Solution

Section A

1. $kx(x-2)+6=0$

$\Rightarrow kx^2-2kx+6=0$

On Comparing $ax^2+bx+c=0$

$a=k, b=-2k, c=6$

$\because$ Roots of the given equation are equal,

$\therefore D=0$ [½]

$b^2-4ac=0$

$\Rightarrow (-2k)^2-4\times k\times 6=0$

$\Rightarrow 4k^2-24k=0 \Rightarrow 4k(k-6)=0$

$\Rightarrow k-6=0$

$\Rightarrow \boxed{k=6}$ [½]

2. Given, first term (a) = 18

last term (l) = – 47

$\text{Common difference (d)} = 15\frac{1}{2}-18$

$= \frac{31}{2}-18 = \frac{31-36}{2} = \frac{-5}{2}$ [½]

Let the number of terms in A.P be n

$\because l = a+(n-1)d$

$-47 = 18+(n-1)\times\left(\frac{-5}{2}\right)$

$\Rightarrow -47 = 18-\frac{5}{2}n+\frac{5}{2}$

$\Rightarrow -47 = -\frac{5}{2}n+\frac{41}{2}$

$\Rightarrow -\frac{5}{2}n = -47-\frac{41}{2}$

$\Rightarrow -\frac{5}{2}n = \frac{-94-41}{2}$

$\Rightarrow -5n = -135 \Rightarrow n = \frac{135}{5} = 27$ [½]

Hence, the number of terms in A.P = 27

3. $\frac{\tan 65°}{\cot 25°} = \frac{\tan(90-25)°}{\cot 25°}$ [½]

$\frac{\cot 25°}{\cot 25°}$ $[\because \tan(90-\theta) = \cot\theta]$

$= 1$ [½]

OR

$\sin 67° + \cos 75° = \sin(90 - 23)° + \cos(90 - 15)°$ [½]

$= \cos 23° + \sin 15°$ [$\because \cos(90 - \theta) = \sin\theta$ & $\sin(90 - \theta) = \cos\theta$

Hence, required value is

$\cos 23° + \sin 15°$ [½]

4. $\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{\text{area of}(\Delta ABC)}{\text{area of}(\Delta DEF)} = \frac{(BC)^2}{(EF)^2}$ (by conversion of thales theorem) [½]

Given, area of $(\Delta ABC) = 64$ sq cm,

area of $(\Delta DEF) = 121$ sq. cm and $EF = 15.4$ cm

$\therefore \frac{64}{121} = \frac{(BC)^2}{(15.4)^2}$

$\Rightarrow \frac{BC}{15.4} = \sqrt{\frac{64}{121}}$

$\Rightarrow \frac{BC}{15.4} = \frac{8}{11} \Rightarrow BC = \frac{8 \times 15.4}{11} = 11.2\,\text{cm}$ [½]

Hence, BC = 11.2 cm.

5.

Distance between two points (x_1, y_1) and (x_2, y_2) [½]

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(a+a)^2 + (b+b)^2}$

$= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2 + b^2)} = 2\sqrt{a^2 + b^2}$ [½]

6. $\because \sqrt{2} = 1.414$

and $\sqrt{7} = 2.645$ [½]

$\therefore$ Rational number between $\sqrt{2}$ and $\sqrt{7}$ = 2 [½]

OR

$\Rightarrow 2^2 \times 5^3 \times 3^2 \times 17 = 2 \times 2 \times 5 \times 5 \times 5 \times 3 \times 3 \times 17$ [½]

$= 10 \times 10 \times 15 \times 51 = 76500$

Hence, the number of zeroes in the end = 2 [½]

SECTION-B

7. Let the number of multiples of 4 lie between 10 and 205 be n.

$\therefore$ first multiples $(a) = 12$

& last multiples $(l) = 204$

Common difference $(d) = 4$

$\because l = a + (n - 1)d$

$204 = 12 + (n - 1)4$ [1]

$\Rightarrow 4(n - 1) = 204 - 12$

$\Rightarrow 4(n - 1) = 192 \Rightarrow (n - 1) = \frac{192}{4} = 48$

$\Rightarrow n = 48 + 1 = 49$

Hence, required number of multiples = 49. [1]

OR

$\Rightarrow$ we know that n^{th} term of AP

$a_n = a + (n - 1)d$

where a = first term & d = common difference

So, $a_3 = a + (3 - 1)$

$a_3 = a + 2d$

$16 = a + 2d$ (Given 3^{rd} term is 16)

$a + 2d = 16$ ...(i)

Also, $a_7 = a + (7 - 1)d$

$a_7 = a + 6d$ ...(ii)

Similarly $a_5 = a + (5 - 1)d$

$a_5 = a + 4d$ ...(iii)

Given that

7^{th} term exceed the 5^{th} term by 12

7^{th} term – 5th term = 12

$a_7 - a_5 = 12$

$\Rightarrow a + 6d - a - 4d = 12$ [from equation (ii) & (iii)]

