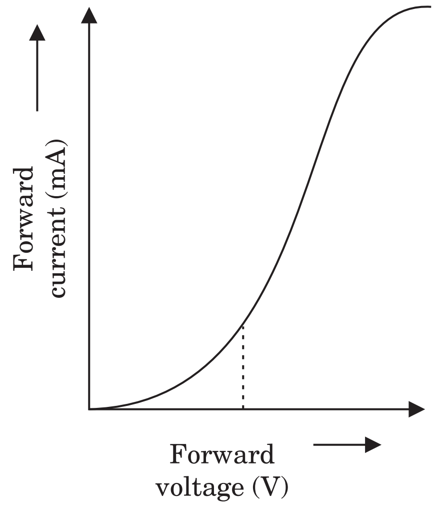


A p-n junction diode is said to be forward biased if the positive terminal of the external battery *B* is connected to p-side and the negative terminal to the n-side of p-n junction.

The applied voltage of battery mostly drops across the depletion region and the voltage drop across the p-side and n-side of the p-n junction in negligible small. The resistance of depletion region is very high as it has no free change carriers. [1]

Electron in n-region moves towards the p-n junction and holes in the p-region move towards the junction. The width of the depletion layer decreases and hence, it offers less resistance. Diffusion of majority carriers takes place across the junction. This leads to the forward current.

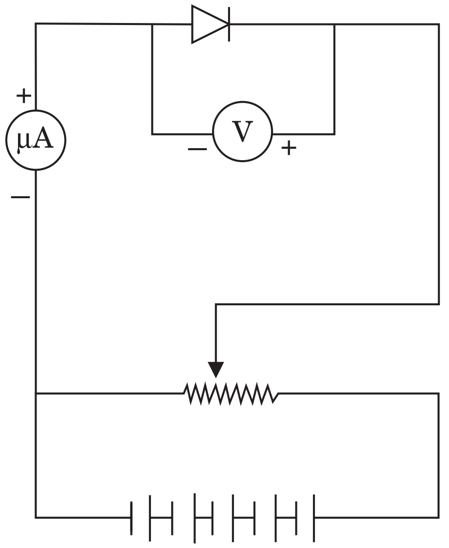
The V-I characteristics of p-n junction is forward bias is shown below:



[1]

(ii) The p-n junction under reverse bias Positive terminal of battery is connected to n-side and negative terminal to p-side.

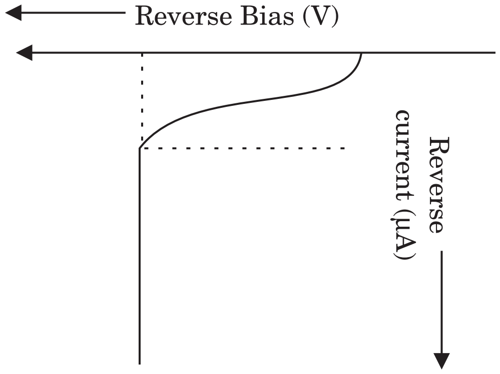
Reverse bias supports the potential barrier. Therefore, the barrier height increases and the width of depletion region also increases. Due to the majority carriers, there is no conduction across the junction. A few minority carriers cross the junction after being accelerated by high reverse bias voltage.



Reverse Biasing

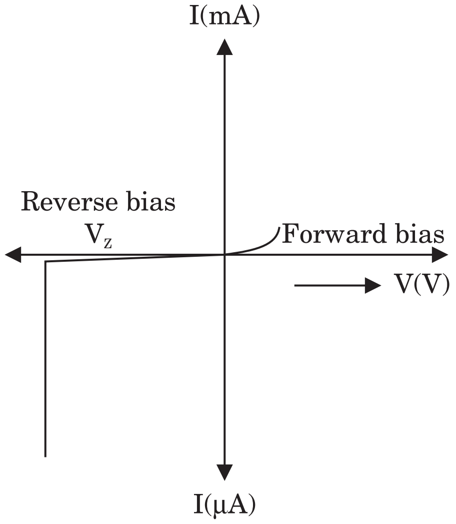
This constitutes a current that flows in opposite direction, which is called reverse current.

The V-I characteristics of p-n junction diode in reverse bias is shown on previous page:



p-n junction diode is used as a half-wave rectifier. Its working is based on the fact that the resistance of p-n junction becomes low when forward biased and becomes high when reverse biased. These characteristics of diode are used in rectification. [1]

16.



I–V Characteristics [1]

Zener has a sharp breakdown voltage and this property of zener is used for voltage regulation.

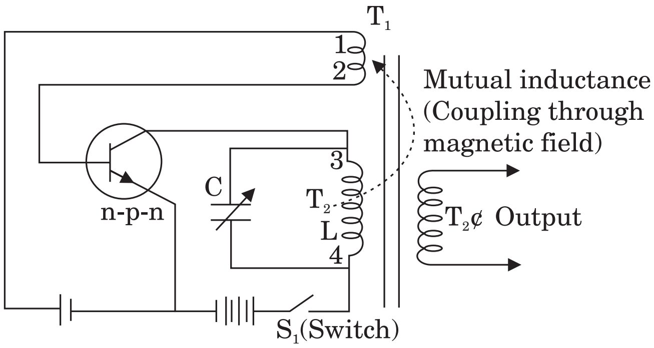
An ac current has a positive half cycle and a negative half cycle. A p-n junction allows current to pass only in one direction and that is when it is forward biased.

When a positive half-cycle occurs, the p-side has a lower potential. Therefore, the diode is now forward biased and therefore conducts and this positive cycle is available for the load. [1]

When a negative half cycle occurs, the n-side has a higher potential than the p-side. Hence, the diode is now reverse biased and thus, does not conduct. As a result, this positive half cycle also does not conduct. Therefore, it does not appear at the load and is cut-off. [1]

We obtain a waveform, which has only positive half cycles and therefore it is called half-wave rectifier.

17. (a) The circuit diagram for a transistor amplifier as an oscillator is represented as:

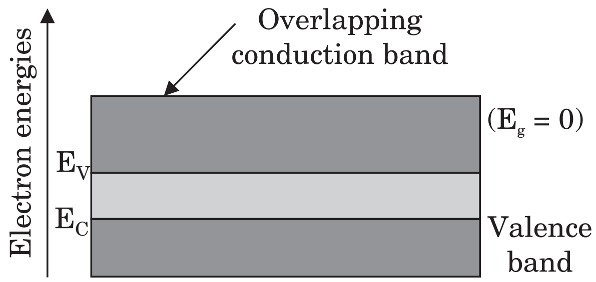


[1]

In an oscillator, a sustained *A.C.* output is obtained without any input oscillation. For this to happen, the output of a transistor amplifier is fed back into its input. This is achieved by coupling the winding T_1 to winding T_2.

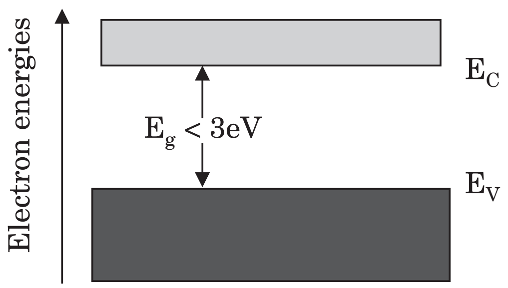
When key S_1 is closed, the collector current begins to increase, which supports the forward bias of the emitter-base circuit. Collector current increases until it reaches saturation. When the saturation is reached, the magnetic flux linked to winding T_1 becomes steady. Hence, the forward bias of the emitter-base circuit is no longer supported. The transistor is now driven into cut-off. This cycle repeats itself and an oscillating output is obtained. [2]

(b) The energy-band diagram of a conductor is shown as below:

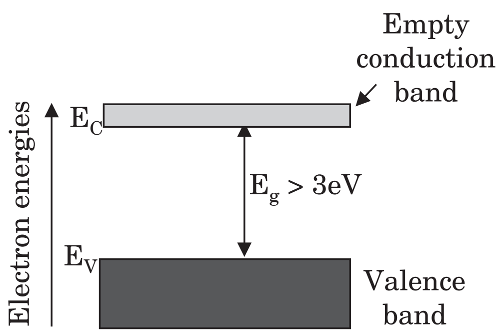


[2]

The energy-band diagram of a semiconductor is as below:



The energy-band diagram of an insulator is as below:



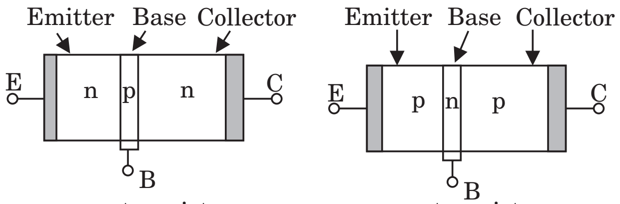
[1]

[Topic 2] Transistors, its application and logic gates

Summary

- A thin layer of one type of semiconductor is added between two thick layers of other semiconductor of same type and this forms a transistor.

  It can be done in two ways, i.e. adding a p-type semiconductor between two n-type semiconductors forming n-p-n transistor or by adding an n-type semiconductor between two p-type semiconductors forming p-n-p transistor.

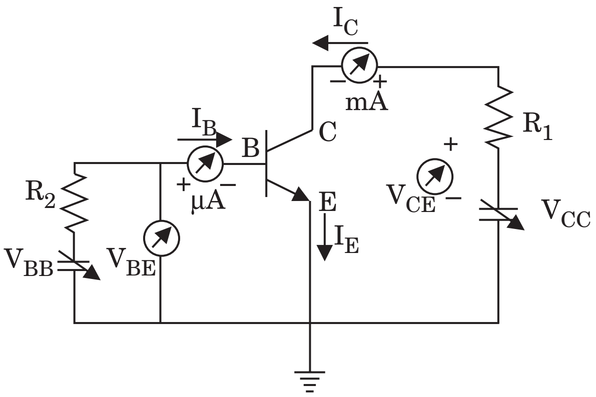


n-p-n transistor p-n-p transistor

➢ Any transistor has 3 parts: Base (central block), Emitter and Collector (two electrodes). Therefore the three parts of the transistor can be connected in three ways: Common Emitter (CE), Common Collector (CC) and Common Base (CB).

➢ For fixed I_B, the plot between I_C gives output characteristics and for fixed V_{CE}, the plot between I_B and V_{BE} gives input characteristics.

- ***Common emitter transistor:*** The input is between the base and the emitter and output is between the collector and the emitter.



n-p-n transistor in CE configuration

➢ ***Input resistance*** is the ratio of change in base emitter voltage to the resulting change in base current at constant collector emitter voltage and is given by

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

➢ Output resistance is the ratio of change in collector emitter voltage to the change in collector current at a constant base current and is given by

$$r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B}$$

- ***Current gain:*** There are two low current gains defined as follows:

➢ ***Common base current amplification factor (α) :*** Ratio of the small change in collector current to the small change in emitter current at constant collector-base voltage.

$$\alpha = \left[\frac{\delta I_C}{\delta I_E}\right]_{V_{CB}=\text{constant}}$$

➢ ***Common emitter current amplification factor (β)***: Ratio of the small change in collector current to the small change in base current at constant collector-emitter voltage.

$$\beta = \left[\frac{\delta I_C}{\delta I_B}\right]_{V_{CE}=\text{constant}}$$

Terms α and β are related as: $\alpha = \dfrac{\beta}{1+\beta}$ and

$$\beta = \frac{\alpha}{1-\alpha}$$

- A transistor can be used as an amplifier to increase voltage, current or power. Voltage gain of an amplifier can be defined as the ratio of small change in output voltage to small change in input voltage. Ratio of the small change in collector current to the small change in base current at constant collector-emitter voltage is called current gain.

  Voltage gain of amplifier is given by,

$$A_v = -\left(\frac{\beta_{ac} R_L}{r}\right)$$

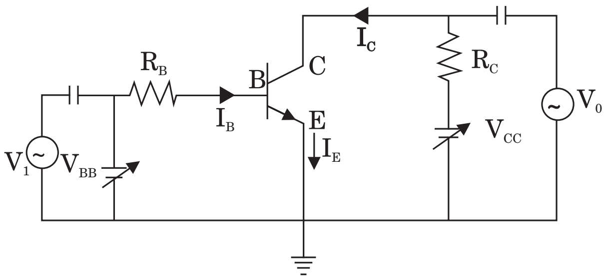


Fig.: C-E transistor amplifier

- A transistor can be used as a switch by analyzing the behavior of the base-biased transistor in CE configuration. When transistor works as a switch a low input to the transistor gives high output and a high input gives a low output. In this case the transistor does not remain in active state.

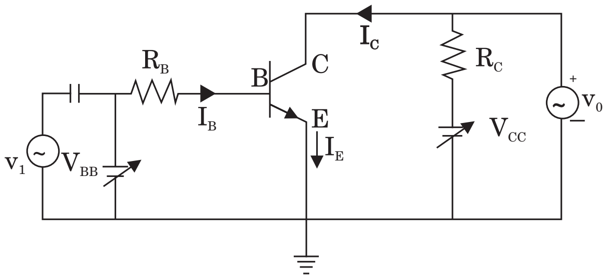


Fig.: Base-biased transistor in CE configuration to work as a switch

- ***Transistor oscillator:*** When we get ac output without any external input signal then the transistor works as an oscillator.

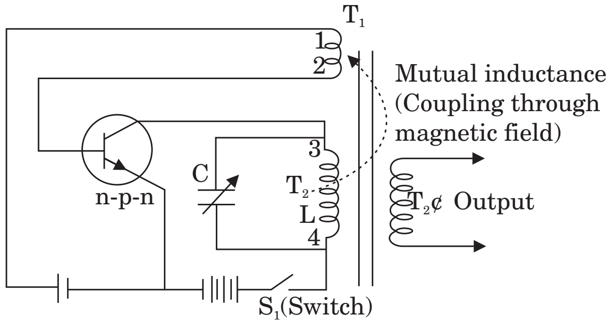
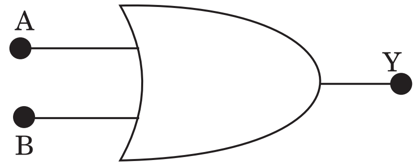


Fig.: Tuned collector oscillator

Frequency at which the oscillator will work is given by, $v = \dfrac{1}{2\pi\sqrt{LC}}$

- Logic gates are digital circuits which perform special; logic operations. These logic gates can be described as OR, AND, NOT, NAND, and NOR. Different logic gates are integrated in a single chip called Integrated circuits (IC).

