Test Booklet Code

A

JEE Main – 2017
Answers & Explanations

Physics				Chemistry				Mathematics			
1	1	**16**	3	**31**	1	**46**	1	**61**	2	**76**	3
2	3	**17**	4	**32**	4	**47**	2	**62**	3	**77**	4
3	3	**18**	1	**33**	1	**48**	2	**63**	4	**78**	3
4	1	**19**	1	**34**	4	**49**	1	**64**	1	**79**	2
5	1	**20**	3	**35**	2	**50**	2	**65**	4	**80**	1
6	1	**21**	1	**36**	3	**51**	1	**66**	4	**81**	1
7	4	**22**	1	**37**	2	**52**	3	**67**	3	**82**	1
8	2	**23**	2	**38**	2	**53**	3	**68**	1	**83**	1
9	1	**24**	2	**39**	2	**54**	2	**69**	4	**84**	1
10	3	**25**	4	**40**	4	**55**	4	**70**	1	**85**	4
11	4	**26**	2	**41**	3	**56**	4	**71**	4	**86**	1
12	4	**27**	4	**42**	4	**57**	3	**72**	1	**87**	4
13	3	**28**	1	**43**	3	**58**	2	**73**	2	**88**	3
14	3	**29**	2	**44**	1	**59**	3	**74**	1	**89**	2
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PART A – PHYSICS

1. 1 $\text{Stress} = \frac{\text{Weight}}{\text{Area}} = \frac{\text{Density} \times \text{Volume}}{\text{Area}}$

If Dimensions increase by a factor of 9 the stress becomes 9 times

2. 3 Velocity decreases linearly

3. 3 $\frac{mdv}{dt} = -kv^2$

$$\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = \frac{-k}{m}\int_0^t dt$$

$$-\frac{1}{v_0} = \frac{-k}{m}t$$

at t = 10 s

$$k = \frac{m}{v_0 t} = 10^{-4}\ \text{kg}\,\text{m}^{-1}$$

4. 1 $m\frac{dv}{dt} = 6t$

$$m\int dv = 6\int t\,dt$$

$mv = 3t^2$

$v = \frac{3}{m}t^2$

Work = Change in K.E.

$$= \frac{1}{2}m\left[v_1^2 - v_0^2\right] = 4.5\ \text{J}$$

5. 1 $I = \frac{MR^2}{4} + \frac{ML^2}{12}$

$$\frac{dI}{dL} = \frac{MR}{2}\frac{dR}{dL} + \frac{ML}{6} = 0 \qquad ...(1)$$

Now R^2L = constant

$$2RL\frac{dR}{dL} + R^2 = 0$$

or $\frac{dR}{dL} = \frac{-R}{2L}$ ...(2)

using (1) & (2)

$$\frac{MR}{2}\left(\frac{-R}{2L}\right) + \frac{ML}{6} = 0$$

$$\Rightarrow \frac{R^2}{4} = \frac{L^2}{6}$$

or $\frac{L}{R} = \sqrt{\frac{3}{2}}$

6. 1 $\tau = I\alpha$

$$mg\ \frac{l}{2}\sin\theta = \frac{Ml^2}{3}\alpha$$

$$\alpha = \frac{3g}{2l}\sin\theta$$

7. 4 Inside:

$$g' = g\left(1 - \frac{d}{R}\right) = g\frac{x}{R}$$

Outside:

$$g' = \frac{GM}{x^2}$$

8. 2 Using concept at calorimetry

100 × 0.1 × (T – 75°)

= 100 × 0.1 × (75° – 30°) + 170 × 1 × (75° – 30°)

Solving we get T = 885° C

9. 1 $K = \frac{P}{\frac{\Delta V}{V}}$

$$\therefore \frac{\Delta V}{V} = \frac{P}{K}$$

Coefficient of volume expansion = 3α

$\therefore 3\alpha T = \frac{P}{K}$,

$\therefore T = \frac{P}{3\alpha K}$

10. 3 $C_P - C_V = \frac{R}{2}$ for hydrogen gas and

$C_P - C_V = \frac{R}{28}$ for nitrogen gas

Here, C_P and C_V are defined as specific heat per unit mass
Thus, the answer will be (3).
Note:
If C_P and C_V are taken as molar specific heat then $C_P - C_V$ for the both the gases will be R.

11. 4 Using PV = nRT

$n_i = \frac{PV}{R \times 290}, n_f = \frac{PV}{R \times 300}$

$\therefore \Delta n = n_f - n_i = \frac{PV}{R}\left\{\frac{1}{300} - \frac{1}{290}\right\}$

The number of molecules
= Δn × Avogadro number = $\Delta n \times 6.02 \times 10^{23}$
Solving we get = -2.5×10^{25}.