$\Rightarrow 2d = 12 \Rightarrow d = \frac{12}{2} = 6$ [1]

putting the value of 'd' in equation (i), we get

$a + 2 \times 6 = 16$

$\Rightarrow a + 16 - 12 = 4$

Hence, first term of A.P. = 4

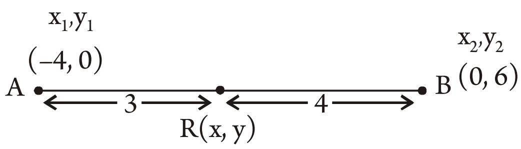
Second term of A.P. = First term + Common difference = 4 + 6 = 10

Third term of A.P. = Second term + Common difference = 10 + 6 = 16

And So On,

So, the A.P. is 4, 10, 16, ----- [1]

8. Given, $AR = \frac{3}{4}AB$



$\frac{AR}{AB} = \frac{3}{4}$

$R(x, y) = \left(\frac{mx_2 - nx_1}{m+n}, \frac{my_2 - ny_1}{m+n}\right) = \left(\frac{3\times 0 - 4\times -4}{3+4}, \frac{3\times 6 - 4\times 0}{3+4}\right)$ [1]

$= \left(\frac{16}{7}, \frac{18}{7}\right)$

$\therefore$ Required coordinate of point $R = \left(\frac{16}{7}, \frac{18}{7}\right)$ [1]

9. By using Euclid's division leema

$a = bq + r$

where, a > b
So, a = 867 and b = 255
$867 = 255 \times 3 + 102$
here, $r \neq 0$, Hence, a = 255 and b = 102 [1]

Now, $255 = 102 \times 2 + 51$

Here, $r \neq 0$, Hence, a = 102 and b = 51

$102 = 51 \times 2 + 0$

Here, r = 0

So, HCF of (867, 251) = 51 [1]

10. Number of possible out comes = 8

{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T) (T, T, H) & (T, T, T)}

Number of favourable outcomes = 3 [1]

{(H, T, T), (T, H, T), (T, T, H)}

$\therefore$ Probability (p) $= \dfrac{\text{favourable outcomes}}{\text{possible outcomes}} = \dfrac{3}{8}$ [1]

Hence, probability of getting exactly one head = $\dfrac{3}{8}$ [1]

11. Number of possible outcomes = 52

Number of favourable outcomes = 13 + 3 = 16 [1]

$\therefore$ Probability (p) $= \dfrac{\text{favourable outcomes}}{\text{possible outcomes}} = \dfrac{16}{52} = \dfrac{4}{13}$

Hence, probability of card which is neither a spade nor a king = $\dfrac{4}{13}$ [1]

12. $\dfrac{3}{x} + \dfrac{8}{y} = -1$ ...(i)

$\dfrac{1}{x} - \dfrac{2}{y} = 2$ ...(ii)

On multiplying equation (ii) by 4 and adding equation (i)

$$\begin{array}{l} \dfrac{3}{x} + \dfrac{8}{y} = -1 \\ +\dfrac{4}{x} - \dfrac{8}{y} = 8 \\ \hline \dfrac{3}{x} + \dfrac{4}{x} = -1 + 8 \end{array} = \dfrac{7}{x} = 7$$ [1]

$\Rightarrow$ $x = \dfrac{7}{7} = 1$

Putting the value of x in equation (ii), we get

$\dfrac{1}{1} - \dfrac{2}{y} = 2$

$\frac{-2}{y} = 2 - 1 \Rightarrow \frac{-2}{y} = 1 \Rightarrow y = -2$

Hence, x = 1 and y = –2 [1]

OR

Given :

$\Rightarrow$ kx + 2y = 3

kx + 2y –3 = 0 ...(i)

Comparing with $a_1x + b_1y + c_1 = 0$

$\therefore$ $a_1 = k$, $b_1 = 2$ and $c_1 = -3$ [½]

Also, 3x + 6y = 10

3x + 6y – 10 = 0 ...(ii)

Comparing with $a_2x + b_2y + c_2 = 0$

$a_2 = 3$, $b_2 = 6$ and $c_2 = -10$ [½]

As per question

Equations has unique solution

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{k}{3} \neq \frac{2}{6}$

$\Rightarrow \frac{k}{3} \neq \frac{1}{3}$

$\Rightarrow k \neq 1$

So, for all values of k except 1. [1]

SECTION-C

13. If possible, let a = $(3 + 2\sqrt{5})$ be a rational number

On squaring both sides, we get

$a^2 = (3 + 2\sqrt{5})^2$

$\Rightarrow a^2 = 9 + 20 + 12\sqrt{5}$

$\Rightarrow a^2 = 29 + 12\sqrt{5}$

$\Rightarrow \sqrt{5} = \frac{a^2 - 29}{12}$ ...(i) [1]

since 'a' is a rational number,

$\therefore \frac{a^2 - 29}{12}$ is also a rational number

$\Rightarrow \sqrt{5}$ is a rational number [1] but It is given that $\sqrt{5}$ is an irrational number.