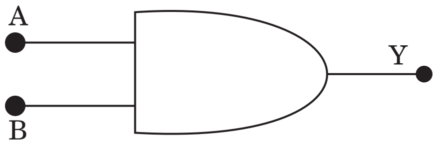
➢ Boolean expression for ***OR gate:*** Y = A + B



The truth table for OR gate is shown below:

A	*B*	*Output Y*
0	*0*	*0*
0	*1*	*1*
1	*0*	*1*
1	*1*	*1*

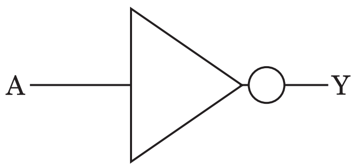
➢ Boolean expression for ***AND gate***: Y = A.B



The truth table for AND gate is shown below:

A	*B*	*Output Y*
0	*0*	*0*
0	*1*	*0*
1	*0*	*0*
1	*1*	*1*

➢ Boolean expression for ***NOT gate:*** $Y = \overline{A}$



The truth table for NOT gate is shown below:

A	*Output Y*
0	*1*
1	*0*

- ***Integrated circuits:*** When a entire circuit (including all passive components and active devices) is fabricated on a single chip or block of a semiconductor then it is known as integrated circuit(IC). The most widely used technology for making IC's is monolithic Integrated circuit.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

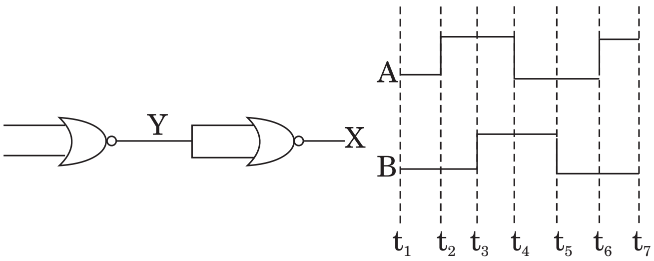
1 Mark Questions

1. In a transistor, doping level in base is increased slightly. How will it affect (i) collector current and (ii) base current?

[ALL INDIA 2011]

2 Mark Questions

2. Draw the output waveform at X, using the given inputs A and B for the logic circuit shown below. Also, identify the logic operation performed by this circuit.



[ALL INDIA 2011]

3. Draw the transfer characteristic curve of a transistor in CE configuration. Explain clearly how the active region of the V_0 versus V_i– curve in a transistor is used as an amplifier.

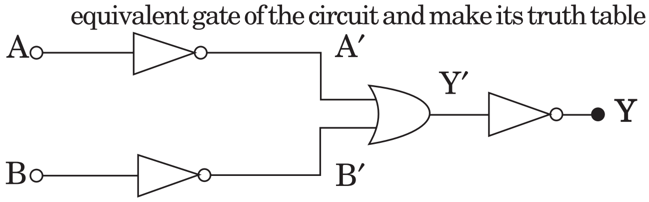
[ALL INDIA 2011]

4. Draw the transfer characteristic curve of a base biased transistor in CE configuration. Explain clearly how the active region of the V_0 versus V_i curve in a transistor is used as an amplifier.

[ALL INDIA 2011]

5. Draw typical output characteristics of an n-p-n transistor in CE configuration. Show how these characteristics can be used to determine output resistance. **[ALL INDIA 2013]**

6. In the circuit shown in the figure, identify the equivalent gate of the circuit and make its truth table

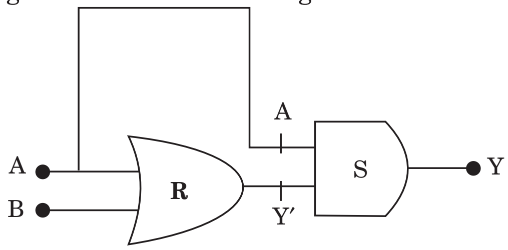


[ALL INDIA 2013]

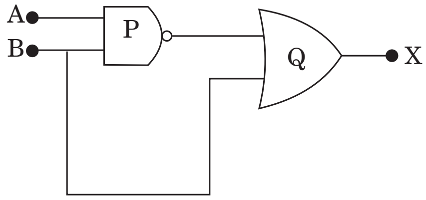
7. Draw a circuit diagram of n-p-n transistor amplifier in CE configuration. Under what condition does the transistor act as an amplifier?

[DELHI 2014]

8. Write the truth table for the combination of the gates shown. Name the gates used.



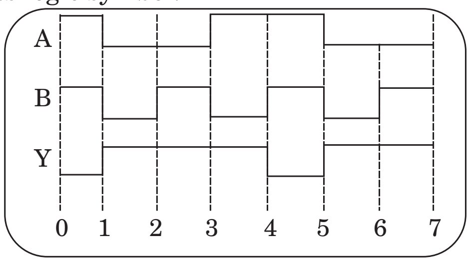
Identify the logic gates marked 'P' and 'Q' in the given circuit. Write the truth table for the combination.



[DELHI 2014]

3 Mark Questions

9. The following figure shows the input waveforms (A. B) and the output waveform (Y) of a gate. Identify the gate, write its truth table and draw its logic symbol.



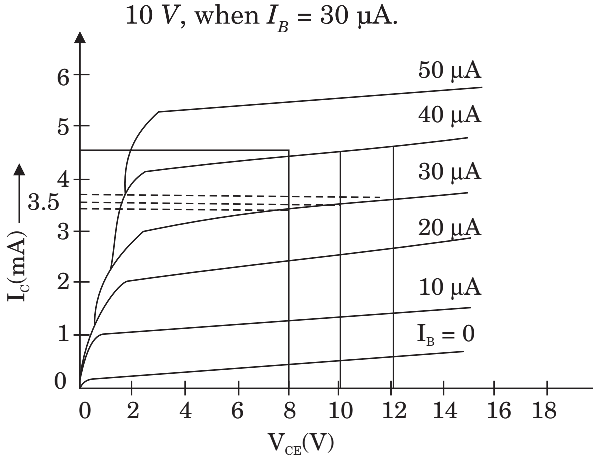
[DELHI 2017]

10. Output characteristics of a n-p-n transistor in CE configuration is shown in the figure. Determine:

(i) Dynamic output resistance

(ii) DC current gain and

(iii) AC current gain at an operating point V_{CE} = 10 V, when I_B = 30 μA.



[DELHI 2013]

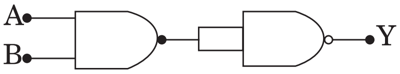
11. Draw a circuit diagram of a transistor amplifier in *CE* configuration. Define the terms:
(i) Input resistance and (ii) Current amplification factor. How are these determined using typical input and output characteristics?

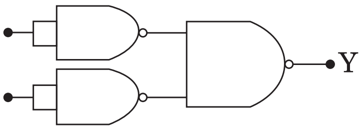
[DELHI 2015]

12. The outputs of two NOT gates are fed to a NOR gate. Draw the logic circuit of the combination of gates. Write its truth table. Identify the gate equivalent to this circuit.

Or

You are given circuits (a) and (b) as shown in the figures, which consists of NAND gates. Identify the logic operation carried out by the two. Write the truth tables for each. Identify the gates equivalent to the two circuits.

(a) 

(b) 

[ALL INDIA 2015]

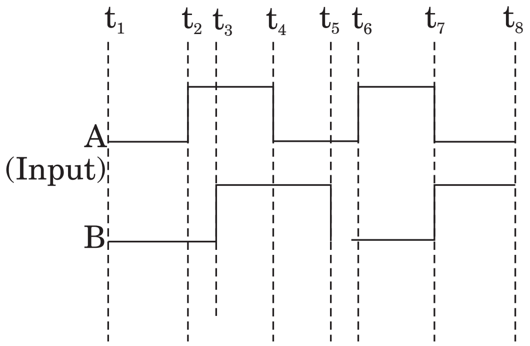
13. (i) Write the functions of three segments of a transistor.

(ii) Draw the circuit diagram for studying the input and output characteristics of n-p-n transistor in common emitter configuration. Using the circuit, explain how input, output characteristics are obtained.

[DELHI 2016]

14. (a) Write the functions of the three segments of a transistor.

(b) The figure shows the input waveforms A and B for 'AND' gate. Draw the output waveform and write the truth table for this logic gate.



[ALL INDIA 2017]

15. Draw the typical input and output characteristics of an n-p-n transistor in *CE* configuration. Show how these characteristics can be used to determine (*a*) the input resistance (r_i), and (*b*) current amplification factor.

[ALL INDIA 2018]

5 Mark Questions

16. Draw a simple circuit of a *CE* transistor amplifier. Explain its working. Show that the voltage gain, Av, of the amplifier is given by $A_v = -\frac{\beta_{ac} R_L}{r_i}$, where β_{ac} is the current gain, R_L is the load resistance and r_i is the input resistance of the transistor. What is the significance of the negative sign in the expression for the voltage gain?

[ALL INDIA 2012]

17. (a) Differentiate between three segments of a transistor on the bias of their size and level of doping.

(b) How is a transistor biased to be in active state?

(c) With the help of necessary circuit diagram, describe briefly how n-p-n transistor in CE configuration amplifies a small sinusoidal input voltage. Write the expression for the ac current gain.

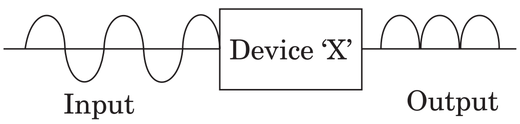
[DELHI 2014]

18. (i) Draw a circuit diagram to study the input and output characteristics of an n-p-n transistor in its common emitter configuration. Draw the typical input and output characteristics.

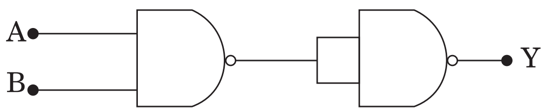
(ii) Explain, with the help of a circuit diagram, the working of n-p-n transistor as a common emitter amplifier. **[DELHI 2017]**

19. (a) Explain the formation of depletion layer and potential barrier in p-n junction.

(b) In the given figure below the input waveform is converted into the output waveform by a device 'X'. Name the device and draw its circuit diagram.



(c) Identify the logic gate represented by the circuit as shown and write its truth table.



[DELHI 2018]

Solutions

1. If the doping level in the base of a transistor is increased:

(i) Collector current will decrease. [½]

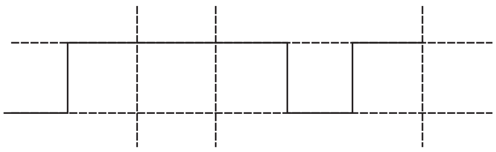
(ii) Increased base doping lowers its resistance hence its base current should increase. [½]

2. The truth table will be

A	B	Y	X
0	0	1	0
1	0	0	1
1	1	0	1
0	1	0	1
0	0	1	0
1	0	0	1

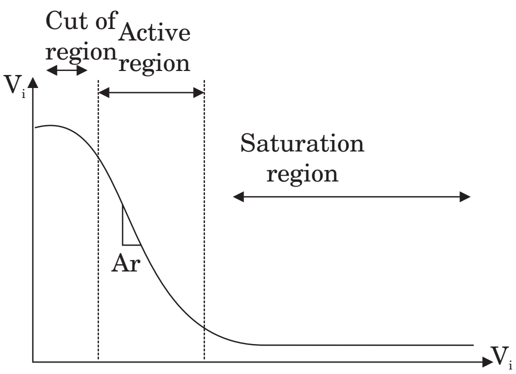
[½]

The output waveform will be



[½]

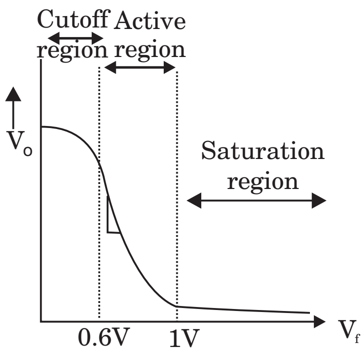
3.



[½]

In the active region, a (small) increase of V_i results in a (large, almost linear) increase in I_c. This results in an increase in the voltage drop across R_c. [½]

4.



If we plot V_o, V_s, V_i, we get a graph as shown in figure; this characteristic curve is also called transfer characteristic curve of a base biased transistor in *CE* configuration.

The curve shows that there are three non-linear regions:

(i) between cut-off stage and active stage

(ii) between active stage and saturation stage; [½]

For using the transistor as an amplifier we will use the active region of the V_o versus V_i curve. The slope of the linear part of the curve represents the rate of change of the output with the input. It is negative because the output is $V_{cc} - I_c R_c$ and $_c$ not $I_c R$. That is why as input voltage of the *CE* amplifier increases its output voltage decreases and the output is said to be out of phase with the input. If we consider ΔV_o and ΔV_i as small changes in the output and input voltages then $\frac{\Delta V_o}{\Delta V_i}$ is called the small signal voltage gain A_V of the amplifier. If the V_{BB} voltage has a fixed value corresponding to the midpoint of the active region, the circuit will behave as a *CE* amplifier with voltage gain $\frac{\Delta V_o}{\Delta V_i}$. We can express the voltage gain A_v in terms of the resistors in the circuit and the current gain of the transistor as follows.

We have, $V_o = V_{cc} - I_c R_c$

Therefore, $\Delta V_o = 0 - R_c \Delta I_c$

Similarly, from $V_i = I_B R_B + V_{BE}$

$\Delta V_i = R_B \Delta I_B + \Delta V_{BE}$

But ΔV_{BE} is negligibly small in comparison to $\Delta I_B R_B$ in this circuit

So, the voltage gain of this *CE* amplifier is given by

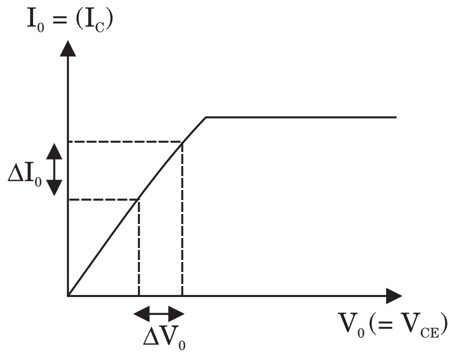
$$A_V = -\frac{R_C \Delta I_C}{R_B \Delta I_B}$$

$$= -\beta_{ac}\left(\frac{R_c}{R_B}\right) \quad (14.14)$$

Where β_{ac} is equal to $\frac{\Delta I_C}{\Delta I_B}$ [½]

Thus the linear portion of the active region of the transistor can be exploited for the use in amplifiers.

5.



[½]

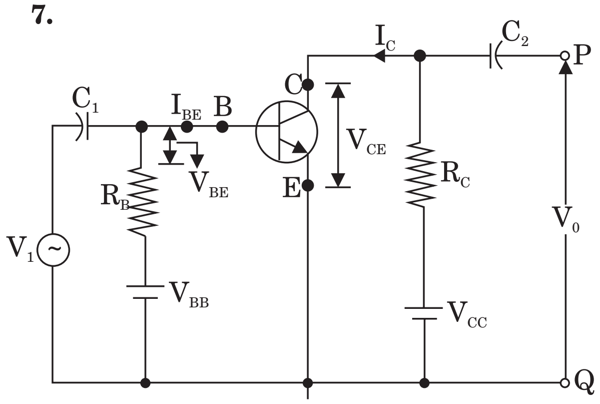
output resistance $= \dfrac{\Delta V_o}{\Delta I_o}$ [½]

6.

Input		Output
1	0	0
0	1	0
0	0	0
1	1	1

[1]

7.



[½]

The condition necessary for the amplifier to work is that the base emitter junction should be forward biased and collector base junction should be reverse biased. [½]

8.