12. 4 K.E is max at t = 0, Then it decreases to zero at $t = \frac{T}{4}$.

13. 3 For light the doppler effect formulas is

$$n = n_0\sqrt{\frac{c+v}{c-v}} = n_0\sqrt{\frac{c+\frac{c}{2}}{c-\frac{c}{2}}} = n_0\sqrt{3}$$

14. 3 $\vec{T}_1$ = PE sinθ

$\left|\vec{T}_2\right| = P\sqrt{3}E\sin\left(90° - \theta\right)$

Because if θ is the angle with x-axis, 90° – θ will be the angle with y-axis.

Since $\left|\vec{T}_1\right| = \left|\vec{T}_2\right|$

$\therefore$ PE sinθ = $\sqrt{3}\,PE\cos\theta$

ten θ = $\sqrt{3}$
$\therefore$ θ = 60°.

15. 4 We can connect 32 capacitors, with 4 capacitors in series & 8 such series in parallel.

16. 3 $i = \frac{E}{r + r_2}$

There will be no current through the branch of capacitor.

$$\Delta V_{r_2} = ir_2 = \frac{Er_2}{r + r_2} = \Delta V_c$$

$$\therefore q_c = C\,\Delta V_C = \frac{CEr_2}{r + r_2}$$

17. 4 The p.d. across each resistor is zero.

18. 1 $T = 2\pi\sqrt{\frac{I}{MB}}$

M → Magnetic moment
B → Magnetic field
I → Moment of Inertia

$$T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}}$$

$$= 2\pi\sqrt{\frac{7.5}{6.7} \times 10^{-2}} = 2\pi \times 1.06 \times 10^{-1}$$

= 0.6675.
Time for 10 oscillations = 6.67 s

19. 1 $10 = 5 \times 10^{-3}\,(15 + R)$
2000 Ω = 15 Ω + R
R = 1985 Ω
= 1.985×10^3 Ω

20. 3 From the graph:

$i = 10 - 20t$.

$$\Delta\phi = R\int_0^{0.5} i\,dt = 100\int_0^{0.5}(10-20t)\,dt$$

$$= 100\left(10t - 10t^2\right)_0^{0.5}$$

= 100 (10 × 0.5 – 10 × 0.25)

= 100 (5 – 2.5) = 250 Wb.

21. 1 $\frac{hc}{\lambda_{min}} = eV$

$\log(hc) - \log(\lambda_{min}) = \log e + \log V$.

$$\log(\lambda_{min}) = \log\left(\frac{hc}{e}\right) - \log V$$

This is a straight line with positive y-intercept and slope negative.

22. 1

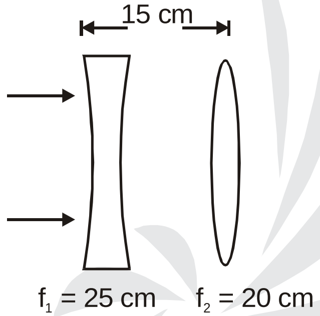


Image by diverging lens is formed at 25 cm, to the left of the lens. This image serves as object for the converging lens.

$\therefore$ u = – 40 cm; f = +20 cm.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{-40} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$$

v = +40 cm

$\therefore$ Image is Real and at a distance of 40 cm from convergent lens.

23. 2 $n_1\lambda_1\frac{D}{d} = n_2\lambda_2\frac{D}{d}$ for coinciding of bright fringes.

or $n_1\lambda_1 = n_2\lambda_2$

$n_1\,650 = n_2\,520$

$5n_1 = 4n_2$

$n_1 = 4$ and $n_2 = 5$

$\therefore$ Minimum distance $= n_1\frac{\lambda_1 D}{d}$

$$= \frac{4\times 650\times 10^{-9}\times 150\times 10^{-2}}{0.5\times 10^{-3}}$$

$= 600 \times 650 \times 2 \times 10^{-8}$

$= 7.8 \times 10^{+5} \times 10^{-8} = 7.8$ mm

24. 2 Before Collision

Because P_{sys} = constt

$\Rightarrow mv = -mv_1 + \frac{m}{2}v_2$.