Hence, it is a contradiction

So, 3 + $2\sqrt{5}$ is an irrational number. [1]

14. Distance travelled by train = 480 km

Let the usual speed of train be x km/h

$\therefore$ Time taken for the journey $= \frac{480}{x}$ hours

Given, speed is decreased by 8 km/h.

So, the new speed of train = (x – 8) km/h

$\therefore$ Time taken for the journey $= \frac{480}{x-8}$ hours [1]

According to the question,

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left(\frac{x-x+8}{x(x-8)}\right] = 3$$

$$\Rightarrow \frac{8}{x^2-8x} = \frac{1}{160}$$

$\Rightarrow x^2 - 8x = 1280$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow x\,(x - 40) + 32\,(x - 40) = 0$

$\Rightarrow (x - 40)\,(x + 32) = 0$

$x - 40 = 0$ & $x + 32 = 0$

$x = 40$ $x = -32$ (Not possible) [2]

$\therefore$ Usual speed of train = 40 km/h

15. $f(x) = x^2 - 4x + 3$

$\because$ α & β are the Zeros of f(x)

$\therefore$ f(x) = 0

$x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow x\,(x - 3) - 1(x - 3) = 0$

$\Rightarrow (x - 3)\,(x - 1) = 0$

$\therefore x - 3 = 0 \Rightarrow x = 3$

and $x - 1 = 0 \Rightarrow x = 1$

$\therefore \alpha = 3$ and $\beta = 1$ or $\alpha = 1$ and $\beta = 3$ [1]

when $\alpha = 3$ & $\beta = 1$

$\therefore \alpha^4\beta^2 + \alpha^2\beta^4 = (3)^4(1)^2 + (3)^2(1)^4 = 81 + 9 = 90$ [2]

16. L.H.S. : $(\sin\theta + \cos\theta)(\sin\theta - 1 + \cos\theta).\sec\theta\ \text{cosec}\,\theta$

$= \{(\sin\theta + \cos\theta)+1\ (\sin\theta + \cos\theta)-1\}.\sec\theta\ \text{cosec}\,\theta$

$= \{(\sin\theta + \cos\theta)^2 - (1)^2\}\sec\theta\ \text{cosec}\,\theta$ $[\because (a-b)(a+b) = a^2 - b^2]$ [1]

$= \{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\}\dfrac{1}{\cos\theta}\ \dfrac{1}{\sin\theta}$ $\left[\because \text{cosec}\theta = \dfrac{1}{\sin\theta} \text{ and} \because \sec\theta = \dfrac{1}{\cos\theta}\right]$

$= \dfrac{\{1 + 2\sin\theta\cos\theta - 1\}}{\sin\theta\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$ [1]

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{R.H.S}$ [1]

OR

L.H.S. : $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}}$

$= \sqrt{\dfrac{(\sec\theta - 1)}{(\sec\theta + 1)} \times \dfrac{(\sec\theta - 1)}{(\sec\theta - 1)}} + \sqrt{\dfrac{(\sec\theta + 1)}{(\sec\theta - 1)} \times \dfrac{(\sec\theta + 1)}{(\sec\theta + 1)}}$ [1]

$= \sqrt{\dfrac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\dfrac{(\sec\theta + 1)^2}{\tan^2\theta}}$ $[\because \sec^2\theta + \tan^2\theta = 1]$

$= \dfrac{(\sec\theta - 1)}{\tan\theta} + \dfrac{(\sec\theta + 1)}{\tan\theta}$ [1]

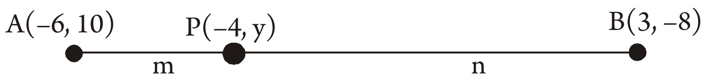
$= \dfrac{\sec\theta - 1 + \sec\theta + 1}{\tan\theta} = \dfrac{2\sec\theta}{\tan\theta} = \dfrac{2 \times \cos\theta}{\sin\theta \times \cos\theta}$

$= 2 = 2\ \text{cosec}\,\theta = \text{R.H.S}$ [1]

17. Let the point P divide the line segment A B into m : n

Using section formula

$\therefore\ -4 = \dfrac{3 \times m + (-6) \times n}{m + n}$



$\Rightarrow -4(m + n) = 3m - 6n$

$\Rightarrow -4m - 4n = 3m - 6n$

$\Rightarrow -4m - 3m = -6n + 4n$

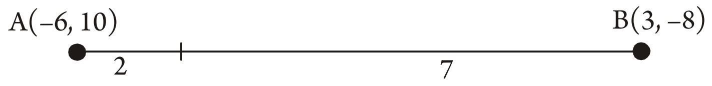
$\Rightarrow -7m = -2n$

$\Rightarrow \dfrac{m}{n} = \dfrac{2}{7}$

m : n = 2 : 7

Using section formula,

$$y = \frac{2 \times -8 + 7 \times 10}{2+7}$$



$$= \frac{-16+70}{9} = \frac{54}{9} = 6$$

OR

Given three points :