R-OR GATE

S-AND GATE

A	B	Y' = A+B	Y = Y'.A
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

[1]

OR

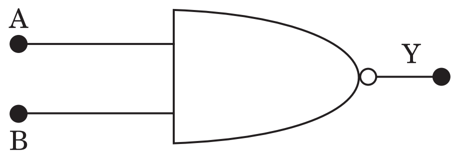
P-NAND GATE

Q-OR GATE

A	B	$\overline{AB}$	$\overline{AB}+B$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	1

[1]

9. The gate is the NAND gate.



[1]

Input		Output
0	0	1
0	1	1
1	0	1
1	1	0

[1]

10. (i) Dynamic output resistance is given as:

$$R_o = \left(\frac{\Delta V_{CE}}{\Delta l_C}\right)_{I_b \text{constant}}$$

$$= \frac{12-8}{(3.6-3.4)\times 10^{-3}} = \frac{4}{0.2\times 10^{-3}}$$

[1]

$R_o = 20\ K\Omega$

(ii) DC current gain,

$$\beta_{dc} = \frac{I_C}{I_B} = \frac{3.5\times 10^{-3}}{30\times 10^{-6}} = \frac{350}{3}$$

[½]

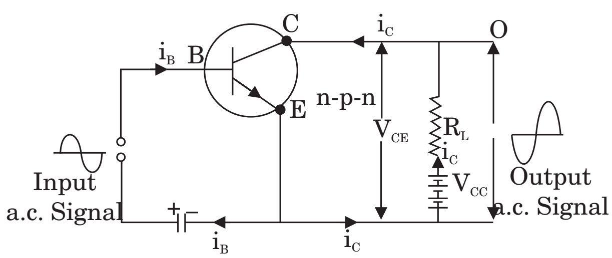
(iii) AC current gain,

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{(4.7-3.5)\times 10^{-3}}{(40-30)\times 10^{-6}} = \frac{1.2\times 10^{-3}}{10\times 10^{-6}}$$

[½]

$\beta_{ac} = 120$

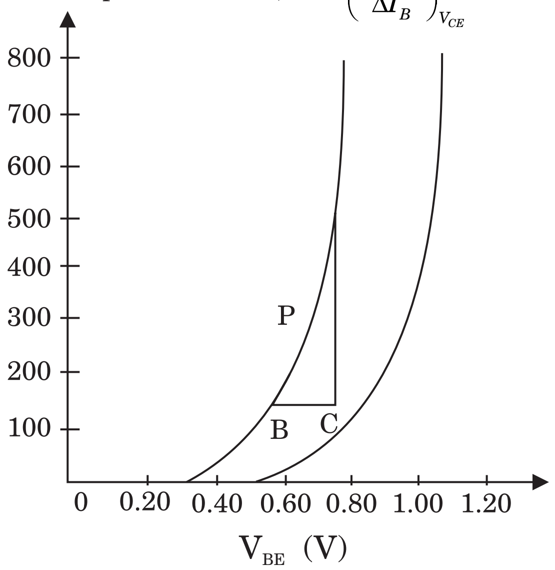
11.



[½]

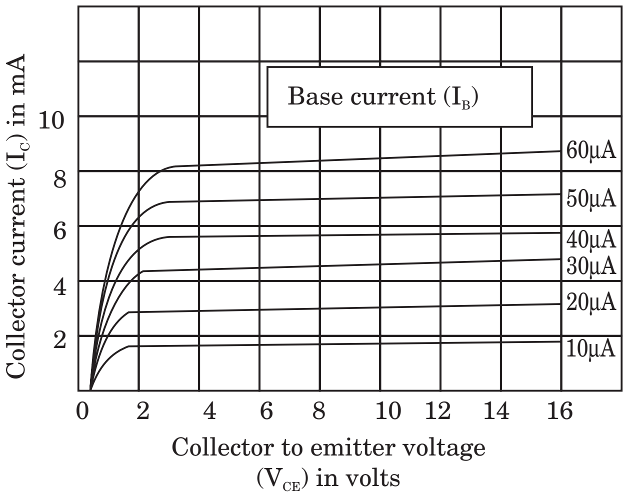
(i) Input resistance may be defined as the ratio of small change in the base-emitter voltage (ΔV_{BE}) to the resulting change in the base current (I_B) at constant collector-emitter voltage (V_{CE}) [½]

Input resistance, $R_i = \left(\dfrac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$



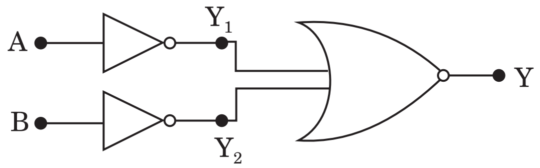
To find the input resistance mark a point *P* on the input characteristic. Now, draw a tangent at point *P*. The reciprocal of slope of *AB* will give the input resistance.

(ii) Current amplification factor $r = \left(\dfrac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$ is defined as the ratio of change in collector current (ΔI_C) to the change in base current. [½]



[½]

12.



$Y_1 = \overline{A}$

$Y_2 = \overline{B}$

$Y = \overline{Y_1 + Y_2} = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$

The equivalent gate is AND gate.

Truth table

A	**B**	**Y_1**	**Y_2**	**Y**
0	0	1	1	0
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

Or

(b) $Y_1 = \overline{A \cdot A} = \overline{A}$

$Y_2 = \overline{B \cdot B} = \overline{B}$

$Y = \overline{Y_1 Y_2} = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

The equivalent gate is OR gate

Truth table

A	**B**	**Y_1**	**Y_2**	**Y**
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

13. (i) Three segments of transistor are:

(1) Emitter (2) Base

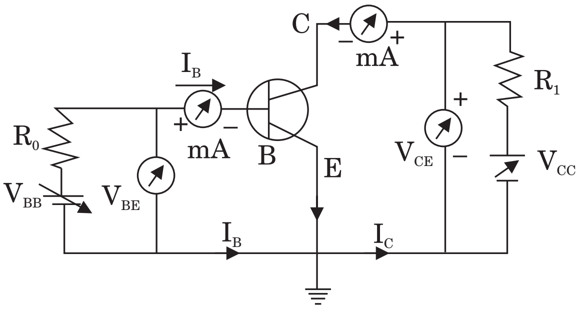
(3) Collector

Emitter: It is of moderate size and heavily doped, it supplies a large number of majority carriers which flow through the transistor.

Base: It is very thin and lightly doped and it separates emitter and collector region of transistor and controls the flow of charge carriers.

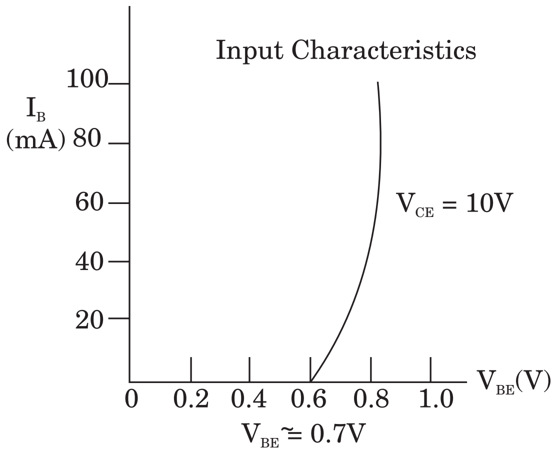
Collector: This segment is moderately doped and larger in size as compared to emitter. It collects a major portion of majority carriers supplied by the emitter. [½]

(ii)



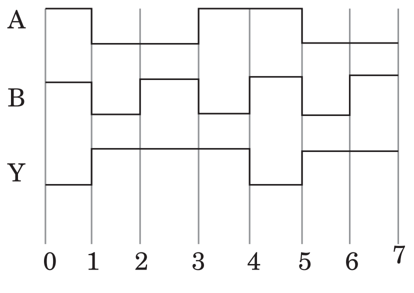
Circuit to study characteristics of a transistor

For input characteristics, base current I_B V_S base emitter voltage V_{BE} is plotted while collector base voltage V_{CE} is kept constant. V_{CE} is kept large 3V to 20V. Input characteristics for various values of V_{CE} give almost same curves.



[½]

Output characteristics are obtained by varying I_C with V_{CE} keeping I_B constant. Different curves are obtained for different values of I_B.



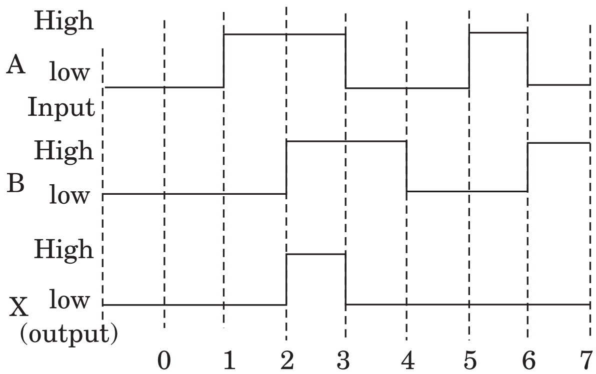
[½]

14. (a) These segments of a transmitter are called emitter (E), Base (B) and collector (C) Emitter: It is of moderate size and heavily doped. It supplies a large number of majority carrier for the current flow through the transistor

Base: Base is the control segment and it is very thin and lightly doped. [½]

Collector: It is the segment that collects major portion of the majority carries supply by the emitter. It is moderately doped and large in size as compared to the emitter. Input of AND gate is $Y = A.B$ in this case output, will be 1 only when both inputs are 1 [½]

(b)

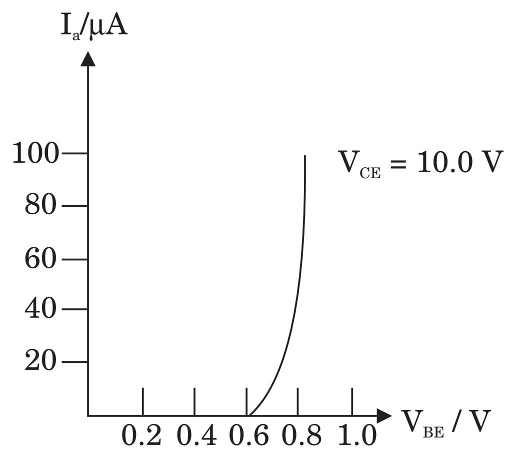


[½]

Input		Output
A	B	$Y = A.B$
0	0	0
0	1	0
1	0	0
1	1	1

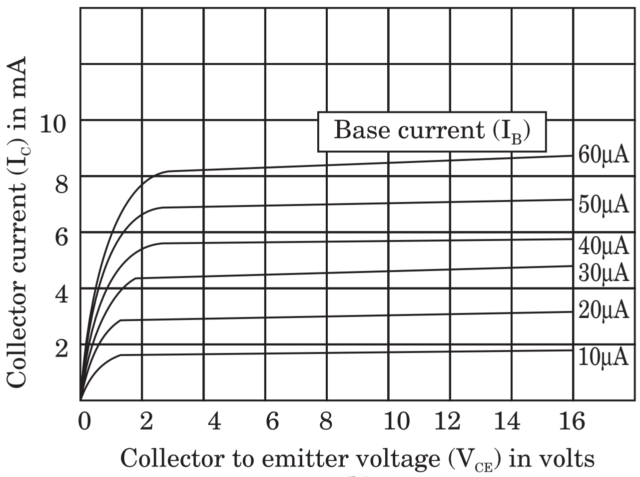
[½]

15.



(a)

[½]



(b)

[½]

The above plots show the typical input output characteristics of an n-p-n transistor in *CE* combination.

$$R_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

[½]

here V_{BE} is voltage across base emitter, I_b is base current

$$R_o = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B}$$

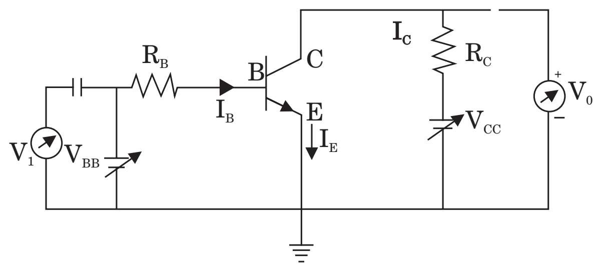
[½]

here V_{CE} is voltage across collector emitter and I_c is collector current.

$$\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$$

[½]

16. Circuit diagram of *CE* transistor amplifier:



[1]

Working: If a small sinusoidal voltage is applied to the input of a *CE* configuration, the base current and collector current will also have sinusoidal variations. Because the collector current drives the load, a large sinusoidal voltage v_o will be observed at the output.

The expression for voltage gain of the transistor in *CE* configuration is:

$$A_\nu = \frac{\nu_o}{\nu_2} = \frac{-\beta_{ac} R_L}{r_i}$$

[1]

Where, β_{ac} = *ac* current gain

R_L = Load resistance

r = $R_B + r_i$

r_i = input resistance

R_B = Base resistance

Current gain of the transistor will decrease if the base is made thicker because current gain,

$$\beta = \frac{I_c}{I_b}$$

[1]

If the base of an n-p-n transistor is made thicker, then more and more electrons will recombine with the p-type material of the base. This results in a decrease in collector current I_c. Furthermore, I_b also increases.

Hence, ac current gain $\beta = \frac{I_c}{I_b}$ decreases.

Finding expression for voltage gain of the amplifier:

Applying Kirchhoff's law to the output loop,

$V_{CC} = V_{CE} + I_C R_C$

$V_{CC} = V_{CE} + I_C R_C$

$V_{CC} = V_{CE} + I_C R_C$

$V_{BB} = V_{BE} + I_B R_B$

$v_i \neq 0$

Then, $V_{BB} + v_i = V_{BE} + I_B R_B + \Delta I_B (R_B + r_i)$

$$\therefore r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

[1]

$$\therefore v_i = \Delta I_B (R_B + r_i) = r\Delta I_B$$

$$\beta_{ac} = \frac{\Delta I_c}{\Delta I_B} = \frac{i_c}{i_b}$$

It is a current gain denoted by A_i

Change I_c due to change in I_B causes a change in V_{CE} and the voltage drop across resistance R_c, because V_{CC} is fixed.

$$\Delta V_{CC} = \Delta V_{CE} + R_C \Delta I_C = 0$$

$$\Delta V_{CE} = -R_C \Delta I_C$$

Change in V_{CC} is the o/p voltage V_o.

Voltage gain of amplifier,

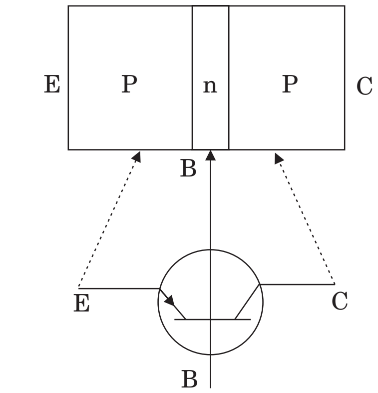
$$A_V = \frac{V_o}{V_i} = \frac{\Delta V_{CE}}{r\Delta I_B}$$

$$= -\beta_{ac} = \frac{R_C}{r}$$

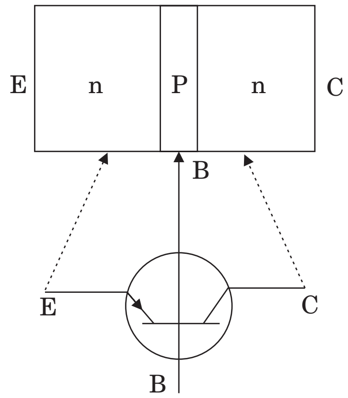
(Negative sign represents that o/p voltage is in opposite direction to i/p voltage. [1]

17. (a) **Emitter (E)**: It is the left hand side thick layer of the transistor, which is heavily doped.