$\Rightarrow 2v = v_2 - 2v_1$ ...(1)

Further $e = \frac{v_1 + v_2}{v} = 1 \Rightarrow v_1 + v_2 = v$ ...(2)

Solving the 2 equations we get,

$$v_1 = -\frac{v}{3};\ v_2 = \frac{4v}{3}.$$

$$\therefore p_A = \frac{mv}{3};\ p_B = \frac{2mv}{3}$$

$$\therefore \lambda_A = \frac{h}{p_A} = \frac{3h}{mv}$$

$$\lambda_B = \frac{h}{p_B} = \frac{3h}{2mv};\ \therefore \frac{\lambda_A}{\lambda_B} = 2$$

25. 4 $\frac{hc}{\lambda_1} = 2E - E = E \quad \Rightarrow \lambda_1 = \frac{hc}{E}$

$\frac{hc}{\lambda_2} = \frac{4E}{3} - E = \frac{E}{3} \quad \Rightarrow \lambda_2 = \frac{3hc}{E}$

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{1}{3} = r.$

26. 2 $A \rightarrow B$

$N_A = N_0\, e^{-\lambda t}$

$N_B = N_0\,(1 - e^{-\lambda t})$

$\frac{N_B}{N_A} = \frac{(1-e^{-\lambda t})}{e^{-\lambda t}} = e^{\lambda t} - 1 = 0.3$ (given)

$e^{\lambda t} = 1.3$

$\Rightarrow \lambda t = \log 1.3$ or $t = \frac{\log 1.3}{\lambda}$

Also; $\lambda = \frac{\log 2}{T}$

$\therefore t = \frac{T \log 1.3}{\log 2}$

27. 4 conceptual.

28. 1 conceptual.

29. 2 conceptual.

30. 2 $T = \frac{dhg}{4}$

$d \rightarrow$ diameter

$h \rightarrow$ rise of water

$g \rightarrow$ acceleration due to gravity

$\frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} + \frac{\Delta g}{g}$

$= \frac{0.01}{1.25} + \frac{0.01}{1.45} \simeq 1.5\%$

PART B – CHEMISTRY

31. 1 $890.3 = [\Delta H_r(CH_4) + 2 \times 0] - [(-393.5) + (2 \times (-285.8)]$

$890.3 = \Delta H_r(CH_4) + 965.1$

$\Delta H_r(CH_4) = 890.3 - 965.1$

$= -74.8$ KJ mol^{-1}

32. 4 $M_2CO_3 + 2HCl \rightarrow 2MCl + H_2O + CO_2$

M g of M_2CO_3 gives $\rightarrow$ 1 mole of CO_2

1 g of M_2CO_3 gives = $\frac{1}{M}$ moles

So, $\frac{1}{M} = 0.01186$

$M = \frac{1}{0.01186}$

$M = 84.3$

33. 1 $\Delta U = q + w$

for adiabatic

$q = 0$

$\Delta U = w$

34. 4 Two conditions should be satisfied regarding tyndall effect:

(i) The diameter of the dispersed particles should be smaller but not much than the wavelength of light used.

(ii) The refractive indices of dispersion medium and the dispersed phase must vary in magnitude to a large scale.

35. 2 For F.C.C. lattice

$r = \frac{a}{2\sqrt{2}}$

closest approach in $2r = \frac{a}{\sqrt{2}}$

36. 3 Strongest reducing agent will be Cr as

$E^\circ_{Cr/Cr^{3+}} = +0.74V$

37. 2 $\Delta T_f = 0.45$

$$i = 1 - \alpha + \frac{\alpha}{2} = 1 - \frac{\alpha}{2}$$

$$\Delta T_f = iK_f \frac{w_A \times 1000}{M_A \times w_B}$$

$$\frac{0.45}{100} = \frac{i \times 5.12 \times 0.2 \times 1000}{1000 \times 60 \times 20}$$

$$i = \frac{45 \times 6}{512} = \frac{270}{512} = 0.52$$

$$0.52 = 1 - \frac{\alpha}{2}$$

$\frac{\alpha}{2} = 1 - 0.52$ or $\alpha = 0.48 \times 2$

$= 0.96$

38. 2 $r_1 = \frac{0.529 \times 2^2}{Z} A^\circ$

$$r_2 = \frac{0.529 \times 4}{1} = 2.198A^\circ \approx 2.12A^\circ$$

39. 2 $K = Ae^{-\frac{Ea}{RT}}$

For R_1, $K_1 = Ae^{-\frac{Ea_1}{RT}}$

For R_2, $K_2 = Ae^{-\frac{Ea_2}{RT}}$

$\ln k_1 = \ln A - \frac{Ea_1}{RT}$ ... (i)

$\ln k_2 = \ln A - \frac{Ea_2}{RT}$ ... (ii)

(ii) – (i) gives,

$$\ln \frac{K_2}{K_1} = \frac{1}{RT}(Ea_1 - Ea_2)$$

$$\ln \frac{K_2}{K_1} = \frac{1 \times 10}{8.314 \times 300} = \frac{100}{8.314 \times 3} = 4.009$$

40. 4 $pH = \frac{1}{2}[Pk_w + Pk_a - Pk_b]$

$$pH = \frac{1}{2}[14 + 3.2 - 3.4] = 6.9$$

41. 3 Lithium and Magnesium, both form basic carbonates.