$x_1 y_1$, $x_2 y_2$ and $x_3 y_3$

(–5, 1), (1, p) and (4, –2)

Three points are collinear if area of triangle = 0 [1]

$$\Rightarrow \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$$

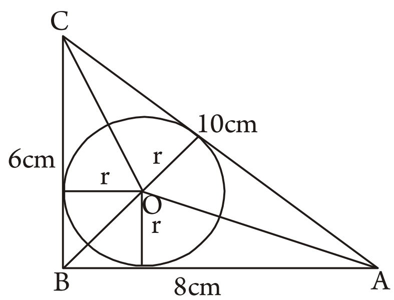
–5(p + 2) + 1(–2 – 1) + 4(1–p) = 0

⇒ –5p – 10 – 3 + 4 – 4p = 0

⇒ –9p – 9 = 0

⇒ –9p = 9 ⇒ p $= -\frac{9}{9} = -1$ [2]

18. In Δ ABC,



By pythagoras theorem,

$(AB)^2 = (AB)^2 + (BC)^2$

$$AC = \sqrt{(8)^2 + (6)^2} = \sqrt{64+36}$$

$$= \sqrt{100} = 10 \text{ cm}$$ [1]

∴ Ar(Δ ABC) = Ar(Δ AOB) + Ar(Δ BOC) + Ar(Δ COA)

$$\Rightarrow \frac{1}{2} \times AB \times BC = \frac{1}{2} \times AB \times r + \frac{1}{2} \times BC \times r + \frac{1}{2} \times AC \times r$$

$$\Rightarrow \frac{1}{2} \times 8 \times 6 = \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$$

⇒ 48 = 8r + 6r + 10r

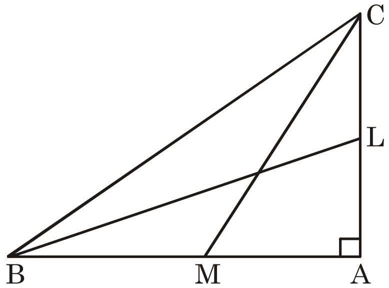
$\Rightarrow 48 = 24r \Rightarrow r = \frac{48}{24} = 2$

radius of circle = 2cm

$\therefore$ diameter of circle = 2r = 2 × 2 = 4 cm [2]

19. **Given**: Δ ABC, right angled at A.

BL and CM are medians.



To prove: $4(BL^2 + CM^2) = 5\,BC^2$

proof In Δ ABL,

By Pythagoras theorem, $(BL)^2 = (AB)^2 + (AL)^2$

$\Rightarrow BL^2 = AB^2 + \left(\frac{AC}{2}\right)^2$ ($\because$ BL is median)

$\Rightarrow BL^2 = AB^2 + \frac{AC^2}{4}$ ...(i) [1]

In Δ ACM,

By Pythagoras theorem.

$CM^2 = AC^2 + AM^2$

$\Rightarrow CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$ ($\because$ CM is median)

$\Rightarrow CM^2 = AC^2 + \frac{AB^2}{4}$ ...(ii) [1]

Adding equation (i) and (ii), we get

$BL^2 + CM^2 = AB^2 + \frac{AC^2}{4} + AC^2 + \frac{AB^2}{4}$

$\Rightarrow BL^2 + CM^2 = \frac{5AB^2}{4} + \frac{5AC^2}{4}$

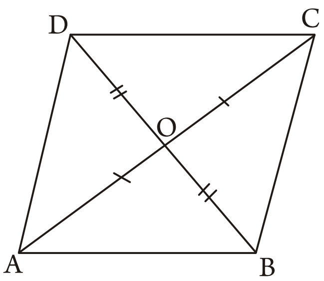
$\Rightarrow 4(BL^2 + CM^2) = 5(AB^2 + AC^2)$

$\Rightarrow 4(BL^2 + CM^2) = 5\,BC^2$

[In Δ ABC, by pythagoras theorem $BC^2 = AB^2 + AC^2$] [1]

OR

$\Rightarrow$ **Given:** Rhombus ABCD with diagonals AC & BD intersecting at = O.



To prove: Sum of the square of all sides = sum of the square of it's diagonals.

$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proof: Since, side of a rhombus are equal

$\therefore$ AB = BC = CD = AD

We know that, diagonals of a rhombus bisect each other at a right angles. [1]

Therefore,

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

Also $AO = CO = \frac{1}{2}AC$ ...(i)

and $BO = DO = \frac{1}{2}BD$ ...(ii) [1]

Now, AOB is a right angle triangle

By, Pythagoras theorem.