Base (B): It is the central thin layer of the transistor, which is lightly doped. [1]



p-n-p transistor



n-p-n transistor [1]

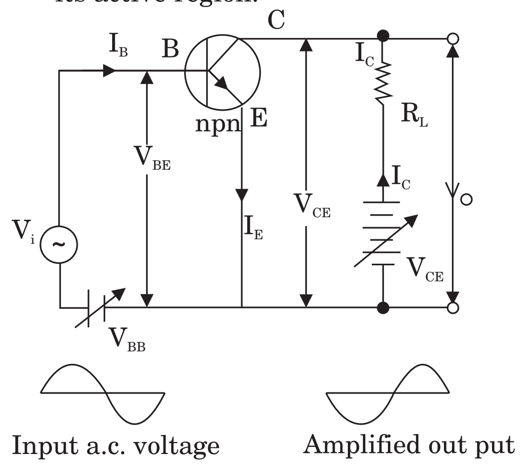
Collector (C): It is the right hand side thick layer of the transistor, which is moderately doped.

(b) There are two conditions for a transistor to be into an active region.

(1) The input circuit should be forward biased by using a low voltage battery.

(2) The output circuit should be reverse biased by using a high voltage battery.

(c) n-p-n transistor as an amplifier:
The operating point is fixed in the middle of its active region. [1]



An a.c. signal v_1 is superimposed on bias $V_{BB}(dc)$. The o/p is taken between the collector and the ground.

Applying Kirchhoff's law to the output loop:

$I_E = I_B + I_C$

If V_{CE} is the collector voltage then,

$V_{CE} = V_{CC} - I_C R_L$ [1]

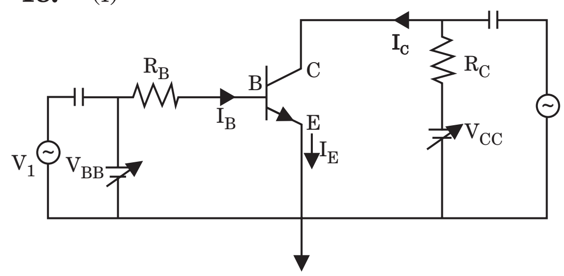
When the input signal voltage is fed to the emitter base circuit, it will change the emitter base voltage and hence the emitter current, which in turn will change the collector current. Due to this the collector voltage V_{CE} will vary in accordance with relation (A). This variation in collector voltage appears as amplified output.

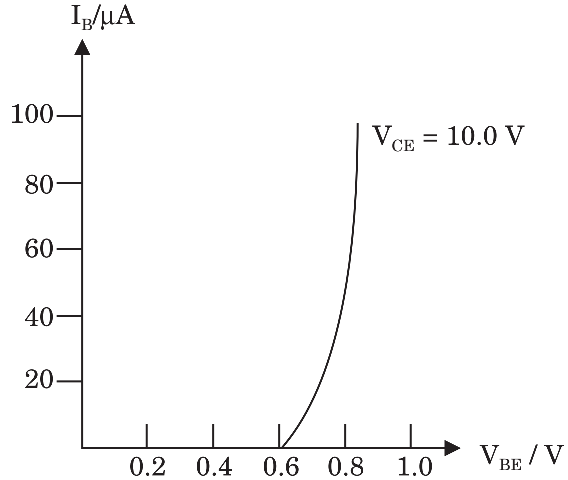
Let R_i be the input resistance of emitter-base circuit. When an input voltage V_i is applied to the emitter base circuit, let ΔI_B, ΔI_C be the change and collector current respectively.

Then, $V_i = \Delta I_B \times R_i$

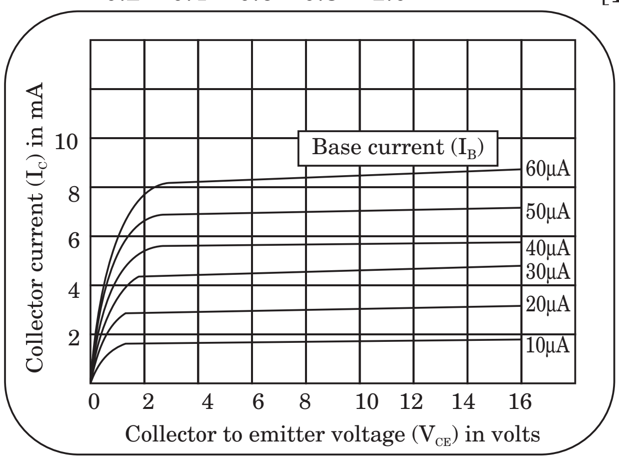
The a.c. current gain of transistor is, $\beta_{ac} = \dfrac{\Delta I_C}{\Delta I_B}$

18. (i)

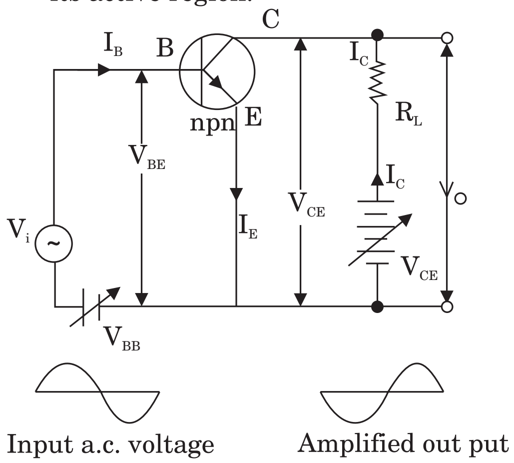




[1]



(c) n-p-n transistor as an amplifier:
The operating point is fixed in the middle of its active region. [1]



An a.c. signal v_1 is superimposed on bias $V_{BB}(dc)$. The o/p is taken between the collector and the ground.

Applying Kirchhoff's law to the output loop:

$I_E = I_B + I_C$

If V_{CE} is the collector voltage then,

$V_{CE} = V_{CC} - I_C R_L$ [1]

When the input signal voltage is fed to the emitter base circuit, it will change the emitter base voltage and hence the emitter current, which in turn will change the collector current. Due to this the collector voltage V_{CE} will vary in accordance with relation (*A*). This variation in collector voltage appears as amplified output.

Let R_i be the input resistance of emitter-base circuit. When an input voltage V_i is applied to the emitter base circuit, let ΔI_B, ΔI_C be the change and collector current respectively.

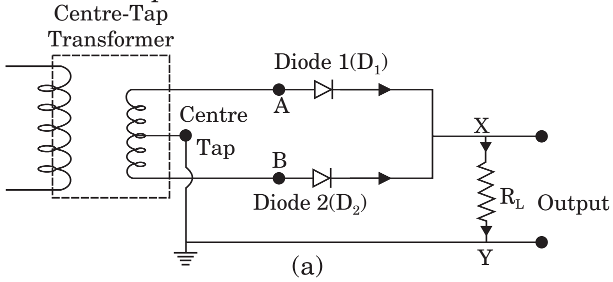
Then, $V_i = \Delta I_B \times R_i$

The a.c. current gain of transistor is, $\beta_{ac} = \frac{\Delta I_C}{\Delta I_B}$

19. In a p-n junction, a p-type and an n-type material are joined together. The concentration of holes is higher in p-type material as compared to that in n-type material. Therefore, there is a concentration gradient between the p-type and n-type materials. As a result of this concentration gradient, holes move from p-side to n-side ($p \to n$) by the process of diffusion. Similarly, electrons move from n-side to p-side ($n \to p$). [1]

As the holes diffuse from p-side, they leave ionized spaces (negatively charged) on p-side near the junction. These ionized spaces are immobile. Hence, a negative space-charge region is formed on the p-side near the junction. Similarly, a positive space-charge region is formed on the n-side. These two space-charge regions on either sides of the junction constitute what is called a depletion layer. Since the n-side loses electrons and p-side gains electrons, a potential difference is developed across the junction of the two regions. This potential difference tends to oppose further motions of electron from the n-region into the p-region. The same happens for holes too. The reverse polarity of this potential opposes further flow of carriers and is thus called the barrier potential. [2]

(a) The device is a full-wave rectifier. The circuit diagram of a full-wave rectifier is represented as:



(a)

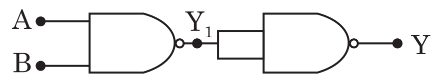
[1]

(b) The logic gate represented by the circuit is an AND gate. The truth table of the AND gate is represented as:

A	**B**	**A.B**
0	0	0
1	1	1
1	0	0
0	1	0

[1]

(c)



Y_1 = Error! Objects cannot be created from editing field codes.

$$Y = \overline{(\overline{A\cdot B})\cdot(\overline{A\cdot B})} = \overline{\overline{(A\cdot B)}}\cdot\overline{\overline{(A\cdot B)}} = (A\cdot B)+(A\cdot B)$$

$Y = A.\ B$

The equivalent gate is AND Gate.

Truth table

A	**B**	**Y_1**	**Y**
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

Smart Notes ....

CHAPTER 15

Communication System

Chapter Analysis with respect to Last 3 Years' Board Exams

List of Topics	2016		2017		2018
	Delhi	**All India**	**Delhi**	**All India**	**Delhi/All India**
Elements of a communication system				1Q (3 marks)	
Bandwidth of signals (speech, TV and digital data)			1Q (2 marks)		
Bandwidth of transmission medium					
Propagation of electromagnetic waves in the atmosphere, sky & space wave propagation, satellite communication		1Q (2 marks), 1Q (3 marks)			
Need for modulation, amplitude modulation and frequency modulation, advantages of frequency modulation over amplitude modulation	1Q (1 mark), 1Q (3 marks)	1Q (1 mark)	1Q (3 marks)	1Q (3 marks)	1Q (2 marks), 1Q (3 marks)

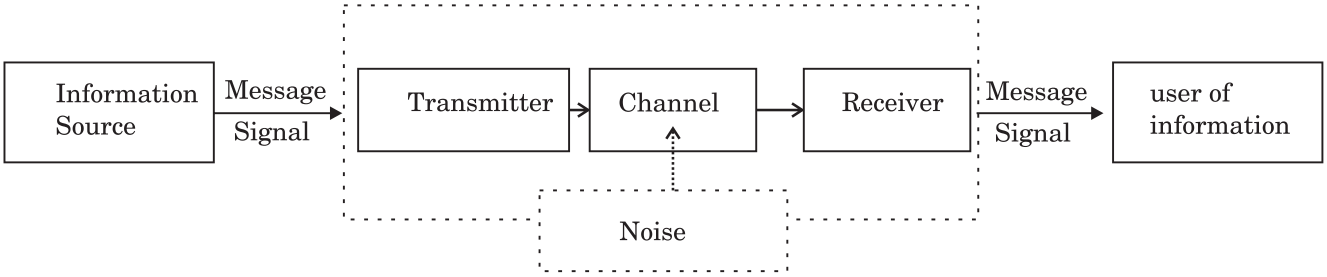
On the basis of above analysis, it can be said that from exam point of view Mode of Communication for Telephonic Communication, Attenuation and Demodulation, Component of Communication System, Mode of Communication in Satellite Communication System, Sight Communication and TV Signals are most important concepts of the chapter.

[Topic 1] Communication

Summary

- Communication is a two way process in which exchange of information takes place either in verbal or written form.
- ***Elements of communication system:*** There are three essential elements of communication transmitter, medium/channel and receiver.

Communication System



Transmitter transmits the signals through channel which is a physical medium and the receiver receives the signals.

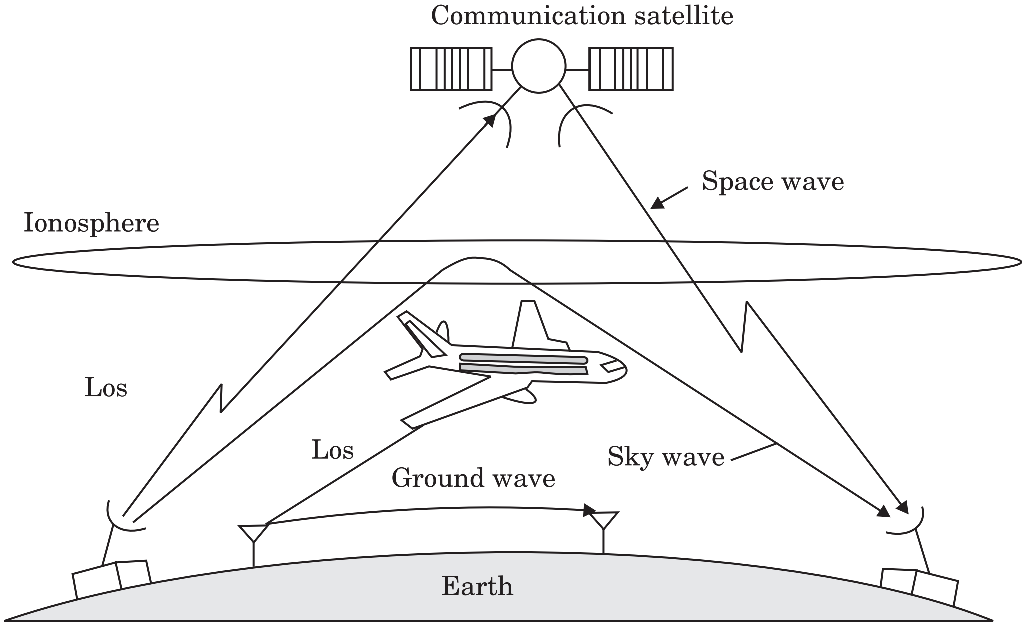
The two basic types of communication modes are point-to-point and broadcast.