42. 4 CO = 14

$= \sigma 1s^2, \sigma * 1s^2, \sigma 2s^2, \sigma * 2s^2,$

$\pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

It has no unpaired electrons so diamagnetic.

43. 3 $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

Oxidation no. of Xe changes from +4 to +6

Oxidation no. of O changes from +1 to 0.

As both oxidation and reduction takes place so a redox reaction.

44. 1 Permissible value is 1ppm for Flouride ions.

45. 3 O^{2-} = 10 electrons

Na^+ = 10 electrons

F^- = 10 electrons

Mg^{2+} = 10 electrons

46. 1 $\underset{\text{(cold and dilute)}}{2NaOH} + Cl_2 \rightarrow NaCl + NaOCl + H_2O$

47. 2 $\underset{\text{(acid)}}{ZnO} + \underset{\text{(base)}}{Na_2O} \longrightarrow Na_2ZnO_2$

$$\underset{\text{(base)}}{ZnO} + \underset{\text{(acid)}}{CO_2} \longrightarrow ZnCO_3$$

48. 2 $Na_2C_2O_4 \xrightarrow{2H^+} H_2C_2O_4 \xrightarrow{\text{Conc } H_2SO_4} CO + CO_2\uparrow + H_2O$

$$H_2C_2O_4 \xrightarrow{CaCl_2} \underset{\text{(White ppt.)}}{CaC_2O_4}$$

49. 1 % of Hydrogen is = 10
The weight of person is = 75 kg
Weight of Hydrogen in that is =7.5 kg
If ${}_1^1H$ is replaced by ${}_1^2H$
Weight gain = 7.5 kg

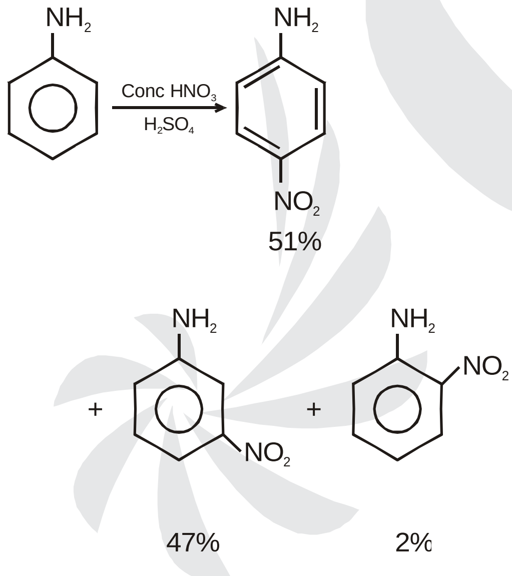
50. 2 Number of milli moles of $CoCl_3.6H_2O$
= 0.1×100 = 10
Number of milli moles of ions

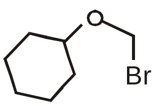
$$= \frac{1.2\times10^{22}}{6\times10^{23}}\times1000 = 20$$

$$[Co(H_2O)_5Cl]Cl_2.H_2O + 2AgNO_3 \longrightarrow$$

$$\left[Co(H_2O)_5Cl\right]^{2+} + 2AgCl + 2NO_3^- + H_2O$$

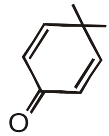
51. 1



52. 3 

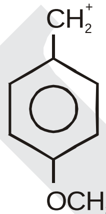
$\longrightarrow$ Elimination reaction not possible.

53. 3 Nylon – 6

54. 2 

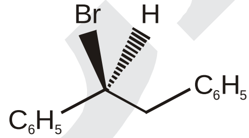
has least resonating structures.