$(AB)^2 = (OA)^2 + (OB)^2$

$(AB)^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$ [from equation (i) & (ii)]

$AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$

$4\,AB^2 = AC^2 + BD^2$

$\Rightarrow AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$

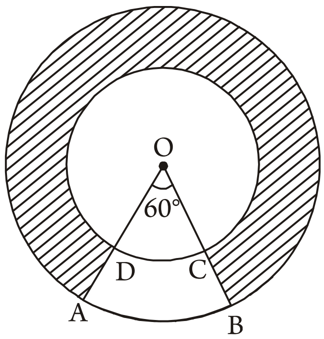
$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$ [1]

(Since sides of a rhombus are equal)

20. We have,

Area of shaded region =

Area of the circular region – Area of region ABCD.



Now, are of circular region

$= \pi\,(R^2 - r^2) = \left[\frac{22}{7}\left(42^2 - 21^2\right)\right]$ [1]

$= \left[\frac{22}{7}(1764 - 441)\right] = \left[\frac{22}{7} \times 1323\right] = 4158\,\text{cm}^2$

and Area of the region ABCD = Area of sector OABO – Area of sector ODCO

$= \frac{\theta}{360} \times \pi R^2 - \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times \pi\left(R^2 - r^2\right)$ [1]

$=\frac{60}{360}\times\frac{22}{7}\times1323 = \frac{22}{6\times7}\times1323 = 693\,cm^2$

$\therefore$ Required shaded are = $(4158 - 693)$ cm^2 = 3465 cm^2 [1]

21. Given, Height of Cone (h) = 24 cm

Radius of Cone (r) = 6 cm

Volume of Cone $=\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\pi\times(6)^2\times24$

$=\frac{1}{3}\times\pi\times36\times24 = 288\,\pi\,cm^2$ [1]

Let the radius of sphere be R cm

According to the question,

Volume of sphere = Volume of Cone

$\frac{4}{3}\pi R^3 = 288\pi$

$\Rightarrow \frac{4}{3}R^3 = 288$

$\Rightarrow R^3 = \frac{288\times3}{4} = 216$

$\Rightarrow R = \sqrt[3]{216} = 6\,cm$ [1]

Hence, radius of sphere = 6 cm.

$\because$ Surface area of sphere = $4\pi r^2$

= $4\times\pi\times(6)^2$

= $144\,\pi$ sq. cm

Hence, radius of sphere = 6 cm

and surface area of sphere = $144\,\pi\,cm^2$ [1]

OR

Given, Radius of tank R = $\frac{10}{2}$ = 5 m

Height of tank (H) = 2 m

Radius of pipe (r) = $\frac{20}{2}$ = 10 cm = 0.1 m

Speed of water (h) = 3 km/h = 3000 m/h

Volume of cylindrical tank = $\pi R^2 H$

$= \pi\times5\times5\times2 = 50\,\pi\,m^3$ [1]

$\therefore$ Volume of water in 1 hour through pipe = $\pi r^2 h$

$= \pi\times0.1\times0.1\times3000 = 30\,\pi\,m^3$ [1]

Time taken to fill the tank $=\frac{\text{Volume of tank}}{\text{Volume of water in 1 hour}}$

$=\frac{50\pi}{30\pi}=\frac{5}{3}$ hour = 1 hr 40 minutes [1]

22. Here, the maximum frequency is 20 and the Corresponding class is 20 – 25, So, modal class is 20 – 25

We have, Lower class boundary of modal group (l) = 20

group width (h) = 25 – 20 = 5

Frequency of the modal group $(f_1) = 20$

Frequency of the group before the modal group $(f_0) = 7$

Frequency of the group after the modal group $(f_2) = 8$ [1]

$$\text{Mode} = l + \frac{f_1 - f_2}{2f_1 - f_0 - f_2} \times h = 20 + \frac{20-8}{2\times 20 - 7 - 8} \times 5$$ [1]

$$= 20 + \frac{12}{40-15} \times 5 = 20 + \frac{12}{25} \times 5 = 20 + 2.4 = 22.4$$ [1]

SECTION-D

23. $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$ [½]

$\frac{2x-(2a+b+2x)}{(2a+b+2x)2x} = \frac{b+2a}{(2a)b}$ [½]

$\Rightarrow \frac{2x-2a-b-2x}{(2a+b+2x)2x} = \frac{b+2a}{2ab}$

$\Rightarrow \frac{-(b+2a)}{(2a+b+2x)2x} = \frac{(b+2a)}{2ab}$ [½]

$\Rightarrow -ab = (2a + b + 2x)x$

$\Rightarrow -ab = 2ax + bx + 2x^2$ [½]

$\Rightarrow 2x^2 + 2ax + bx + ab = 0$

$\Rightarrow 2x(x + a) + b(x + a)0$

$\Rightarrow (x + a)(2x + b) = 0$

$\Rightarrow x + a = 0$ and $2x + b = 0$

or $x = -a$ or $x = \frac{-b}{2}$

Hence, $x = -a$ and $x = \frac{-b}{2}$ [2]

OR

$\Rightarrow$ Let side of first square be x and that of another be y.