- ***Transducer:*** A device which transforms the energy from one form into another. Example: Loudspeaker.
- ***Signal:*** An information transformed into electrical form for suitable transmission is termed as signal. Signals can be of two types: analog or digital.
- ***Noise:*** The unwanted signals which have a tendency to create the disturbancein the transmission and processing of message is called noise.
- ***Transmitter:*** The device that processes the incoming message signal in order to make it suitable for transmission through a channel and subsequent reception is known as transmitter.
- ***Receiver:*** In order to extract the appropriate message signals from the received signals at the channel output, receiver is used.
- ***Attenuation:*** When signals are propagated through a medium, some of their strength is lost which is known as attenuation.
- ***Amplification:*** The process of increasing the amplitude and the strength of a signal using an electronic circuit is called amplification.
- ***Range:*** The largest distance between a source and a destination is called range up to which the signal is received with sufficient strength.
- ***Bandwidth:*** The range of frequency over which an equipment operates or the portion of the spectrum occupied by the signal is called bandwidth.
- ***Repeater:*** A combination of transmitter and receiver is the repeater which amplifies the signals picked up from the transmitter and then retransmits those signals to the receiver. In order to extend the range of the communication system, the repeaters are used.
- ***Bandwidth of signals:*** The difference between the upper and lower frequencies of the signals is termed as bandwidth of signals. The different bandwidths of the different kinds of signals is shown in the following table:

Types of Signals	*Bandwidth*
Speech signal	*2800 Hz*
Music signal	*20 KHz*
Video signal	*4.2 MHz*
TV signal	*6 MHz*

- ***Bandwidth of transmission medium:*** Free space, wire, fibre optic cable and optical fibre are the common transmission media. The bandwidths are different for various transmission media.
- ***Propagation of Electromagnetic Waves:*** In radio waves communication, the EM waves are radiated at the transmitter by antenna.
- ***Ground wave propagation:*** The ground wave propagation is also termed as surface wave propagation.The radio waves are travelled along the earth surface in this type of propagation. It is necessary for the antenna to be of a size which is comparable to the wavelength of the signal so that the signals can be radiated with high efficiency. As the frequency increases, the attenuation also increases.
- ***Sky wave propagation:*** It is used for long distance communication in the frequency range from few MHz to 40MHz. It uses the phenomenon of bending of EM waves so that they are diverted towards the earth is similar to total internal reflection in optics.
- ***Space wave propagation:*** For long distance transmission, antennas are used to radiatesignals into space. In order to travel from transmitting antenna to the receiving antenna, space wave takes the straight line path.They are useful for line-of-sight (LOS) communication and satellite communication.
- The range d_T of an antenna of height h_T that radiates electromagnetic waves is given by $\sqrt{2Rh_T}$; R = radius of the earth.
- To find out the maximum distance of line of sight (d_M) between antennas with heights hT and h_R:

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

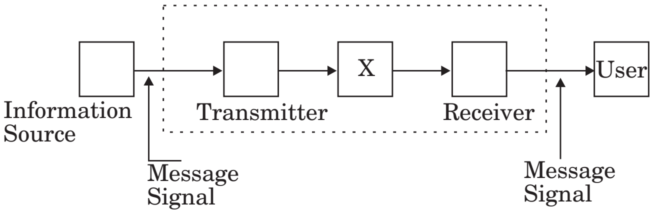


PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The figure given below-shows the block diagram of a generalized communication system. Identify the element labeled '*X*' and write its function.



[CBSE 2014]

2. How are side bands produced? **[CBSE 2015]**

3. What is the function of a band pass filter used in a modulator for obtaining *AM* signal? **[CBSE 2015]**

4. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz what are the frequencies of the side bands produced? **[CBSE 2016]**

5. What is sky wave propagation? **[CBSE 2017]**

6. Which mode of propagation is used by short wave broad cast services? **[CBSE 2018]**

2 Mark Questions

7. Distinguish between 'Analog and Digital signals'.

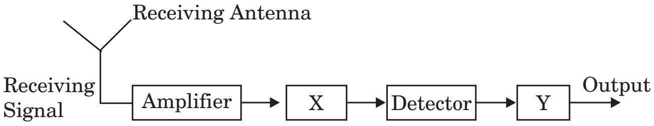
Or

Mention the function of any two of the following used

i. Transducer
ii. Repeater
iii. Transmitter
iv. Bandpass Filter **[CBSE 2011]**

8. Block diagram of a receiver is shown in the figure:

(a) Identify '*X*' and '*Y*'
(b) Write their functions.



[CBSE 2014]

9. Write the functions of the following in communication systems:

(i) Receiver
(ii) Demodulator **[CBSE 2014]**

10. Explain the terms (i) Attenuation and (ii) Demodulation used in Communication System. **[CBSE 2016]**

11. Why is base band signal not transmitted directly? Give any example **[CBSE 2016]**

12. Explain the function of a repeater in a communication system. **[CBSE 2018]**

13. What range of frequencies used in satellite communication? What is common between these waves and light waves? **[CBSE 2018]**

3 Mark Questions

14. Draw a schematic diagram showing the (i) ground wave (ii) sky wave and (iii) space wave propagation modes for em waves.

Write the frequency range for each of the following:

(i) Standard AM broadcast
(ii) Television
(iii) Satellite communication **[CBSE 2011]**

15. What is space wave propagation? State the factors which limit its range of propagation. Derive an expression for the maximum line of sight distance between two antennas for space wave propagation. **[CBSE 2011]**

16. Draw a schematic diagram showing the (i) ground wave (ii) sky wave and (iii) space wave propagation modes for em waves.

Write the frequency range for each of the following:

(i) Standard AM broadcast
(ii) Television
(iii) Satellite communication **[CBSE 2011]**

17. Distinguish between 'sky waves' and 'space waves' modes of propagation in communication system.

(a) Why is sky wave mode propagation restricted to frequencies up to 40 MHz?
(b) Give two examples where space wave mode of propagation is used. **[CBSE 2013]**

18. Name the type of waves which are used for line of sight (LOS) communication. What is the range of their frequencies? A transmitting antenna at the top of a tower has a height of 45 m and the receiving antenna is on the ground. Calculate the maximum distance between them for satisfactory communication in LOS mode. (Radius of the Earth = $6.4 \times 10^6 m$) **[CBSE 20136]**

19. Draw a block diagram of a detector for *AM* signal and show, using necessary processes and the wave forms, how the original message signal is detected from the input *AM* wave. **[CBSE 2015]**

20. Name the three different modes of propagation in a communication system. State briefly why do the electromagnetic waves with frequency range from a few MHz up to 30 MHz can reflect back to the earth. What happens when the frequency range exceeds this limit? **[CBSE 2015]**

21. Draw a block diagram of a generalized communication system. Write the functions of each of the following (a) Transmitter (b) Channel (c) Receiver. **[CBSE 2017]**

22. Why are high frequency carrier waves used for transmission? **[CBSE 2017]**

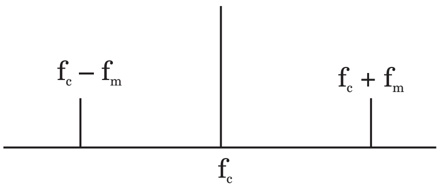
23. By what percentage will the transmission ranges of TV tower be affected when the height of the tower is increased by 21%? **[CBSE 2017]**

24. What is space wave propagation? Give two examples of communication system which use space wave mode. A TV tower is 80 m tall. Calculate the maximum distance up to which the signal transmitted from the tower can be received. **[CBSE 2018]**

Solutions

1. The element labeled '*X*' is called 'channel'. The function of the channel is to connect the transmitter and the receiver. A channel may either be wireless or in the form of wires connecting the transmitter and the receiver.[1]

2. Side bands are produced during the process of modulation. During modulation the audio frequency modulating signal wave is super imposed on a high frequency wave called carrier wave. Any form of modulation produces frequencies that are the sum and difference of the carrier and modulating frequencies. These frequencies are called as side bands. [½]



Lower side band frequency = $f_c - f_m$

Upper side band frequency = $f_c + f_m$

Where, $f_c \rightarrow$ Carrier wave frequency

$f_m \rightarrow$ Modulating signal frequency [½]

3. The output produced by square law device is passed to band pass filter which rejects the dc and the sinusoids of frequencies w_m, $2w_m$ and $2w_c$ and retains the frequencies w_c, $w_c - w_m$, and $w_c + w_m$. The output of band pass filter is an *AM* wave. [1]

4. Given frequency of carrier wave

$f_c = 2$ MHz $= 2 \times 10^3$ KHz

Frequency of modulating signal $f_m = 5$ kHz

Frequency of lower side band (LSB) = $f_c - f_m$

LSB = $(2 \times 10^3 - 5)$kHz [½]

LSB = (1995) kHz

Frequency of upper side band (USB) $f_c + f_m$

USB = $(2 \times 10^3 + 5)$kHz

USB = 2005 kHz [½]

5. The type of propagation in which radio waves are transmitted towards the sky and are reflected by the ionosphere towards the desired location on earth is called sky wave propagation. [1]

6. Sky wave propagation is used by short wave broadcast services having frequency range from a few MHz upto 30 MHz. Sky wave can travel very long distances and can even travel round the earth. [1]

7.

	Analog Signal	Digital Signal
1.	It is continuous signal, which varies continuously with variable may be time or distance etc.	It is a type of signal which has only two values high or low. In digital high mean 1 and low means 0 (zero)
2.	Example: Sound of human	Example: Temperature of day

[1 + 1]

Or

1. **Transducer:** It is an electric device which converts energy from one form to another form. e.g. microphone, which converts sound energy into electric energy and vice — versa. [½]
2. **Repeater:** It is an electronic device used in transmission system to regenerate the signal. It picks up a signal amplifies it and transmits it to receiver. [½]
3. **Transmitter:** Transmitter is an electronic device which is used to radiate electromagnetic waves. The purpose of the transmitter is to boost up the signal to be radiated to the required power level, so that it can travel long distances. The most

familiar transmitters are mobile transmitter antennas, radio and T.V. broadcasting antennas etc. [½]

4. **Band pass filter:** It is an electronic filter, which pass the certain band (range) of frequency and reject rest of all. [½]

8. From the given block diagram of demodulator of a typical receiver, we can conclude the following things:

(a) *X* represents Intermediate Frequency (*IF*) stage while *Y* represents an amplifier. [½]

(b) At *IF* stage, the carrier frequency is transformed to a lower frequency then in this process, the modulated signal is detected. The function of amplifier is to amplify the detected signal which may not be strong enough to be made use of and hence is essential. [½]

9. Receiver: Receiver separates the message signal from the carrier signal. It reconstructs actual signal using output transducers. [1]

Demodulator: The process of retrieval of information from the carrier wave at the receiver is called demodulation and Electronic Circuit used for it is called demodulator. [1]

10. (i) Attenuation: The loss of strength of a signal while propagating through a medium is known as attenuation. [1]

(ii) Demodulation: The process of retrieval of information from the carrier wave at the receiver is termed demodulation. This is the reverse process of modulation. [1]

11. The needs of modulation for transmission of a signal are given below

(i) The transmission of low frequency signal needs antenna of height 4-5 km which is impossible to construct. So, there is need to modulate the wave in order to reduce the height of antenna to a reasonable height. [1]

(ii) Effective power radiated by antenna for low wavelength or high frequency wave as

$$P \propto \frac{1}{\lambda}$$ [1]

So, for effective radiation by antenna, there is need to modulate the wave.

12. A repeater is used for extending the range of a communication system. It consists of a receiver and a transmitter. The receiver of a repeater collects the signal from the transmitter of another repeater and after amplifying, it retransmits the signal. Sometimes, it also changes the carrier frequency of the pick-up signal before transmitting it to the receiver. [1 + 1]

13. The waves used for satellite communication lie in the following two frequency ranges:

(i) 3.7 – 4.2 GHz for downlink

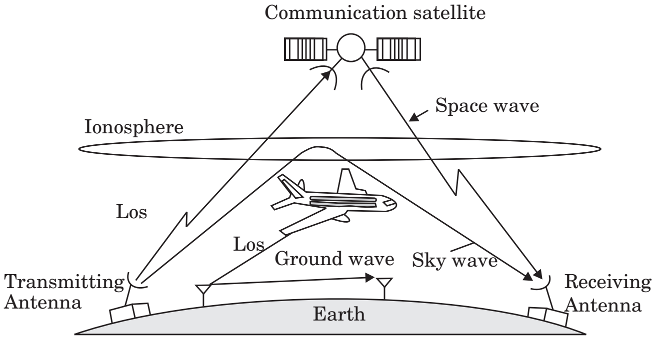
(ii) 5.9 – 6.4 GHz for uplink [1]

These waves and light waves both are electromagnetic waves. They both travel in a straight line. [1]

14. (i) Standard AM Broadcast:
540 – 1600 kHz

(ii) Television: 54 – 890 MHz

(iii) Satellite communication: 5.925 – 6.425 GHz uplink and 3.7 – 4.2 GHz downlink [2]

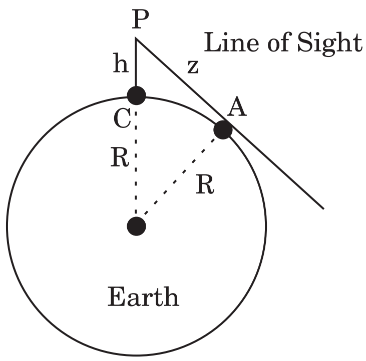


[1]

15. When a wave propagates in a straight line, from the transmitting antenna to the receiving antenna, its mode of propagation is called space wave communication. Frequency range: Above 40 MHz. Space waves are used for the Line of Sight (LOS) communication.

Space wave communication involves the transmission from transmitter, traveling along a straight line in space, reaches to receiving antenna. The range of their frequencies is 40 MHz and above.

The range of space wave propagation is limited by line of sight distance between transmissions to receiver /repeater antenna. [1]



$h < R$

h = Height of antenna

R = Radius of Earth

Range

As h is very small, $CA \approx PA$

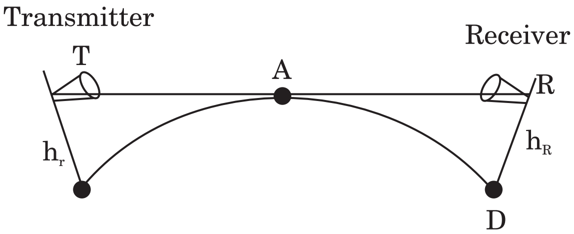
Using Pythagoras theorem

$PA^2 + R^2 = (R + h)^2$

$PA^2 + R^2 = R^2 + 2Rh + h^2$

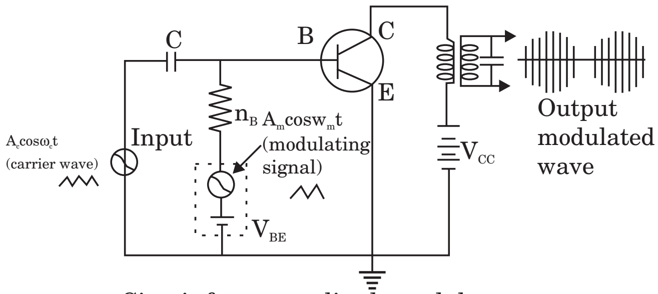
$PA = \sqrt{2Rh}$ (neglecting h^2)

Range $= \sqrt{2Rh}$ on ground surface



Total range on ground

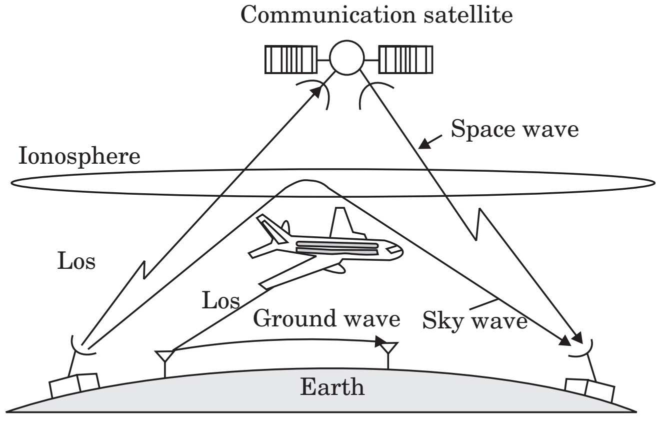
$= CA + AD = \sqrt{2Rh_r} + \sqrt{2Rh_R}$ [1]



Circuit for an amplitude modulator

[1]

16.