55. 4 Due to stability of carbocations



$$> CH_3 - \overset{(+)}{C}H - CH_2 - CH_3$$

$$> CH_3 - CH_2 - \overset{(+)}{C}H_2$$

56. 4 

$\xrightarrow[\Delta]{t-BuOK}$

$C_6H_5CH = CH - C_6H_5$

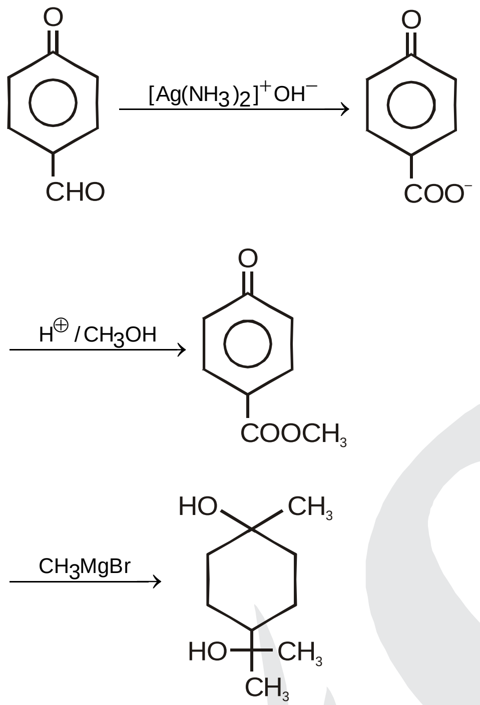
57. 3 As on C –1, – $OCOCH_3$ will get converted to –OH group so due to this free group it will act as reducing sugar.

58. 2 $CH_3 - CH = \underset{\displaystyle CH_3}{\underset{|}{C}} - CH_2 - CH_3 \xrightarrow[\text{Peroxide}]{HBr}$

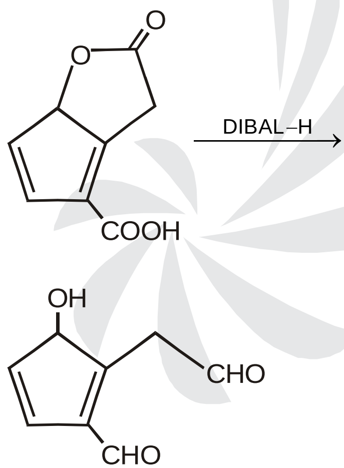
$$CH_3 - \overset{\displaystyle Br}{\overset{|}{C}H} - \underset{\displaystyle CH_3}{\underset{|}{C}H} - CH_2 - CH_3$$

(Two chiral center in the product.)
∴ Number of stereoisomers = 4

59. 3



60. 4



PART C – MATHEMATICS

61. 2 $y = \dfrac{x}{1+x^2}$

$y + yx^2 = x$

$yx^2 - x + y = 0$

$D \geq 0$

$1 - 4y^2 \geq 0$

$4y^2 - 1 \leq 0$

$(2y + 1)(2y - 1) \leq 0$

$-\dfrac{1}{2} \leq y \leq \dfrac{1}{2}$

Range = Co-domain therefore function is surjective $\dfrac{dy}{dx} = \dfrac{1-x^2}{(1+x^2)^2}$

Function is increasing & decreasing therefore function is not injective

62. 3 $x(x+1) + (x+1)(x+2) + \ldots\ (x+\overline{n-1})(x+n) = 10n$

$$nx^2 + \sum_{n=1}^{n}(2n-1)x + \sum_{n=1}^{n}(n-1)(n) = 10n$$

$$nx^2 + n^2x^2 + \frac{n(n^2-1)}{3} - 10n = 0$$

$$x^2 + nx + \frac{n^2-1}{3} - 10 = 0$$

$$x^2 + nx + \frac{n^2-1}{3} - 10 = 0$$

$$x^2 + nx + \frac{n^2-31}{3} = 0,$$

Let roots are k & k + 1

$k + k + 1 = -n$

$$k(k+1) = \frac{n^2-31}{3}$$

$$\frac{-n-1}{2}\left(\frac{-n-1}{2} + 1\right) = \frac{n^2-31}{3}$$

$$\frac{n^2-1}{4}=\frac{n^2-31}{3}$$

$3n^2 - 3 = 4n^2 - 124$

$n^2 = 121$

$n = 11.$

63. 4 $2\omega + 1 = z$

$z=\sqrt{-3}$

$2\omega=-1+i\sqrt{3}$

$$\omega=-\frac{1}{2}++\frac{i\sqrt{3}}{2},\omega^2=-\frac{1}{2}-\frac{i\sqrt{3}}{2}$$

$$\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=\begin{vmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}$$

$$=\begin{vmatrix}3 & 1 & 1\\ 0 & \omega & \omega^2\\ 0 & \omega^2 & \omega\end{vmatrix} \qquad C_1\to C_1+C_2+C_3$$

$= 3 (\omega^2 - \omega^4) = 3 (\omega^2 - \omega)$

$= -3i\sqrt{3} = -3z = 3k$

$\boxed{k=-z}$

64. 1 $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$

$$3A^2+12A=3\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}^2+12\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$$

$$=3\left(\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix}+\begin{bmatrix}8 & -12\\ -16 & 4\end{bmatrix}\right)$$

$$=3\begin{bmatrix}24 & -21\\ -28 & 17\end{bmatrix}=\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$$

$$\text{Adj}\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}=\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$$