In I[st] Case,

sum of areas = 640

$x^2 + y^2 = 640$ ...(i) [1]

II[nd] Case,

Difference of their perimeters = 64

$4x - 4y = 64$

$\Rightarrow 4(x - y) = 64$

$\Rightarrow x - y = \frac{64}{4} = 16$

$\Rightarrow x = 16 + y$ ...(ii) [1]

putting the value of x in equation (i), we get

$(16 + y)^2 + y^2 = 640$

$256 + y^2 + 32y + y^2 = 640$

$\Rightarrow 2y^2 + 32y - 640 + 256 = 0$

$\Rightarrow 2y^2 + 32y - 384 = 0$

$\Rightarrow y^2 + 16y - 192 = 0$

$\Rightarrow y^2 + 24y - 8y - 192 = 0$

$\Rightarrow y^2 + 24y - 8y - 192 = 0$

$\Rightarrow y(y + 24) - 8(y + 24) = 0$

$\Rightarrow (y + 24)(y - 8) = 0$

$\Rightarrow y + 24 = 0 \Rightarrow y = -24$ (Not Possible)

and $y - 8 = 0 \Rightarrow y = 8$ [1]

putting the value of y in equation. (ii), we get

$x = 16 + 8 = 24$

$\therefore$ side of first square = 24m

and side of second square = 16m [1]

24. Let a and d be the first term and common difference

respectively of the given A.P.

using, $Sn = \frac{n}{2}[2a + (n-1)d]$ [1]

$Sp = Sq$ (given)

$\frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$\Rightarrow 2ap + p(p - 1)d = 2aq + q(q - 1)d$

$\Rightarrow 2a(p - q) + \{p(p - 1) - q(q - 1)\}d = 0$

$\Rightarrow 2a(p - q) + \{p^2 - p - q^2 + q\}d = 0$

$\Rightarrow 2a(p - q) + \{(p^2 - q^2) - (p - q)\}d = 0$

$\Rightarrow 2a(p - q) + \{(p + q)(p - q) - (p - q\}d = 0$

$\Rightarrow (p - q)[2a + \{(p + q) - 1\}d\} = 0$ [1]

$\Rightarrow 2a + (p + q - 1)d = 0$ $[\because p \neq q]$ ...(i)

Now $Sp + q = \frac{p+q}{2}[2a + (p + q - 1)d]$

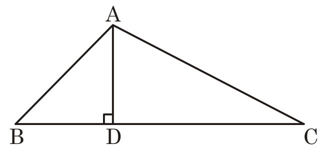
$= \frac{p+q}{2} \times 0$ [from equation(i)]

$= 0$

Thus, $Sp + q = 0$ Hence proved. [2]

25. **Given:** ABC is a triangle where

$\angle ABC < 90°$ and $AD \perp BC$



To prove: $AC^2 = AB^2 + BC^2 - 2\ BC \times BD$

Proof: In Δ ADB, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ...(i) [1]

In Δ ADC, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

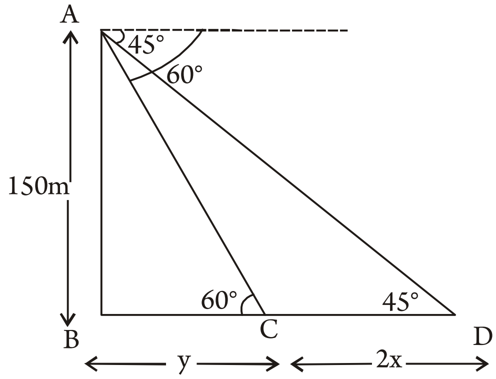
$\Rightarrow AC^2 = AD^2 + (BC - BD)^2$ [$\because$ DC = BC – BD] [1]

$\Rightarrow AC^2 = AD^2 + BC^2 + BD^2 - 2\ BC \times BD$

$\Rightarrow AC^2 = (AD^2 + BD^2) + BC^2 - 2\ BC \times BD$

$\Rightarrow AC^2 = AB^2 + BC^2 - 2\ BC \times BD$ [from eqn. (i)] [2]

26. Let the speed of the boat be 'x' m/min.



$\therefore$ Distance covered in 2 minute = 2 x m

Let BC = 'y' m

In Δ ABC, By Pythagoras theorem,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{150}{y}$$

$$\Rightarrow y = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 50\sqrt{3}\ \text{m} \quad ...(i)$$ [1]

In Δ ABD, By Pythagoras theorem.