[1]

Frequency range:

(i) Standard AM broadcast: 540 – 1600kHz

(ii) Television: 54 – 72kHz [1]

76-88 MHz: VHF (very high frequencies)

174-216 MHz

420-890 MHz: UHF (ultra high frequencies)

(iii) Satellite communication: 5.925-6.425 GHz (uplink)

3.7 – 4.2 GHz (downlink) [1]

17. **Sky wave:** Sky waves are the *AM* radio waves, which are received after being reflected from the ionosphere. The propagation of radio wave signals from one point to another via reflection from ionosphere is known as sky wave propagation. The sky wave propagation is an important consequence of the total internal reflection of radio waves. As we go higher in the ionosphere, there is an increase in the free electron density. Consequently, there is a decrease of refractive index. Thus, as a radio wave travels up in the ionosphere, it finds itself travelling from denser to rarer medium. It continuously bends away from its path till it suffers total internal reflection to reach back the Earth. [1½]

Space waves: Space waves are the waves which are used for satellite communication and line of sight path. The waves have frequencies up to 40 MHz provides essential communication and limited the line of sight paths. [½]

(a) The e.m. waves of frequencies greater than 40 MHz penetrate the ionosphere and escape so, the sky wave propagation is restricted to the frequencies up to 40 MHz.

(b) In television broadcast and satellite communication, the space wave mode of propagation is used. [1]

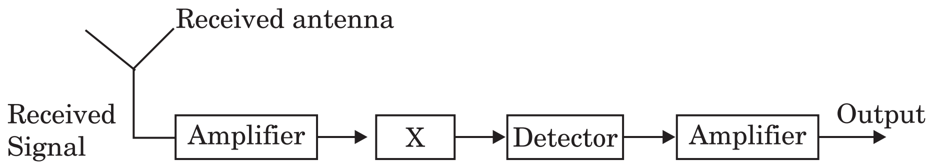
18. High frequency waves (above 40 MHz) called space waves can be transmitted from transmitting to receiving antenna and the mode for travelling of these waves through space is known as space wave propagation. [2]

$d = \sqrt{2hR_e}$

$d = \sqrt{2 \times 45 \times 6.4 \times 10^6}$

$d = 24000m = 24km$ [1]

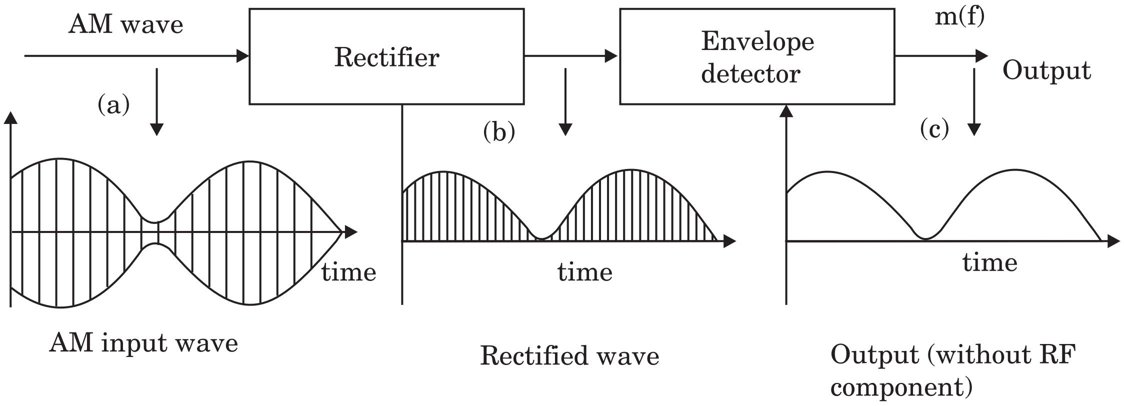
19.



[Block diagram for detection of AM signal] [1]

When a message is received, it gets attenuated through the channel therefore,receiving antenna is to be followed by an amplifier and a detector. The camera frequency is usually changed to a lower frequency in an Intermediate Frequency (*IF*) stage. The detected signal may not be strong enough to be made use of and hence is required to be amplified.

In order to obtain the original message signal $m(t)$ of angular frequency a simple method is used which is shown below in the form of a block diagram. [½]



[1]

When the received modulated signal is passed through a rectifier, an envelope signal is produced. This envelope signal is the message signal. In order to retrieve the message, the signal is passed through an envelope detector. [½]

20. The three different modes of propagation in a communication system are

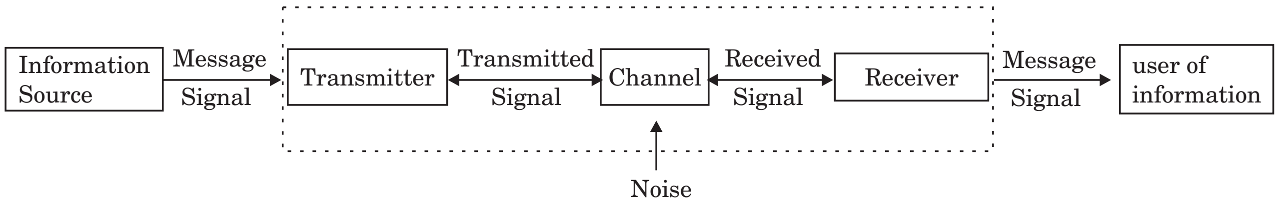
(1) Ground wave

(2) Sky wave

(3) Space wave

In the frequency range from a few MHz up to 30 to 40 MHz, long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. This mode of propagation is called sky wave propagation and is used by short wave broadcast services. The ionosphere is so called because of the presence of a large number of ions or charged particles. It extends from a height of ~ 65 Km to about 400 Km above the earth's surface. Ionization occurs due to the absorption of the ultraviolet and other high-energy radiation coming from the sun by air molecules. The ionosphere is further subdivided into several layers. The degree of ionization varies with the height. The density of atmosphere decreases with height. At great heights the solar radiation is intense but there are few molecules to be ionized. Close to the earth, even though the molecular concentration is very high, the radiation intensity is low so that the ionization is again low. However, at some intermediate heights, there occurs a peak of ionization density. The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape. The phenomenon of bending of em waves so that they are diverted towards the earth is similar to total internal reflection in optics.

21. Block diagram of communication system:



[1]

Transmitter: A transmitter is an arrangement that converts the message signal to a form suitable for transmission and then transmits it through some suitable communication channel.

[1]

Channel: Channel is the medium through which the signal is transmitted for transmitter to receiver. Receiver: A receiver extracts the desired message signals from the received signals at the channel output.

[1]

22. For transmitting a signal, the antenna should have a size comparable to the wavelength of the signal (at least—A in dimension), where A is the wavelength. If the frequency of the signal is small, then its wavelength becomes very large and it is impractical to make that large antennas for the corresponding large wavelengths. For higher frequencies, Wavelength is smaller, which is the reason why high frequency carrier waves are used for transmission.

[1+ 1]

23. $d_t = \sqrt{2Rh_T}$

Let the transmission of the tower before transmission be $= d_{T_1}$

Range after increase in height $= d_{T_2}$

$$\text{Ratio} = \frac{d_{T_1}}{d_{T_2}} = \sqrt{\frac{2Rh_{t_2}}{2Rh_{t_1}}} = \sqrt{\frac{h_{t_2}}{h_{t_1}}}$$

Height increase = 21% [1]

Thus, $\frac{121}{100} h_{t_1} = h_{t_2}$ [1]

$$\frac{d_{T_2}}{d_{T_1}} = \sqrt{\frac{121h_{t_2}}{100h_{t_1}}} = 1.1$$

$d_{T_2} : d_{T_1} = 11:10$

[1]

24. Space wave propagation is the propagation of waves whose frequencies lie above 40 MHz. Examples of communication systems which use space wave mode are:

(i) Television broadcast

(ii) Microwave links

[1]

The maximum distance up to which signals can be received,

$d = \sqrt{2R_E h_T}$

$= \sqrt{2 \times 6400000 \times 80}$

= 32000 m

= 32 km

[2]

[Topic 2] Modulation

Summary

- ***Modulation*** is the process by which a low frequency is superimposed on a high frequency carrier signal so that the low frequency can be transmitted to long distance.
- ***Demodulation:*** The reverse process of modulation is called as demodulation in which the information from the carrier wave is retrieved at the receiver.
- ***Need of Modulation:***

➢ As there is a need of a very large antenna for low frequency signals, signals from different stations mixes up and the attenuation is large, so the modulation is needed.

➢ The size of antenna is given by $\frac{\lambda}{4}$ and low frequency implies larger wavelength so the size of antenna is not achievable.

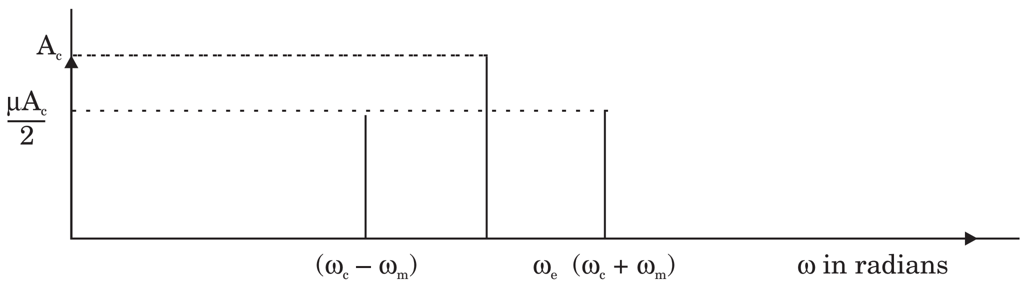
➢ There are 4 types of modulation: Amplitude modulation, Frequency modulation, Pulse modulation and Phase modulation

- ***Amplitude Modulation:*** The alteration of the amplitude of the carrier in accordance with the information signal is amplitude modulation. The following expression represent the AM of a carrier wave having amplitude and frequency f_c:

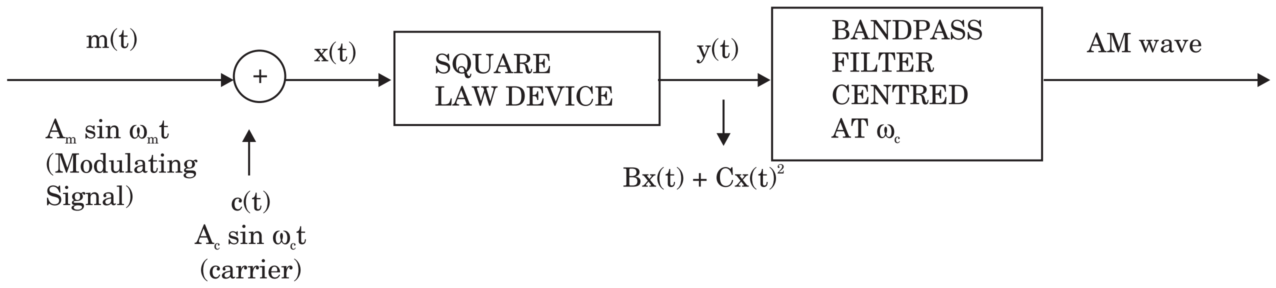
$$C_m(t) = A_c A \sin \omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$$

The amplitude of the modulating wave is A_m and the frequency is f_m.

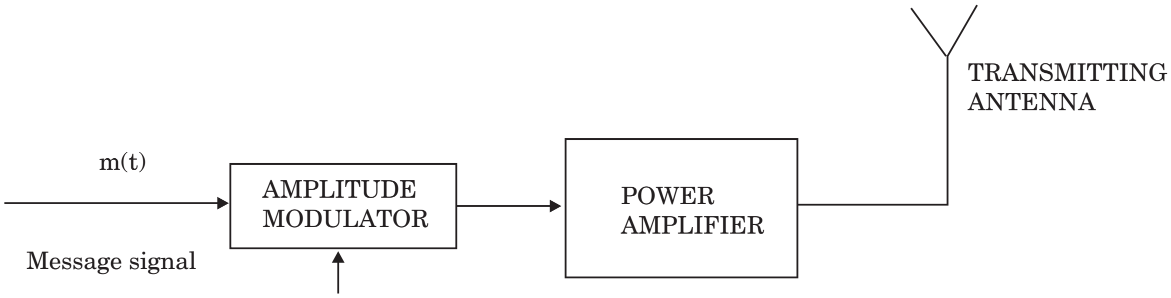
Modulation index $\mu = \frac{A_m}{A_c}$; $\mu \le 1$.



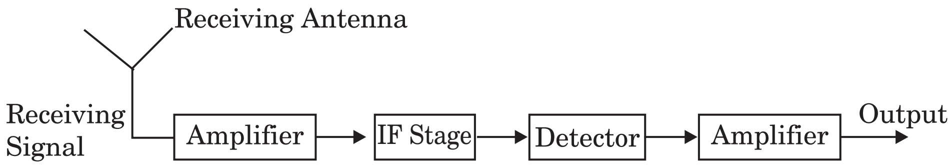
- ***Production of AM wave:*** The following block diagram shows the production of AM wave:



The block of transmitter is as follows:



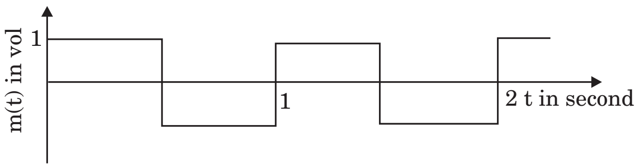
- ***Detection of AM wave:*** Detected signals need modification as they may not be strong enough to use. The block diagram of receiver is given below:



PREVIOUS YEARS'
EXAMINATION QUESTIONS
TOPIC 2

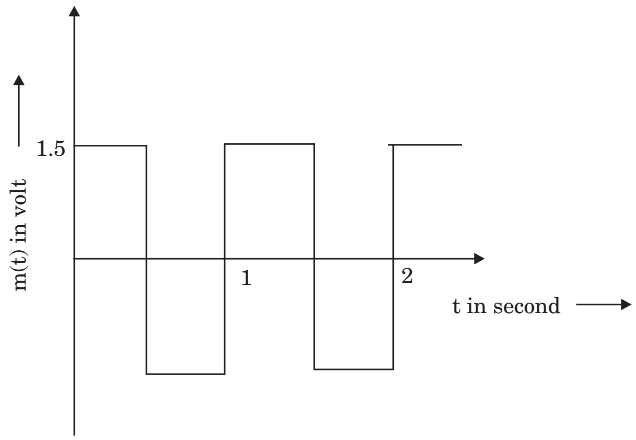
1 Mark Questions

1. The carrier wave is represented by $C(t) = 5 \sin (10\pi t)V$. A modulating signal is a square wave as shown.



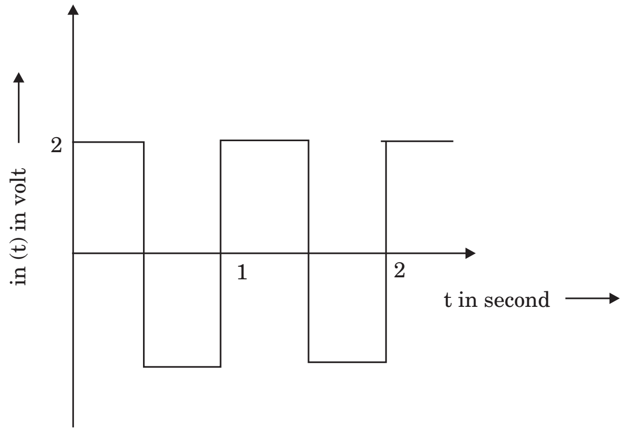
Determine modulation index. **[CBSE 2014]**

2. The carrier, wave of a signal is given by $C(t) = 3 \sin (8\pi t)$ volt. The modulating signal is a square wave as shown. Find its modulation index.