65. 4 $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & a & 1\\ a & b & 1\end{vmatrix}=0$

$$\begin{vmatrix}1 & 0 & 0\\ 1 & a-1 & 0\\ a & b-a & 1-a\end{vmatrix}=0 \qquad \begin{matrix}C_2\to C_2-C_1\\ C_3\to C_3-C_1\end{matrix}$$

$\boxed{a=1}$

$$\Delta_1=\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 0 & b & 1\end{vmatrix}=0$$

$$\Delta_2=\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 1\end{vmatrix}=0$$

$$\Delta_3=\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & b & 0\end{vmatrix}=0$$

No value of b

66. 4 Let X invites a ladies and b men
Y invites c ladies and d men
so a + c = 3, b + d = 3, a + b = 3, c + d = 3

X	Y
(a ladies, b men)	(c ladies, d men)
(0, 3)	(3, 0)
(1, 2)	(2, 1)
(2, 1)	(1, 2)
(3, 0)	(0, 3)

$({}^4C_0 \times {}^3C_3)^2 + ({}^4C_1 \times {}^3C_2)^2 + ({}^4C_2 \times {}^3C_1)^2 + ({}^4C_3 \times {}^3C_0)^2$

$= 1 + 144 + 324 + 16 = 485.$

67. 3 $({}^{21}C_1+{}^{21}C_2+\ldots+{}^{21}C_{10})-$

$({}^{10}C_1+{}^{10}C_2+\ldots{}^{10}C_{10})$

$=(2^{20}-1)-(2^{10}-1)=2^{20}-2^{10}.$

68. 1 $225a^2 + 9b^2 + 25c^2 - 7cac - 45ab - 15bc = 0$

Multiply two both sides

$(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2 = 0$

$15a - 3b = 0$

$3b - 5c = 0$

$5c - 15a = 0$

$b = 5a$

$c = 3a$

b, c, a are in AP.

5a, 3a, a

$$\frac{5a + a}{2} = 3a.$$

69. 4 $f(x + y) + f(x) + f(y) + xy \ \forall \ x.u \in R$

$a(x + y)^2 + b(x + y) + c = (ax^2 + bx + c) + (ay^2 + by + c) + c + xy$

$$\Rightarrow a = \frac{1}{2}, c = 0 \quad \text{so } b = \frac{5}{2}$$

$$f(x) = \frac{1}{2}x^2 + \frac{5}{2}x$$

$$\sum_{n=1}^{10}\left(\frac{1}{2}n^2 + \frac{5}{2}n\right) = 330.$$

70. 1 $$\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

Put $x = \left(\frac{\pi}{2} - h\right)$

$$\lim_{h \to 0} \frac{\tan h - \sin h}{(2h)^3}$$

$$\lim_{h \to 0} \frac{(1 - \cos h)\sin h}{(2h)^3} = \lim_{h \to 0} \frac{1 - \cos h}{8h^2}$$

$$= \lim_{h \to 0} \frac{2\sin^2\left(\frac{h}{2}\right)}{4.8\left(\frac{h}{2}\right)^2} = \frac{1}{16}.$$

71. 4 $$\tan^{-1}\left(\frac{6 \times \sqrt{x}}{1 - 9 \times 3}\right)$$

$$= 2\tan^{-1}\left(3x^{3/2}\right) \forall x \in \left(0, \frac{1}{4}\right)$$

$$\frac{d}{dx}\left(2\tan^{-1}(3x^{3/2})\right) = \frac{9\sqrt{x}}{1 + 9x^3}.$$

72. 1 Curve intersects y-axis at :

$x = 0, y(-2)(-3) = 0 + 6$

$y = 1$

Differentiating

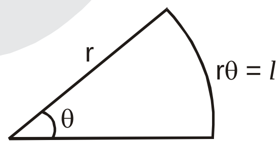
$$\frac{dy}{dx}(x - 2)(x - 3) + y[2x - 5] = 1$$

$$\frac{dy}{dx} \text{ at } x = 0, y = 1 \text{ is } \frac{dy}{dx} = 1$$

Normal at (0, 1) in $y - 1 = -1 (x - 0)$

$\Rightarrow x + y = 1.$

73. 2



Given $2r + r\theta = 20$

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}r(20 - 2r)$$

$$\frac{d(\text{Area})}{dr} = \frac{1}{2}(20 - 4r) = 0$$

$\Rightarrow r = 5$

$$\text{Maximum area} = \frac{1}{2}5 \cdot (20 - 10) = 25 \text{ sq. m.}$$

74. 1 $$I_n = \int \tan^n \times dx = \int \tan^{n-2} x \cdot (\sec^2 x - 1)\, dx$$

$$I_n + I_{n-2} = \int \tan^{n-2} x \cdot \sec^2 x\, dx$$

$$= \frac{(\tan x)^{n-1}}{n - 1} + C$$

If n = 6

$$I_6 + I_4 = \frac{(\tan x)^5}{5} + C$$

$$a = \frac{1}{5}, b = 0.$$

75. 1 $$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos(\pi - x)} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$$