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{150}{y + 2x}$$

$\Rightarrow y + 2x = 150$ ...(ii) [1]

Substituting the value of y from equation (i) in equation (ii),

we get

$50\sqrt{3} + 2x = 150$

$\Rightarrow 2x = 150 - 50\sqrt{3}$

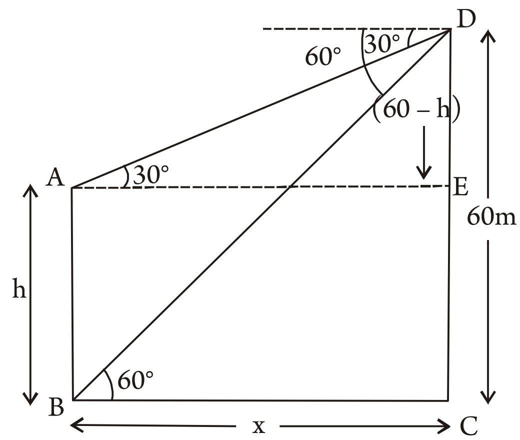
$\Rightarrow x = \frac{50}{2}(3-\sqrt{3}) = 25(3 - \sqrt{3})$

Hence, the speed of boat = $25(3 - \sqrt{3})$ m/min [2]

OR

Let the width of river be x m and height of other pole be h m.

In Δ DCB,



By Pythagoras theorem, [1]

$\tan 60° = \frac{DC}{BC}$

$\sqrt{3} = \frac{60}{x}$

$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$ m

$\because$ DE = DC – EC = (60 – h)m [1]

In Δ AED, By Pythagoras theorem,

$\tan 30° = \frac{DE}{AE}$

$\frac{1}{\sqrt{3}} = \frac{60-h}{x}$ $(\because AE = BC)$

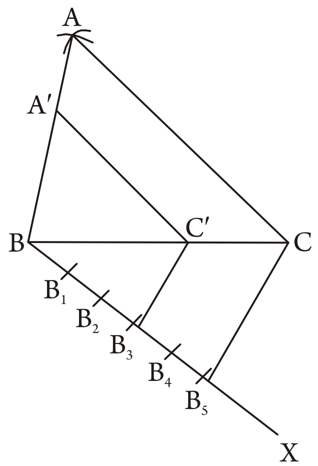
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{20\sqrt{3}}$

$\Rightarrow 20 = 60 - h \Rightarrow h = 60 - 20 = 40$ m

Hence, the width of river be $20\sqrt{3}$ m and height of other pole be 40 m. [2]

27. Stop of construction

1. Draw a line regiment BC = 6 cm
2. With B as a centre and radius = AB = 5 cm, draw an arc. [1]
3. With C as centre and radius = AC = 7 cm, draw another arc, intersecting the arc drawn in step 2 at the point A.
4. Join AB and AC to obtain Δ ABC. [1]
5. Below BC, mark an acute angle ∠CBX.
6. Along BX mark off five points B_1, B_2, B_3, B_4, B_5 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$
7. Join B_5C. [1]
8. From B_3, draw $B_3C' \parallel B_5C$.
9. From C, draw C′A′ || CA, meeting BA at the point A′



Then A′ BC′ is the required triangle. [1]

28. LHS : $\sin^8\theta - \cos^8\theta = (\sin^4\theta)^2 - (\cos^4\theta)^2$

By using $a^2 - b^2 = (a - b)(a + b) = (\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$ [1]

$= \{(\sin^2\theta)^2 - (\cos^2\theta)^2\}(\sin^4\theta + \cos^4\theta)$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta)$ [1]

By using $\sin^2\theta + \cos^2\theta = 1$

$= (1 - \cos^2\theta - \cos^2\theta). 1. \{(\sin^2\theta)^2 + (\cos^2\theta)^2\}$

$= (1 - 2\cos^2\theta)\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta \cdot \cos^2\theta\}$ [1]

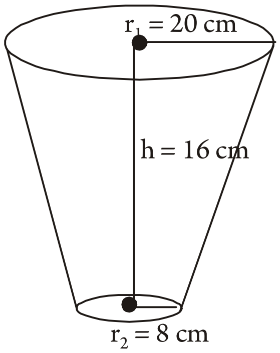
$[\because a^2 + b^2 = (a + b)^2 - 2ab]$

$= (1 - 2\cos^2\theta)\{(1)^2 - 2\sin^2\theta \cdot \cos^2\theta\}$

$= (1 - 2\cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$ [1]

= R.H.S

29. In order to find cost of milk which can completely fill container, we need to find volume (in litres)



Volume of container = Volume of frustum $=\frac{1}{3}\pi h\left(r_1^2+r_2^2+r_1 r_2\right)$ [½]

Here, h = height = 16 cm,

r_1 = radius of upper end = 20 cm

r_2 = radius of lower end = 8 cm

Now, Volume of container $=\frac{1}{3}\times 3.14\times 16\left[20^2+8^2+20\times 8\right]$

$=\frac{3.14\times 16}{3}\times[400+64+160]$

$= 16.74 \times 624 = 10445.76 \text{ cm}^2$

$=\frac{10445.76}{1000}$ litre $\left[\because 1000\text{ cm}^2 = 1\text{ litre}\right]$ [½]

= 10.44576 litre.

