CBSE
Solved Paper 2019

Mathematics
Class XII

Time : 3 hrs | **Maximum Marks : 100**

General Instructions

(i) All questions are compulsory.

(ii) This question paper contains **29** questions divided into four sections A, B, C and D. Section A comprises of 4 questions of **one mark** each, Section B comprises of 8 questions of **two marks** each, Section C comprises of 11 questions of **four marks** each and Section D comprises of 6 questions of **six marks** each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only **one** of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask logarithmic tables, if required.

Section A

Question numbers 1 to 4 carry 1 mark each.

1. Find the order and the degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$.

(1)

2. If f(x) = x + 7 and g(x) = x – 7, $x \in R$, then find $\frac{d}{dx}(fog)(x)$. (1)

3. Find the value of x – y, If

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$ (1)

4. If a line makes angle 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines. (1)

OR

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i}+2\hat{j}-2\hat{k}$.

SECTION B

Question numbers 5 to 12 carry 2 marks each.

5. Examine whether the operation * defined on R by a * b = ab + 1 is (i) a binary or not. (ii) if a binary operation, is it associative or not ? (2)

6. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, then find (A^2-5A). (2)

7. Find: $\int\sqrt{1-\sin 2x}\,dx, \frac{\pi}{4}<x<\frac{\pi}{2}$

OR (2)

Find: $\int \sin^{-1}(2x)\,dx$.

8. From the differential equation representing the family of curves $y=e^{2x}(a+bx)$, where 'a' and 'b' are arbitrary constants. (2)

9. A die is thrown 6 times. If " getting an odd number" is a "success", what is the probability of (i) 5 successes ? (ii) atmost 5 successes ?

OR (2)

The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X=x)=\begin{cases}k, & \text{if } x=0\\ 2k, & \text{if } x=1\\ 3k, & \text{if } x=2\\ 0, & \text{otherwise}\end{cases}$$

Determine the value of 'k'.

10. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not. (2)

11. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR (2)

If $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}, \vec{b}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{c}=-3\hat{i}+\hat{j}+2\hat{k}$, find $[\vec{a}\vec{b}\vec{c}]$.

12. Find: $\int\frac{\tan^2 x\sec^2 x}{1-\tan^6 x}dx$. (2)

SECTION C

Question numbers 13 to 23 carry 4 marks each.

13. Solve for $x:\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$. (4)

14. If $\log\left(x^2+y^2\right)=2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx}=\frac{x+y}{x-y}$.

OR (4)

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

15. Find : $\int \frac{3x+5}{x^2+3x-18}dx$. (4)

16. Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$, hence evaluate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$. (4)

17. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C and D then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not. (4)

18. Using properties of determinants, prove the following :

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c+ \end{vmatrix} = 2(a+b)(b+c)(c+a).$$ (4)

19. If $x = \cos t + \log\tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$.

(4)

20. Show that the relations R on $\mathbb{R}$ defined as $R = \{(a, b) : a \le b\}$, is reflexive, and transitive but not symmetric.

OR (4)

Prove that the function $f : N \to N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f : N \to S$, where S is range of f.

21. Find the equation of tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

(4)

22. Solve the differential equation : $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$, given that $y = 0$ when $x = 1$.

OR (4)

Solve the differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

23. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not. (4)

SECTION D

Question numbers 24 to 29 carry 6 marks each.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}\cdot$ Also find the maximum volume of cone. (6)

25. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, then find A^{-1}. Hence solve the following system of equations : $2x-3y+5z=11$, $3x+2y-4z=-5$, $x+y-2z=-3$.

OR (6)

Obtain the inverse of the following matrix using elementary operations : $A=\begin{bmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

26. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ? (6)

27. Find the vector and Cartesian equation of the plane passing through the points (2, 2 –1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR (6)

Find the vector equation of the plane that contains the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and the point (–1, 3, –4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane thus obtained.

28. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

OR (6)

Find the area of the region lying about x-axis and included between the circle $x^2+y^2=8x$ and inside of the parabola $y^2=4x$.

29. A manufactures has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi -skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit. (6)