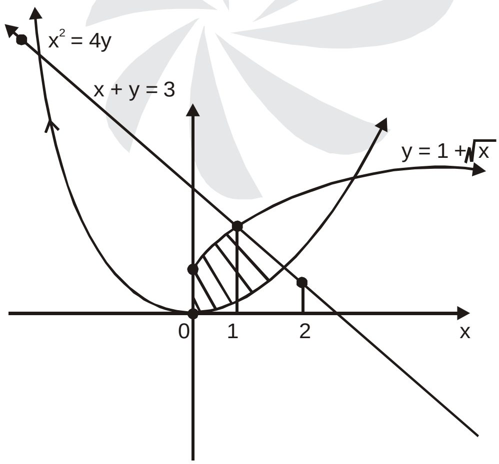
$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2\,dx}{1-\cos^2 x} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\,\mathrm{cosec}^2 \times dx$$

$$= -2\cot x \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= -2\cot\left(\frac{3\pi}{4}\right) + 2\cot\left(\frac{\pi}{4}\right) = 4$$

I = 2.

76. 3



Area enclosed

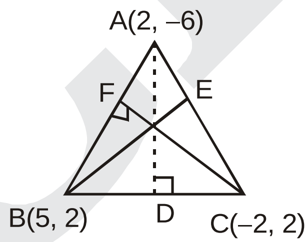
$$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= 1 + \frac{2}{3} + 3 - \frac{3}{2} - \frac{2}{3} = \frac{5}{2}.$$

77. 4 2 dy + sin xdy + y cos xdx + cos xdx = 0
Integrating on both sides
2y + sin x · y +sin x = c
x = 0, y = 1 ⇒ c = 2

$\therefore$ 9t $x = \frac{\pi}{2}, y = \frac{1}{3}.$

78. 3



$$\text{Area} = \frac{1}{2}\begin{vmatrix} k-5 & 2k \\ -4k & -3k-2 \end{vmatrix} = 28$$

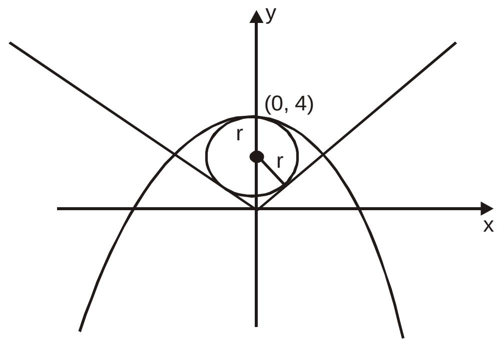
$$\Rightarrow 3k^2 + 13k + 10 = \pm 56$$

$$\Rightarrow k = 2, \frac{-23}{5},$$

BE : x – 2y – 1 = 0

$\therefore$ Ortho centre : $\left(2, \frac{1}{2}\right)$

79. 2



$4 - r = \sqrt{2}\,r$

$r = 4\left(\sqrt{2} - 1\right)$

80. 1 $e = \frac{1}{2}, \frac{-a}{e} = -4 \Rightarrow a = 2$

$b^2 = a^2(1 - e^2) = 3$

$\therefore$ Equation of elipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Equation of normal is $\frac{4x}{4} - \frac{3y}{\frac{3}{2}} = 1$

$\Rightarrow 4x - 2y = 1$

81. 1 Let Hyerperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Passes through $P(\sqrt{2}, \sqrt{3})$

$\therefore \quad \frac{2}{a^2} - \frac{3}{b^2} = 1$ ... (1)

foci is $(\pm 2, 0)$

$ae = 2$

$\therefore \quad b^2 = a^2(e^2 - 1)$

$b^2 = a^2e^2 - a^2$

$a^2 + b^2 = 4$ ... (2)

(2) in (1)

$\frac{2}{a^2} - \frac{3}{4 - a^2} = 1$

$2(4 - a^2) - 3a^2 = 4a^2 - a^4$

$a^4 - 9a^2 + 8 = 0$

$\therefore \quad a^2 = 1$ (or) $a^2 = 8$.