Now, Cost of 1 litre milk = Rs. 50

$\therefore$ Cost of 10.44576 litre milk = Rs.(50 × 10.44576)

= Rs. 522.28 = Rs. 522 [1]

Now, we need to find the cost of metal.

To find the cost of metal, we need to find the area of container.

Since container is closed from bottom,

Area of container = Area of frustum + Area of bottom circle = $\pi(r_1 + r_2)l + \pi r_2^2$

Here, r_1 = 20 cm, r_2 = 8 cm

$l=\sqrt{h^2+(r_1-r_2)^2}$

$l=\sqrt{(16)^2+(20-8)^2}=\sqrt{256+144}$

$=\sqrt{400}$ = 20 cm [1]

$\therefore$ Area of container = 3.14 (20 + 8) × 20 + 314 × 8^2

= 3.14 [28 × 20 + 64] = 3.14 [560 + 64]

= 3.14 × 624 = 1959.36 cm^2

$\because$ Cost of making 100 cm^2 metal sheet = Rs. 10

Cost of making 1 cm^2 metal sheet = Rs. $\frac{10}{100}$

Cost of making 1959.36 cm^2 metal sheet = Rs. $\frac{10}{100} \times 1959.36$

$= \text{Rs. } \frac{1959.36}{10} = \text{Rs. } 195.93 = \text{Rs. } 196$

Hence, Cost of milk = Rs. 522 and cost of metal sheet = Rs. 196 [1]

Class	Frequency(f_i)	x_i	$d_i = xi - A$	$\mu_i = \frac{x_i - A}{h}$	$f_i \mu_i$
10 – 30	5	20	–60	–3	–15
30 – 50	8	40	–40	–2	–16
50 – 70	12	60	–20	–1	–12
70 – 90	20	A $\boxed{80}$	0	0	0
90 – 110	3	100	20	1	3
110 – 130	2	120	40	2	4
	$\Sigma f_i = 50$				$\Sigma f_i \mu_i = -36$

Mean $= A + \frac{\Sigma f_i \mu_i}{\Sigma f_i} \times h$

Here, A = 80, $\Sigma f_i \mu_i = -36$, $\Sigma f_i = 50$ and h = 30 – 10 = 20

$\therefore$ Mean $= 80 + \frac{(-36)}{50} \times 5 = 80 + \frac{(-36)}{10} = 80 - 3.6 = 76.4$ [2]

OR

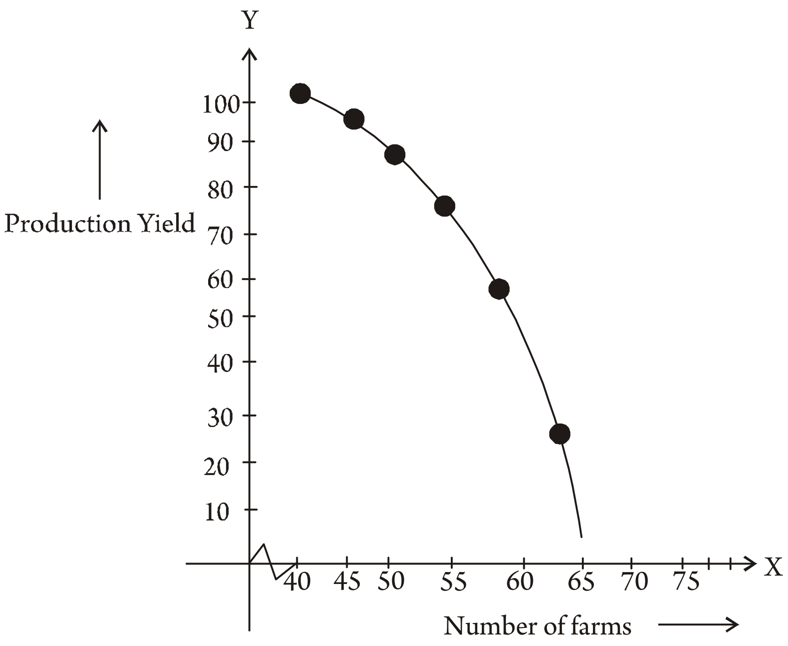
$\Rightarrow$

Production yield (in kg/ha)	Number of farms (f_i)
40 - 45	4
45 – 50	6
50 – 55	16
55 – 60	20
60 – 65	30
65 – 70	24

$\Sigma f_i = 100$ [1]

Production yield (X –axis)	Number of farms (y-axis)
More than 40	100
More than 45	100 – 4 = 96
More than 50	90 – 6 = 90
More than 55	90 – 16 = 74
More than 60	74 – 20 = 54
More than 65	54 – 30 = 24

[1]



[2]