[CBSE 2014]

3. The carrier wave is given by $C(t) = 2 \sin (\omega t)$ volt. The modulating signal is a square wave as shown. Find modulation index.



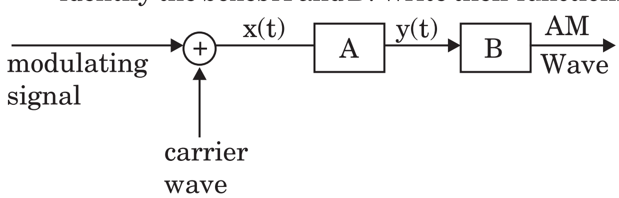
[CBSE 2014]

2 Marks Questions

4. (i) Define modulation index.

(ii) Why is the amplitude of modulating signal kept less than the amplitude of carrier wave? **[CBSE 2011]**

5. In the block diagram of a simple modulator for obtaining an *AM* signal, shown in the figure, identify the boxes *A* and *B*. Write their functions.



[CBSE 2012]

6. Differentiate between amplitude modulated (*AM*) and frequency modulated (*FM*) waves by drawing suitable diagrams. Why is *FM* signal preferred over *AM* signal?

[CBSE 2015]

7. A carrier wave of peak voltage 15V is used to transmit a message signal. Find the peak voltage of the modulating signal in order to have a modulation index of 60%.

[CBSE 2018]

3 Marks Questions

8. Write any two factors which justify the need for modulating a signal. Draw a diagram showing an amplitude modulated wave by superposing a modulating signal over a sinusoidal carrier wave.

[CBSE 2012]

9. Write two basic modes of communication. Explain the process of amplitude modulation. Draw a schematic sketch showing how amplitude modulated signal is obtained by superposing a modulating signal over a sinusoidal carrier wave.

[CBSE 2014]

10. (a) Explain any two factors which justify the need of modulating a low frequency signal.

(b) Write two advantages of frequency modulation over amplitude modulation.

[CBSE 2017]

11. What is meant by term 'modulation'? Draw a block diagram of a simple modulator for obtaining an AM signal.

[CBSE 2017]

12. (a) How is amplitude modulation achieved?

(b) The frequencies of two side bands in an *AM* wave are 640 kHz and 660 kHz respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation?

[CBSE 2017]

13. (a) Give three reasons why modulation of a message signal is necessary for long distance transmission.

(b) Show graphically an audio signal, a carrier wave and an amplitude modulated wave.

[CBSE 2018]

Solutions

1. Modulation index 'μ' is the ratio of the amplitude of the modulating signal to the amplitude of the carrier wave

The generalized equation of a carrier wave is given below:

$C(t) = A_C \sin \omega_C t$

The generalized equation of a modulating wave is given below:

$CM(t) = A_C \sin \omega_C t + \mu A_C \sin \omega_m t \sin \omega_C t$ [½]

Here, μ is defined as $\frac{A_m}{A_c}$.

On comparing this with the equations of carrier wave and modulating wave, we get:

Amplitude of modulating signal, $A_m = 2V$

Amplitude of carrier wave, $A_C = 5V$

Hence, $\mu \frac{A_m}{A_c} = \frac{2}{5}$ [½]

2. The generalized equation of a carrier wave is given by:

$c(t) = A_C \sin \omega_C t$

The generalized equation of a modulating wave is given by:

$Cm(t) = A_C \sin \omega_C t + \mu A_C \sin \omega_C t \sin \omega_C t$

Here, μ is given as $\frac{A_m}{A_c}$ [½]

On comparing this with the equations of modulation wave and carrier:

Amplitude of a modulating signal, $A_m = 1.5\ V$

Amplitude of a carrier wave, $A_c = 3V$

$\mu = \frac{A_m}{A_c} = \frac{1.5}{3} = \frac{1}{2}$

$\mu = 0.5$ [½]

3. Modulation index (μ) is the ratio of the amplitude of the modulating signal to the amplitude of the carrier wave. The generated equation of a carrier wave is given below:

$C(t) = 2\sin \omega_c t$

The generalized equation of a modulating wave is given below:

$CM(t) = 2\sin \omega_C t + \mu A_C \sin \omega_m t \sin \omega_C t$

Here, (μ) is defined as $\frac{A_m}{A_c}$. [½]

On comparing this with the equations of carrier wave and modulating wave, we get, amplitude of modulating signal, $A_m = 1V$

Amplitude of carrier wave, $A_c = 2V$

$$\mu = \frac{A_m}{A_c} = \frac{1}{2}$$

$\mu = 0.5$ [½]

4. (i)Modulation index is the ratio of amplitude of modulating signal and amplitude of carrier wave.

$$\mu = \frac{A_m}{A_c}$$ [1]

(ii) $\mu = \frac{A_m}{A_c} < 1$ Modulation index is $A_m < A_c$ kept less than 1, that is in order to avoid distortion. [1]

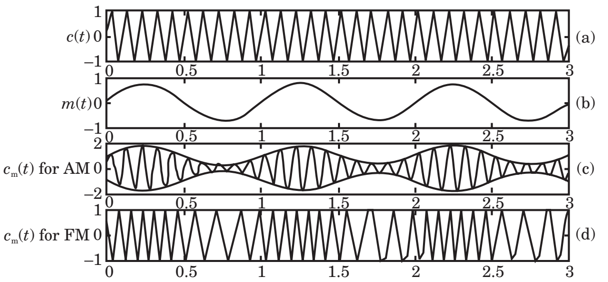
5. In the given block diagram, block (A) is modulator and Block (B) is power amplifier [½]

(A) = modulator changes the amplitude of carrier wave according to modulating signals

(B) = Power amplifier enhances the voltage and enhances power of modulated signals [½]

6. When the amplitude of carrier wave is changed in accordance with the intensity of the signal, it is called amplitude modulation. [½]

When the frequency of carrier wave is changed in accordance with the intensity of the signal, it is called frequency modulation.



[½]

FM signal is preferred over AM signal because

(i) Various electrical machines and noises cause amplitude disturbance in the transmission of amplitude modulated wave. This makes the reception noisy. So, there is a need for Frequency Modulation which can reduce the noise factor. [½]

(ii) Fidelity or audio quality of amplitude modulated transmission is poor. This type of transmission is also not good for musical programs. There is a need to eliminate amplitude-sensitive noise. This is possible if we eliminate amplitude variation. In other words, there is a need to keep the amplitude of the carrier constant. This is precisely what we do in frequency modulation. [½]

7. Modulation index $\frac{A_m}{A_c}$

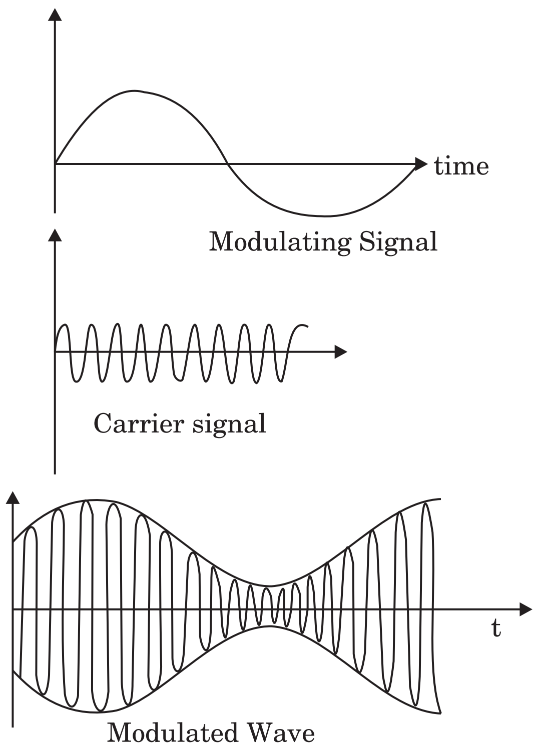
$$60\% = \frac{15}{A_c}$$

$$A_c = \frac{15}{60} \times 100$$ [1]

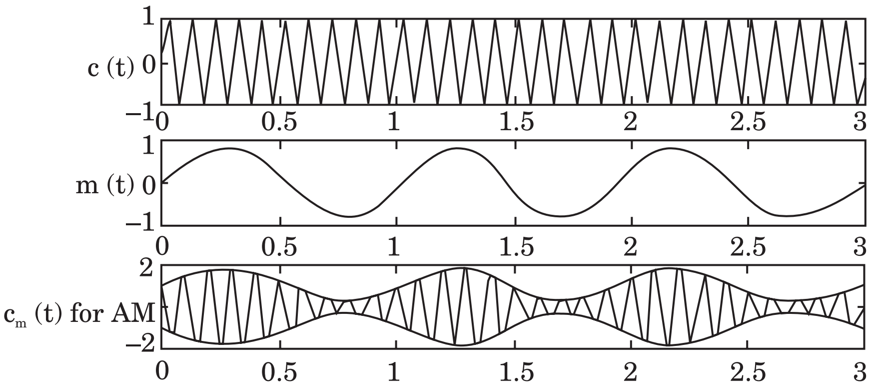
$A_c = 25V$ [1]

8. Factors needed for modulating a signal:

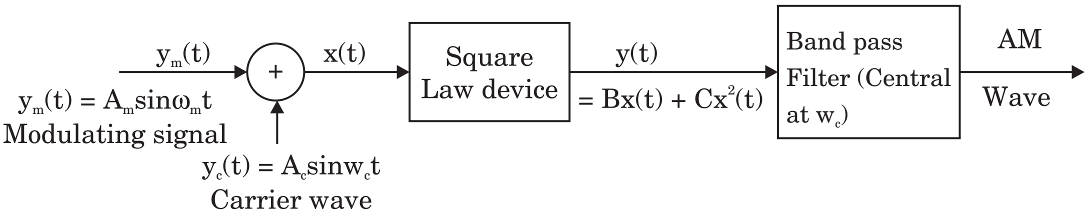
1. To send the signal over large distance for communication.
2. Practical size of antenna. [1]



9. There are two basic modes of communication: point-to-point and broadcast. In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver. Telephony is an example of such a mode of communication. In broadcast mode, there are a large number of receivers for a single transmitter. Radio and television are examples of broadcast mode of communication. If 8the amplitude of the carrier wave is varied in accordance with the amplitude of the signal, it is called amplitude modulation. Frequency and phase are kept constant. [2]



A conceptually simple method of production of amplitude modulated wave is shown in the block diagram below.



[1]

10. (a) (i) Size of Antenna: The size of antenna required will be of order of $\frac{\lambda}{4}$. When frequency is small, the height of antenna will be large, so audio frequency signal should be modulated over a high frequency carrier wave. [1]

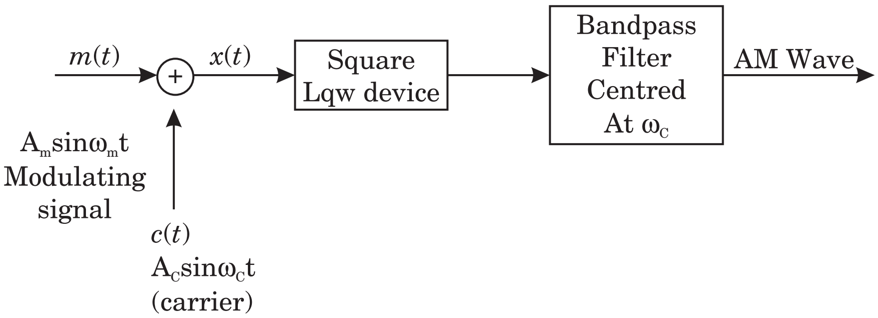
(ii) Effective power radiated by an Antenna: As power radiated $\propto \frac{1}{\lambda^2}$, hence when frequency is increased then the power radiated will be more. [1]

(b) Advantage of frequency modulation over amplitude modulation.

(i) Noise can be reduced.

(ii) Transmission efficiency is more because the amplitude of an Fm wave is constant. [1]

11. The process of superimposing information contained in a low frequency signal on a high frequency signal is called modulation. [1]

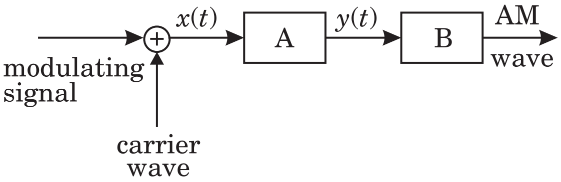


[1]

12. (a) Amplitude modulation

In AM, the modulating wave is superimposed on a carrier wave in such a manner that the frequency of the modulated wave is the same as that of the carrier wave but its amplitude varies in accordance with the instantaneous amplitude of the modulating wave.

Fig. Block diagram for a simple modulator for obtaining an AM signal.



[1]

add this block diagram

Let the modulating signal be represent by

$m(t) = A_m \sin\omega_m t$

and carrier wave

$c(t) = A_m \sin\omega_m t$

when they added the resultant wave

$x(t) = A_m \sin\omega_m t + A_c \, \text{sn}\omega_c t$

This can be further written as

$x(t) = A_C \sin\omega_c t \, \mu \, A_m \sin\omega_m t \times \sin\omega_c t$

$\mu = \dfrac{A_m}{A_c}$ is the modulation index [1]

In practice, $\mu \le 1$ to avoid distortion and it is represented in percent.

Using trigonometric relation

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

So,

$$s\,x(t) = A_C \sin\omega_c t + \frac{\mu A_C}{2}\cos(\omega_C - \omega_m)t$$

$$-\frac{\mu A_C}{2}\cos(\omega_C + \omega_m)t$$

(b) Given:

$\omega_c + \omega_m = 660$ kHz-------- (1)

And $\omega_c - \omega_m = 660$ kHz-------- (2)

Adding equations (1) and (2)

$2\omega_c = 660 + 640$k Hz

$2\omega_c = 1300$ kHz

$\omega_c = \dfrac{1300}{2} = 650\text{kHz}$

Then $\omega_c = 650 - 600$ kHz

$2\omega_c = 10$ kHz

$\omega_c = 2\pi f_c = 650$ kHz ($\omega_c = 2\pi f$)

$\Rightarrow f_c = \dfrac{650}{2\pi}$ kHz

$\Rightarrow \omega_m = 2\pi f_m = 10$ kHz

$\Rightarrow f_m = \dfrac{10}{2\pi}$ kHz

Band width required for amplitude modulation = upper side band – lower side band

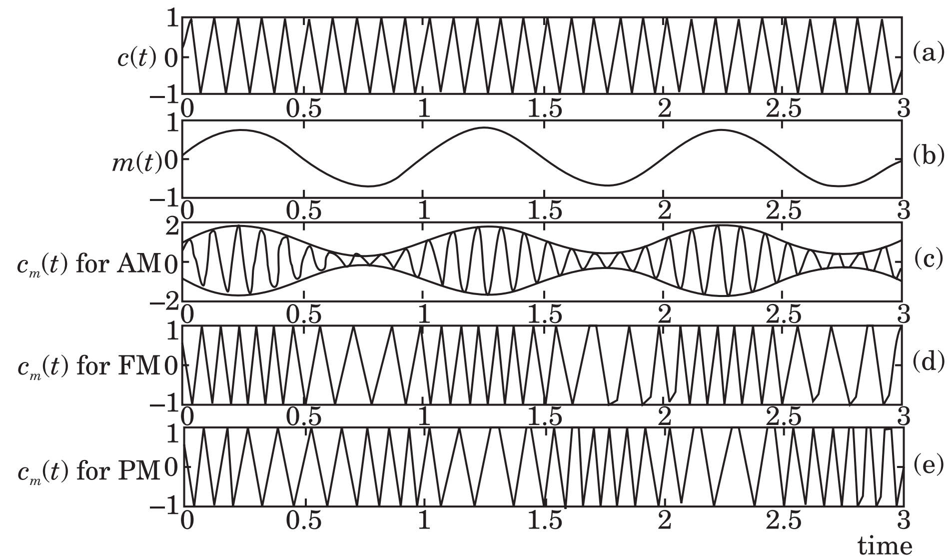
$(f_c + f_m) - (f_c - f_m) = 2\,f_m$ [1]

13. (a) (i) Height of antenna

(ii) Utilization of frequency

(iii) Power of signal [1]

(b)



[1]

Smart Notes ....