$a^2 \neq 8 \quad \because \; a^2 + b^2 = 4$

$\therefore \quad a^2 = 1$ tangent at $(\sqrt{2}, \sqrt{3})$ is

$\therefore \quad x^2 - \frac{y^2}{3} = 1 \qquad \sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

Option 1 satisfies.

82. 1 Normal vector is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$

$= \hat{i}(5) - \hat{j}(-7) + \hat{k}(3) = 5\hat{i} + 7\hat{j} + 3\hat{k}$.

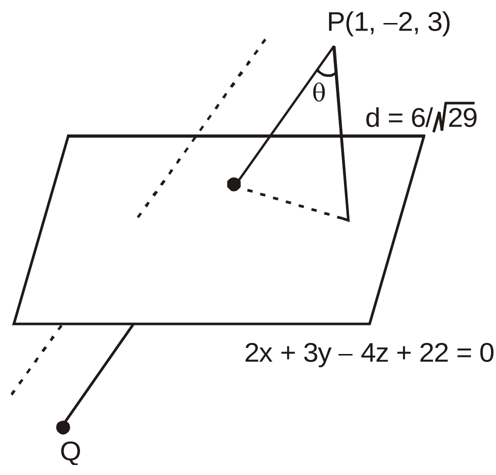
$\therefore$ Equation of a plane is

$5(x - 1) + 7(y + 1) + 3|z + 1| = 0$

$\therefore$ distance from (1, 3, – 7) to plane is

$d = \left|\frac{5 + 21 - 21 + 5}{\sqrt{25 + 49 + 9}}\right| = \frac{10}{\sqrt{83}}$

83. 1 distance = $\frac{6}{\sqrt{29}}$



$$\cos\theta = \frac{|2+12-20|}{\sqrt{29}.\sqrt{42}} = \frac{6}{\sqrt{29}.\sqrt{42}}$$

$$PQ = \frac{2d}{\cos\theta} = 2\sqrt{42}$$

84. 1 $|\vec{a}\times\vec{b}||\vec{c}|\sin\theta = 3$ $\theta = 30°$

$|\vec{a}\times\vec{b}||\vec{c}| = 6$ ... (1)

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$2\vec{i} - 2\vec{j} + \vec{k}$

$|\vec{a}\times\vec{b}| = 3$

$|\vec{c}| = 2$

$\therefore |\vec{c}\times\vec{a}| = 3 \Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{e} = 9$

$\vec{a}.\vec{c} = 2$

85. 4 P(g) = $\frac{3}{5}$, P(y) = $\frac{2}{5}$

Its a Binomial distribution whose variance is

$npq = 10\times\frac{3}{5}\times\frac{2}{5} = \frac{12}{5}$

86. 1 $P(A\cap B) = P(B\cap C) = P(A\cap C) = \frac{1}{4}$

$$P(A\cap B\cap C) = \frac{1}{16}$$

$$P(A\cup B\cup C) = \frac{1}{2}[P(A\cap B) + P(A\cap C) + P(B\cap C)] + P(A\cap B\cap C)$$

$$= \frac{1}{2}\cdot\left[\frac{3}{4}\right] + \frac{1}{16} = \frac{7}{16}.$$

87. 4 $4n \to \{0, 4, 8\} \to$ Any 2

$4n + 1 \to \{1, 5, 9\}$

$4n + 2 \to \{2, 6, 10\} \to$ Any 2

$4n + 3 \to \{3, 7\}$

$$\text{Probability} = \frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{55}.$$

88. 3 Let cos 2x = t

$$5\left(\frac{1-t}{1+t} - \frac{1+t}{2}\right) = 2t + 9$$

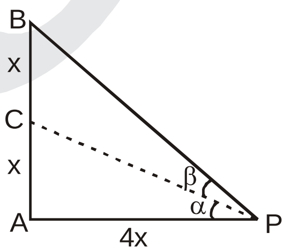
$5(1 - 4t - t^2) = 4t^2 + 22t + 18$

$\Rightarrow 9t^2 + 42t + 13 = 0$

$\Rightarrow t = \frac{-1}{3}x\frac{-13}{3}x$

$\therefore \cos 2x = \frac{-1}{3}, \cos 4x = \frac{-7}{9}.$

89. 2



$\tan\beta = \tan((\alpha + \beta) - \alpha)$

$$= \frac{\tan(\alpha+\beta) - \tan\alpha}{1 + \tan(\alpha+\beta)\cdot\tan\alpha}$$

$$= \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2}\cdot\frac{1}{4}} = \frac{2}{9}.$$

90. 4

p	q	p→q	~p	~p→q	(~p→q)→q	s
T	T	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T