CBSE
Sample Question Paper 1

Physics
Class XII

Time : 3 hrs **Maximum Marks : 70**

General Instructions

General guidelines given in the paper.

Please check that this question paper contains 5 printed pages

Code number given on the right hand side of the question paper should be written on title page of the answer-book by the candidate.

Please check that this question paper contains 26 questions.

Please write down the Serial Number of the question paper before attempting it.

15 minutes time has been allotted to read this paper. The question paper will be distributed at 10:15 a.m. to 10:30 a.m., the student will read the question paper only and will not write any answer on the answer-book during this period.

All questions are compulsory. There are 26 questions in all.

This question paper has five sections - Section A, Section B, Section C, Section D and Section E

Section A contains five questions of one mark each,

Section B contains five questions of two marks each,

Section C contains twelve questions of three marks each,

Section D contains one value based question of four marks and

Section E contains three questions of five marks each.

You may use the following values of physical constants wherever necessary

$$c = 3 \times 10^{8} \text{ m/s}$$

$$h = 6.34 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\{1/(4\pi\varepsilon_0)\} = 9 \times 10^{9} \text{ N m}^2 \text{ C}^{-2}$$

Mass of electron = 9.1×10^{-31} kg

Mass of neutron = 1.675×10^{-27} kg

Mass of proton = 1.673×10^{-27} kg

Avogadro's number = 6.023×10^{23} per gram mole

Boltzmann constant = 1.38×10^{-23} JK^{-1}.

SECTION A

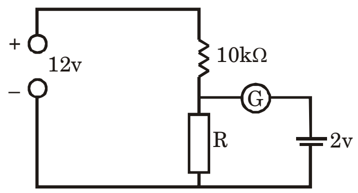
(1 × 5 = 5)

1. The speed of a charged particle moving in a magnetic field does not change. Why?
2. Which electromagnetic radiation plays an important role in maintaining the earth's warmth or average temperature through the green house effect?
3. For a fixed frequency of incident radiation how does photoelectric current vary with intensity of incident light?
4. Why is it experimentally difficult to detect neutrinos in nuclear decay?
5. What is meant by the term Attenuation in communication systems?

SECTION B

(2 × 5 = 10)

6. In the given circuit the 12V source is resistance free; If the galvanometer in the given circuit reads zero, then find the value of the resistor.



7. When a high power heater is connected to electric mains, the bulbs lightening in the house become dim, Why?

OR

A 10 m long potentiometer wire carries a steady current. A 1.018V standard cell is balanced at a length of 850 cm. Then, what is the maximum emf that can be measured?

8. The optical effect in electromagnetic wave is produced by which vector?
9. From photoelectric effect the equation for maximum energy is given by $E_{max} = h\nu - \Phi$. This equation is stated for one photon. If an electron absorbs two photons each of frequency ν, what will be the maximum energy of emitted electron?
10. By what percentage will the transmission range of a TV tower be affected, when the height of tower is increased by 21%?

SECTION C

(3 × 12 = 36)

11. Two identical spheres having unequal, opposite charges are placed at a distance of 0.90m apart. After touching them, they are again placed at the same distance apart. Now they repel each other with a force of 0.025N. Answer the following questions:

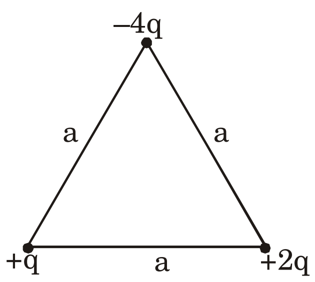
 (a) After touching what will be the ratio of charges on both the spheres?

 (b) Final charge on each sphere.

OR

Three charges are arranged as shown. What is the electric potential energy of the system?

(Given $q = 1 \times 10^{-7}$C and $a = 0.10$ m)



12. (a) Define the term electrical resistance. What is its SI unit?

 (b) A cylindrical wire is stretched to increase its length by 10%. Find the percentage increase in resistance.

13. When a coil of magnetic moment 2.5×10^{-8}Am^2 is placed in a magnetic field such that its plane is parallel to the field, then the moment of the couple acting on the coil is 7.5×10^{-9}Nm. If the area of the coil be 1.5 cm^2 and the number of turns in it be 12, find the following :

 (a) Magnitude of magnetic field.

 (b) Current in it.

14. Let a magnetic dipole of moment M be rotated in a uniform magnetic field of magnitude B through an angle θ from the field direction

(a) Derive an expression for the work done in rotating the dipole through an angle θ from the field direction.

(b) What will be the work done if the dipole is rotated through 90^0 from the direction of field?

15. (a) State law of Malus.

(b) Draw a graph showing the variation of intensity (I) of polarized light transmitted by an analyser with angle θ between the polarizer and the analyser.

(c) What is the value of refractive index of a medium of polarizing angle 60^0?

16. (a) What are the basic conditions for interference to occur?

(b) How does the fringe width of interference fringe change when whole apparatus is dipped in a liquid of refractive index 1.3?

17. A ray of light incident at an angle of θ on the refracting face of the prism emerges from the other face normally. If the angle of the prism is 5^0 and the prism is made of a material of refractive index 1.5, what is the angle of incidence?

18. In a hydrogen atom, a transition takes place from n = 3 to n = 2 orbit. ($R = 1.097 \times 10^7$ meter^{-1})

(a) Find the wavelength of the emitted photon.

(b) Will the photon be visible?

(c) To which spectral series will this photon belong?

19. (a) What is meant by the term half life in radioactivity? Derive a relation between half life and decay constant.

(b) The half life of radium is 1600 years. How long will a sample of radium take for 75% of its initial mass to disintegrate?

20. (a) How is a transistor used as an amplifier?

(b) The input and output resistances in a common base amplifier circuit are 400 h and 400 kh respectively. If the current gain α is 9.8, then find the voltage gain.

21. Give the circuit symbol and truth table of NOR gate.

22. What is the function of emitter, base and collector in the transistor?

Section D

(4 × 1 = 4)

23. Sanjeev belongs to a rural area of UP. One day a storm came and the high power lines came too close to each other; also their height got decreased. This immediately caught the attention of sanjeev, who, keeping the safety aspect in mind, immediately informed the electricity department. It being a Sunday, officials were not available, hence he and his friends made a large warning sign and placed it near the wires.

(1) What are the values possessed by Sanjeev?

(2) (a) Who would have been the biggest victim of the lines coming close to each other?

(b) What solution will the authorities give for this problem?

Section E

(5 × 3 = 15)

24. (a) What is meant by the term restoring couple?

(b) In what direction should a dipole be placed to electric field so that the torque acting on it is maximum?

25. (a) Derive expression for electric field just outside a charged conductor.

(b) Does the field depend upon the shape of the conductor?

OR

(a) What is a transformer; Write a short note on step down and step up transformer.

(b) Derive an expression for efficiency of a transformer.

26 . (a) What is the advantage of reflecting telescope over refracting telescope?

(b) Draw a ray diagram to illustrate the refraction of light at convex spherical surface.

OR

(a) When light passes from one medium to another, how does its frequency vary?

(b) What is the effect on wavelength of light in going from one medium to another?

CBSE
Sample Question Paper 2

Physics
Class XII

Time : 3 hrs **Maximum Marks : 70**

General Instructions

General guidelines given in the paper.

Please check that this question paper contains 5 printed pages

Code number given on the right hand side of the question paper should be written on title page of the answer-book by the candidate.

Please check that this question paper contains 26 questions.

Please write down the Serial Number of the question paper before attempting it.

15 minutes time has been allotted to read this paper. The question paper will be distributed at 10:15 a.m. to 10:30 a.m., the student will read the question paper only and will not write any answer on the answer-book during this period.

All questions are compulsory. There are 26 questions in all.

This question paper has five sections - Section A, Section B, Section C, Section D and Section E

Section A contains five questions of one mark each,

Section B contains five questions of two marks each,

Section C contains twelve questions of three marks each,

Section D contains one value based question of four marks and

Section E contains three questions of five marks each.

You may use the following values of physical constants wherever necessary

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.34 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\{1/(4\pi\varepsilon_0)\} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

Mass of electron = 9.1×10^{-31} kg

Mass of neutron = 1.675×10^{-27} kg

Mass of proton = 1.673×10^{-27} kg

Avogadro's number = 6.023×10^{23} per gram mole

Boltzmann constant = 1.38×10^{-23} JK^{-1}.

SECTION A

(1 × 5 = 5)

1. A charged particle enters a uniform magnetic field obliquely. What will be the trajectory of the particle?

2. Which radiations are used in medicine to destroy cancer cells?

3. For a fixed frequency and intensity of incident light, how does photoelectric current vary with increase in potential applied to collector?

4. What is the atomic number of nucleus produced when U_{92}^{238} produces α decay?

5. In order to extend the range of a communication system what is used?

SECTION B

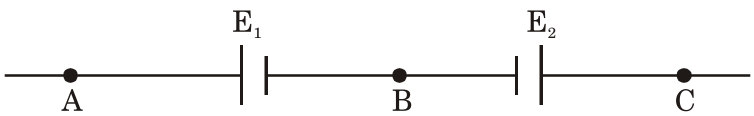
(2 × 5 = 10)

6. Six lead accumulators each of emf 2V and internal resistance 0.015Ω are joined in series to an external resistance of 8.5Ω. Find the current drawn from the supply.

7. In order to drive a current of 3A for 5 minutes in an electric circuit, 1350J of work is to be done. The emf of source in the circuit is how much?

OR

Two cells of emf E_1 and E_2 ($E_1 > E_2$) are connected as shown. When a potentiometer is connected between A and B, the balancing length of the potentiometer wire is 300 cm. On connecting the same potentiometer wire between A and C , the balancing length is 100 cm. Compute E_1/E_2.



8. What is the basic source of electromagnetic wave?

9. When monochromatic radiation of wavelength 200 Å falls upon a nickel plate, the latter acquires a positive charge. When the wavelength is increased, at 3400 Å the effect is found to cease, however intense the incident radiation may be. Explain it.

10. A TV tower has a height of 300 m, what is the maximum distance upto which this TV transmission can be received. (R = 6400 km) ?

SECTION C

(3 × 12 = 36)

11. A small ball of mass m is suspended by an inextensible , insulated , light thread of length l from a hook. Each of the ball and the hook is given a charge q.

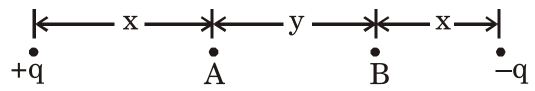
Answer the following:

(a) Draw a diagram showing the various forces acting on the ball.

(b) Find an expression for time period of the ball for small oscillations.

OR

Calculate the value of $V_A - V_B$ in the given arrangement.



12. (a) What do you mean by the term specific resistance. Give its SI unit.

(b) Let a wire of area of cross section A, length l and resistance R be taken. Find an expression for specific resistance.

13. A bar magnet is suspended by a thin wire in a uniform magnetic field. On twisting the upper end of the wire by 150°, the magnet is displaced from its initial position by 30˚. How much should the upper end of the wire be twisted so that the magnet is displaced by 90 cm from its initial position?

14. What do you mean by :

(a) What do you mean by Magnetic susceptibility and relative permeability?

(b) Derive a relation between relative permeability and magnetic susceptibility.

15. (a) State Brewster's law.

(b) Show that reflected and refracted rays are perpendicular rays.

16. Two waves of amplitude a_1 and a_2 interfere at a point where phase difference is Φ.

(a) Write the expression of resultant amplitude.

(b) What will be the values for constructive interference and destructive interference?

17. The optical density of turpentine is higher than that of water, while its mass density is lower. A layer of turpentine is made to flow over water in a container. Trace the path of incident ray.

18. For Lyman series of hydrogen spectrum, find the following :

(a) Wavelength of the first line.

(b) Wavelength of the limit of this series.

19. (a) State Rutherford and Soddy law for radioactive decay.

(b) Find the half life period of a radioactive material if its activity drops to $1/16^{th}$ of its initial value.

OR

(a) If the number of atoms in a radioactive substance be N_0 and N at time t = 0 and after time t, and λ be the decay constant, then derive the relation

$$N = N_0 e^{-\lambda t}$$

20. Draw the circuit diagram of common base amplifier; What is the primary use of this amplifier?

21. Give the circuit symbol and truth table of NOT gate.

22. (a) What is linearly polarized light? Describe briefly using a diagram how sunlight is polarized.

(b) Unpolarized light is incident on a Polaroid. How the intensity of would transmitted light change when the Polaroid is rotated?

SECTION D

(1 × 4 = 15)

23. Last night a thunderstorm occurred in a small town of Uttar Pradesh. Because of it many trees fell on the road and at many places water got filled. Rohan's neighbor has a son aged three years; He was busy playing with his ball and all of a sudden his ball went into water in which two broken electric poles were lying. The child rushed to pick his ball from there. Rohan saw the scene and ran to catch hold of the child before he reaches near the water.

(a) What qualities of Rohan are depicted by this act?

(b) Why did Rohan not allow the child to go near the water?

Section E

(5 × 3 = 15)

24. (a) Define the term electric dipole . Is it a vector quantity?

(b) An electric dipole is situated in a uniform electric field. Is a net force acting on the dipole? Explain.

OR

(a) Derive an expression for electric field E at any point inside a spherically symmetric uniformly charged sphere, applying Gauss's Law.

(b) Draw a graph to show the variation of E with distance.

25. (a) What is a choke coil. Discuss the construction of the same.

(b) Prove that average power dissipated in a choke coil is nearly zero.

OR

(a) Derive an expression for power in a circuit containing both inductance and capacitance.

(b) An alternating current of 1.5 mA rms and angular frequency = 100rad/s flows through a 10k Ω resistor and 0.50μ F capacitor in series. Find the rms voltage across the capacitor.

26. (a) Draw a ray diagram for the formation of image by a compound microscope.

(b) Derive an expression for total magnification when image is formed at infinity.

OR

(a) State Huygen's principle and also explain their propagation of waves.

(b) What type of wavefront will emerge from a (i) point source (ii) distant light source?